

Determination of the Threshold for Extreme Load Extrapolation Based on Multi-Criteria Decision-Making Technology

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The peak over threshold (POT) model is commonly used in extreme load extrapolation. Due to the important role that the threshold plays in establishing a POT model, a method to select the suitable threshold is designed based on multi-criteria decision-making (MCDM) technology. The mean deviation in probability distribution function, Kolmogorov-Smirnov test, and Chi-Square test are employed as the test criteria. An entropy method is applied to obtain the weight values of these tests. The VIKOR method is adopted to obtain compromise solutions according to the corresponding criteria and weight values. The POT requirement is used to determine a suitable threshold. The effectiveness and feasibility of this method are validated by the load time history measured in experiment and generated by simulation. A comparative analysis between this method and two other common methods is also conducted. The proposed method based on MCDM shows better performance.

Keywords: peak over threshold (POT), extreme load extrapolation, threshold, multi-criteria decision-making (MCDM), entropy method, VIKOR

Highlights

- Multi-criteria decision-making technology is adopted.
- Simulation is used to show the good performance of the method.
- The result is closer to the true threshold value with more intervals.
- The superiority of the MCDM method is shown through comparative analysis.

0 INTRODUCTION

Engineering vehicle components often suffer from random loads because they work in harsh environments and on bumpy roads [1]. Frequent fatigue failures often occurred [2]. The fatigue behaviour assessment for the mechanical components is traditionally conducted in time-domain [3]. The ultimate limit state analysis caused by the extreme working conditions is often considered [4]. Thompson estimated the return level associated with a particular return period to improve the coastal defence structure design.[5]. Wang determined that the extreme loads have significant impact on damage [6]. Because acquiring the actual load time history is time-consuming and costly, only limited time periods of load can be performed [7]. The extreme loads cannot be measured through limited time [8]. Therefore, in the above procedures, extrapolation is used to transfer limited periods load to longer periods load, especially to a design life [9]. During the process of extrapolation, extreme loads occurred more than before [10]. Through the extreme load extrapolation, the load amplitude and frequency can be properly estimated, which provides reasonable data for fatigue analysis.

The extreme value theory is applied to solve the problem of extreme load extrapolation. There are three models to determine the extreme values. The first model divides the load time history into blocks

and picks the extreme value in each block for analysis; this is called block maximum method (BMM) [11]. The second is named peak over threshold (POT) [12], which is based on the exceedances over a threshold. The third is called the method of independent storm (MIS) [13], which is the combination of the former two methods. The data above the threshold are divided into blocks, and the extreme values of each block are extracted. The BMM and MIS both need large sample data. In this paper, the POT model is applied; the main idea of that model for extreme loads extrapolation is based on a random simulation of the high maxima and low minima. Only the maxima above a level u_{max} , and the minima below a low load level u_{min} are randomly regenerated. The level u_{max} and u_{min} are collectively called the threshold. In this paper, we propose a method based on multi-criteria decision-making (MCDM) technology to select a suitable threshold.

The concept of the POT model was first introduced by Pickands [14]. Set a sample of load data $X=(x_1, x_2, \dots, x_n)$. They are independent and identically distributed and belong to an unknown continuous distribution. Suppose u is the threshold, the extreme exceedances over the level u are modelled: $y_i = x_i |_{x_i > u} - u$. The distribution function of the exceedances $F_u(y)$ is given by:

$$F_u(y) = P(x - u < y | x > u) = \frac{F(u + y) - F(u)}{1 - F(u)}. \quad (1)$$

When u is set high enough, then under certain conditions, the distribution function of these exceedances $F_{u(y)}$ approximately follow the Generalized Pareto Distribution (GPD). The distribution is defined in Eq. (2). Where ξ and σ are the shape parameter and scale parameter, respectively.

$$G(x|u, \xi, \sigma) = \begin{cases} 1 - (1 + \xi \frac{x-u}{\sigma})^{-\frac{1}{\xi}}, & \xi \neq 0 \\ 1 - \exp(-\frac{x-u}{\sigma}), & \xi = 0 \end{cases} \quad (2)$$

The traditional threshold selection method uses graphical diagnostics to make an initial threshold choice. The mean residual life (MRL) [15] method chooses the lowest level as the threshold because the mean exceedance over a level u is a linear function of the level u . The hill plot [16] method prefers the largest value of the hill estimator after it becomes stabilized, which means the lowest data would be chosen as the threshold. The advantage of these two approaches is that the results can be graphically inspected and assessed. The subjectivity and the rich experience are required in these methods.

In contrast to the above graphical methods, the numerical approaches are objective and automatic. Beirlant et al. [17] and Ferreira et al. [18] used the mean square error (MSE) method and set the value corresponding to the minimum MSE as the best threshold. Johannesson [10] proposed a load extrapolation method based on the POT model, and the threshold is determined by the exceedance number $\sqrt{N_0}$ and N_0 is the load cycles of the load time history. Thompson et al. [5] proposed an automatic threshold selection method, which builds a new parameter, and selects the appropriate threshold according to the distribution of the parameter. In order to obtain the extreme wave values, Mazas and Hamm [19] used the dual-threshold method to calculate wave heights. The first threshold is used to extract load peaks from the load time history, and the extracted data follow the GPD. The second threshold is determined when the corresponding shape parameter and the improved scale parameter maintain a constant.

The results (threshold) obtained through the above methods may be different. Many numerical test criteria are used to verify the fitting accuracy of the GPD with the corresponding selected threshold. Each of the methods may perform well under certain criteria and may be assumed as the best method. In order to select a threshold establishing a POT model, the joint influence of multi-criteria should be taken into consideration. MCDM works when the practical

problems are characterized by several conflicting criteria.

In this paper, a method named MCDM is proposed to select a threshold. An entropy method is used to measure the weight value of each criterion for its objectivity [20] and [21]. The VIKOR [22] to [24] is an MCDM technology. It proposes compromise solutions from a series of candidates by ranking them based on several conflicting criteria. According to the POT requirement, the minimum value of the compromise solutions is set as the suitable result. The method using MCDM is presented in detail in Section 1. In Section 2, the validity of the MCDM method is illustrated by the load time history and simulated data. Finally, comparison analysis and conclusions are presented.

