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Parameter Identification of Multistorey Frame Structure from Uncertain Dynamic Data

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This paper investigates the identification procedure of the column stiffness of multistorey frame structures by using the prior (known) uncertain parameters and dynamic data. Uncertainties are modelled through triangular convex normalized fuzzy sets. Bounds of the identified uncertain stiffness are obtained by using a proposed fuzzy based iteration algorithm with Taylor series expansion. Example problems are solved to demonstrate the reliability and efficiency of the identification process.
Keywords: stiffness matrix, mass matrix, Taylor series expansion, triangular fuzzy number, natural frequency

0 INTRODUCTION

Structural dynamics problems may be categorized as direct or inverse problems. The direct problem consists of finding the response for a specified input or excitation. In the inverse problem the response is known then to develop a mathematical model of the system. The modelling problem may also be divided into two categories. In the first category the nature of the process is completely unknown. But in the second category, a considerable knowledge of the nature of the system may be available, whereas the particular values of the system parameters are unknown. In this paper the second category has been studied, where system equations are known or deducible from the physics of the system, with coefficients remaining to be estimated and modified as per the known initial dynamic characteristics.

In this context various workers [1] to [3] have reviewed the state of the art of system identification in structural dynamics. Developments and various methods for studying this important field are available in literature [4]. More recent methods and practical guidelines for linear systems may be found in the work of Schoukens and Pintelon [5]. Few researchers have studied the above issues but, continuous efforts are being made to refine and develop new models for identification problems. Some representative works on the subject are available in [6] to [9]. Accordingly related works done are discussed in the subsequent paragraphs.

Loh and Ton [10] have studied a system identification approach to detect changes in structural dynamic characteristics on the basis of measurements. They used the recursive instrumental variable method and extended Kalman filter algorithm for the identification procedure. Potential of using neural network to identify the internal forces of typical systems has been investigated by Chassiakos and Masri [11]. A localized identification of many degrees of freedom structures is investigated by Zhao et al. [12] and a memory-matrix based identification methodology for structural and mechanical systems is studied by Udwadia and Proskurowski [13]. Notable studies in this field have also been done by other workers [14] to [18]. Recently various authors [19] to [22] studied different procedures for the parameter identification problems of different structures and buildings. Budipriyanto [19] addresses the application of blind source separation technique for identifying dynamic parameters of a seismic-excited multi-story building from its measured response. A new technique based on second order blind identification, called the modified cross-correlation method for the identification of the structures has been studied by Hazra et al. [20]. Rahmani and Todorovska [21] presented two new algorithms for 1D system identification of buildings during earthquakes by seismic interferometry using waveform inversion of impulse responses. Wave travel time analysis and layered shear beam models are used by Todorovska and Rahmani [22] for the system identification of buildings. Also an interesting and important review paper related to vibration based damage identification methods has been written by Fan and Qiao [23].

The objective of the structural dynamic analysis related to identification is to develop an analytical model of a structure which can be verified and adjusted by actual test results. However, this adjustment is not easy and can be done by computer with good convergence algorithm in terms of iterative cycles.

As discussed above usual method of identification uses the values of the parameters initially given to the structure by an engineer. It then modifies the original parameter values as per the observed values from test by an iteration process. The parameters involved in the said problems are traditionally considered as crisp or defined exactly. But, rather than

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particular value, we may have only the uncertain or incomplete information about the parameters being a result of errors in measurements, observations, applying different operating conditions or it may be maintenance induced error, etc. So, for various scientific and engineering problems, it is an important issue how to deal with variables and parameters of uncertain value. Recently some effort has been made by various researchers throughout the globe to handle these uncertainties in terms of probabilistic, interval or fuzzy approach. Unfortunately, probabilistic methods may not deliver reliable results with required precision without sufficient data. Hence interval and fuzzy theory are becoming powerful tools for handling these uncertainties in recent decades.

Recently, few authors [24] to [27] have studied the solution methods for fuzzy and interval system of linear equations. They have considered the system of linear equations with fuzzy and interval numbers. Also vibration analysis of structures with imprecise material properties is also done by few authors. As such papers that are related to interval and fuzzy eigenvalue problems are discussed here. An excellent paper by Chen et al. [28] who has presented a new method for calculating the upper and lower eigenvalue bound of structures with interval parameter. Uncertain bunds of eigenvalue are also studied by Friswell et al. [29]. Dimarogonas [30] discussed the vibration problem using interval analysis. Cechlarova [31] investigated the eigenvectors of the interval matrix using max-plus algebra. Recently, modal analysis of structures by using interval analysis is studied by Sim et al. [32]. Qui et al. [33] presented a paper which gives detailed analysis for exact bounds for the static response of structures with uncertain-but-bounded parameters. Xia and Yu [34] studied modified interval and subinterval perturbation methods for the static response analysis of structures with interval parameters. Dynamic Analysis of structures with interval uncertainty has been explained by Modares and Mullen [35].

