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System Model Modes Developed from Expansion of Uncoupled Component Dynamic Data

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System models are often developed from component models either as modal components or as reduced order models. The resulting system model data is only available at the reduced order space unless some expansion is performed. Of course this can be achieved using system model mapping matrices but that requires the development of the full space system model which defeats the purpose of the component synthesis approach.

However, the individual uncoupled component mapping matrices can be utilized to expand to the full space of the system model provided the modes of the components span the space of the full system model. This paper shows the results of using component modes from the unconnected components as projection matrices to identify the system level full field response. Multiple analytical cases are presented to show how the selection of component modes affects the expansion results. The results show accurate system model expansion using a sufficient set of component modes that span the space of the system model.

Keywords: component modeling, system modeling, expansion

0 INTRODUCTION

Reduced order models are often used as component representations in a system model to reduce computation time while including appropriate structural dynamic characteristics. The model reduction is performed to lower the total number of degrees of freedom (DOF) while retaining important dynamic characteristics. The system response can then be computed with very efficient reduced order system models. Once the system level response is obtained from the reduced component system model, the full space solution is often desired for computation of the overall system dynamic stress and strain, in the individual components, at the element level.

However, in order to obtain the full field response, expansion procedures are needed. Generally, to find the system model transformation matrix, the full DOF mode shapes must be computed for the full system model. This procedure is counterproductive because the purpose of model reduction is to avoid computing the full DOF model. This paper shows that the system modes can be expanded using the transformation matrices of the original uncoupled component modes and that the final system full shapes are actually not needed to predict the system level full field characteristics.

To perform this proposed expansion, the system shapes are separated into component shapes and expanded on a component by component basis using the component transformation matrices obtained from the uncoupled, original component modes. The system equivalent reduction expansion process (SEREP) [1] is used for the development of the transformation matrix because this technique exactly preserves the dynamics of the model regardless of which degrees of freedom are retained. The SEREP process used here is augmented with the variability improvement of key inaccurate node groups (VIKING) [2].

The VIKING paper [2] showed that over specifying the number of modes used in the reduction/expansion process that span the space of the system model modes significantly improves the expansion results. Therefore, if the modes used to reduce the component span the space of the component's shapes in the system modes, accurate results can be achieved; the error in the expanded shapes actually results from mode truncation, not from the expansion process itself.

This paper shows how to obtain full field, expanded system level characteristics from the individual, uncoupled component modes of the individual components. The following sections present some background theory along with the expansion methodology proposed followed by test cases to demonstrate the technique.

1 THEORETICAL BACKGROUND

Some of the pertinent equations of importance are described below relating to the modal space representation, the structural dynamic modification process and the reduction expansion process as well as correlation tools used to verify the results obtained.

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1.1 Equations of Motion for Multiple Degree of Freedom System

The general equation of motion for a multiple degree of freedom system written in matrix form is:

$$[M_1]\{\ddot{x}\}+[C_1]\{\dot{x}\}+[K_1]\{x\}=\{F(t)\}. \tag{1}$$

Assuming proportional damping, the eigensolution is:

$$\left[[K_1]-\lambda[M_1]\right]\{x\}=\{0\}. \tag{2}$$

The results of the eigensolution yield the eigenvalues (natural frequencies) and eigenvectors (mode shapes). The eigenvectors are arranged in column fashion to form the modal matrix $[U_1]$. Usually a subset of modes is included in the modal matrix to save computation time. Exclusion of modes results in truncation error which can be serious if key modes are excluded. Truncation error will be discussed in further detail in the structural dynamic modification section.

The physical system can be transformed to modal space using the modal matrix as:

$$[U_1]^T[M_1][U_1]\{\ddot{p}_1\}+[U_1]^T[K_1][U_1]\{p_1\}= \\ =[U_1]^T\{F(t)\}. \tag{3}$$

Scaling to unit modal mass yields:

$$[I_1]\{\ddot{p}_1\}+\left[\Omega_1{}^2\right]\{p_1\}=\left[U_1^n\right]^T\{F(t)\}, \tag{4}$$

where $[I_1]$ is the identity matrix and Ω_1 is the diagonal natural frequency matrix. More detailed information on the equation development is contained in ref. [3].

1.2 Structural Dynamic Modification

Structural dynamic modification (SDM) is a technique that uses the original mode shapes and natural frequencies of a system to estimate the dynamic characteristics after a modification of mass and/or stiffness is made. First, the change of mass and stiffness are transformed to modal space as shown:

$$\left[\Delta\bar{M}_{12}\right]=[U_1]^T[\Delta M_{12}][U_1], \tag{5}$$

$$\left[\Delta\bar{K}_{12}\right]=[U_1]^T[\Delta K_{12}][U_1]. \tag{6}$$

The modal space mass and stiffness changes are added to the original modal space equations to give:

$$\left[\begin{bmatrix}\ddots & & \\ & \bar{M}_1 & \\ & & \ddots\end{bmatrix}+\left[\Delta\bar{M}_{12}\right]\right]\{\ddot{p}_1\}+ \\ +\left[\begin{bmatrix}\ddots & & \\ & \bar{K}_1 & \\ & & \ddots\end{bmatrix}+\left[\Delta\bar{K}_{12}\right]\right]\{p_1\}=[0]. \tag{7}$$

The eigensolution of the modified modal space model is computed and the resulting eigenvalues are the new frequencies of the system. The resulting eigenvector matrix is the $[U_{12}]$ matrix, which is used to transform the original modes to the new modes and is given as:

$$[U_2]=[U_1][U_{12}]. \tag{8}$$

The new mode shapes are $[U_2]$. The new mode shapes are formed from linear combinations of the original mode shapes. The $[U_{12}]$ matrix shows how much each of the $[U_1]$ modes contributes to forming the new modes. Fig. 1 shows the formation of the new mode shapes as seen in Eq. (8).