1 METHODOLOGY

For a load data set, once a value is set as the threshold, the corresponding mathematical expression of GPD can be obtained after parameter estimation. Test criteria are used, and test values show the fitting accuracy of the GPD. The entropy method is used to determine the weight value of each criterion. Based on the MCDM, the compromise solutions are obtained by integrating the criteria into a synthetic indicator. Through the POT requirement, the suitable threshold is determined. The process chart of obtaining a suitable threshold u^* is shown in Fig. 1.

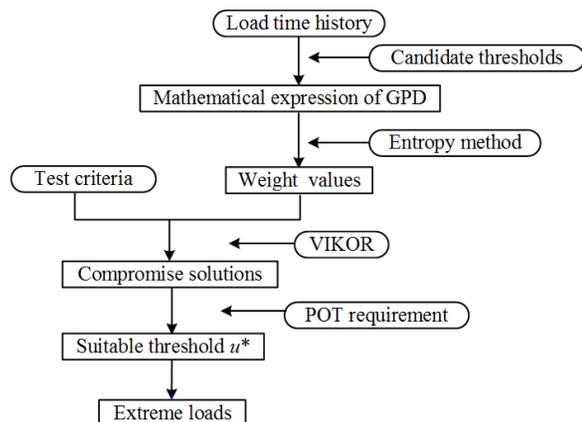


Fig. 1. Process chart of the MCDM method

1.1 Determination of the Initial Scope

The load time history can be viewed as a sequence of turning points [10]. According to the character of the load distribution and with the aim of improving the efficiency of calculation, candidate thresholds

(u_1, \dots, u_m) are set in ascending order. Set the load data value in an ascending order. u_1 is the value in the middle place of the data length, and u_m is the 98 % quantile of the data. m equally spaced candidate thresholds exist between u_1 and u_m . The corresponding ξ_i ($i=1, \dots, m$) and σ_i ($i=1, \dots, m$) are the parameters obtained by maximum likelihood estimation (MLE). Then the mathematical expressions of GPD are obtained for candidate thresholds.

1.2 Determination of the Criteria

The principles of test criteria are different and can be reflected by different results. Thus, three test criteria are used in the present study: mean deviation in probability distribution function (PDF), Kolmogorov-Smirnov (K-S) test and Chi-Square (χ^2) test [25] and [26].

1.2.1 Mean Deviation in PDF

The mean deviation between the theoretical PDF and the empirical PDF are shown by δ_{pdf} in Eq. (3).

$$\delta_{pdf} = \frac{1}{l} \sum_{i=1}^l |f(x_i) - g(x_i)|. \tag{3}$$

1.2.2 K-S test

The test value is the greater deviation value of the theoretical cumulative distribution function (CDF) and empirical CDF, which is calculated in Eq. (4).

$$K_{max} = \max_{1 \leq j \leq l-1} \{|F(x_j) - G(x_j)|, |F(x_{j+1}) - G(x_j)|\}. \tag{4}$$

1.2.3 χ^2 test

Assume O_i is the observed frequency and E_i is the expected frequency. Then χ^2 represents the deviation between them.

$$\chi^2 = \sum_{i=1}^l \frac{(O_i - E_i)^2}{E_i}. \tag{5}$$

1.3 Determination of the Weight Values of Criteria

For each candidate threshold, a test value will be obtained under a certain criterion. For a series of candidate thresholds and three test criteria, there will be many test values. Each of them has its role in evaluating fitting accuracy. To find a suitable threshold under three criteria, these criteria should be taken into

consideration simultaneously. Taking account of the different principle of test criteria, the larger the weight value, the more important will the test criterion be. Thus, the weight values of criteria should be settled and the entropy method is used for its objectivity.

The number of candidate threshold is m , and n shows the number of test criterion. Thus, the test values r_{ij} of candidate threshold i under the criterion j form a decision matrix RR .

$$RR = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{2j} & \dots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{m1} & r_{m2} & \dots & r_{mn} \end{bmatrix}. \tag{6}$$

Then, the normalized matrix RR , r_{ij} is transformed by the Eq. (7). And H is obtained by keeping the effect of each criterion.

$$h_{ij} = r_{ij} / \sum_{i=1}^m r_{ij} \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n, \tag{7}$$

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1n} \\ h_{21} & h_{2j} & \dots & h_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ h_{m1} & h_{m2} & \dots & h_{mn} \end{bmatrix}. \tag{8}$$

The relative importance of each criterion is expressed by E_j , which is the j^{th} entropy value and can be obtained from the following formulas:

$$E_j = -\frac{1}{\ln m} \sum_{i=1}^m h_{ij} \ln h_{ij}. \tag{9}$$

Suppose $h_{ij} \ln h_{ij} = 0$ when $h_{ij} = 0$. Then the weight of entropy of j^{th} criterion is defined as:

$$\omega_j = (1 - E_j) / (n - \sum_{j=1}^n E_j), \tag{10}$$

where $0 \leq \omega_j \leq 1$ and $\sum_{j=1}^n \omega_j = 1$.

1.4 Determination of the Compromise Solutions by MCDM

Among the MCDM technologies, VIKOR is used. Another commonly used MCDM method, the technique for order preference by similarity to an ideal solution (TOPSIS), finds a solution that satisfies the constraints that the distance from the positive-ideal solution (PIS) is the nearest while from the negative-ideal solution (NIS) is the farthest. However, it does not take the relative importance of these distances into consideration [22] and [24]. In contrast, VIKOR redresses this shortage and introduces a multi-

criterion ranking index based on a measurement of “closeness” to the “ideal” solution. According to the VIKOR method, the best feasible solution is also called compromise solutions, which contains one or more values.

1.4.1 Normalization

Through the vector normalization, shown in Eq. (11), the matrix T is obtained.

$$t_{ij} = r_{ij} / \sqrt{\sum_{i=1}^m r_{ij}^2} \quad i = 1, 2, \dots, m \quad j = 1, 2, \dots, n, \quad (11)$$

$$T = \begin{bmatrix} t_{11} & t_{12} & \dots & t_{1n} \\ t_{21} & t_{2j} & \dots & t_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{m1} & t_{m2} & \dots & t_{mn} \end{bmatrix}. \quad (12)$$

1.4.2 PIS t_j^* and NIS t_j^-

Determine PIS t_j^* and NIS t_j^- of all criterion functions:
The PIS is the best value and the NIS is the worst value of all candidate thresholds on every criterion.