Fuzzy material and geometric properties have also been considered by various authors for finite element analysis. Both static and dynamic analyses of structures are excellently explained by Akpan et al. [36] using fuzzy finite element analysis. An important paper is that of Hanss et al. [37] who proposed the application of fuzzy arithmetic in the finite element analysis. Behera and Chakraverty [38, 39] investiagted various solution procedures for the static analysis of structures with fuzzy parameters. Very recently Sahoo and Chakraverty [40] presented fuzzified data based neural network modeling for health assessment of multistorey shear buildings. Also soft computing methods for model updating of multistory shear buildings for simultaneous identification of mass, stiffness and damping matrices have been investigated by Khanmirza et al. [41].

In view of the above, the present study proposes a systematic mathematical model for the identification of uncertain structural parameters using the vibration characteristics consistent with the uncertain experimental data. The method first uses the values of the uncertain structural parameters (viz. as triangular fuzzy numbers) initially given to the structure by an engineer. It then modifies the original parameter values as per the observed values from test by an iteration process using Taylor series expansion. It gives uncertain fuzzy bound of modified values of the parameters to have a better estimation of structural safety. In the following sections, first preliminaries are described, followed by the mathematical modelling and identification process. Then numerical examples for two storey frame structures are described. Finally discussion and conclusions are drawn.

1 PRELIMINARIES

In the following paragraph some definitions related to the present work are given.

Definition 1.1 Fuzzy number [42] and [43]

A fuzzy number U is convex normalised fuzzy set U of the real line R such that:

$$\{\mu_U(x): R \to [0,1], \forall\, x \in R\},$$

where, μ_U is called the membership function of the fuzzy set and it is piecewise continuous.

Definition 1.2 Triangular fuzzy number (TFN)

A fuzzy number U is said to be triangular if:

i. There exists exactly one $x_0 \in R$ with $\mu_U(x_0)=1$ (x_0 is called the mean value of U), where μ_U is called the membership function of the fuzzy set.
ii. $\mu_U(x)$ is piecewise continuous.

The membership function μ_U of an arbitrary triangular fuzzy number $U=(a,b,c)$ may be defined as follows:

$$\mu_U(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a \le x \le b \\ \dfrac{c-x}{c-b}, & b \le x \le c \\ 0, & x \ge c \end{cases}.$$

Any arbitrary triangular fuzzy number $U=(a,b,c)$ can be represented with an ordered pair of functions through α–cut approach as:

$$[\underline{u}(\alpha),\overline{u}(\alpha)]=[(b-a)\alpha+a,-(c-b)\alpha+c],$$

where $\alpha\in[0,1]$.

This satisfies the following requirements:

i. $\underline{u}(\alpha)$ is a bounded left continuous non-decreasing function over $[0,1]$.

ii. $\overline{u}(\alpha)$ is a bounded right continuous non-increasing function over $[0,1]$.

iii. $\underline{u}(\alpha)\le\overline{u}(\alpha),\ 0\le\alpha\le1$.

Definition 1.3 Fuzzy arithmetic [24] and [25]

As discussed above, fuzzy numbers may be transformed into an interval through α–cut approach. So, for any arbitrary fuzzy number $x=[\underline{x}(\alpha),\overline{x}(\alpha)]$, $y=[\underline{y}(\alpha),\overline{y}(\alpha)]$ and scalar k, we have, $x=y$ if and only if $\underline{x}(\alpha)=\underline{y}(\alpha)$ and $\overline{x}(\alpha)=\overline{y}(\alpha)$.

Addition: $x+y=[\underline{x}(\alpha)+\underline{y}(\alpha),\overline{x}(\alpha)+\overline{y}(\alpha)]$.

Subtraction: $x-y=[\underline{x}(\alpha)-\overline{y}(\alpha),\overline{x}(\alpha)-\underline{y}(\alpha)]$.