Unless $[U_1]$ includes all of the original 'n' system modes, there will be truncation error due to the 'n-m' missing modes. The severity of truncation error depends on which modes are missing from $[U_1]$. Some original system modes are more important than others for forming the new modes. A $[U_{12}]$ calculated using

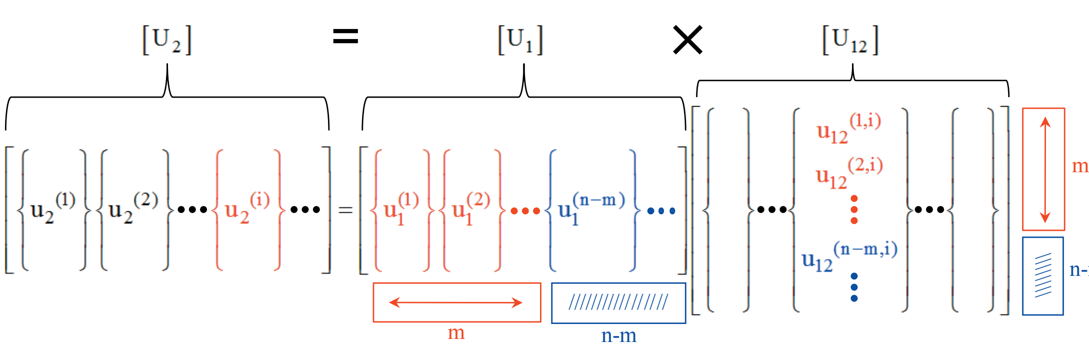


Fig. 1. *Structural dynamic modification, mode contribution identified using U_{12} [2]*

all the original system modes would show the correct contributions of all modes. More detailed information on SDM is contained in ref. [3].

1.3 Physical Space System Modeling

To form a physical system model, the mass and stiffness matrices of each component (A and B) are assembled in stacked form into the system mass and stiffness matrices. In physical space, these are coupled with a stiffness tie matrix; a mass tie can also be included if desired but not included in this work (see [3] for further development of the mass tie case).

$$\begin{bmatrix} [M^A] & \\ & [M^B] \end{bmatrix}\{\ddot{x}\} + \left[\begin{bmatrix} [K^A] & \\ & [K^B] \end{bmatrix} + K_{TIE}\right]\{x\} = \{F\}. \tag{9}$$

This can be cast in a modal space representation as:

$$\left[\begin{bmatrix} [\bar{M}^A] & \\ & [\bar{M}^B] \end{bmatrix}\right]\begin{Bmatrix} \{\ddot{p}^A\} \\ \{\ddot{p}^B\} \end{Bmatrix} + \begin{bmatrix} [\bar{K}^A] & \\ & [\bar{K}^B] \end{bmatrix}\begin{Bmatrix} \{p^A\} \\ \{p^B\} \end{Bmatrix} + \left[[U]^T[\Delta K][U]\right]\begin{Bmatrix} \{p^A\} \\ \{p^B\} \end{Bmatrix} = \{0\}, \tag{10}$$

where $\bar{M}$ and $\bar{K}$ are diagonal matrices and with the mode shapes of each component stacked as:

$$[U] = \begin{bmatrix} [U^A] & \\ & [U^B] \end{bmatrix}.$$

Eq. (9) is a general equation of motion; Eq. (10) is used for the eigensolution in which the force is never included.

1.4 General Reduction/Expansion Technique

Model reduction is used to reduce the number of degrees of freedom in an analytical model to reduce computing time while attempting to preserve the full DOF characteristics. The relationship between the full space and reduced space model can be written as:

$$\{x_n\} = \begin{Bmatrix} x_a \\ x_d \end{Bmatrix} = [T]\{x_a\}, \tag{11}$$

where subscript 'n' signifies the full set of DOF (n DOF), 'a' signifies the reduced set of DOF (a DOF) and 'd' is the deleted DOF. The transformation matrix [T] relates the full set of DOF to the reduced set of DOF.

The transformation matrix is used to reduce the mass and stiffness matrices as:

$$[M_a] = [T]^T[M_n][T] \text{ and } [K_a] = [T]^T[K_n][T]. \tag{12}$$

The eigensolution of these 'a' set mass and stiffness matrices are the modes of the reduced model. These modes can be expanded back to full space using the transformation matrix:

$$[U_n] = [T][U_a]. \tag{13}$$

If an optimal 'a' set is not selected when using methods such as Guyan condensation [4] or improved reduced system technique [5], the reduced model may not perfectly preserve the dynamics of the full space model. If system equivalent reduction expansion process (SEREP) is used, the dynamics of selected modes will be perfectly preserved regardless of the 'a' set selected.