Benefit index $\begin{cases} t_j^* = \max_{1 \leq i \leq m} t_{ij} \\ t_j^- = \min_{1 \leq i \leq m} t_{ij} \end{cases}, j = 1, 2, \dots, n, \quad (13)$

Cost index $\begin{cases} t_j^* = \min_{1 \leq i \leq m} t_{ij} \\ t_j^- = \max_{1 \leq i \leq m} t_{ij} \end{cases}, j = 1, 2, \dots, n. \quad (14)$

1.4.3 Group Utility S and Individual Regret R

Calculate the group utility S and individual regret R using Eqs. (15) and (16):

$$S_i = \sum_{j=1}^n [\omega_j (t_j^* - t_{ij}) / (t_j^* - t_j^-)] \quad i = 1, 2, \dots, m, \quad (15)$$

$$R_i = \max_{1 \leq j \leq n} [\omega_j (t_j^* - t_{ij}) / (t_j^* - t_j^-)] \quad i = 1, 2, \dots, m, \quad (16)$$

where ω_j obtained by the entropy method in VIKOR is the weight of j^{th} criteria.

1.4.4 Interest Rate Q

The interest rate Q is calculated with the equation below:

$$Q_i = \frac{\nu(S_i - S^*)}{S^- - S^*} + \frac{(1-\nu)(R_i - R^*)}{R^- - R^*}, i = 1, 2, \dots, m, \quad (17)$$

where $S^* = \min_{1 \leq i \leq m} S_i$ is the minimum of group utility, and $S^- = \max_{1 \leq i \leq m} S_i$ is the maximum group utility. The opponent of group utility is the individual regret. The minimum of individual regret is $R^* = \min_{1 \leq i \leq m} R_i$, and $R^- = \max_{1 \leq i \leq m} R_i$ the maximum individual regret. The value ν is the decision-making mechanism factor. In order to balance the relationship between group utility and individual regret, set $\nu = 0.5$.

1.4.5 Conditions

The Q values are arranged in ascending order with the corresponding S and R . The candidate X_1 corresponding to the minimum Q is the best solution when the following two conditions are satisfied:

C1: Acceptable advantage: $Q_2 - Q_1 \geq 1 / (m - 1)$ where Q_2 is the corresponding Q value of X_2 , and X_2 is the alternative with second place in the ranking Q and m is the number of candidates.

C2: Acceptable stability in decision making: the S value or R value of X_1 is the best ranked.

1.4.6 Compromise Solutions

Compromise solutions can be obtained if one of the conditions is not satisfied. If the results only conform to C1, both alternatives are suitable. If only C2 is satisfied, X_1, X_2, \dots, X_M are the compromise solutions. The subscript M is the number of the solution, and X_M is determined by Eq. $Q_M - Q_1 < 1 / (m - 1)$ for maximum M value.

1.5 Determination of Suitable Threshold by POT Requirement

According to 1.4.6, the compromise solutions could be concluded as X_1, X_2, \dots, X_M .

The compromise solutions include one or more values. If the maximum value is selected as the suitable threshold within the limited load sample, some information will be lost. According to the concept of the POT model, it is advantageous to select the minimum value on the premise of satisfying the fitting accuracy of GPD. Because preserving more data is conducive to the effective use of the data information, which is called the POT requirement in this paper. Thus, the suitable threshold u^* is shown in Eq. (18):

$$u^* = \min_{1 \leq i \leq M} (X_i). \quad (18)$$

2 CASE STUDY

2.1 Load Time History Analysis of Wheel Loader Axle Shaft

Fig. 2 is the torque data acquisition process of a wheel loader axle shaft. Fig. 2a is the operation mode. It can be seen as the letter “V”. Six sections are known as V1, V2, V3, V4, V5 and V6. The spading section V2 is studied because of the obvious load fluctuation. Fig. 2b is the load-time data of wheel loader axle shaft of the V2 section. Assume that loads are independent and identically distributed.

There are 1049 turning points obtained by filtered rain flow, and the load value is from 71.33 Nm to 4123.63 Nm. The range of the initial threshold $[u_1, u_m]$ is $[1806.32, 3498.61]$. Set $m=50$ and pick every alternative as a candidate threshold, and the data points above the threshold are left to obtain mathematical expression of GPD with the help of MLE. The test values of mean deviation in PDF, K-S test, and χ^2 test are shown in Fig. 3. The trends of curves for different test criteria are different.

According to the procedure in Section 1.3, the weight values of the corresponding test criterion

obtained by entropy method are $[0.001 \ 0.358 \ 0.641]$. According to the procedure in Section 1.4. The weight value is obtained based on Q_i, S_i and R_i ($i=1, 2, \dots, m$) and they are obtained using the VIKOR method.

Table 1. Rank order of Q_i, S_i and R_i (50×6)

u	Q_i	Rank	S_i	Rank	R_i	Rank
2517.082	0.016147	1	0.147023	2	0.094584	1
2957.077	0.018547	2	0.130509	1	0.11486	8
2855.54	0.035569	3	0.166259	6	0.095254	2
2923.231	0.041232	4	0.161069	5	0.106992	5
...
3464.764	0.466029	48	0.359967	48	0.358786	48
1840.166	0.679164	49	0.472336	49	0.471677	49
1806.32	1	50	0.64187	50	0.641205	50

The rank number of each value of Q, S and R are marked, which can aid in obtaining the compromise solutions. The values in Table 1 are displayed in ascending order of Q . Next, the number of thresholds is 50, i.e. $m=50$. According to C1 and C2, the result of Eq. $1/(m-1)$ is 0.0204. Seen in Table 1, $Q_2-Q_1 < 0.0204$ and $Q_3-Q_1 < 0.0204$, while