Multiplication:

$$x\times y=\left[\min\,(a_1,a_2,a_3,a_4),\max\,(a_1,a_2,a_3,a_4)\right],$$

where,

$$a_1=\underline{x}(\alpha)\times\underline{y}(\alpha),\quad a_2=\underline{x}(\alpha)\times\overline{y}(\alpha),$$
$$a_3=\overline{x}(\alpha)\times\underline{y}(\alpha),\quad a_4=\overline{x}(\alpha)\times\overline{y}(\alpha),$$

$$\text{and }kx=\begin{cases}[k\overline{x}(\alpha),k\underline{x}(\alpha)], & k<0,\\ [k\underline{x}(\alpha),k\overline{x}(\alpha)], & k\ge0.\end{cases}$$

2 MATHEMATICAL MODELLING AND METHOD OF IDENTIFICATION

To investigate the present method, a two-storeyed frame structure, as shown in Fig. 1 is considered. However the general multistorey frame structure modeling may easily be extended from this example of two storey frame. This is investigated for the sake of demonstration of the procedure. The uncertain floor mass, $\tilde{m}$ is assumed to be the same and the uncertain column stiffnesses $\tilde{k}_1,\tilde{k}_2,\tilde{k}_3$ and $\tilde{k}_4$ (as labelled in Fig. 1) are the structural parameters which are to be identified. Corresponding uncertain dynamic equation of motion in matrix form for two degrees of freedom system may be written as:

$$[\tilde{M}]\{\ddot{\tilde{x}}\}+[\tilde{K}]\{\tilde{x}\}=\{0\},\tag{1}$$

where $[\tilde{M}]=\begin{bmatrix}\tilde{m} & 0\\ 0 & \tilde{m}\end{bmatrix}$ is a 2×2 fuzzy mass matrix,

$$[\tilde{K}]=\begin{bmatrix}(\tilde{k}_1+\tilde{k}_2+\tilde{k}_3+\tilde{k}_4) & -(\tilde{k}_3+\tilde{k}_4)\\ -(\tilde{k}_3+\tilde{k}_4) & (\tilde{k}_3+\tilde{k}_4)\end{bmatrix},$$

is a 2×2 fuzzy stiffness matrix and $\{\tilde{X}\}=2\times1$ is a fuzzy vector of displacements.

Considering the simple harmonic motion, Eq. (1) can be written as a fuzzy eigenvalue problem:

$$[\tilde{K}]\{\tilde{X}\}=\tilde{\lambda}[\tilde{M}]\{\tilde{X}\}.\tag{2}$$

By using the parametric form of fuzzy numbers, Eq. (2) will be:

$$[\underline{K}(\alpha),\overline{K}(\alpha)]\{\underline{X}(\alpha),\overline{X}(\alpha)\}=$$
$$=[\underline{\lambda}(\alpha),\overline{\lambda}(\alpha)][\underline{M}(\alpha),\overline{M}(\alpha)]\{\underline{X}(\alpha),\overline{X}(\alpha)\}.$$

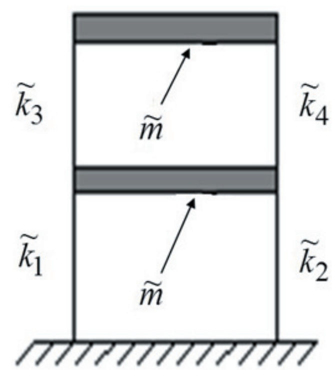

Fig. 1. *Two storey frame structure*

Now our aim is to solve the above fuzzy eigenvalue problem to get the lower and upper bounds of the fuzzy eigenvalues.

With the above in mind, let us proceed now with the identification procedure which can handle the uncertain data. Let us assume that the uncertain structural parameters to be identified are denoted by $\tilde{P}_i$, for $i=1,2,3,4$. The uncertain value of the structural parameters of the prior original structure given initially are denoted by $\hat{\tilde{P}}_i$, for $i=1,2,3,4$ and the corresponding fuzzy eigenvalues are symbolized as, $\hat{\tilde{\lambda}}_i(\hat{\tilde{P}})$.

Next the well-known Taylor's series expansion of the fuzzy modal parameters about the initial estimates of the parameters give;

$$\{\tilde{\lambda}(\tilde{P})\} = \left\{\hat{\tilde{\lambda}}(\hat{\tilde{P}})\right\} + [\tilde{S}]\left(\{\tilde{P}\} - \left\{\hat{\tilde{P}}\right\}\right), \tag{3}$$

where

$\{\tilde{P}\} = \left[\tilde{P}_1, \tilde{P}_2, \tilde{P}_3, \tilde{P}_4\right]^T$, $\left\{\hat{\tilde{P}}\right\} = \left[\hat{\tilde{P}}_1, \hat{\tilde{P}}_2, \hat{\tilde{P}}_3, \hat{\tilde{P}}_4\right]^T$ and

$[\tilde{S}]$ is the fuzzy eigenvalue partial derivative matrix, $[\partial(\tilde{\lambda}) / \partial(\tilde{P})]$.

