Nov pristop k preračunu aritmetičnega srednjega odstopanja profila pri kopirnem frezanju

A New Approach to Calculating the Arithmetical Mean Deviation of a Profile during Copy Milling

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Hrapavost površine, kot posledica frezanja s krogelnim frezalom, je v strokovni literaturi in univerzitetnih učbenikih le redko opisana. Problem je pogosto poenostavljen. Kar pomeni, da so uporabljena poenostavljena razmerja za teoretične izračune hrapavosti površine pri kopirnem frezanju s krogelnim frezalom: računski parameter Rz in največja višina valovitosti profila. V tem prispevku je predstavljen izpopolnjen izračun, ne samo parametra Rz, ampak tudi parametra Ra, aritmetičnega srednjega odstopanja profila. V prispevku so predstavljene nove enačbe za neposredni izračun aritmetičnega srednjega ostopanja prečnih in vzdolžnih profilov pri kopirnem frezanju ravne površine in poševne površine.
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(Ključne besede: frezanje kopirno, finiširanje, hrapavost površin, oblike proste)

Surface roughness as a result of milling with a cylindrical ball-end cutter is determined in the technical literature, as it is in university textbooks, relatively infrequently. The problem is usually simplified, which means simplified relations for the calculations of theoretical surface roughness during copy milling with cylindrical ball-end cutters are introduced: the calculation of parameter Rz, and the maximum height of the undulation profile. In this contribution an improved calculation, not only for parameter Rz, but also for parameter Ra, the arithmetical mean deviation of the profile, is presented. The paper presents new equations for the direct calculation of the arithmetical mean deviation of the transverse and longitudinal profiles during copy milling on a plane (face) surface and an oblique surface.
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0 PREFACE

Reference [1] describes various possibilities for calculating theoretical surfaces roughness. The surface roughness can be evaluated using different parameters. Theoretically, it is best to calculate ten points for the height of the irregularities, *Rz*. But this parameter is not mentioned in a drawing. This function fulfils the arithmetical mean deviation of the profile *Ra*. An alternative is an empirically determined equation between *Ra* and *Rz*, but this equation is assigned in the large enough interval [2] (see Equation (3), it depends on other parameters too, mainly on the cutting conditions) and its sum arithmetic value does not need to match the actual relation between Ra and Rz during the copy milling using copy tools. The copy-milling tools are mainly use in the CAD/ CAM systems branch [3], in order to manufacture oblique surfaces and free-form surfaces [4].

1 ROUGHNESS ON THE PLANE SURFACE

This section will show the calculation of roughness for two cases: for parameter *Rz* [5] and *Ra*, and for the transverse and longitudinal roughness.

1.1 The parameter *Rz* on the plane surface

Fig. 1. shows the situation for the origin of theoretical transverse roughness during copy milling on a plane surface. Equation (1) is valid from Fig. 1:

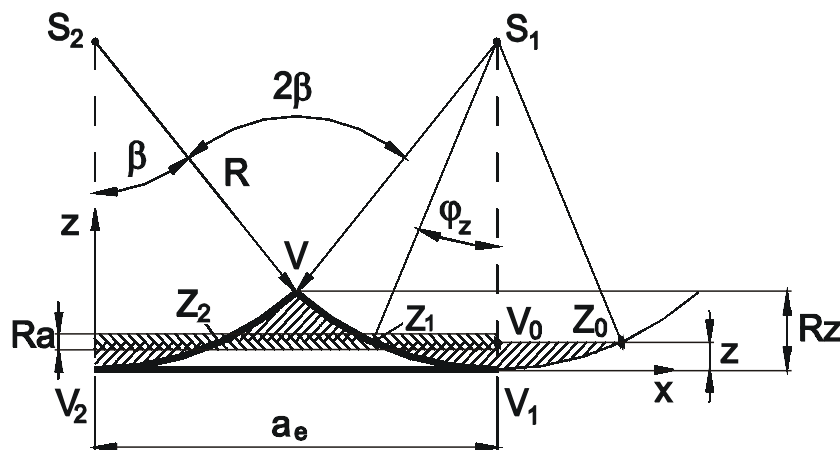$$Rz = R - \frac{1}{2}\sqrt{4R^2 - a_e^2} \cong \frac{a_e^2}{8R} \qquad (1),$$

Fig. 1. *The origin of the transverse roughness during copy milling on a plane surface*

where
R is the tool radius
a_e is the stepover (path interval),
or the same equation for parameter R_t is published in [6]:

$$R_t = \frac{D_C}{2} - \sqrt{\frac{D_C^2 - a_e^2}{4}} \qquad (2),$$

where
D_c is the tool diameter
a_e is the stepover (path interval).

