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Analiza in določanje lastnosti parametrov nadomestnega sistema vzvojnih nihanj, ki se pojavljajo tudi na dizelskih motorjih za pogon tovornih vozil

Modeling and the Analysis of Parameters in the Torsional-Oscillatory System Equivalent to the Diesel Engines in Heavy-Duty Vehicles

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V prispevku je opisan postopek modeliranja parametrov sistema vzvojnih nihanj na sistemu, ki je podoben dejanskemu sistemu na večvaljnem, vrstnem, vodno hlajenem dizelskem motorju, ki poganja tovorna vozila. Na temelju rezultatov lastnih raziskav in rezultatov drugih avtorjev so predstavljeni modeli in algoritmi za določitev masnih vztrajnostnih momentov nihajočih mas, togosti delov ročične gredi, zunanjega in notranjega dušenja v sistemu vzvojnih nihanj, ki ga povzročajo plinske in vztrajnostne sile. Nekoliko več pozornosti je namenjeno določitvi karakteristik viskoelastičnega jedra ustreznega dušilnika vzvojnih nihanj, ki je pritrjen na ročično gred motorja. Nadomestni sistem in izračun vzvojnih nihanj je modeliran z izvirnim računalniškim programom. Podan je primer izračuna vzvojnih amplitud na posameznih delih motorja s prigrajenim dušilnikom vzvojnih nihanj in brez njega. Računski rezultati so primerjani z rezultati preizkusov na preizkuševališču za motorje. Podobnost rezultatov nas vodi k sklepu, da je predlagani postopek izračuna na nadomestnem modelu vzvojnih nihanj ustrezen za reševanje in analizo dejanskih problemov na tem področju.

(Ključne besede: motorji dizelski, vozila tovorna, nihanja vzvojna, modeli računski)

This paper describes the procedure for modeling and calculating the parameters of the torsional-oscillatory system equivalent to multi-cylinder, in-line, water-cooled diesel engines in heavy-duty vehicles. As a result of a literature search and our own investigation we present models and algorithms for defining the following parameters: the moments of inertia of oscillatory masses, the stiffnesses of engine-crankshaft segments, the external and internal damping in the torsional-oscillatory system, and the excitation of gas and inertia forces. Due to the specific characteristics of the viscoelastic element, somewhat more emphasis is placed on the viscoelastic damper of torsional oscillations in internal combustion engines. For the proposed equivalent torsional-oscillatory system, a suitable model and our own calculation program were developed. The analysis of the amplitude of twisting of individual system components in an actual engine, with and without a torsional-oscillations damper, is given. These results are compared to the experiments conducted on an engine test-bench. The agreement between the analytical and experimental results suggests that the proposed procedure for modeling and computing the parameters of the equivalent torsional-oscillatory systems can be successfully applied in practice.

(Keywords: diesel engine, heavy-duty vehicles, torsional oscillations, mathematical models)

0 INTRODUCTION

The internal combustion (IC) engine is a typical example of the occurrence of varying crankshaft torque during constant rotation speed. Varying crankshaft torque ($\mathcal{M}$) creates torsional oscillations around the state of the constant rotation speed, which in some cases presents the limiting factor in crankshaft modeling.

The IC engine system with crank gears and all other subsystems driven by the crankshaft, including the valve gear of the engine, the fuelinjection system, the oil pump, the alternator, etc., presents a highly complex system. The complexity of the system is manifested in the variable stiffness along the length of the crankshaft, in the need for different joints necessary to drive the subsystems (such as the indented joint, belt joint, hydrodynamic joints, etc.), in the hydrodynamic support of the crankshaft with extremely variable support conditions per single cycle, in the crank-gear variable mass moment of inertia ($\mathcal{J}$) under constant angular velocity (ω), and in the dependence of the lubrication requirements on the engine operation regime. We also need to consider the non-steady operational regimes as well as various applications of the engine, including vehicles, power units, locomotives, ships, etc.