1.5 System Equivalent Reduction Expansion Process (SEREP)

The SEREP modal transformation relies on the partitioning of the modal equations representing the system DOFs relative to the modal DOFs [1]. The SEREP technique utilizes the mode shapes from a full finite element solution to map to the limited set of master DOF. SEREP is not performed to achieve efficiency in the solution but rather is intended to perform an accurate mapping matrix for the transformation. The SEREP transformation matrix is formed using a subset of modes at full space and reduced space as:

$$[T_U] = [U_n][U_a]^g, \tag{14}$$

where the modal vectors with subscripts n and a are as described in Section 1.4 and the superscript g indicates the generalized inverse.

When the SEREP transformation matrix is used for model reduction/expansion as outlined in the previous section, the reduced model perfectly preserves the full space dynamics of the modes in $[U_n]$ as presented in more detail in [1].

1.6 Modal Assurance Criterion (MAC)

Modal assurance criterion is a correlation tool commonly used to compare mode shapes [6]. MAC compares two vectors (u_i and e_j) and calculates a value from 0 to 1 that quantifies the degree of similarity between the vectors. The equation is:

$$MAC_{ij} = \frac{\left[\{u_i\}^T\{e_j\}\right]^2}{\left[\{u_i\}^T\{u_i\}\right]\left[\{e_j\}^T\{e_j\}\right]}. \quad (15)$$

A MAC value of 1 signifies perfect correlation and 0 signifies no correlation.

1.7 Pseudo Orthogonality Check

Pseudo orthogonality check (POC) is a mass weighted orthogonality tool used to compare mode shapes and is given as:

$$POC = [U]^T[M][E]. \quad (16)$$

The POC is mass weighted. If the shapes are scaled to unit modal mass, POC ranges from 0 to 1, similar to MAC. See [7] for further information on model correlation.

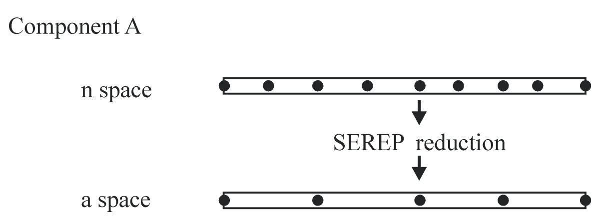
2 METHODOLOGY

The procedure for expanding reduced system models using limited sets of component modal information can be described in the four steps as follows.

a) Reduce models of system components individually using SEREP. Connection degrees of freedom must be retained during reduction in all cases for the assembly process. The modes selected for reduction are very important for minimizing truncation error; Fig. 2 schematically shows the component reduction.
b) Assemble component mass and stiffness matrices into system mass and stiffness matrices and link connection DOF; Fig. 3 schematically shows the formation of the system's matrices.

Component A:
SEREP reduction

$$\left[M_a^A\right] = \left[T_u^A\right]^T\left[M_n^A\right]\left[T_u^A\right],$$
$$\left[K_a^A\right] = \left[T_u^A\right]^T\left[K_n^A\right]\left[T_u^A\right].$$



Component B:
SEREP reduction

$$\left[M_a^B\right] = \left[T_u^B\right]^T\left[M_n^B\right]\left[T_u^B\right],$$
$$\left[K_a^B\right] = \left[T_u^B\right]^T\left[K_n^B\right]\left[T_u^B\right].$$

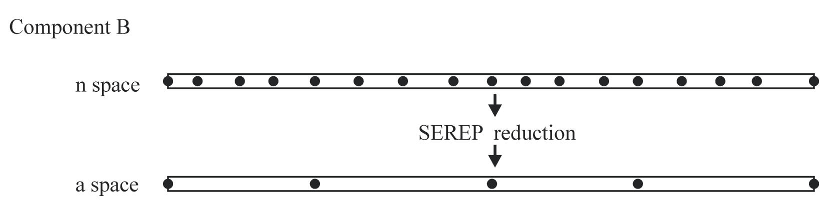
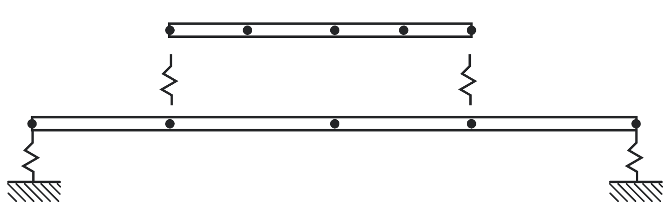


Fig. 2. *Reduce system components using SEREP*



a space

$$\left[M_{sys}\right] = \begin{bmatrix} \left[M^A\right] & \\ & \left[M^B\right] \end{bmatrix},$$
$$\left[K_{sys}\right] = \begin{bmatrix} \left[K^A\right] & \\ & \left[K^B\right] \end{bmatrix} + \left[K_{tie}\right].$$

Fig. 3. *Assemble reduced components to form system*

$$\text{a Space}\quad \left[\begin{bmatrix} [K^A] & \\ & [K^B] \end{bmatrix} + [K_{tie}] - \lambda \begin{bmatrix} [M^A] & \\ & [M^B] \end{bmatrix}\right]\{x\} = \{0\}\quad \text{yields:}$$

$$\text{natural frequencies:}\ \begin{bmatrix} \omega_1^2 & & \\ & \omega_2^2 & \\ & & \ddots \end{bmatrix},\quad \text{system mode shapes: } [U_a] = \begin{bmatrix} [U^A] \\ [U^B] \end{bmatrix}.$$

