

Dynamic Model for the Stress and Strain State Analysis of a Spur Gear Transmission

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A gear transmission dynamic model for the gear dynamic contact loading, dynamic contact stress state and dynamic contact strain state analysis is presented. Dynamic model of the transmission with four degrees of freedom is used. The transmission is analyzed using nonlinear finite elements contact formulation, using a novel software modules developed by the author used for the generalized analysis of the geared transmissions in the environment of the open source finite elements framework CODE-ASTER/SALOME.

Keywords: Stress and strain state analysis, tensor invariants, Finite Element Method, Geared transmissions.

0 INTRODUCTION

Gear transmissions have a long history dating back since the time of the first engineering systems. Their practical usage in the present day modern engineering systems is enormous. In accordance with contemporary development of mechanical engineering technics ever growing requirements have been imposed concerning characteristics and working specifications. The machines which utilize high-power duty gear transmissions (excavating machines, crushing machines, rolling machines, ships, etc.) operate under non-stationary conditions so that the loads of the elements of these gear transmissions are variable. For example, abrupt accelerations and abrupt decelerations of machine parts, that is, masses of the gear transmissions cause inertial forces which, in addition to the conditions of operation, influence the magnitude of actual loads of the elements of gear transmissions. All this, together with the changes of the torque of drive and operating machine, the forces induced by dynamic behaviours of the complete system, etc., lead to the situation where the stresses in the gears are higher than critical stresses; after certain time this may result in breakage of the teeth.

Internal dynamic forces are present, too. During the teeth meshing action, internal dynamic forces occur because of the elastic tooth deformation and manufacturing errors (for example: tooth profile form tolerance errors and circular gear pitch difference). During the

meshing action, tooth deformation can cause impact between gear teeth. Intensity of impact between the teeth is dependent of circular pitch difference size. The bigger circular pitch difference, **the more intensive is the** impact between the gear teeth during the meshing action. The gear vibrations can also influence the internal dynamic forces between the gear teeth. The teeth in meshing action can be modelled as an oscillatory system [1, 2, 3]. This model consists of concentrated masses (each of which represent one gear) connected with elastic and damp element. Applying the basic principles of analytical mechanics and taking the initial and boundary conditions into consideration, it is possible to obtain the system of equations representing physical meshing process between the two or more gears. In order to obtain better results, it is possible to model the elastic element as a nonlinear spring.

Although gear dynamics has been studied for decades, few studies present a formulation intended for the dynamic response of full gear-train systems that contain multiple gear meshes, flexible shafts, bearings, and housing structures. There are few reliable computational tools for the dynamic analysis of general gear configurations. Some models exist, but they are limited by simplified modeling of gear tooth mesh interfaces, two-dimensional models that neglect out-of-plane behavior, and models specific to a single gear configuration. General three-dimensional finite element models for dynamic response are rare because they require significant computa-

tional effort. This is accomplished by many time steps required for the transient response to diminish so that steady state data can be obtained. This study attempts to fill this gap with a general finite element formulation that can be used for full gearbox dynamic analyses.

Dynamic transmission error is taken as the parameter for modelling of noise in geared transmission. In the last two decades there is plenty of work concentrated on modelling of the dynamic transmission error for spur and helical gears and representing the influence of the dynamic transmission errors on the level of noise in the geared transmission. Lately, there has been experiments conducted in order to isolate particular noise effects like noise coming from bearing, housing noise, meshing action noise and backlash noise simply by measuring the dynamic transmission error. Some of the earliest models came from, Kahraman and Blankenship, (1988), [4] and Kahraman, A., Singh, R., (1990), [5].

For different analysis purposes, there are several modelling choices such as a simple dynamic factor model, compliance tooth model, torsional model, and geared rotor dynamic model, Blankenship and Singh, (1995), [6]. Using the free vibration analysis one calculates critical parameters such as natural frequencies and vibration modes that are essential for almost all dynamic investigations. The free vibration properties are very useful for further analyses of planetary gear dynamics, including eigen-sensitivity to design parameters, natural frequency veering, planet mesh phasing, and parametric instabilities from mesh stiffness variations, LIN and Parker, (1999, 2001), [7,8]. It is also necessary to systematically study natural frequency and vibration mode sensitivities and their veering characters to identify the parameters critical to gear vibration. In addition, practical gears may be mistuned by mesh stiffness variation, manufacturing imperfections and assembling errors. For some symmetric structures, such as turbine blades, space antennae, and multispan beams, small disorders may dramatically change the vibration, LIN and PARKER, (2002), [9]. The following articles Parker, G.R., Vijayakar, M.S., Imajo, T., (2000), [10] and Parker, G.R., Agashe, V., Vijayakar, M.S., (2000), [11], related to the nonlinear analysis of dynamic behavior of gears, using experimental methods and the application of FEM.

Based on the results of the experiments conducted during the gear vibration research, it is to conclude that the excitation is restored every time when a new pair of teeth enters the mesh. Vibrations with natural frequencies dominate the vibration spectrums. The internal dynamic forces in teeth mesh, vibration and noise are consequences of the: change in teeth deformation, teeth impact, gear inertia due to measure and teeth shape deviation, Agemi and Ognjanović, (2004), [12].