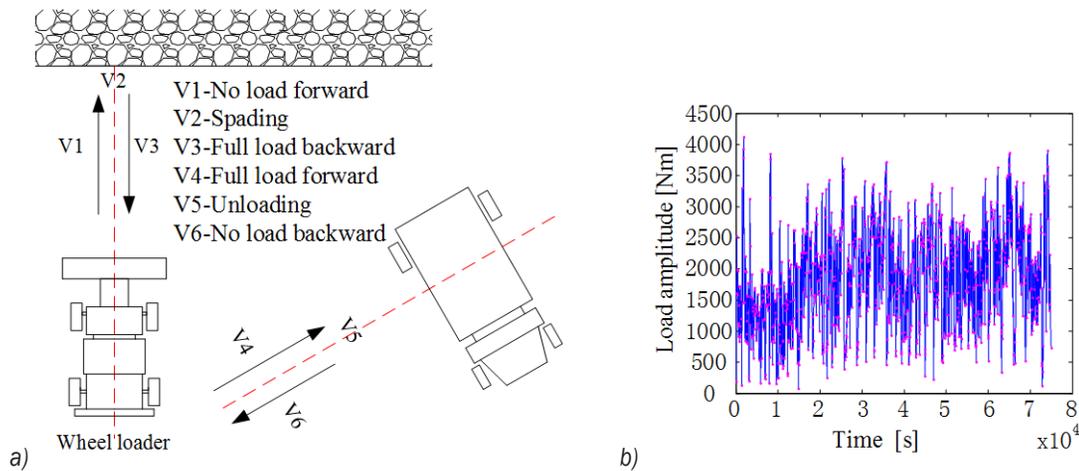


Fig. 2. Data acquisition of wheel loader axle shaft; a) operation mode of wheel loader; b) load data of wheel loader axle shaft from spading section

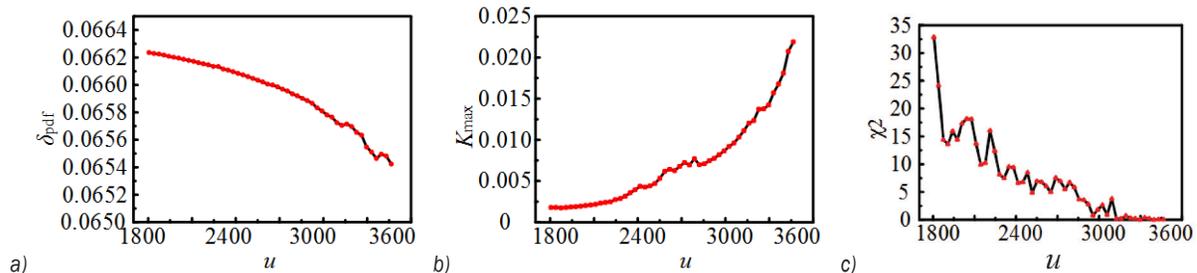


Fig. 3. Test values of different u under different test criteria; a) Mean deviation in PDF; b) K-S test; c) χ^2 test

$Q_4 - Q_1 > 0.0204$. At the same time, both R_2 and R_3 are smaller than R_1 . Thus, three candidates are finally picked as compromise solutions of the case. The results are shown in Table 2.

Table 2. Compromise solutions of u

u	Q_i	Rank	S_i	Rank	R_i	Rank
2517.082	0.016147	1	0.147023	2	0.094584	1
2957.077	0.018547	2	0.130509	1	0.11486	8
2855.54	0.035569	3	0.166259	6	0.095254	2

Table 3. Suitable solution

u	Q_i	Rank	S_i	Rank	R_i	Rank
2517.082	0.016147	1	0.147023	2	0.094584	1

According to the POT requirement, in order to have enough available data, the minimum value of 2517.082 of the compromise solutions is finally chosen as the suitable threshold.

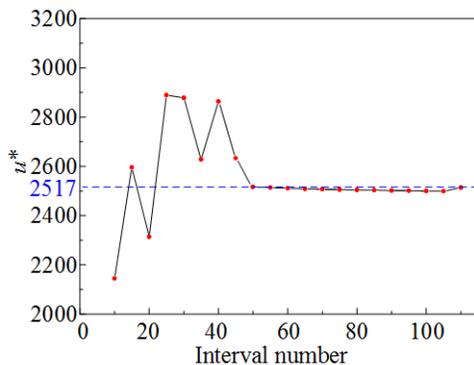


Fig. 4. Compromise solutions in different intervals

The interval number m above is a fixed value 50. If the value of m changed, the final result may be influenced. Now set the interval number $m=10, 15, \dots, 105, 110$. Compromise solutions of each interval are obtained at different intervals. Furthermore, the smallest value of each solution is

connected with a line. The results are shown in Fig. 4. When the number of intervals is smaller than 50, the curve of minimal solution values fluctuates fiercely. And the result u^* will near to the value 2517 when m is bigger than 50.

2.2 Simulation Study

In this section, the performance of the MCDM method is investigated using simulated data. Firstly, a group of data with a known threshold is used to verify the feasibility of this method. After that, the process is repeated 500 times to verify the effectiveness statistically. The tail part of the mixture distribution is made up of GPD, and the threshold is 3.2816. The histogram of 500 simulated data groups is shown in Fig. 5, and the PDF (the solid line) is also shown together with the threshold $u^*=3.2816$ (the dotted line).

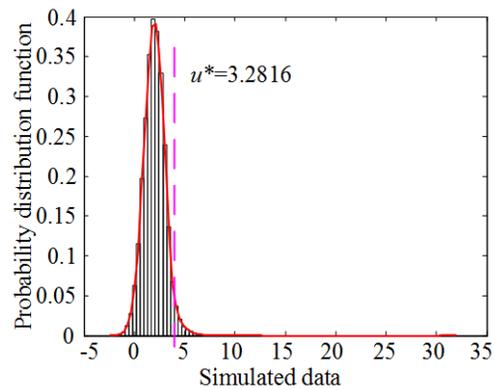


Fig. 5. Histogram and PDF of 500 simulated data sets

For mean deviation in PDF, K-S test and χ^2 test, each candidate threshold will obtain a test value under the corresponding test criteria. Firstly, the primary threshold range is divided into 100 sections. With the help of MLE, the test values with changing threshold values under different test criteria are shown in Fig. 6.