Fig. 4. *Perform eigensolution to calculate frequencies and mode shapes*

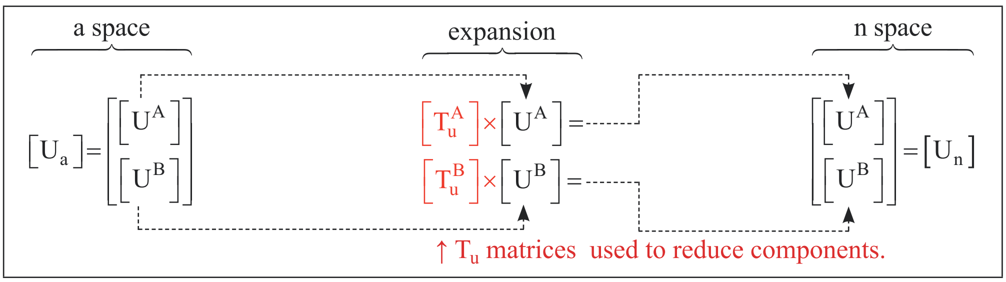


Fig. 5. *Expand reduced system shapes using component transformation matrices*

c) Perform eigensolution on system mass and stiffness matrices to calculate frequencies and mode shapes; Fig. 4 schematically shows the solution process.

d) Partition the system mode shapes into matrices of shape information for each component. Use the SEREP transformation matrices of each component to expand the component shape matrices to full space; Fig. 5 schematically shows the expansion of the reduced system matrices.

The results of this expansion process are the system mode shapes at the full set of DOF whose accuracy is only affected by mode truncation error that occurred during the reduction process. This will be demonstrated by the following cases.

3 MODEL DESCRIPTION

A simple two beam system model was used to illustrate the expansion technique. The beams were modeled using 2D Euler Bernoulli beam elements with only in plane motion considered. Fig. 6 shows the two beam model. The bottom beam is connected to ground at each end by a 175,126.84 N/m translational spring. The two beams are connected with two translation springs.

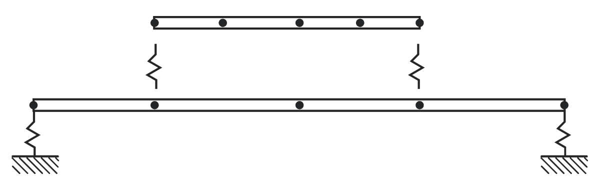


Fig. 6. *Configuration of two beam system model*

Table 1 shows the parameter used for the two beam system of Fig. 6.

Table 1. *Two beam system model parameters*

Parameter	Top beam	Bottom beam
Length [m]	1.27	3.56
Height [m]	0.038	0.038
Width [m]	0.076	0.076
Wall thickness [m]	0.0048	0.0048
Area moment of inertia [m^4]	2.22×10^{-7}	2.22×10^{-7}
Elastic Modulus [GPa]	69	69
Density [kg/m^3]	2712	2712
Nodes	15	29
Beam elements	14	28
Full space DOF	30	56
Connector spring stiffness [N/m]	175,126.835	
Ground support spring stiffness [N/m]	175,126.84	

4 CASES STUDIED

In order to evaluate the expansion methodology, two cases are presented here; other cases were also explored but are not included in this paper due to space restrictions and yield similar results supporting the accuracy and correctness of the expansion methodology presented in this paper. All cases were reduced and expanded following the procedure outlined in the methodology section. For comparison, a full space physical model and a structural dynamic modification model were also developed and compared. The SDM model is comprised of the full DOF using the same modes used for reduction. MAC and POC are used to compare mode shapes. Two cases were studied here and are described as:

- Case 1: 5 modes each from component sA & B;
- Case 2: 10 modes each from components A & B.

These cases are described in the following sections. Fig. 7 shows the entire expansion process schematically to further describe the overall procedure. The component model modes are extracted from the full space component models (which are typically available in the design process) as shown in the upper portion of the figure. These component models are then reduced with the component transformation matrices and used to develop the reduced order system model from the reduced order component models as shown in the center portion of the figure. The individual, uncoupled component models are used to develop the transformation/expansion matrix shown on the left and right middle section of the figure; this transformation/expansion matrix already exists because it was used to reduce the components. These expansion matrices are used to expand the reduced order system model modes to obtain the full space system model modes for the assembled system model shown at the bottom of the figure.

4.1 Case 1: 5 Modes from Component A & 5 Modes from Component B

Each beam was reduced to 5 translational DOF using the first 5 modes of each component beam to form the system model. The expanded results are compared to the full space physical and SDM models in Table 2 and Fig. 8. The table is broken down into the left and right sides as discussed next; the left side compares the full space physical system model results and the right side presents comparison checks to a SDM model to confirm accuracy of the results.

On the left side, the full space system model lists the results obtained from a full space physical model and is referred to as the reference solution. Based on the $[U_{12}]$ matrix generated for this case (not shown for brevity), only 6 system modes are expected to be accurate; this is due to the fact that the 5 component modes of each component only accurately span that space of the system model. The $[U_{12}]$ from the structural dynamic modification system modeling process identifies this and these results are anticipated.