Paper, Hedrih (Stevanovic), K., Nikolic–Stanojevic, V. (2010), [13], presents two models of the geared transmission with two or more shafts. First approach give a model based on the rigid rotors coupled with rigid gear teeth, with mass distributions not balanced and in the form of the mass particles as the series of the mass debalances of the gears in multistep gear transmission. By very simple model is possible and useful investigation of the nonlinear dynamics of the multistep gear transmission and nonlinear phenomena in free and forced dynamics. This model is suitable to explain source of vibrations and big noise, as well as no stability in gear transmission dynamics. Layering of the homoclinic orbits in phase plane is source of a sensitive dependence nonlinear type of regime of gear transmission system dynamics. Second approach give a model based on the two-step gear transmission taking into account deformation and creeping and also visco-elastic teeth gears coupling. Our investigation was focused to a new model of the fractional order dynamics of the gear transmission. For this model we obtain analytical expressions for the corresponding fractional order modes like one frequency eigen vibrational modes. Generalization of this model to the similar model of the multistep gear transmission is very easy.

Model in this paper represents typical transmission model with three shafts. Starting with first gear pair, a meshing action is observed for all gear meshes simultaneously using no additional assumptions concerning the gear geometry or the meshing action boundary and load conditions. The boundary conditions on the gear-shaft connections are assumed to be equal to zero. The internal dynamic forces are obtained as a result of the first step of the nonlinear quasi-static analysis and are taken as input for further analysis steps. Procedure described can be

modified for the purpose of calculation of different types of geared systems simply by combining different steps in described analysis and different configurations of the gears and shafts. It is optional which of the mentioned steps will be of the highest priority, and depends, of course, of the purpose of the concrete analysis.

1 FINITE ELEMENT MODEL OF THE SPUR GEAR TRANSMISSION

Starting with one gear pair, a meshing action is observed using no additional assumptions concerning the gear geometry or the meshing action boundary and load conditions. Numeric experiment is developed simply by using the boundary conditions on the gear-shaft connection, which are assumed to be equal zero. The software that is developed for the purpose of dynamic analysis of the gear-boxes can be used to obtain following results from such model:

- a) forces that arise in gear contact during the gear meshing action as a result of the incremental quasi-static series of analysis [8, 9, 10], for each meshing action time step,
- b) the comprehensive modal analysis for the gear pair, and the
- c) forces obtained in a) will be used further as an input for the nonlinear dynamic analysis. For the purpose of this paper, it is assumed that the obtained force is harmonic function and in form:

$$F_n = F_o \sin \omega t, \tag{1}$$

where F_o is the resultant force for the observed meshing action time step.

- d) Finally, the dynamic analysis of the gear meshing action is done, [10]. The contact load as inner dynamic load is taken as an input value obtained in a) and approximated using harmonic function (as it is described in c)).

Developed software can be modified for the purpose of calculation of different types of geared systems. It is optional which of the mentioned steps will be of the highest priority, and depends, of course, on the purpose of the concrete analysis. That is not of interest here, and therefore will not be discussed.

The gears analyzed in this paper are the one from the deep drilling machine gear set, [14]. The gear geometries are generated using the software developed by the authors, [13, 14, 15,

16, 17, 18, 19], and adapted for the SALOME platform [40]. The obtained geometries are transferred to SALOME mesh generation software and discretized using four node tetrahedral elements there (Figure 1.).

- Mesh parameters of the geared system are:
- Number of nodes: 19209
 - Number of elements: 102868
 - Number of elements of the shaft 1 (staring from left hand side): 23618
 - Number of elements of the shaft 2 (middle): 30034
 - Number of elements of the shaft 3 (right bottom): 1586.

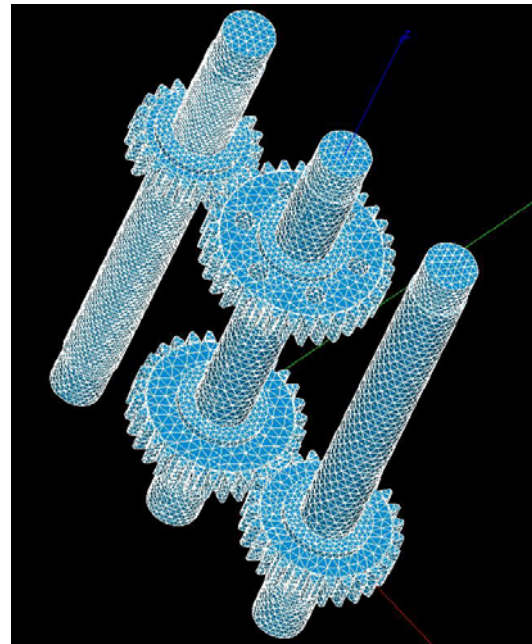


Fig. 1. Gear transmission model

Gear geometry parameters are given in Table 1.

Table 1. Constructive transmission parameteres

Parameter	Gear 1	Gear 2	Gear 3	Gear 4
Number of teeth	27	30	33	22
Modul (m)	0.003	0.003	0.003	0.003
Pitch circle diameter (m)	0.081	0.090	0.099	0.066
Addendum circle diameter (m)	0.087	0.096	0.106	0.072
Base circle diameter (m)	0.073	0.0828	0.0918	0.0588
Measurement teeth number (m)	0.004	0.004	0.004	0.003
Standard angle (degrees)	20	20	20	20
Axial distance (m)	0.081	0.081	0.081	0.099

Displacement boundary conditions (Figure 2) are set on the shaft-bearing connections nodes. The displacement boundary conditions on the axial bearing connection nodes are set to be zero in all degrees of freedom, and for the radial bearing connection nodes this displacement are set to be zero only in X,Y directions.