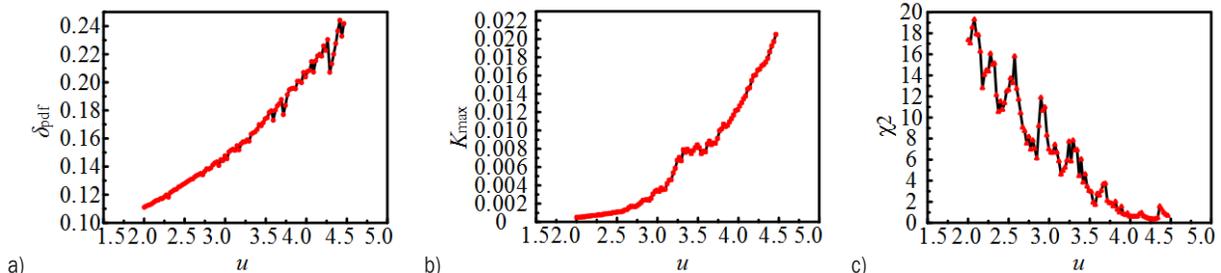


Fig. 6. Test values of different u under different test criteria; a) Mean deviation in PDF; b) K-S test; c) χ^2 test

A decision matrix of 100×6 is formed. The value of Q_i , S_i and R_i ($i=1, 2, \dots, m$) are displayed in Table 4.

Table 4. Rank of Q_i , S_i and R_i (100×6)

u	Q_i	Rank	S_i	Rank	R_i	Rank
3.14577	0.027952	1	0.21751	4	0.106447	1
2.84749	0.049371	2	0.198777	1	0.144548	6
3.170626	0.054168	3	0.227194	6	0.115527	2
...
4.438314	0.946539	99	0.514978	99	0.472797	98
4.46317	1	100	0.533865	100	0.492306	100

Here $m=100$, which means $1/(m-1)=1/(100-1)=0.0101$. According to Table 4 and the two conditions (C1, C2), the value of Q_2-Q_1 is larger than 0.0101. The candidate threshold $X_1=3.14577$ is chosen as the compromise solution. Only one value is available, so 3.14577 is the suitable solution, which is about 4% smaller than 3.2816.

The number of intervals selected above is 100. Set the interval number $m=10, 20, \dots, 140, 150$. The suitable solution u^* for each interval can be seen in Fig. 7. The suitable solution changes when the interval number increases. When the interval number is no less than 60, the suitable solution is close to the known value of 3.2816.

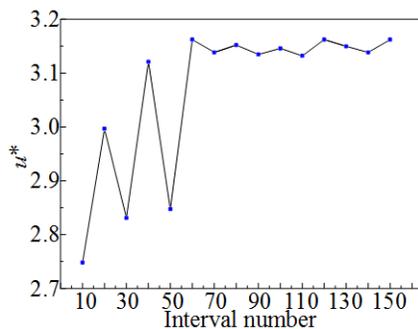


Fig. 7. Suitable solution u^* in different interval

In order to further verify the feasibility of the method, 500 groups of simulated data are conducted through the calculation process. A histogram of these calculated thresholds is shown in Fig. 8, together with the quantiles (dotted line) of 2.5% (2.5377) and 97.5% (3.6501). The mean value of these thresholds is 2.9853.

As seen in Fig. 8, most of the calculated thresholds normally distributed around 3.2816 in the interval of $[-22\%, 11\%]$. By comparing the mean value of thresholds with the known threshold u^* , the difference is less than 10%. Therefore, it shows that the MCDM method reaches a good fitting accuracy.

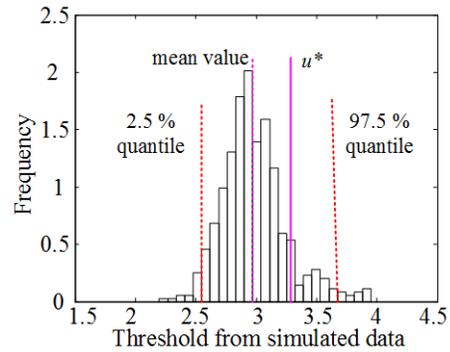


Fig. 8. Histogram of thresholds obtained from 500 simulated data groups

3 RESULTS AND DISCUSSIONS

As shown in Table 3 and Table 5, the suitable thresholds are selected under a certain interval number m and three test criteria. The performance of the proposed method using MCDM is compared with the MRL method and automatic method. The results of the two existing methods are shown as follows.

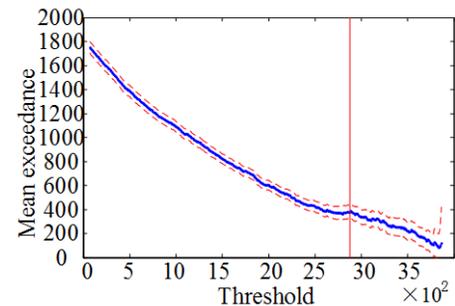


Fig. 9. MRL plot in confidence interval of 95%

Fig. 9 is the MRL plot in the confidence interval of 95%, which plots the mean exceedance as a function of u to decide a proper threshold. The curve on the right side of the vertical line seems to approximate a linear relationship when $u \geq 2800$. The critical value 2800 is chosen as the threshold.