Table 2 shows that the first 6 system modes are accurately represented by the reduced component system model as evidenced by the accurate frequency prediction (<2.00% difference). The reduced order system model mode shapes are expanded to full space using the original unmodified individual component modes as described in this paper. These expanded system model modes are then checked with the full space reference model using both MAC and POC; these two columns of values on the left side of the table clearly show that the expansion of the reduced order system model using the original unmodified component modes are an accurate representation of the mode shapes. Any modes beyond the 6 system model modes are not accurately predicted because the modes of the component models do not span the space of the higher order system modes and therefore do not accurately predict those frequencies.

The results on the right side of the table are presented as a check to make sure that the results are as expected. The SDM results are those obtained using the full space modal model and are compared to the expanded results; these two results must produce identical results for both the accurately predicted system modes as well as the truncated higher frequency results. The MAC and POC also substantiate this. This perfect correlation shows there is no distortion when expanding from ‘a’ space to ‘n’ space; the only source of error for the modal models is mode truncation.

The results of this case clearly show that if the original unmodified component modes are sufficient to span the space of the reduced order system model to predict accurate system model frequencies, then these same component modes are also sufficient to be used as expansion matrices to obtain the full system model combined response; but this is only true for the system modes that can be accurately represented by the individual component modes.

4.2 Case 2: 10 Modes from Component A & 10 Modes from Component B

In Case 1, the first 6 system modes were accurately represented by 5 component modes for each individual component. These 5 component modes are only capable of predicting the first 6 system model modes accurately. To show that truncation limits the number of accurate system modes, Case 2 uses 10 component modes for each of the individual components and based on the $[U_{12}]$ then the system model can only predict the first 12 system model modes accurately. Table 3 and Fig. 9 show these results. Case 2 shows that more system modes are accurately predicted due to the inclusion of more component modes to describe each reduced order component and further substantiate the results in Case 1.

4.3 Observations for Cases Studied

Results of the case studies show that expanding system model modes using the component transformation matrices yields exactly the same results as a full space structural dynamic modification model, assuming the same modes are used.

In the full space SDM model, mode truncation is the only source of error. There is no expansion error because the SDM is done at full space. The SDM and expansion results were shown to be equivalent. Therefore mode truncation is the only source of error and differences are not a result of the expansion

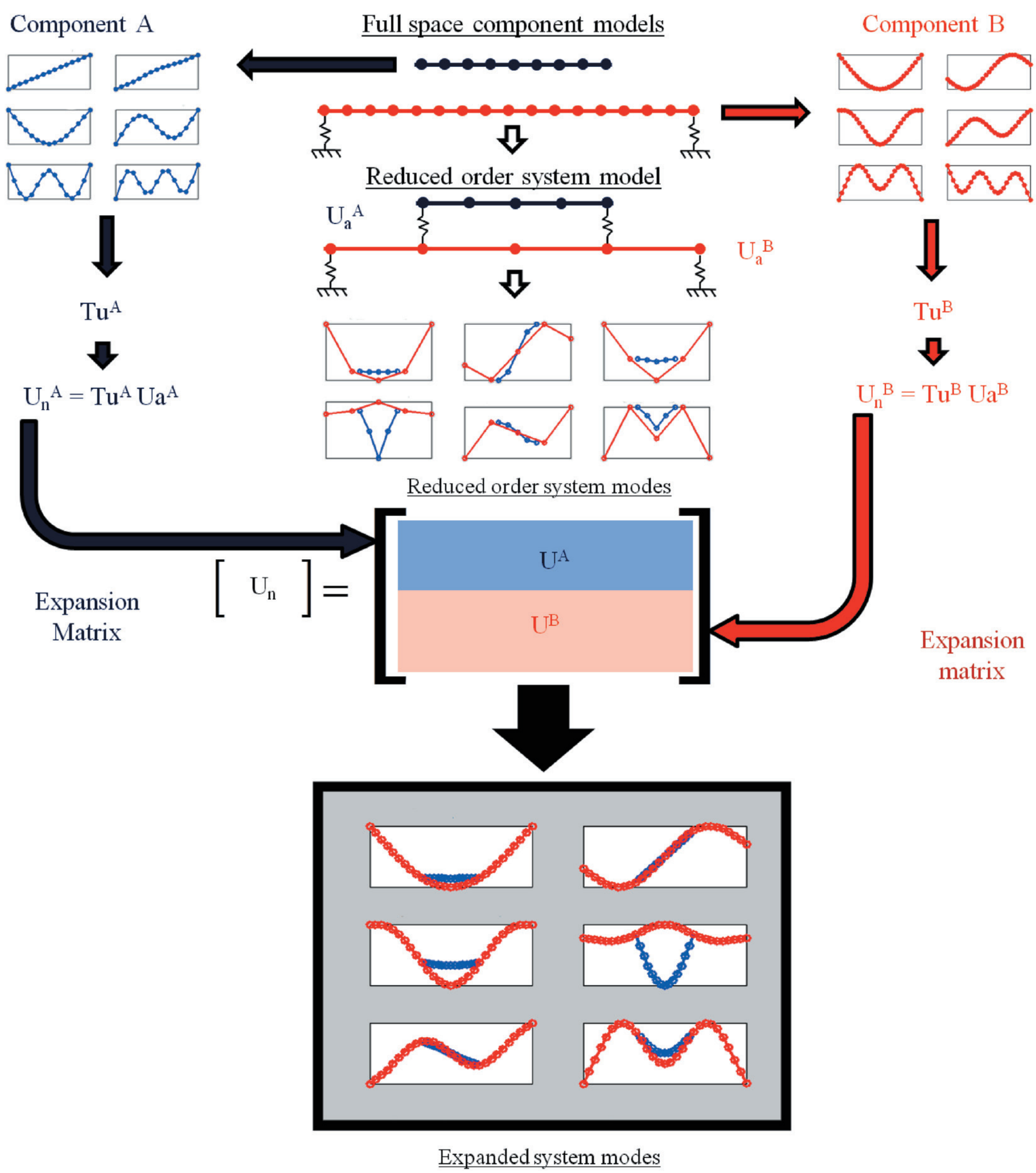
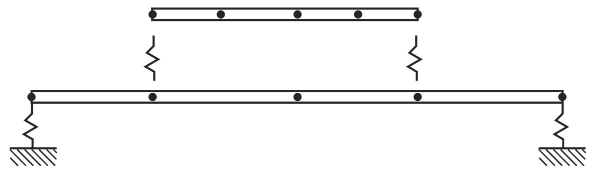