Next, we can calculate the *Ra* parameter from its dependence on *Rz* parameter using different empirical equations. The reference example, according to [2], is shown in the following empirical equation (3) for conventional machining (turning, milling, drilling):

$$Ra \cong \frac{Rz}{(3-5)} \qquad (3).$$

The selection of the denominator value from the interval (3 to 5) depends on the methods of cutting and on the other experimental cutting parameters. The question is can we calculate directly the arithmetical mean deviation of the profile *Ra* using the tool radius *R* and the stepover (path interval) a_e (or feed per tooth f_z)? The answer is yes.

1.2 The direct calculation of *Ra* on the plane surface

In this section we will show the derivation for the direct calculation of the arithmetical mean deviation of the profile during copy milling on a plane surface. From Fig.1 the angle β is:

$$\beta = \arcsin \frac{a_e}{2R} \qquad (4).$$

The area of profile A_r (butted and bounded with points V_2VV_1) between two surfaces of the ball-shaped miller above stepover a_e will be the area of the rectangle $V_1V_2S_2S_1$ reduced to the area of the

triangle S_1S_2V and to the area of the circular segments VV_2S_2 a V_1S_1V, Equation (5):

$$A_r = a_e \cdot R - R^2 \sin \beta \cos \beta - R^2 arc\beta \qquad (5).$$

Next, it is possible to express from Fig.1. the area of the profile valley A_p that is bounded by the points $Z_1V_1Z_0$. The calculation of this area requires Equation (6):

$$A_p = \frac{1}{2} R^2 \left(arc2\varphi_z - \sin 2\varphi_z \right) \qquad (6).$$

In the next step we need to present the area of the profile peak A_v that is butted and bounded by the points Z_2VZ_1. This area we can calculate using the following Equation (7):

$$A_v = A_r - \left[a_e \cdot z - \frac{1}{2} R^2 \left(arc2\varphi_z - \sin 2\varphi_z \right) \right] \qquad (7).$$

The definition of the mean line of the profile needs to compare the area of the profile valley and the profile peak, $A_v = A_r$. After the substitution and editing we obtain Equation (8):

$$a_e \cdot z = a_e \cdot R - \frac{1}{2} a_e \cdot R \cos \beta - R^2 arc\beta \qquad (8)$$

and from equation (8) we can find the position of the mean line of the profile (see the Fig.1) of surface roughness:

$$z = R - \frac{1}{2} R \cos \beta - \frac{R^2}{a_e} arc\beta \qquad (9)$$

or:

$$z = R \left(1 - \frac{1}{2} \cos \beta - \frac{R}{a_e} arc\beta \right) \qquad (10).$$

From the definition of the arithmetical mean deviation of the profile *Ra* we can calculate this deviation as the width of the strip (of length a_e) on which it is possible to transform the double area of the profile valley (or the double area of the profile

peak, or the area sum of the profile valley and the profile, depending on which is the better to calculate). For the area of the profile valley we need the angle φ_z, which from Fig.1. is:

$$\cos\varphi_z = \frac{R-z}{R} = \frac{1}{2}\cos\beta + \frac{R}{a_e}arc\beta \qquad (11)$$

from Equation (11) we can obtain the angle φ_z:

$$\varphi_z = \arccos\left(\frac{1}{2}\cos\beta + \frac{R}{a_e}arc\beta\right) \qquad (12)$$

and the parameter R_a we designate from the following condition (13):

$$Ra \cdot a_e = 2A_p \qquad (13),$$

after the substitution and editing we obtain the equation:

$$Ra = \frac{R^2}{a_e}\left(arc2\varphi_z - \sin 2\varphi_z\right) \qquad (14),$$

where:
Ra is the arithmetical mean deviation of the profile,
R is the radius of the copy milling tool,
a_e is the stepover (path interval),
φ_z is the angle,
and the total equation for the calculation of the transverse roughness on the plane surface by milling with a ball-end cutter will be:

$$Ra = \frac{R^2}{a_e}\left\{arc2\left[\arccos\left(\frac{1}{2}\cos\arcsin\frac{a_e}{2R} + \frac{R}{a_e}arc\arcsin\frac{a_e}{2R}\right)\right] - \right.$$
$$\left. -\sin 2\left[\arccos\left(\frac{1}{2}\cos\arcsin\frac{a_e}{2R} + \frac{R}{a_e}arc\arcsin\frac{a_e}{2R}\right)\right]\right\} \cdot 1000$$
$$(15).$$

Equation (15) is exactly the final relation. Now we can calculate directly the arithmetical mean deviation of the transverse profile Ra from the radius of the copy milling tool and from the technological parameter a_e, the milling width, which the technology recommends in the CAM system.