According to [5], ξ_j and σ_j are the shape parameter and scale parameter when the threshold is u_j , $j=1, \dots, m$. The initial scope of the threshold is the same as Section 2.1. This scope is divided into 100 equal intervals. The load data of the wheel loader axle shaft is used and the relationship between $\tau_{u_j} - \tau_{u_{j-1}}$ and threshold u_{j-1} is shown in Fig. 10. A new parameter τ_{u_j} is established, and $\tau_{u_j} = \sigma_{u_j} - \xi_{u_j} u_j$. Exclude unsuitable u_1, u_2, \dots, u_{j-1} until there is a suitable value u_j to make the distribution of $(\tau_{u_j} - \tau_{u_{j-1}})$ follow the normal distribution. Fig. 11 shows the data

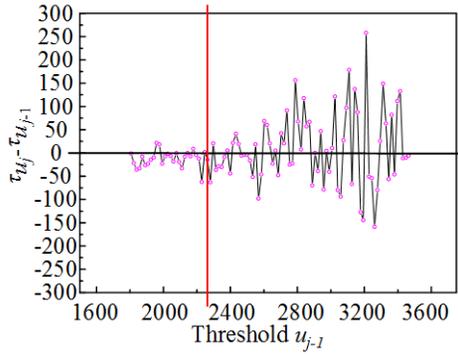


Fig. 10. Relationship between $\tau_{u_j} - \tau_{u_{j-1}}$ and u_{j-1}

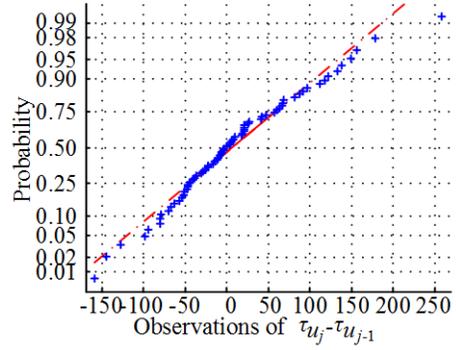


Fig. 11. Normal fitting test of observations ($\tau_{u_j} - \tau_{u_{j-1}}$)

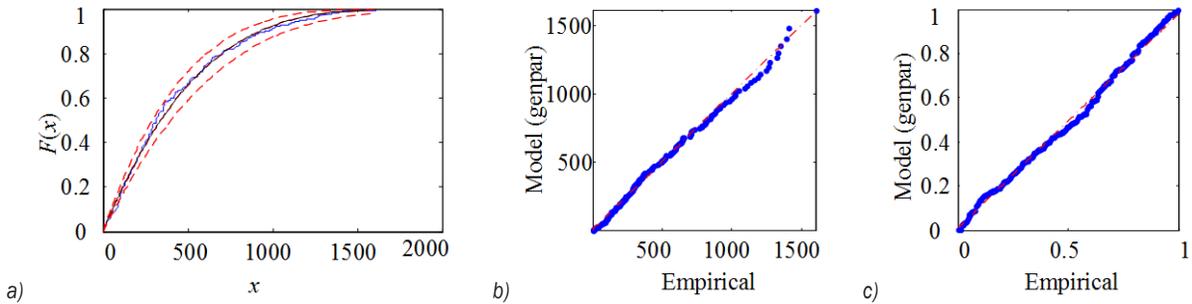


Fig. 12. Threshold $u = 2517.082$ by the MCDM method, MLE; a) CDF plot; b) P-P plot; c) Q-Q plot

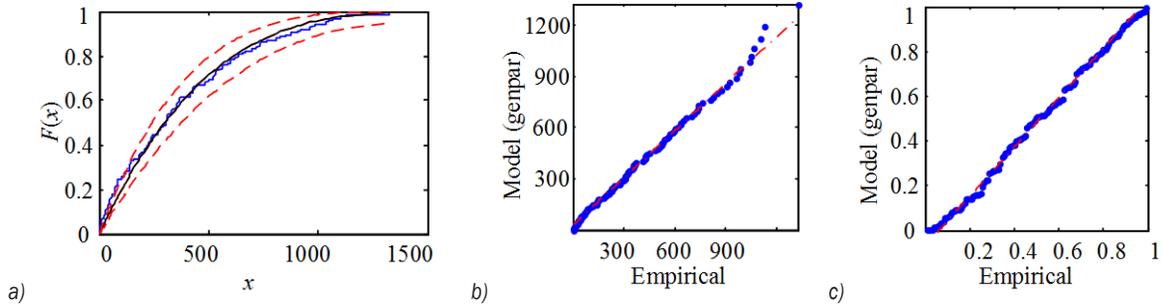


Fig. 13. Threshold $u = 2800$ by the MRL method, MLE; a) CDF plot; b) P-P plot; c) Q-Q plot

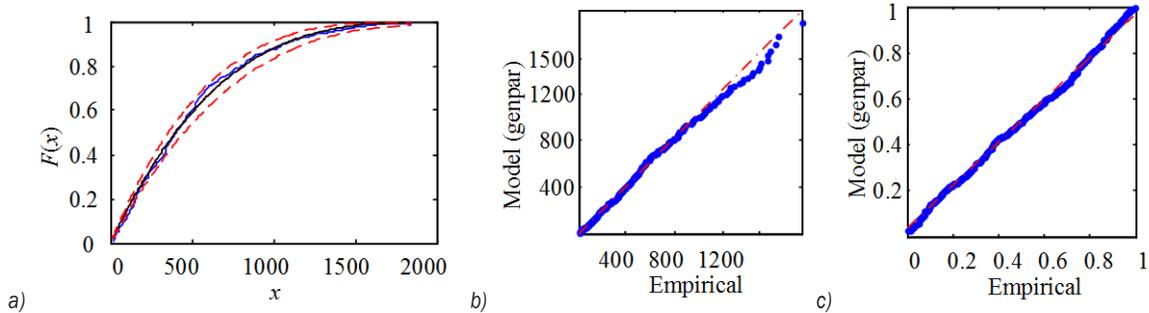


Fig. 14. Threshold $u = 2280.161$ by the automatic method, MLE; a) CDF plot; b) P-P plot; c) Q-Q plot

points almost form a nearly straight line which verifies they belong to a normal distribution. Thus, the

corresponding value 2280.161 is the suitable threshold.

The graphic method is used as a method of qualitative analysis to show the fitting effect of the selected threshold. Figs. 12 to 14 are the results corresponding to the MCDM method, the MRL, and the automatic method, respectively. Fig. 15 is the return level curves from the above three methods.

Figs. 12 to 14 all show good agreement, since the CDF of the exceedances are located within the confidence interval (the dotted lines) in Figs. 12a, 13a and 14a and most of the data points are distributed in a straight line in the P-P and Q-Q plots. As seen from these figures, all of the results obtained by the three methods show good fitting effects except a few small differences, which could be seen clearly in graphics.