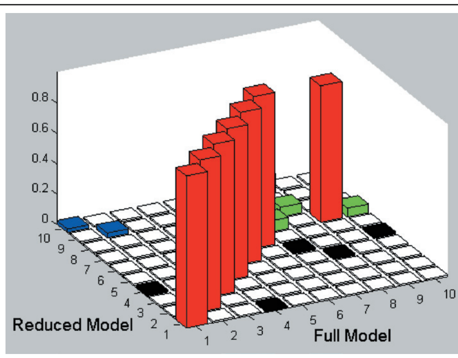
Fig. 7. *Overall expansion process schematic for models studied*

Table 2. *Case 1, expanded results compared to full Space physical and SDM model*

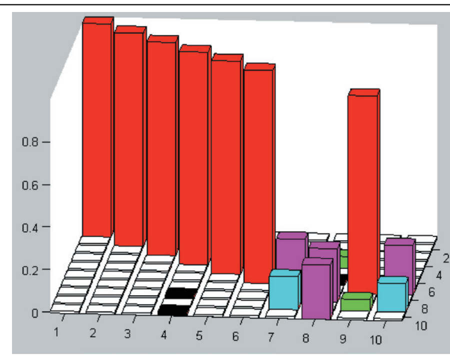
Case 1: 5 Modes each from component A & B with symmetric mounting



Mode	Full space system model comparison					Structural dynamic modification comparison				
	Frequency [Hz]			MAC	POC	Frequency [Hz]			MAC	POC
	Full Physical 'n' space	Expanded model from 'a' space	% Diff			SDM at 'n' space	Expanded model from 'a' space	% Diff		
1	7.3	7.2	0.01	1.000	1.000	7.2	7.2	0.00	1.000	1.000
2	29.5	29.5	0.02	1.000	1.000	29.5	29.5	0.00	1.000	1.000
3	58.7	58.7	0.01	1.000	1.000	58.7	58.7	0.00	1.000	1.000
4	81.7	83.36	2.00	0.997	0.998	83.36	83.36	0.00	1.000	1.000
5	83.3	83.37	0.10	1.000	0.999	83.37	83.37	0.00	1.000	1.000
6	132.8	132.8	0.00	1.000	0.999	132.8	132.8	0.00	1.000	1.000
7	191.8	316.4	65.0	0.073	0.253	316.4	316.4	0.00	1.000	1.000
8	273.8	680.8	148	0.064	0.252	680.8	680.8	0.00	1.000	1.000
9	306.2	4461.2	1356	0.002	0.058	4461.2	4461.2	0.00	1.000	1.000
10	398.7	4977.3	1148	0.000	0.000	4977.3	4977.3	0.00	1.000	1.000

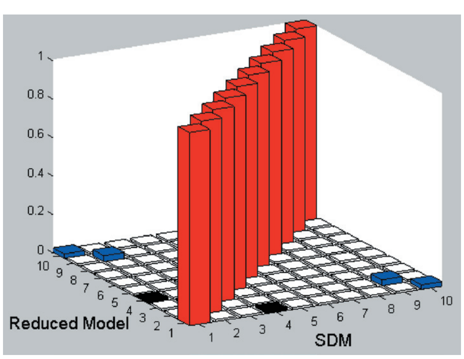


MAC

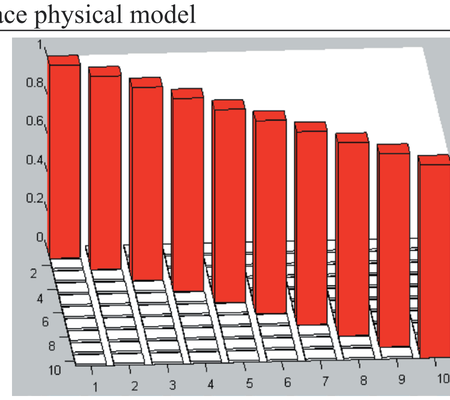


POC

Expanded compared to Full space physical model



MAC



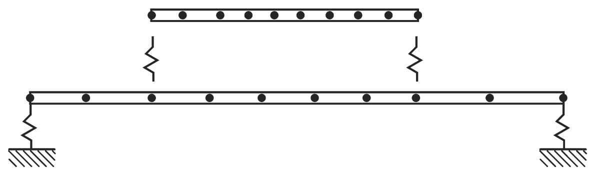
POC

Expanded compared to Structural dynamic modification model

Fig. 8. *Case 1, expanded results compared to full space model and SDM*

Table 3. *Case 2, expanded results compared to full Space physical and SDM model*