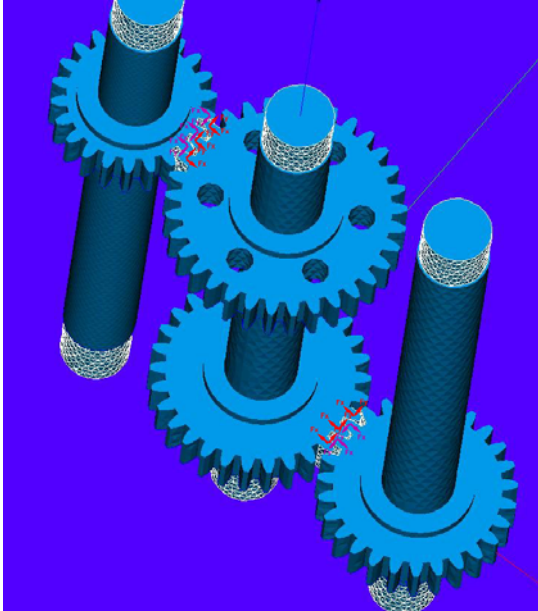


Fig. 2. Displacement and force boundary conditions (right)

Load boundary conditions are generated on the meshing teeth on edges that are opposite to the contact surfaces of each tooth, like [26]. The form of the load function is taken according to (1) with amplitude obtained from the nonlinear static analysis. This analysis helps determine the form of the amplitude force distribution in gear mesh. Detailed procedure on how the force is taken from nonlinear static analysis will not be discussed in detail here. The force distribution on other transmission elements is calculated on the same way.

All other meshing conditions are not taken into consideration. Following assumptions are made:

- It is assumed that the contact is without friction
- There is no sliding between the teeth during the meshing action.

- The transmission is functioning in constant temperature field. All other thermodynamic effects can be neglected.

- The proportional damping is used according to

$$C = 2 \beta M + \alpha K.$$

The continuum discretization is done using tetrahedral finite elements with linear approximation of the finite element geometry.

2 ANALYTIC MODEL OF THE SPUR GEAR TRANSMISSION

The transmission model on the Figure 1 can be simplified using the transmission model on Figure 3 for dynamic analysis [1,2,3, 10, 27, 31].

This class of dynamic models can be used for the analysis of the oscillation parameters of the transmission, only if meshing functions of the stiffness and dumping are known. As this is not the case, it is necessary to experimentally obtain the form of the mesh stiffness (dumping) of the geared mesh, or to use other means like finite elements method to obtain unknown stiffness/dumping functions.

The solution for the differential equations given in (2) is not possible without explicitly defined mesh stiffness and dumping functions, because of the fact that the equations are non-linear functions of time and the solution can be obtained only using numeric methods.

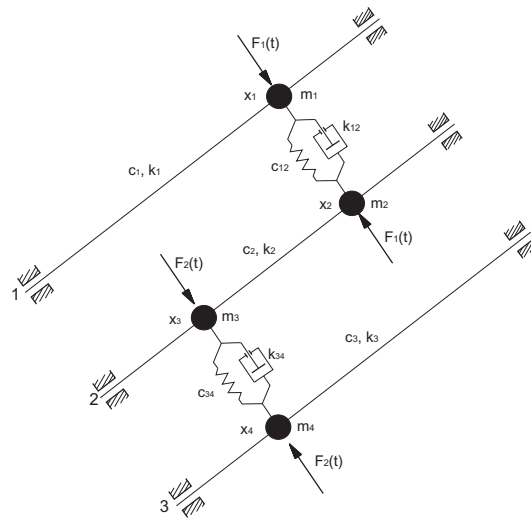


Fig. 3. A simplified dynamic model of the geared transmission on Fig.1

The differential equation [2, 3, 20, 21, 22, 24] of the model on Figure 3 are:

$$\begin{aligned}
 m_1 \ddot{x}_1 + (c_1 + c_{12})x_1 - c_{12}x_2 + (k_1 + k_{12})\dot{x}_1 - \\
 -k_{12}\dot{x}_2 &= F_1(t) \\
 m_2 \ddot{x}_2 + (c_2 + c_{12})x_2 - c_{12}x_1 - c_2x_3 + (k_2 + k_{12})\dot{x}_2 - \\
 -k_{12}\dot{x}_1 - k_2\dot{x}_3 &= -F_1(t) \\
 m_3 \ddot{x}_3 + (c_2 + c_{34})x_3 - c_{34}x_4 - c_2x_2 + (k_2 + k_{34})\dot{x}_3 - \\
 -k_{34}\dot{x}_4 - k_2\dot{x}_2 &= F_2(t) \\
 m_4 \ddot{x}_4 + (c_3 + c_{34})x_4 - c_{34}x_3 + (k_3 + k_{34})\dot{x}_4 - \\
 -k_{34}\dot{x}_3 &= -F_2(t)
 \end{aligned} \tag{2}$$

where:

- m_i ($i=1,4$) are equivalent masses of transmission elements,
- c_i are shaft stiffness ($i=1,3$),
- k_i are shaft dumping ($i=1,3$)
- c_{12}, c_{34} are gear mesh stiffness functions depending on t ,
- k_{12}, k_{34} are gear mesh dumping functions depending on t .

Shaft stiffness/dumping outside of the gears on the shaft 2 are not considered.

3 STRESS AND STRAIN STATE ANALYSIS USING STRESS AND STRAIN TENSOR INVARIANTS

The calculation of the component stresses and strains is done by means of the nonlinear Finite Element method using already described procedure.