Fig. 15 is the return level curves from the three methods, which shows the extreme load value (return value) with the corresponding load cycles (return period). The return values obtained via the MCDM and the automatic method are almost the same. The difference value between the MRL curve and the other two curves is firstly decreasing and then starts increasing after a certain point as the return period increases. Details between the three curves can be seen in the local enlarged drawing. Suppose the extreme load value occurs when the return period is 10^6 . Three extreme load values can be obtained in Table 5. The extreme load obtained by the MRL is more conservative than the other two methods are. While the MCDM method is a little more conservative than the automatic method, the difference value is only 36.7.

Numerical test criteria are employed, and the results are shown in Table 5. For the same load data set, the number of exceedance is determined by the

threshold value. The value of the MCDM method is 2517.082, which is between 2280.161 and 2800. As seen in Table 5, a_i , b_i , c_i are the test values under the criteria of mean deviation in PDF, K-S test, and χ^2 test, respectively. The 'min' means the corresponding minimum value of them. Seen from these numbers, every threshold selection method shows its advantage in a certain criterion. When comparing the three test values under mean deviation in PDF, the test value obtained by MRL method is the minimum. The three values are close to each other, but the MCDM method is better than the automatic method from the deviation of test values a_i/a_{\min} and $(a_i/a_{\min})/a_{\min}$. While little difference has appeared under the K-S test, the difference between the minimum (0.0029) and the maximum value (0.0047) of the criterion is an astonishing 158.6 %, which is a factor of 2.59. The difference between the 0.0047 (by MCDM) and the minimum is about 62.1 %, a factor of 1.62, which shows the effect of MCDM method is little poorer than the automatic method under the criterion of K-S. When the test criterion is χ^2 test, it is clear to see the value obtained from the MCDM is the smallest. The factors have also been analysed. The difference between the minimum (4.8446) and the maximum value (10.3993) of the criterion is about 114.65 %, which is a factor of 2.15. And the difference between the middle place value (8.1110) and the minimum (4.8446) is 67.4 % with a factor of 1.67.

It is a new trial to take account of the fitting effect into the process of threshold selection. In general, the MCDM method shows a better performance than the graphic method does. The results of comparisons

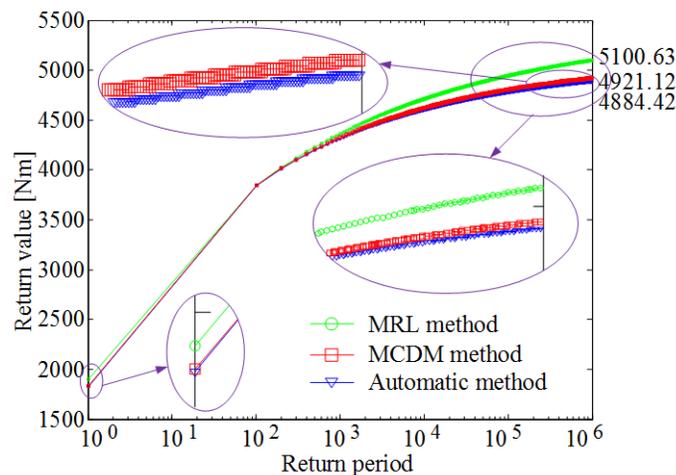


Fig. 15. Return level curves from the MRL, the MCDM and the automatic method

between the MCDM and the automatic method shows that the performance of the MCDM method is better.

Figs. 6 and 10 further illustrate the influence on the result when the interval number changes. It shows that more intervals will be helpful in obtaining a more precise result. In addition, only three test criteria, mean deviation in PDF, K-S test and χ^2 test, are applied in this proposed method. Presently there are no general rules that can be obeyed in selecting the major criteria in final decision-making. Therefore, some personal preference or even prejudice may exist in this process. Further research should be focused on the number and kinds of criteria to build a more realistic POT model.

Table 5. Comparison results of the three methods

	MRL	MCDM	Automatic method
Threshold value	2800	2517.082	2280.161
Number of exceedance	125	224	327
Extreme load	5100.63	4921.12	4884.42
Mean deviation in PDF (a_i)	0.0659	0.0660	0.0661
a_i/a_{\min}	/	0.152 %	0.3 %
$(a_i/a_{\min})/a_{\min}$	/	1.0015	1.003
K-S (b_i)	0.0075	0.0047	0.0029
b_i/b_{\min}	158.6 %	62.1 %	/
$(b_i/b_{\min})/b_{\min}$	2.59	1.62	/
χ^2 (c_i)	10.3993	4.8446	8.1110
c_i/c_{\min}	114.65 %	/	67.4 %
$(c_i/c_{\min})/c_{\min}$	2.15	/	1.67

4 CONCLUSIONS

The determination of threshold is a key point in establishing the POT model for extreme load extrapolation. At present, some methods are used to determine the threshold. The test criteria are used to verify the fitting accuracy of the distribution when the threshold is determined. Each method may perform well under certain criteria and may be assumed to be the best method. Therefore, it is a problem to select a suitable threshold, when several conflicting test criteria are considered together.

To solve this problem, a new method based on MCDM is proposed in this paper. It is a new trial using multiple criteria in threshold selection. The weight values are obtained using the entropy method. The MCDM integrated the test criteria into comprehensive indicators. According to the result of the compromise solutions, the POT requirement is used to determine the suitable threshold value. The actual load time history and simulated data analysis

proved that the MCDM method can create a relatively accurate POT model with a suitable threshold. More intervals lead the results closer to the true threshold value. Finally, the proposed method is compared with two existing methods to show the superiority of the MCDM method. Furthermore, the MCDM method is considered to be a better method to select a proper threshold value. Moreover, the method can be computerized. Each step of the method can be easily modified. For example, test criteria can be added or reduced and the entropy method can also be replaced.