For the arithmetical mean deviation of the longitudinal profile we substitute the stepover, a_e, with feed per cutter tooth, f_z, and the new equation is (16):

$$Ra = \frac{R^2}{f_z}\left(arc2\varphi_z - \sin 2\varphi_z\right) \qquad (16),$$

where:
Ra is the arithmetical mean deviation of the profile,
R is the radius of the copy milling tool,
f_z is the feed per tooth,
φ_z is the angle,
and the total equation for the calculation of the longitudinal roughness on the plane surface when milling with a ball-end cutter can also be used if we change the parameter a_e in Equation (15) to parameter f_z.

2 ROUGHNES ON THE OBLIQUE SURFACE

The described situation in the previous section is for calculating Ra on a plane surface. A similar sequence is possible for calculating Ra on an oblique surface.

2.1 The direct calculation of Ra on the oblique surface

We have here the new parameter α - the ramp of the milling oblique surface Fig.2.

In this case, instead of the stepover a_e for the transverse profile or the feed per cutter tooth f_z for the longitudinal profile, we need to use the new parameter a'_e:

$$a'_e = \frac{a_e}{\cos\alpha} \qquad (17),$$

where
a'_e is the stepover on the oblique surface,
a_e is the stepover on the plane surface,
α is the ramp angle of the oblique surface.

After substituting, Equation (17) into (14) we obtain the following Equation (18) for the transverse profile:
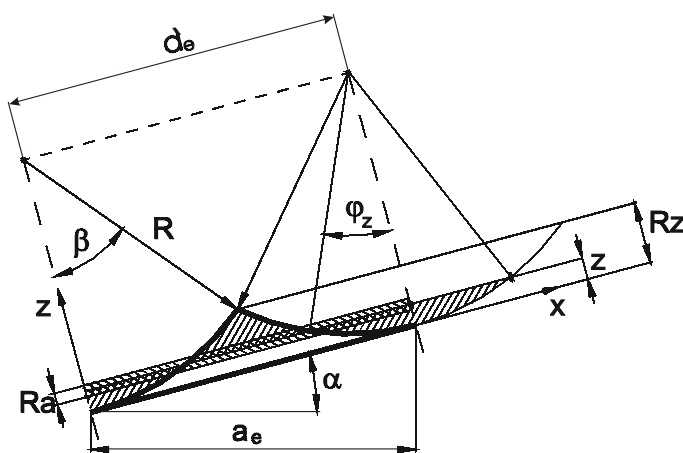

Fig. 2. *The origin of the transverse roughness during copy milling on an oblique surface*

$$Ra = \frac{R^2 \cos\alpha}{a_e}\left(arc2\varphi_z - \sin 2\varphi_z\right) \qquad (18)$$

and the total equation for the calculation of the transverse roughness on the oblique surface during milling with a ball-end cutter will be:

$$Ra = \frac{R^2 \cos\alpha}{a_e}\left\{arc2\left[\arccos\left(\frac{1}{2}\cos\arcsin\frac{a_e}{2R\cos\alpha} + \frac{R\cos\alpha}{a_e}arc\arcsin\frac{a_e}{2R\cos\alpha}\right)\right] - \right.$$
$$\left. - \sin 2\left[\arccos\left(\frac{1}{2}\cos\arcsin\frac{a_e}{2R\cos\alpha} + \frac{R\cos\alpha}{a_e}arc\arcsin\frac{a_e}{2R\cos\alpha}\right)\right]\right\}\cdot 1000$$

$$(19).$$

For the longitudinal profile of roughness the equation (20):

$$Ra = \frac{R^2 \cos\alpha}{f_z}\left(arc2\varphi_z - \sin 2\varphi_z\right) \qquad (20)$$

and the total equation for the calculation of the longitudinal roughness on the oblique surface by milling with a ball-end cutter can also be used if we change the parameter a_e in Equation (19) to parameter f_z.

Equations (15) and (19) are the new equations for the direct calculation of roughness Ra

with the technological parameters R, a_e (or f_z) and α during milling with a ball-end cutter.

3 CONCLUSION

The relations for the calculation of parameter Ra (3) presented in the scientific literature are inexact for milling, because they are only valid with a selected accuracy, depending on the type of machining (turning, milling, drilling, etc. Through relations (15) and (19) it is possible to directly calculate the surface roughness (parameter Ra) during copy milling. These relations will be added to the computer support of technology parameters' optimisation. This article aims to contribute to the possibilities of theoretical surface roughness' determination during milling with a cylindrical ball-end cutter, because the specialized literature has paid only a little attention to this problem.

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