Starting from known component stresses as components of tensor \mathbf{N} , it is possible to obtain stress tensor invariants in every node after the finite elements analysis is done, [3, 23, 21-31]. Since:

$$\begin{aligned}
 f(\sigma_o) &= |\mathbf{N} - \sigma_o \mathbf{I}| = \\
 &= \begin{vmatrix} \sigma_x - \sigma_o & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_o & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_o \end{vmatrix} = 0
 \end{aligned} \tag{3}$$

yields:

$$-(\sigma_o^3 - L_1\sigma_o^2 + L_2\sigma_o - L_3) = 0 \tag{4}$$

Using equation (3) or its algebraic form (4) it is possible to obtain the principal stresses as functions of stress invariants L_1, L_2, L_3 , using equations (5), (6) and (7) as a system of three linear algebraic equations with unknown principal stresses, [25-31]. That yields the functional dependency between principal stress and stress invariants in form: $\sigma_o = f(L_1, L_2, L_3)$. Therefore:

$$L_1 = f(\sigma_1, \sigma_2, \sigma_3) = \sigma_1 + \sigma_2 + \sigma_3 = \text{tr}(\mathbf{G}) \tag{5}$$

$$\begin{aligned}
 L_2 = f(\sigma_1, \sigma_2, \sigma_3) &= \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3 = \\
 &= \frac{1}{2}(\text{tr}(\mathbf{G}^2) - \text{tr}(\mathbf{G})^2)
 \end{aligned} \tag{6}$$

and

$$L_3 = f(\sigma_1, \sigma_2, \sigma_3) = \sigma_1\sigma_2\sigma_3 = \det(\mathbf{G}) \tag{7}$$

where (8) is a principal stress tensor in form

$$\mathbf{G} = \begin{bmatrix} \sigma_1 & & \\ & \sigma_2 & \\ & & \sigma_3 \end{bmatrix} \tag{8}$$

When the component stresses are known, it is also possible to obtain the equations of the main stress directions. The principal stress directions are directions that define coordinate system of principal directions, where there are no shear stresses in defined planes.

When the principal stresses are different, $\sigma_1 \neq \sigma_2 \neq \sigma_3$, the maximal values of the shear stresses are found in the three normal planes, where the principal axis 1 and one of these planes are collinear, and the angles between the other principal planes and the mentioned plane are 45 degrees. The maximum values of the shear stresses are obtained in the following from

$$\begin{aligned}
 \tau_{(I)} &= \pm \frac{\sigma_2 - \sigma_3}{2}, \quad \tau_{(II)} = \pm \frac{\sigma_2 - \sigma_1}{2} \\
 \tau_{(III)} &= \pm \frac{\sigma_1 - \sigma_2}{2}
 \end{aligned} \tag{9}$$

There are no shear stresses in principal planes. On the other hand, in maximal shear stress planes, there are stresses with values given in the form:

$$\sigma_{(I)} = \frac{\sigma_2 + \sigma_3}{2} ; \quad \sigma_{(II)} = \frac{\sigma_1 + \sigma_3}{2}$$

$$\sigma_{(III)} = \frac{\sigma_1 + \sigma_2}{2} \quad (10)$$

As already stated, the principal directions can be obtained from component stresses using the following relations:

$$\frac{\alpha_o}{K_{31}} = \frac{\beta_o}{K_{32}} = \frac{\gamma_o}{K_{33}} = C_o \quad (11)$$

or

$$\frac{\alpha_o}{\begin{vmatrix} \tau_{xy} & \tau_{xz} \\ \sigma_y - \sigma_o & \tau_{yz} \end{vmatrix}} = \frac{\beta_o}{\begin{vmatrix} \sigma_x - \sigma_o & \tau_{xz} \\ \tau_{xy} & \tau_{yz} \end{vmatrix}} =$$

$$= \frac{\gamma_o}{\begin{vmatrix} \sigma_x - \sigma_o & \tau_{xy} \\ \tau_{xy} & \sigma_y - \sigma_o \end{vmatrix}} = C_o \quad (12)$$

the system is defined if condition (13) is satisfied:

$$\alpha_o^2 + \beta_o^2 + \gamma_o^2 = 1 \quad (13)$$

The relations (11) and (12) should be taken for $s=1,2,3$. They give the equations for the three main stress directions represented with three direction cosines, with total of nine direction cosines.

Depending on the value of the principal invariants, the properties of the main stresses are defined:

- a) if $L_3 > 0$, the roots (principal stresses) given in (8) are positive and different.
- b) if $L_3 < 0$, there are negative roots (principal stresses), and
- c) if $L_3 = 0$, one of the roots (principal stresses) is equal zero.

The same discussion can be applied for the strains. Starting from equation (14):

$$\begin{vmatrix} \varepsilon_x - \varepsilon_o & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \varepsilon_y - \varepsilon_o & \gamma_{yz} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_z - \varepsilon_o \end{vmatrix} = 0 \quad (14)$$

yields:

$$-(\varepsilon_o^3 - J_1 \varepsilon_o^2 + J_2 \varepsilon_o - J_3) = 0 \quad (15)$$

where $J_1, J_2, i J_3$ are known invariants of the strain tensor obtained from known strain components of the geared transmission. The solutions of the equation (15) are principal strains given. For the principal directions, the principal strains build the strain tensor in form (16):

$$\mathbf{S} = \begin{pmatrix} \varepsilon_1 & & \\ & \varepsilon_2 & \\ & & \varepsilon_3 \end{pmatrix} \quad (16)$$

$$J_1 = f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = \text{tr}(\mathbf{S})$$

$$J_2 = f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \varepsilon_1 \varepsilon_2 + \varepsilon_2 \varepsilon_3 + \varepsilon_1 \varepsilon_3 =$$

$$= \frac{1}{2}(\text{tr}(\mathbf{S}^2) - \text{tr}(\mathbf{S})^2)$$

$$J_3 = f(\varepsilon_1, \varepsilon_2, \varepsilon_3) = \varepsilon_1 \varepsilon_2 \varepsilon_3 = \det(\mathbf{S}) \quad (17)$$

Analogue to (11), (12) and (13) for the principal stresses, it is possible to obtain the equation of the principal strains by solving the system of equations (17, 18, and 19).