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6 REFERENCES

- [1] Moreno, B., Zapatero, J., Domínguez, J. (2003). An experimental analysis of fatigue crack growth under random loading. *International Journal of Fatigue*, vol. 25, no. 7, p. 597-608, DOI:10.1016/S0142-1123(03)00018-5.
- [2] Wang, J., Zhang, J., Liang, Y., Yang, Y. (2012). A cyclic simulation approach for the generation of the non-stationary load histories of engineering vehicles. *Journal of Mechanical Science and Technology*, vol. 26, no. 5, p. 1547-1554, DOI:10.1007/s12206-012-0315-9.
- [3] Braccisi, C., Cianetti, F., Lori, G., Pioli, D. (2014). Evaluation of mechanical component fatigue behavior under random loads: Indirect frequency domain method. *International Journal of Fatigue*, vol. 61, p. 141-150, DOI:10.1016/j.ijfatigue.2013.11.017.
- [4] Hu, W., Han, L., Park, S.-C., Choi, D.-H. (2012). Multi-objective structural optimization of a HAWT composite blade based on ultimate limit state analysis. *Journal of Mechanical Science and Technology*, vol. 26, no. 1, p. 129-135, DOI:10.1007/s12206-011-1018-3.
- [5] Thompson, P., Cai, Y., Reeve, D., Stander, J. (2009). Automated threshold selection methods for extreme wave analysis. *Coastal Engineering*, vol. 56, no. 10, p. 1013-1021, DOI:10.1016/j.coastaleng.2009.06.003.
- [6] Wang, J., Chen, H., Li, Y., Wu, Y., Zhang, Y. (2016). A review of the extrapolation method in load spectrum compiling. *Strojniški vestnik - Journal of Mechanical Engineering*, vol. 62, no. 1, p. 60-75, DOI:10.5545/sv-jme.2015.2905.
- [7] Bae, H.-R., Ando, H., Nam, S., Kim, S., Ha, C. (2015). Fatigue design load identification using engineering data analytics. *Journal of Mechanical Design*, vol. 137, no. 1, p. 011001, DOI:10.1115/1.4027849.
- [8] Xiang, C.L. (2007). *Dynamics of Armored Vehicle Transmission System*. National defense industry press, Beijing.

- [9] Johannesson, P., Thomas, J.-J. (2001). Extrapolation of rainflow matrices. *Extremes*, vol. 4, no. 3, p. 241-262, DOI:10.1023/A:1015277305308.
- [10] Johannesson, P. (2006). Extrapolation of load histories and spectra. *Fatigue & Fracture of Engineering Materials & Structures*, vol. 29, no. 3, p. 209-217, DOI:10.1111/j.1460-2695.2006.00982.x.
- [11] Marty, C., Blanchet, J. (2012). Long-term changes in annual maximum snow depth and snowfall in Switzerland based on extreme value statistics. *Climatic Change*, vol. 111, no. 3, p. 705-721, DOI:10.1007/s10584-011-0159-9.
- [12] Pickands III, J. (1975). Statistical inference using extreme order statistics. *The Annals of Statistics*, vol. 3, no. 1, p. 119-131, DOI:10.1214/aos/1176343003.
- [13] Harris, R.I. (1999). Improvements to the 'Method of independent storms'. *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 80, no. 1-2, p. 1-30, DOI:10.1016/S0167-6105(98)00123-8.
- [14] Scarrott, C., MacDonald, A. (2012). A review of extreme value threshold estimation and uncertainty quantification. *REVSTAT - Statistical Journal*, vol. 10, no. 1, p. 33-60.
- [15] Beirlant, J., Goegebeur, Y., Segers, J., Teugels, J., De Waal, D., Ferro, C. (2006). *Statistics of Extremes: Theory and Applications*, John Wiley & Sons, Hoboken.
- [16] Drees, H., Resnick, S., de Haan, D. (2000). How to make a Hill plot. *The Annals of Statistics*, vol. 28, no. 1, p. 254-274, DOI:10.1214/aos/1016120372.
- [17] Beirlant, J., Dierckx, G., Goegebeur, Y., Matthys, G. (1999). Tail index estimation and an exponential regression model. *Extremes*, vol. 2, no. 2, p. 177-200, DOI:10.1023/A:1009975020370.
- [18] Ferreira, A., de Haan, L., Peng, L. (2003). On optimising the estimation of high quantiles of a probability distribution. *Statistics*, vol. 37, no. 5, p. 401-434, DOI:10.1080/0233188021000055345.
- [19] Mazas, F., Hamm, L. (2011). A multi-distribution approach to POT methods for determining extreme wave heights. *Coastal Engineering*, vol. 58, no. 5, p. 385-394, DOI:10.1016/j.coastaleng.2010.12.003.
- [20] Wang, J., Wang, N., Wang, Z., Zhang, Y., Liu, L. (2012). Determination of the minimum sample size for the transmission load of a wheel loader based on multi-criteria decision-making technology. *Journal of Terramechanics*, vol. 49, no. 3-4, p. 147-160, DOI:10.1016/j.jterra.2012.02.001.
- [21] Wang, J., Liu, Y., Zeng, X., Zhou, Z., Wang, N., Shen, W. (2013). Selection method for Kernel function in nonparametric extrapolation based on multicriteria decision-making technology. *Mathematical Problems in Engineering*, vol. 2013, art. ID. 391273, DOI:10.1155/2013/391273.
- [22] Opricovic, S., Tzeng, G.-H. (2004). Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS. *European Journal of Operational Research*, vol. 156, no. 2, p. 445-455, DOI:10.1016/S0377-2217(03)00020-1.
- [23] Opricovic, S., and Tzeng, G. H. (2007). Extended VIKOR method in comparison with outranking methods. *European Journal of Operational Research*, vol. 178, no. 2, p. 514-529, DOI:10.1016/S0377-2217(03)00020-1.
- [24] Yazdani, M., Payam, A.F. (2015). A comparative study on material selection of microelectromechanical systems electrostatic actuators using Ashby, VIKOR and TOPSIS. *Materials & Design*, vol. 65, p. 328-334, DOI:10.1016/j.matdes.2014.09.004.
- [25] Wang, J., Wang, Z., Yang, C., Wang, N., Yu, X., (2012). Optimization of the number of components in the mixed model using multi-criteria decision-making. *Applied Mathematical Modelling*, vol. 36, no. 9, p. 4227-4240, DOI:10.1016/j.apm.2011.11.053.
- [26] Wang, J., Hu, J., Wang, N., Yao, M., Wang, Z. (2011). Multi-criteria decision-making method-based approach to determine a proper level for extrapolation of Rainflow matrix. *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, vol. 226, no. 5, p. 1148-1161, DOI:10.1177/0954406211420212.