We examine the complexity of the crank gear of an engine containing the crankshaft and all other auxiliary units. Due to its design, the type of connection with the engine housing**,** and the type of load, the crankshaft is, in addition to torsional oscillations, unquestionably exposed to bending and axial oscillations. Investigations by several authors, as well as practical experiments, demonstrate that the crankshaft bending and axial oscillations do not cause significant fatigue of the crankshaft material, and as such are not separately addressed when testing crankshaft endurance.

Torsional oscillations, whose intensity varies with changing working conditions of the engine, can cause material fatigue and crankshaft breakage. To prevent these oscillations, it is necessary to include all the relevant aspects of potential applications as well as the working conditions of the engine, and then decide whether there is a possibility of significant oscillations under these conditions and constraints, or not. A particular emphasis needs to be placed on the so-called marinized engines, which are in practice obligated for tests of excessive torsional oscillations.

Two approaches are used to understand the impact of torsional oscillations. They are based on:

a) the so-called equivalent systems with disc-shaped concentrated masses ([1] to [4]), or
b) the 3D model of the crankshaft and the application of certain complex numerical methods ([6] and [7]).

In order to effectively apply either of the methods, it is necessary to define the boundary operating values. Especially delicate issues in defining these values in 3D models are the hydrodynamic connections between the crankshaft and its environment, and the transfer of forces and moments under such conditions. For this reason, we chose to study torsional oscillations using the method of the so-called equivalent systems with concentrated masses in the form of discs. This approach to torsional oscillations is not only simpler, but it also leads to the development of the criteria for the risk assessment of torsional oscillations that are nowadays part of engineering practice. This approach to studying torsional oscillations is employed in diagnosing engine parameters [18] and [19], and is normally done by very fast contactless methods for measuring torsional oscillations [8].

In this paper we present an analysis of torsional oscillations using the so-called equivalent system of torsional oscillations. The proposed method includes, first, a definition of all the relevant system parameters for calculating oscillations, and then an experimental validation of the proposed model. In Section 1 we discuss the proposed model of a torsional-oscillatory system, and in Section 2 we describe the relevant parameters of the modeled system. The excitation of the system is presented in Section 3. The analysis is given in Section 4, and the conclusions are summarized in Section 5.

1 MODEL OF A TORSIONAL-OSCILLATORY SYSTEM

Using the model of concentrated masses in the form of discs, two approaches can be used to study the torsional oscillations of an engine crankshaft. They are the equivalent branched system [2], and the in-line equivalent system ([1] to [4]).

For the practical reasons presented in the Introduction, we chose to analyze the torsional oscillations in a so-called in-line equivalent torsional-oscillatory system. A model of a linear system with n degrees of freedom, along with the parameters defining the equivalent torsional-oscillatory system of IC engines, is shown in Figure 1. Since the reduced stiffness of the crankshaft component (c_i) (Figure 1) of the equivalent system must be the same as the stiffness present in the real system, it is calculated by equating the potential energies of real and equivalent systems.

The moments of inertia ($\mathcal{J}_i^*$) of the disc masses of the real and equivalent systems also need to be equal. They are calculated by equating the kinematic energies of the real and equivalent