The principal strain directions can be obtained in the following form:

$$\frac{\alpha_o}{\begin{vmatrix} \gamma_{xy} & \gamma_{xz} \\ \varepsilon_y - \varepsilon_o & \gamma_{yz} \end{vmatrix}} = \frac{\beta_o}{\begin{vmatrix} \varepsilon_x - \varepsilon_o & \gamma_{xz} \\ \gamma_{xy} & \gamma_{yz} \end{vmatrix}} =$$

$$= \frac{\gamma_o}{\begin{vmatrix} \varepsilon_x - \varepsilon_o & \gamma_{xy} \\ \gamma_{xy} & \varepsilon_y - \varepsilon_o \end{vmatrix}} = C_o \quad (18)$$

$$\alpha_o^2 + \beta_o^2 + \gamma_o^2 = 1 \quad (19)$$

Depending on the value of the principal invariants J_1, J_2 and J_3 , the properties of the principal strains are for example:

- a) if $J_3 > 0$, the roots (principal strains) of (7) are positive and different.
- b) if $J_3 < 0$, there are negative roots (principal strains), and
- c) if $J_3 = 0$, one of the roots (principal strains) is equal zero.

4 RESULTS

Using the above procedures, the results are obtained for the geared transmission on the Figure 1, using boundary conditions presented on the Figure 2 using transmission parameters given in Table 1.

Transmission load distribution along the line of contact for each transmission gear pairs given on Figure 4, and distribution of loads on the transmission elements for $t=0.02$ is given on the Figure 5 as an output from the nonlinear dynamic analysis of the geared transmission

Additional results for strain distribution along the line of contact for each gear pair is given on the Figure 8, and Von Mises stress distribution along the line of contact for each transmission gear pair and each time step (Figure 7) together with distribution of Von Mises stress on the transmission elements for $t=0.02$ is presented on the Figure 6.

Work methodology previously tested on simple examples from literature [7,8, 9, 10,11, 28, 30] and obtained a good coincidence of results.

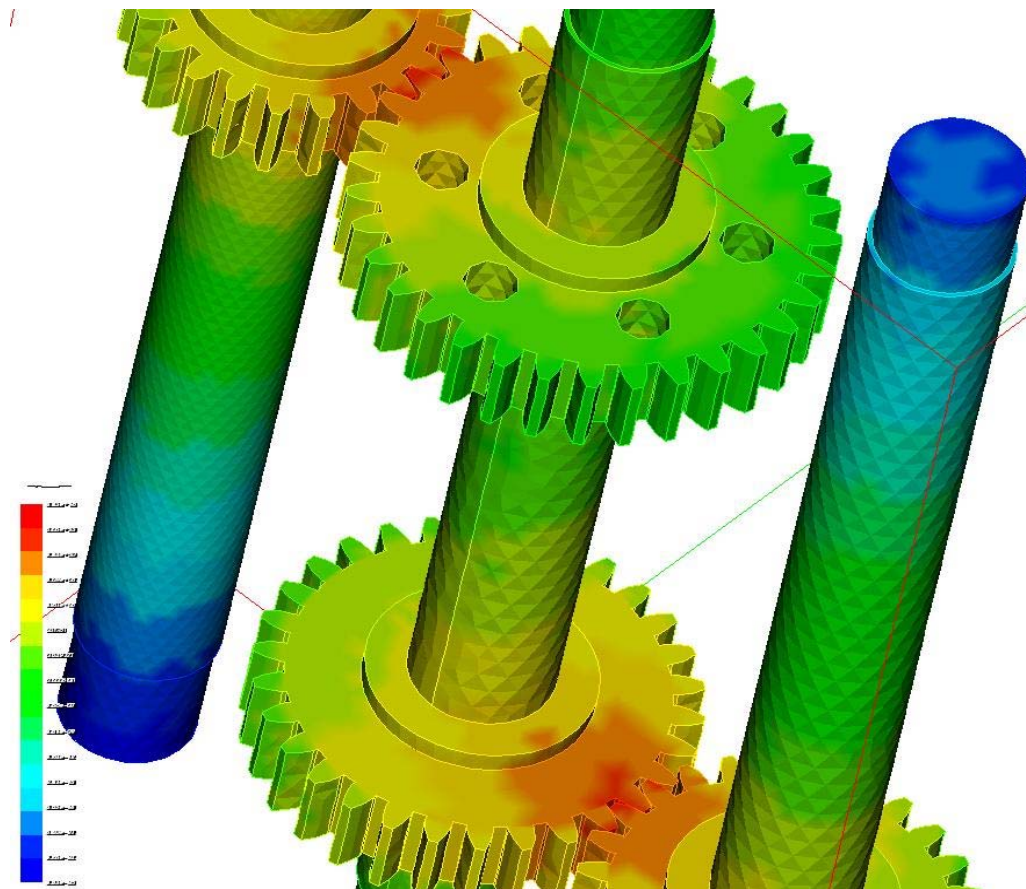


Fig. 4. Transmission load distribution in time step $t=0.02$

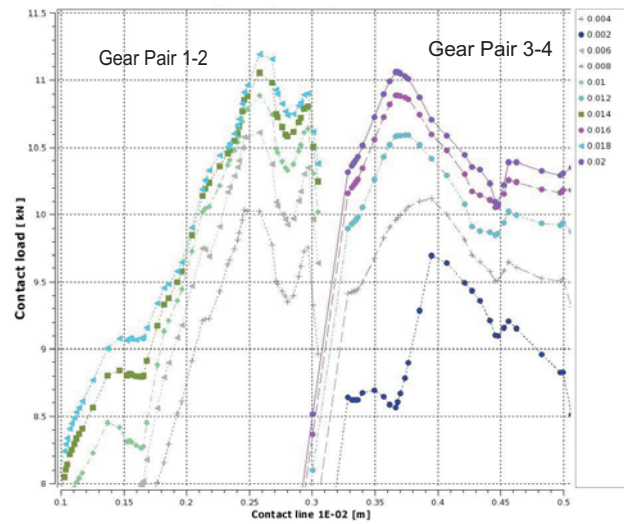


Fig. 5. Load distribution along the line of contact for each transmission gear pair ($t = 0 \div t = 0.02$)