Let us now denote, experimentally measured uncertain eigenvalues by $\{\tilde{\lambda}_E\}$. It is interesting to note here that if the values of the initial and experimental parameters are equal, then no modification is done. But if the values are different then we denote this difference by:

$$\{\delta\tilde{\lambda}\} = \{\tilde{\lambda}_E\} - \left\{\hat{\tilde{\lambda}}\right\}. \tag{4}$$

Next, let us denote the modified parameters as:

$$\{\bar{\tilde{P}}\} = [\bar{\tilde{P}}_1, \bar{\tilde{P}}_2, \bar{\tilde{P}}_3, \bar{\tilde{P}}_4]^T, \tag{5}$$

and, in general, for n–degrees of freedom system the expression for the uncertain modified parameters from Eq. (3) can be written as:

$$\{\bar{\tilde{P}}\} = \left\{\hat{\tilde{P}}\right\} + [\tilde{Q}]\{\delta\tilde{\lambda}\}, \tag{6}$$

where

$$[\tilde{Q}] = \left([\tilde{S}]^T[\tilde{S}]\right)^{-1}[\tilde{S}]^T.$$

In order to have the uncertain bounds of the identified parameters with acceptable accuracy, here an iterative procedure is proposed. After finding the modified parameters from Eq. (6), these are substituted in Eq. (2) to get revised uncertain vibration characteristics viz. $\{\bar{\tilde{\lambda}}\}$.

The new fuzzy eigenvalue partial derivative matrix $\{\bar{\tilde{S}}\}$ is then obtained using the current values of $\{\bar{\tilde{P}}\}$ and $\{\bar{\tilde{\lambda}}\}$. From Eq. (6), the modified parameters $\{\bar{\tilde{P}}_t\}$ are again found by utilizing the above values and then the new (revised) estimates of fuzzy eigenvalues are obtained as $\{\bar{\tilde{\lambda}}_t\}$.

If the vector norm of $\{\bar{\tilde{\lambda}}\}$ and $\{\bar{\tilde{\lambda}}_t\}$ is less than some specified accuracy then the procedure is stopped and the revised parameter viz. $\{\bar{\tilde{P}}_t\}$ is identified, otherwise the next iteration is to be followed.

3 NUMERICAL RESULTS

As mentioned earlier, the procedure is demonstrated for a two storeyed frame structure. Implementing the above procedure with the proposed iterative cycle for the revised uncertain frequencies and parameters, computer programs have been written and tested for the above problem.

In the above example problem, floor masses,

$$m = (3550, 3600, 3650) \text{ kg},$$

and the column stiffnesses:

$$\tilde{k}_1 = \tilde{k}_2 = (5350, 5400, 5450) \text{ N/m},$$
$$\tilde{k}_3 = \tilde{k}_4 = (3550, 3600, 3650) \text{ N/m},$$

have been taken as triangular fuzzy number. Through α–cut these may represented as:

$$m = [50\alpha + 3550, -50\alpha + 3650] \text{ kg},$$
$$\tilde{k}_1 = \tilde{k}_2 = [50\alpha + 5350, -50\alpha + 5450] \text{ N/m},$$
$$\text{and } \tilde{k}_3 = \tilde{k}_4 = [50\alpha + 3550, -50\alpha + 3650] \text{ N/m}.$$

From the prior mass and stiffness parameters, the uncertain vibration characteristics may be computed from Eq. (2) as:

$$\tilde{\lambda}_1 = (0.9314, 1, 1.0703) \text{ and } \tilde{\lambda}_2 = (5.8906, 6, 6.1128).$$

Using the above sets of initial data of the fuzzy parameters with different uncertain experimental (hypothetical) test data for the frequencies, viz. $\tilde{\lambda}_{1E} = (0.65, 0.7, 0.75)$ and $\tilde{\lambda}_{2E} = (5.3, 5.5, 5.7)$ (i.e. first and second experimental eigenvalues of the system) the bounds of the stiffness parameters of the structure have been identified and these are reported in Table 1. Corresponding plot for identified stiffness parameters are depicted in Figs. 2 and 3.

Similarly, another set of experimental (hypothetical) fuzzy data of the natural frequencies are considered as:

$$\tilde{\lambda}_{1E} = (0.88, 0.9, 0.92) \text{ and } \tilde{\lambda}_{2E} = (5.3, 5.5, 5.7).$$

The identified bounds of the stiffness parameters are tabulated in Table 2 and the revised frequencies are also shown in Table 3. Corresponding plot for identified stiffness parameters are given in Figs. 4 and 5.