Case 2: 10 Modes each from component A & B with symmetric mounting



Mode	Full space system model comparison					Structural dynamic modification comparison				
	Frequency [Hz]			MAC	POC	Frequency [Hz]			MAC	POC
	Full Physical 'n' space	Expanded model from 'a' space	% Diff			SDM at 'n' space	Expanded model from 'a' space	% Diff		
1	7.3	7.3	0.00	1.000	1.000	7.3	7.3	0.00	1.000	1.000
2	29.5	29.5	0.00	1.000	1.000	29.5	29.5	0.00	1.000	1.000
3	58.7	58.7	0.00	1.000	1.000	58.7	58.7	0.00	1.000	1.000
4	81.7	81.9	0.21	1.000	1.000	81.9	81.9	0.00	1.000	1.000
5	83.3	83.3	0.01	1.000	1.000	83.3	83.3	0.00	1.000	1.000
6	132.8	132.8	0.00	1.000	1.000	132.8	132.8	0.00	1.000	1.000
7	191.8	191.8	0.03	1.000	1.000	191.8	191.8	0.00	1.000	1.000
8	273.8	273.9	0.06	1.000	1.000	273.9	273.9	0.00	1.000	1.000
9	306.2	307.6	0.44	1.000	1.000	307.6	307.6	0.00	1.000	1.000
10	398.7	398.7	0.01	1.000	1.000	398.7	398.7	0.00	1.000	1.000
11	514.2	515.0	0.15	1.000	0.999	515.0	515.0	0.00	1.000	1.000
12	649.0	649.7	0.10	0.999	1.000	649.7	649.7	0.00	1.000	1.000
13	674.4	694.7	3.02	0.945	0.967	694.7	694.7	0.00	1.000	1.000
14	840.4	1206.6	43.6	0.000	0.000	1206.6	1206.6	0.00	1.000	1.000
15	1011.6	1837.4	81.6	0.000	0.000	1837.4	1837.4	0.00	1.000	1.000

technique. Truncation error is introduced when reducing the component models and forming the system model.

Comparison to the full space model shows that lower order modes can be calculated accurately; higher order modes are significantly affected by mode truncation and cannot be predicted accurately, as expected. If more system modes were desired, more component modes would need to be used in the reduction of the components.

4.4 Important Applications

Results of the case studied clearly show the usefulness for development of linear system models from component data. However another very important application is for the development of nonlinear models interconnected with highly nonlinear connection elements [8] and [9]. In that work, the linear modal components are interconnected with highly nonlinear connection elements and nonlinear response was shown to be very accurately produced with highly efficient models. That work showed that as long as a sufficient number of modes were used to appropriately span the space for the nonlinear solution, then those very efficient models could be deployed without loss of accuracy. That previous nonlinear work coupled with this expansion methodology can accurately expand the nonlinear response to the full space of each of the individual components to provide full field response for complicated dynamic stress-strain distributions with these same very efficient reduced order nonlinear model representations.

5 CONCLUSIONS

The technique proposed in this work uses the expansion matrices of uncoupled component models to expand the modes of an assembled reduced order system model. Expansion to the full space assembled system can be performed to define full space characteristics.

Several reduced order system model cases were presented to demonstrate the validity of the expansion approach developed in this work. This approach works provided that the original unmodified