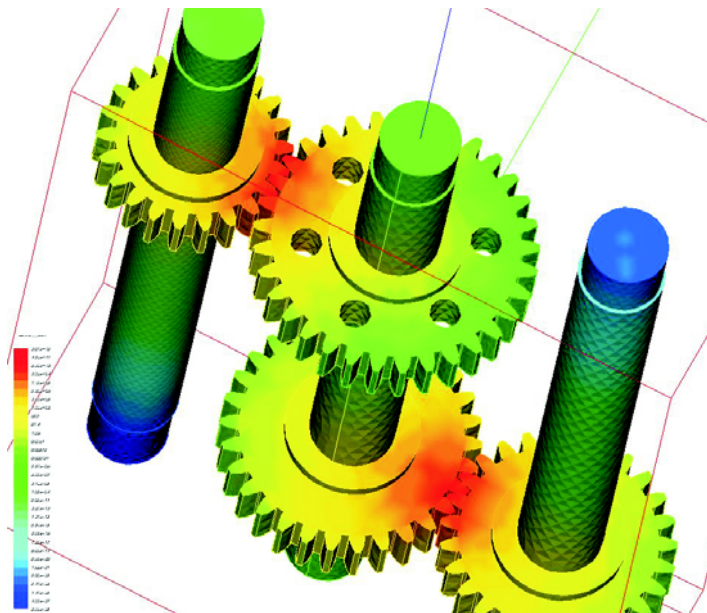


Fig. 6. Von Mises stress distribution on the transmission elements for $t=0.02$

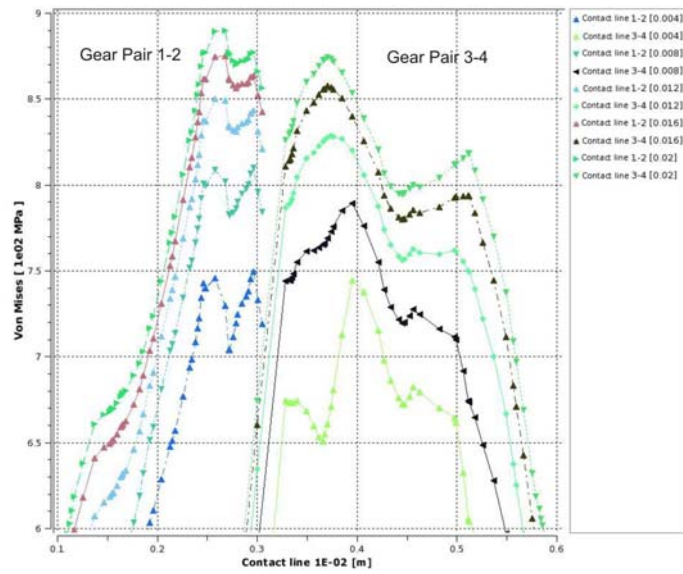


Fig. 7. Von Mises stress distribution along the line of contact for each transmission gear pair ($t = 0 \div t = 0.02$)