Table 1. *Identified lower and upper bounds of stiffness parameters*

Bounds of stiffness parameters [N/m]	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
$\underline{k}_1 = \underline{k}_2$	5232	5274	5316
$\overline{k}_1 = \overline{k}_2$	5400.5	5358	5316
$\underline{k}_3 = \underline{k}_4$	3614.5	3629.8	3645
$\overline{k}_3 = \overline{k}_4$	3674.7	3659.9	3645

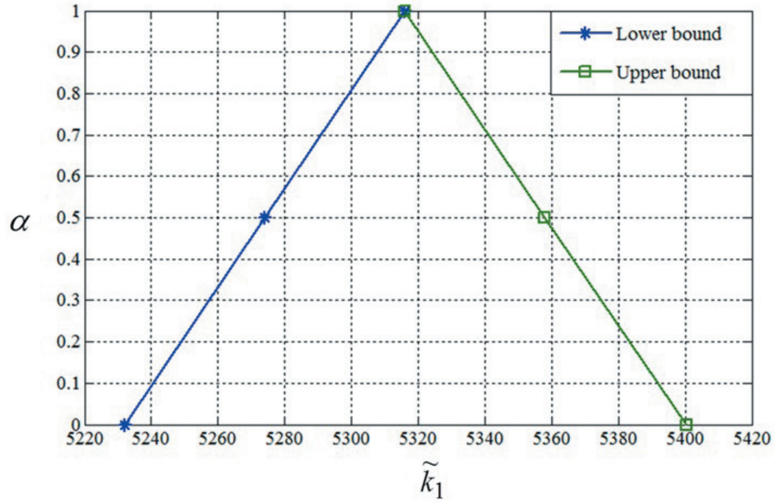

Fig. 2. *Identified lower and upper bounds of stiffness parameter* $\tilde{k}_1$ [N/m]

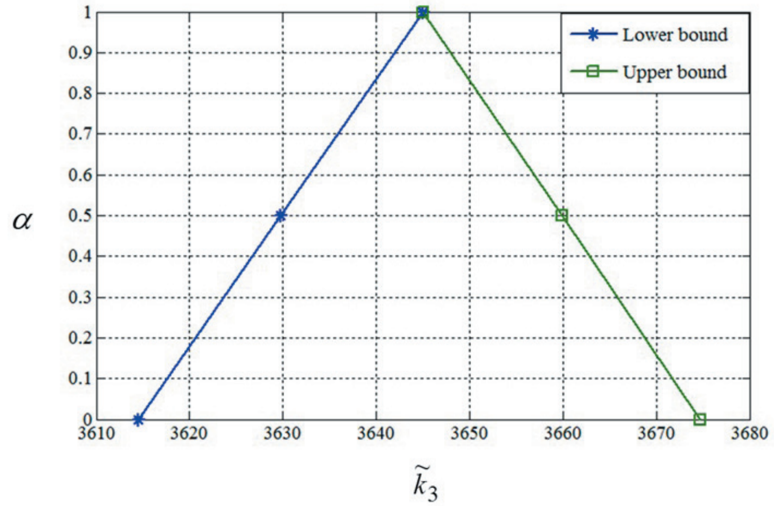

Fig. 3. Identified lower and upper bounds of stiffness parameter $\tilde{k}_3$ [N/m]

4 DISCUSSIONS AND CONCLUSION

The present procedure systematically modifies and finally identifies the uncertain structural parameters, viz. the column stiffness for a frame structure. It uses the prior (known) estimates of uncertain parameters and corresponding uncertain vibration characteristics. Then the algorithm estimates the bounds of present parameters utilizing the known uncertain dynamic data from some experiments. Proposed numerical

Table 2. *Identified lower and upper bounds of stiffness parameters*

Bounds of stiffness parameters [N/m]	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
$\underline{k}_1 = \underline{k}_2$	5286	5324	5362
$\overline{k}_1 = \overline{k}_2$	5440	5401	5362
$\underline{k}_3 = \underline{k}_4$	3590.6	3607.7	3624.5
$\overline{k}_3 = \overline{k}_4$	3657.6	3641	3624.5

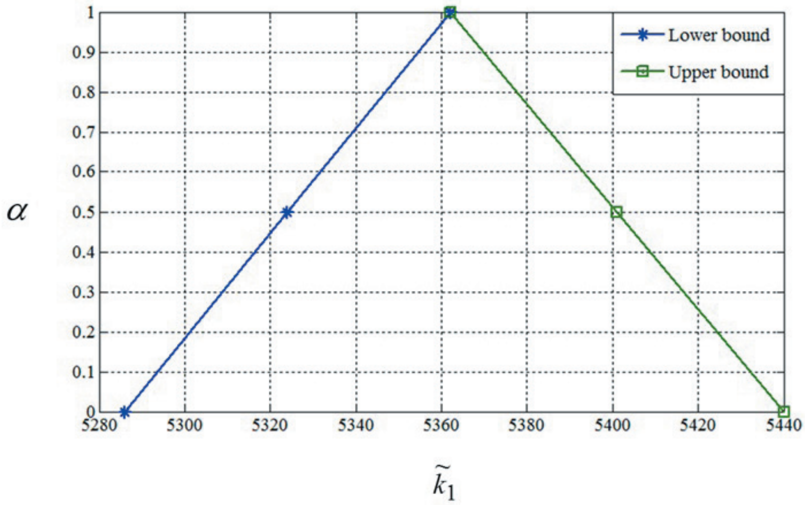

Fig. 4. *Identified lower and upper bounds of stiffness parameter* $\tilde{k}_1$ [N/m]