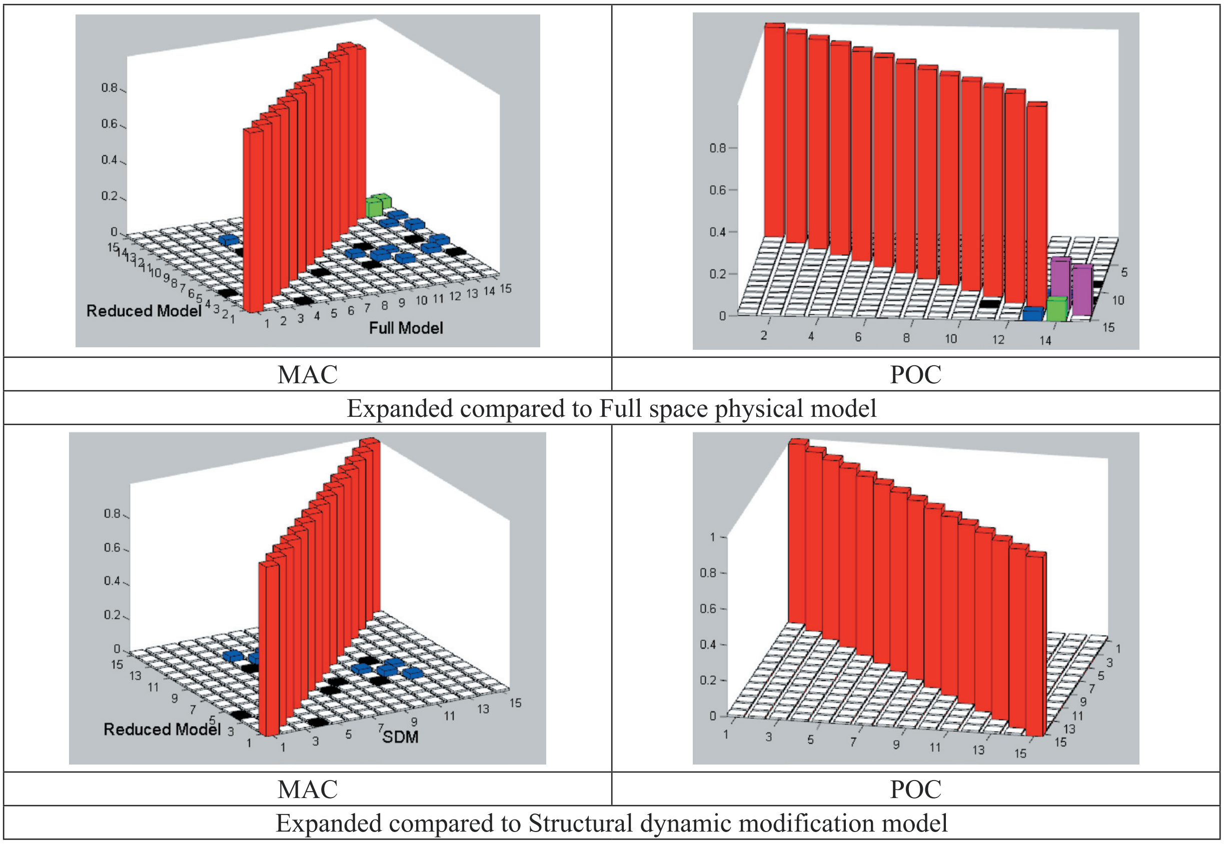


Fig. 9. *Case 2, expanded results compared to full space model and SDM*

component modes retained in the reduced order model are sufficient to span the space of the final system model in order to obtain accurate system modes. The expansion matrices developed in the reduction process, which contains the same modal information as in the reduced models, are also sufficient to be used as expansion matrices to obtain full system model characteristics; but this is only true for the system modes that can be accurately represented by the individual component modes. This allows for full system level identification of modal characteristics without the need for developing the expansion matrices using the fully assembled full space system model and allows for full space prediction of important system level information such as dynamic stress and strain.

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7 NOMENCLATURE

Symbols:

Matrix

[M]	Analytical mass matrix
[K]	Analytical stiffness matrix
[C]	Physical damping matrix
[U]	Analytical modal matrix
$\left[\overline{M}\right]$	Diagonal modal mass matrix
$\left[\overline{K}\right]$	Diagonal modal stiffness matrix
$\left[\overline{C}\right]$	Diagonal modal damping matrix
[T]	Transformation matrix
[I]	Identity matrix
[E]	Matrix of expanded modal vectors

Vector

{p}	Modal displacement
{F}	Force
{u}	Analytical mode shape
{x}	Physical displacement
$\{\dot{x}\}$	Physical velocity
$\{\ddot{x}\}$	Physical acceleration
{t}	Time vector

Subscript

1	State 1
2	State 2
12	State 1-2
i	Row i
j	Column j
n	Full set of finite element DOF
a	Reduced set of DOF
d	Deleted (omitted) set of DOF
U	SEREP
tie	Stiffness tie matrix
sys	System AB of assembled components

Superscript

T	Transpose
g	Generalized inverse
k	k^{th} degree of freedom
-1	Standard inverse
A	Component A
B	Component B

8 REFERENCES

[1] O'Callahan, J.C., Avitabile, P., Riemer, R. (1989). System equivalent reduction expansion process. *Proceedings of the 7th International Modal Analysis Conference*, Las Vegas.

[2] Thibault, L., Butland, A., Avitabile, P. (2012). Variability improvement of key inaccurate node groups – VIKING. *Proceedings of the 13th International Modal Analysis Conference*, Jacksonville.

[3] Avitabile, P. (2003). Twenty years of structural dynamic modification – A review. *Sound and Vibration*, vol. 37, no. 1, p. 14-27.

[4] Guyan, R.J. (1965). Reduction of stiffness and mass matrices. *AIAA Journal*, vol. 3, no. 2, p. 380, DOI:10.2514/3.2874.

[5] O'Callahan, J.C., (1989). A procedure for an improved reduced system (IRS) model. *Proceedings of the 7th International Modal Analysis Conference*, Las Vegas.

[6] Allemang, R.J., Brown, D.L. (2007). A correlation coefficient for modal vector analysis. *Proceedings of the 1st International Modal Analysis Conference*, Orlando.

[7] Avitabile, P., O'Callahan, J. (1988). Model correlation and orthogonality criteria. *6th International Modal Analysis Conference*, Orlando.

[8] Thibault, L., Avitabile, P., Foley, J., Wolfson, J. (2012). Equivalent reduced model technique development for nonlinear system dynamic response. *Mechanical Systems and Signal Processing,* vol. 36, no. 2, p. 422-455, DOI:10.1016/j.ymssp.2012.07.013.

[9] Marinone, T., Avitabile, P., Foley, J., Wolfson, J. (2012). Efficient computational nonlinear dynamic analysis using modal modification response technique. *Mechanical Systems and Signal Processing*, vol. 31, p. 67-93, DOI:10.1016/j.ymssp.2012.02.011.