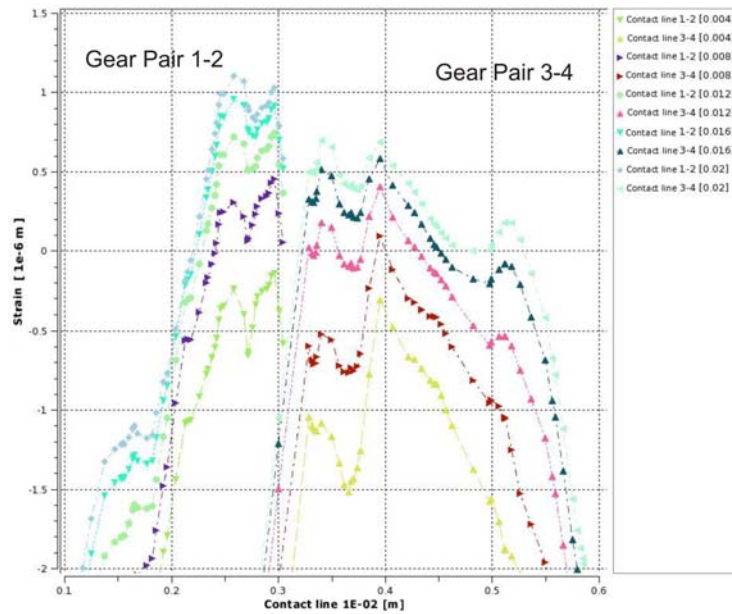
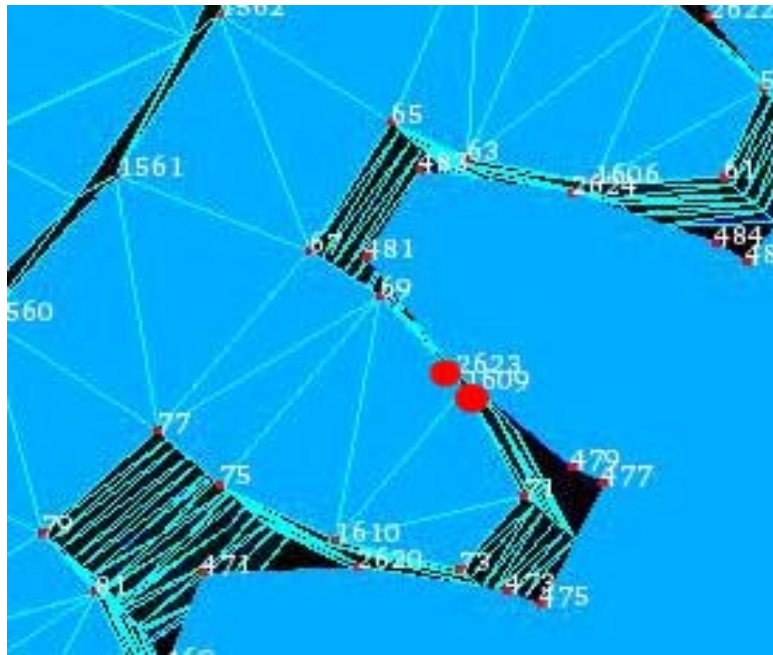
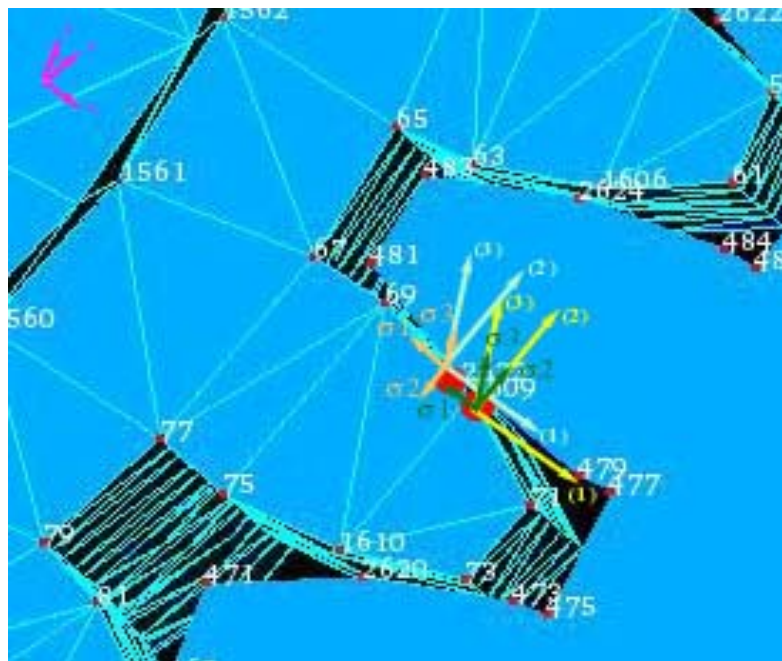


Fig. 8. Strain distribution along the line of contact for each transmission gear pair ($t = 0 \div t = 0.02$)



a)



b)

Fig. 9. Nodes which are used for calculation example of the stress and strain state a) and example of the graphic representation of the principal stress analysis results in chosen nodes b)

The given results are input results for the calculations of principal invariants in each node for each and every element of the presented geared transmission. An example of the principal stress and strain direction calculation for the selected nodes in contact is presented on Fig. 9 and corresponding Tables: Table 2, and Table 3.

The distribution of the principal invariants in the zone of contact for each transmission gear pair is presented on Figure 10, Figure 11, Figure 12 and Figure 13 as a result of the software developed by the authors that can be used for the detailed transmission analysis.

Work methodology previously tested on simple examples from literature [7,8, 9, 10,11, 28, 30] and obtained a good coincidence of results.