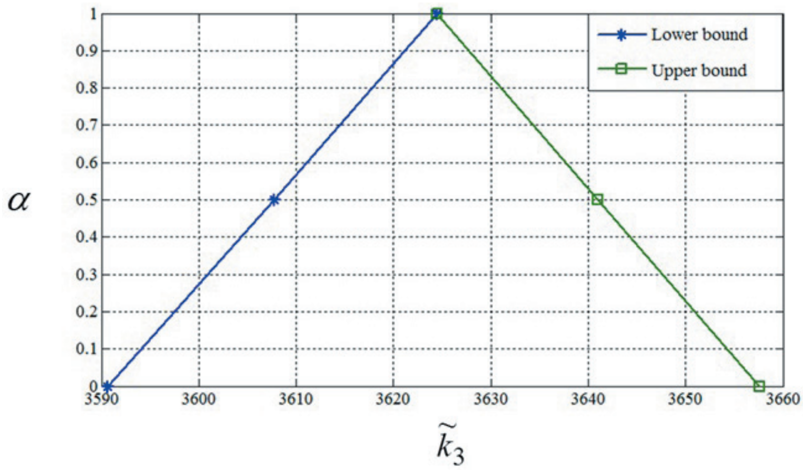

Fig. 5. *Identified lower and upper bounds* of stiffness parameter $\tilde{k}_3$ [N/m]

Table 3. *Experimental and revised lower and upper bounds of frequencies*

α	$\underline{\lambda}_{1E}$	$\overline{\lambda}_{1E}$	$\underline{\lambda}_{2E}$	$\overline{\lambda}_{2E}$	$\underline{\lambda}_{1R}$	$\overline{\lambda}_{1R}$	$\underline{\lambda}_{2R}$	$\overline{\lambda}_{2R}$
0	0.88	0.92	5.3	5.7	0.9430	1.0552	5.8884	6.1306
0.5	0.89	0.91	5.4	5.6	0.9706	1.0267	5.9476	6.0688
1	0.9	0.9	5.5	5.5	0.9985	0.9985	6.0078	6.0078

procedure is tested by incorporating two sets of data. The uncertainties present in the parameters are considered as triangular convex normalized fuzzy sets. It is worth mentioning that if the input data set viz. the design frequency is near to the experimental frequency data then the modified stiffness data bound has less width. This is expected as the design and experimental frequency are close means that the structure has not deteriorated much. On the other hand when the experimental data is taken a bit far from the deigned one then the estimated stiffness parameters give larger bound. These effects may be clearly seen from Tables 1 and 2. It may be noted that the accuracy of the results depends upon many factors viz. on the uncertain bound of the experimental data, initial design values of the parameters, the fuzzy computation, norm as defined etc. The present investigation may be a first of its kind to handle the identification procedure for uncertain data. Although the method has been demonstrated for a simple problem of two storey, but the method may very well be extended to higher storey frames and other structures in a similar fashion.

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6 REFERENCES

[1] Hart, G., Yao, J. (1977). System identification in structural dynamics. *Journal of Engineering Mechanics Division*, vol. 103, no. 6, p. 1089-1104.

[2] Bekey, G.A. (1970). System identification – An introduction and a survey. *Simulation*, vol. 15, no. 4, p. 151-166, DOI:10.1177/003754977001500403

[3] Datta, A.K., Shrikhande, M., Paul, D.K. (1998). System identification of buildings – a review. *Proceeding of the 11th Symposium on Earthquake Engineering*, University of Roorkee, Roorkee,

[4] Natke, H.G. (1982). *Identification of Vibrating Structures*, Springer-Verlag, Berlin, DOI:10.1007/978-3-7091-2896-1.

[5] Schoukens, J., Pintelon, R. (1991). *Identification of Linear Systems*. Pergamon Press, NewYork.

[6] Beck, J.L. (1978). *Determining Models of Structures from Earthquake Records*. Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena.

[7] Ibanez, P. (1979). Review of analytical and experimental techniques for improving structural dynamic models. *Welding Research Council Bulletin*, vol. 249.