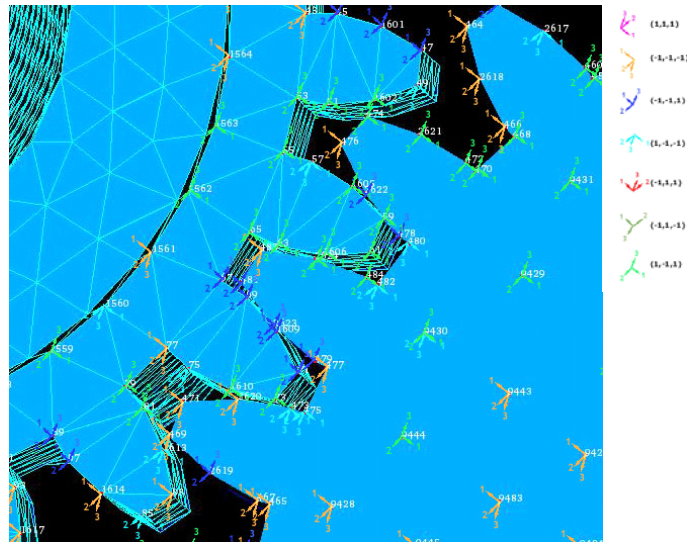


Fig. 10. Contact strain tensor invariants for the gear pair 1-2, for $t=0.02$

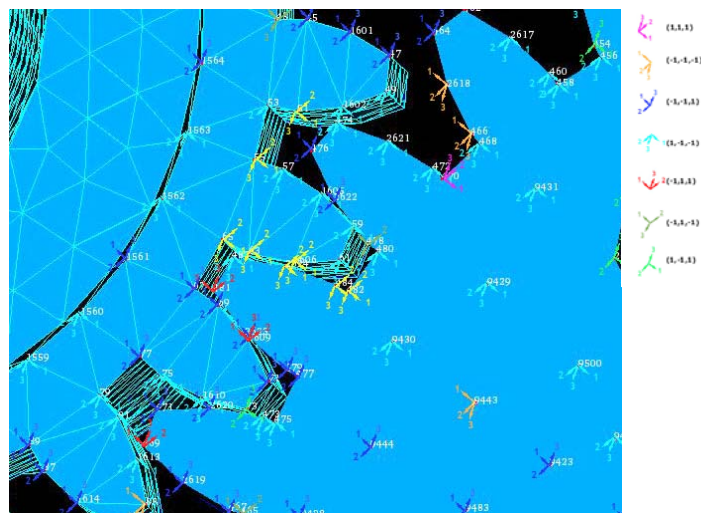


Fig. 11. Contact stress tensor invariants for the gear pair 1-2, for $t=0.02$



Fig. 12. Contact stress tensor invariants for the gear pair 3-4, for $t=0.02$



Fig. 13. Contact strain tensor invariants for the gear pair 3-4, for $t=0.02$

5 CONCLUSIONS

In this paper has been shown how the general finite element method can be applied to study dynamic behavior in the case of the real model of the geared set.

A novel approach for the geared transmission dynamic analysis is developed. Using several incremental finite element procedures, it is possible to obtain a series of states of the geared transmission in every integration time step of the differential equations of motion. Since the conditions in which the dynamic analysis is done are quasistatic, suggested method approximates very well the real working conditions of the transmission. On the other hand, as every dynamic calculation, it is exhaustive and time consuming.

For the observed model of gear transmissions, the analysis distribution of stress and strain invariants has been done.

The selected point that are in contact, are determined by the value of the main stresses and main directions of the observed time-steps.

Based on the results, it can be concluded that there is a change in the directions of principal stresses in the contact zone. This causes arise and changes in the values of the principal stresses, that is stress invariants

In the literature are present similar procedures [7,8,9,10] and they were used as reference material for the composition and verification of models and results .

But, in this paper, a new method for the results analysis, together with developed software, is used for obtaining of the principal stress and strain state of the geared transmission, using principal stress and strain invariants as calculation parameters. The software allows generalized analysis of every system for which this kind of analysis can be used.

On the basis of the results, shown in this paper, it has been concluded that the methodology developed to study the dynamic behaviour of complex systems is very efficient. It gives a lot of possibilities and can be easily upgraded for analysis of other effects [33- 35].

The dynamic behavior and analysis of the stress and strain conditions that suggest that the system of only two gears is very complex and that it is almost impossible to include all the effects by such and similar research. This paper considers two pair gears, which makes the problem more complex.

Further research should be directed at studying the effects of mutual dynamic impact of teeth in engagement, as well as at including connection between the shaft and gear into the dynamic model and the like.

In accordance with the present trend of application of new materials, like in [36-39], authors will, in future studies, simulate the dynamic behavior of a gear made of composite materials and study the life of the gears at the cyclic load.

Based on the numerical results, i.e., on the presented and other diagrams, a general conclusion can be drawn: stress and strain distributions in the case of dynamic behavior of toothed gears is in accord with the results from the aforementioned literature. Thus, these results can be taken as relevant for further research.

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Table 3. Component stresses and strains in node: 1609, principal stress, strain invariants, principal stresses and principal strains and appropriate principal directions, in $t=0.02$

Node: 1609		
σ_x (Pa)	σ_y (Pa)	σ_z (Pa)
2,437E+09	8,759E+08	6,726E+08
τ_{xy} (Pa)	τ_{xz} (Pa)	τ_{yz} (Pa)
1,949E+08	-9,431E+06	6,974E+07
α	β	γ
1,56	1,62	5,10E-02
1,46	2,64	1,09
1,57	1,56	7,13E-03
L_1	L_2	L_3
3,99E+09	-1,38E+19	-6,89E+27
σ_1 (Pa)	σ_2 (Pa)	σ_3 (Pa)
-2,54E+09	4,47E+08	6,08E+09
ϵ_x (μm)	ϵ_x (μm)	ϵ_x (μm)
939,480	-27,195	-153,040
γ_{xy} (μm)	γ_{xz} (μm)	γ_{yz} (μm)
α	β	γ
1,55	2,78	1,21
1,37	2,58	2,09
1,32	2,56	2,09
241,311	-11,676	86,346
J_1	J_2	J_3
7,60E+01	-7,14E+04	2,00E+06
ϵ_1 (μm)	ϵ_2 (μm)	ϵ_3 (μm)
-2,14E+02	-2,93E+01	3,19E+02

Table 2. Component stresses and strains in node: 2623, principal stress, strain invariants, principal stresses and principal strains and appropriate principal directions, for $t=0.02$

Node: 2623		
σ_x (Pa)	σ_y (Pa)	σ_z (Pa)
-2,088E+09	-1,229E+09	-1,053E+09
τ_{xy} (Pa)	τ_{xz} (Pa)	τ_{yz} (Pa)
5,508E+07	-3,036E+07	-3,852E+07
α	β	γ
1,57	1,57	6,26E-03
1,57	1,57	5,77E-03
1,59	1,60	3,61E-02
L_1	L_2	L_3
-4,36E+09	-7,46E+18	2,21E+28
σ_1 (Pa)	σ_2 (Pa)	σ_3 (Pa)
-4,97E+09	-1,83E+09	2,44E+09
ϵ_x (μm)	ϵ_x (μm)	ϵ_x (μm)
-668,191	-136,320	-27,790
γ_{xy} (μm)	γ_{xz} (μm)	γ_{yz} (μm)
α	β	γ
1,52	1,88	3,10E-01
1,77	2,94	1,53
1,88	2,54	1,07E+00
68,191	-37,592	-47,688
J_1	J_2	J_3
-8,30E+01	-5,06E+04	2,66E+06
ϵ_1 (μm)	ϵ_2 (μm)	ϵ_3 (μm)
-2,45E+02	-5,10E+01	2,13E+02

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