[8] Masri, S.F., Werner S.D. (1985). An evaluation of a class of practical optimization techniques for structural dynamics applications. *Journal of Earthquake Engineering and Structural Dynamics*, vol. 13, no. 5, p. 635-649, DOI:10.1002/eqe.4290130506.

[9] Sinha, N. K., Kuszta, B. (1983). *Modelling and Identification of Dynamic Systems*. Van Nostrand Reinhold Co., New York.

[10] Loh, C.H., Ton, I.C. (1995). A system identification approach to the detection of changes in both linear and non-linear structural parameters. *Journal of Earthquake Engineering and Structural Dynamics*, vol. 24, no. 1, p. 85-97, DOI:10.1002/eqe.4290240107.

[11] Chassiakos, A., Masri, S.F. (1996). Modeling unknown structural systems through the use of neural networks. Journal of Earthquake Engineering and Structural Dynamics, vol. 25, no. 2, p. 117-128, DOI:10.1002/(SICI)1096-9845(199602)25:2<117::AID-EQE541>3.0.CO;2-A.

[12] Zhao, Q., Sawada, T., Hirao, K., Nariyuki, Y. (1995). Localized identification of MDOF structures in the frequency domain. *Journal of Earthquake Engineering and Structural Dynamics*, vol. 24, no, 3, p. 325-338, DOI:10.1002/eqe.4290240303.

[13] Udwadia, F.E., Proskurowski, W. (1998). A memory matrix-based identification methodology for structural and mechanical systems. *Journal of Earthquake Engineering and Structural Dynamics*, vol. 27, p. 1465-1481, DOI:10.1002/(SICI)1096-9845(199812)27:12<1465::AID-EQE795>3.0.CO;2-7.

[14] Lus, H., Betti, R., Longman, R. W. (1999). Identification of linear structural systems using eartquake-induced vibration data. *Journal of Earthquake Engineering and Structural Dynamics*, vol. 28, no. 11, p. 1449-1467, DOI:10.1002/(SICI)1096-9845(199911)28:11<1449::AID-EQE881>3.0.CO;2-5.

[15] Sanayei, M., McClain, J.A.S., Wadia-Fascetti, S., Santini, E.M. (1999). Parameter estimation incorporating modal data and boundary conditions. *Journal of Structural Engineering*, vol. 125, no. 9, p. 1048-1055, DOI:10.1061/(ASCE)0733-9445(1999)125:9(1048).

[16] Quek, S.T., Wang, W., Koh, C.G. (1999). System identification of linear MDOF structures under ambient excitation. *Journal of Earthquake Engineering and Structural Dynamics*, vol. 28, no. 1, p. 61-77, DOI:10.1002/(SICI)1096-9845(199901)28:1<61::AID-EQE804>3.0.CO;2-A.

[17] Chakraverty, S. (2004). modelling for Identification of stiffness parameters of multistorey frame structure from dynamic data. *Journal of Scientific and Industrial Research*, vol. 63, p. 142-148.

[18] Chakraverty, S. (2004). Identification of structural parameters of multistore shear buildings from

modal data. Earthquake Engineering and Structural Mechanics, vol. 34, p. 543-554, DOI:10.1002/eqe.431.
[19] Budipriyanto, A. (2013). Blind source separation based dynamic parameter identification of a multi-story moment-resisting frame building under seismic ground motions. *Procedia Engineering*, vol. 54, p. 299-307, DOI:10.1016/j.proeng.2013.03.027.
[20] Hazra, B., Roffel, A.J., Narasimhan, S., Pandey, M.D. (2010). Modified cross-correlation method for the blind identification of structures. *Journal of Engineering Mechanics*, vol. 136, no. 7, p. 889-897, DOI:10.1061/(ASCE)EM.1943-7889.0000133.
[21] Rahmani, M., Todorovska, M.I. (2013). 1D system identification of buildings during earthquakes by seismic interferometry with waveform inversion of impulse responses-method and application to Millikan library. *Soil Dynamics and Earthquake Engineering*, vol. 47, p. 157-174, DOI:10.1016/j.soildyn.2012.09.014.
[22] Todorovska, M.I., Rahmani, M.T. (2013). System identification of buildings by wave travel time analysis and layered shear beam models-Spatial resolution and accuracy. *Structural Control and Health Monitoring*, vol. 20, no. 5, p. 686-702, DOI:10.1002/stc.1484.
[23] Fan, W., Qiao, P. (2011). Vibration-based damage identification methods: a review and comparative study. *Structural Health Monitoring*, vol. 10, p. 83-111, DOI:10.1177/1475921710365419.
[24] Behera, D., Chakraverty, S. (2012). A new method for solving real and complex fuzzy system of linear equations. *Computational Mathematics and Modeling*, vol. 23, no. 4, p. 507-518, DOI:10.1007/s10598-012-9152-z.
[25] Chakraverty, S., Behera, D. (2013). Fuzzy system of linear equations with crisp coefficients. *Journal of Intelligent and Fuzzy Systems*, vol. 25, no. 1, p. 201-207, DOI:10.3233/IFS-2012-0627.
[26] Wang, C., Qiu, Z.P. (2013). Equivalent method for accurate solution to linear interval equations. *Applied Mathematics and Mechanics*, vol. 34, no. 8, p. 1031-1042, DOI:10.1007/s10483-013-1725-6.
[27] Wang, K. (2013). Uzawa method for fuzzy linear system. *Journal of Fuzzy Set Valued Analysis*, vol. 2013, p. 1-7, DOI:10.5899/2013/jfsva-00137.
[28] Chen, S., Qui, Z., Song, D. (1995). A new method for computing the upper and lower bounds on frequencies of structures with interval parameters. *Mechanical Research and Communications*, vol. 22, p. 431-439, DOI:10.1016/0093-6413(95)00045-S.
[29] Friswell, M.I., Prells, U., Penny, J.E.T. (2004). Determining uncertainty bounds for eigenvalues. *Proceeding of ISMA*, p. 3055-3064.
[30] Dimarogonas, A.D. (1995). Interval analysis of vibrating systems. *Journal of Sound and Vibration*, vol. 183, no. 4, p. 739-749, DOI:10.1006/jsvi.1995.0283.
[31] Cechlarova, K. (2005). Eigenvectors of interval matrices over max-plus algebra. *Discrete Applied Mathematics*, vol. 150, no. 1-3, p. 2-15, DOI:10.1016/j.dam.2005.02.016.
[32] Sim, J., Qui, Z., Wang, X. (2007). Modal analysis of structures with uncertain-but-bounded parameters via interval analysis. *Journal of Sound and Vibration*, vol. 303, no. 1-2, p. 29-45, DOI:10.1016/j.jsv.2006.11.038.
[33] Qui, Z., Wang, X., Chen, J. (2006). Exact bounds for the static response set of structures with uncertain-but-bounded parameters. *International Journal of Solids and Structures*, vol. 43, no. 21, p. 6574-6593, DOI:10.1016/j.ijsolstr.2006.01.012.
[34] Xia, B., Yu, D. (2013). Modified interval and subinterval perturbation methods for the static response analysis of structures with interval parameters. *Journal of Structural Engineering*, DOI:10.1061/(ASCE)ST.1943-541X.0000936.
[35] Modares, M., Mullen, R.L. (2013). Dynamic analysis of structures with interval uncertainty. *Journal of Engineering Mechanics*, vol. 140, no. 4, DOI:10.1061/(ASCE)EM.1943-7889.0000660.
[36] Akpan, U.O., Koko, T.S., Orisamolu, I.R., Gallant, B.K. (2001). Practical fuzzy finite element analysis of structures. *Finite Element Analysis and Design*, vol. 38, no. 2, p. 93-111, DOI:10.1016/S0168-874X(01)00052-X.
[37] Hanss, M., Willner, K., Guidati, S. (1998). On applying fuzzy arithmetic to finite element problems. *Conference of the North American Fuzzy Information Processing Society*, p. 365-369, DOI:10.1109/NAFIPS.1998.715607.
[38] Behera, D., Chakraverty, S. (2013). Fuzzy finite element analysis of imprecisely defined structures with fuzzy nodal force. *Engineering Applications of Artificial Intelligence*, vol. 26, no. 10, p. 2458-2466, DOI:10.1016/j.engappai.2013.07.021.
[39] Behera, D., Chakraverty, S. (2013). Fuzzy analysis of structures with imprecisely defined properties. *Computer Modeling in Engineering & Sciences*, vol. 96, p. 317-337.
[40] Sahoo, D.M., Chakraverty, S. (2013). Fuzzified data based neural network modeling for health assessment of multistorey shear buildings. *Advances in Artificial Neural Systems*, vol. 2013, art. no. 3, p. 1-12, DOI:10.1155/2013/962734.
[41] Khanmirza, E., Khaji, N., Majd, V.J. (2011). Model updating of multistory shear buildings for simultaneous identification of mass, stiffness and damping matrices using two different soft computing methods. *Expert Systems with Applications*, vol. 38, no. 5, p. 5320-5329, DOI:10.1016/j.eswa.2010.10.026.
[42] Ross, T.J. (2007). *Fuzzy Logic with Engineering Applications*. Wiley Student Edition, Chichester.
[43] Zimmermann, H.J. (2001). *Fuzzy Set Theory and Its Application*. Kluwer academic publishers, Boston, Dordrecht, London, DOI:10.1007/978-94-010-0646-0.