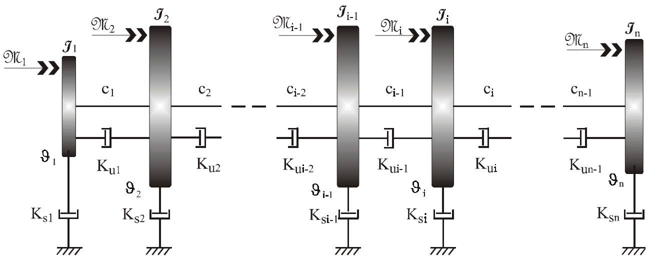

Fig. 1. *General scheme of an in-line equivalent oscillatory system*

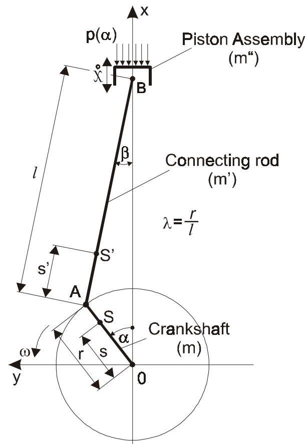

Fig. 2. *Crank gear with torsional-basic parameters*

systems. Other parts of the real system not located on the engine crankshaft axis (valve gear, injection system, water pump, etc.) are replaced with their counterparts on the equivalent system axis, based on equating the kinetic and potential energies of real and equivalent systems. An estimation of the moment of inertia of the crank gear, shown in Figure 2, is the most important aim for further calculation. The parameters of the equivalent torsional-oscillatory system, which include the excitation ($\mathfrak{M}_i$), and internal ($K_{u,i}$) and external ($K_{s,i}$) damping coefficients, are also determined on the basis of the parameters in the real engine.

A system of equations describing the movement of the discs is derived using Lagrange's equations of the second type. For an arbitrary i[th] disc in the system, the equation is:

$$\frac{d}{dt}\left(\frac{\partial E_{ki}}{\partial \overset{\circ}{\alpha}_i}\right)-\frac{\partial E_{ki}}{\partial \alpha_i}+\frac{\partial E_{pi}}{\partial \alpha_i}+\frac{\partial \Phi_i}{\partial \overset{\circ}{\alpha}_i}=\mathfrak{M}_{gi} \qquad (1),$$

where the kinetic energy of the i-th disc is $E_{ki}=1/2\mathcal{J}_i^*(\alpha_i)\dot{\alpha}_i^2$, the potential energy between the *i-1* and the *i*-th crankshaft component is $E_{pi}=1/2\cdot c_{i-1}(\alpha_i-\alpha_{i-1})^2+1/2\cdot c_i(\alpha_i-\alpha_{i+1})^2$, the reduced torsional stiffness per unit length of the crankshaft is c_i, the energy of the internal and external damping of the i^{th} disc is $\Phi_i=1/2\cdot[K_{u,i-1}(\dot{\alpha}_i-\dot{\alpha}_{i-1})^2+K_{u,i}(\dot{\alpha}_i-\dot{\alpha}_{i+1})^2+K_{s,i}\cdot\dot{\alpha}_i^2]$, the moment of the gas forces associated with the equivalent system axis for the i-th disc is $\mathfrak{M}_{gi}$, and the current angle of gyration of the i-th disc is α_i (expressed as $\alpha_i=\omega t+\vartheta_i$ where ωt is the current angle, and ϑ_i is the angle of torsional oscillations around a constant rotation speed).

The set of *n* equations given in (1) describes the movement of *n* discs of the torsional-oscillatory system. Based on these equations we can derive the basic indicators of torsional oscillations, which are the resonant oscillation modes, and the oscillation angles of individual discs (ϑ_i), i.e., amplitudes of disc oscillations (A_i).

The reduced stiffness (c_i) can be regarded as constant when determining the real values of system parameters of the torsional oscillations of the engine, whereas damping can be regarded as being linearly proportional to the relative oscillation velocity. The moments of inertia of the discs masses vary with the crankshaft rotation angle (*ωt)*. Using the above-mentioned assumption, and using the Taylor series expansion around (*ωt),* where we disregard higher terms, Equation (1) can be written as:

$$\mathcal{J}_i^*(\omega t)\cdot\ddot{\vartheta}_i+\omega\frac{d\mathcal{J}_i^*(\omega t)}{d(\omega t)}\cdot\dot{\vartheta}_i+\frac{1}{2}\cdot\omega^2\frac{d^2\mathcal{J}_i^*(\omega t)}{d(\omega t)^2}\vartheta_i+K_{s,i}\cdot\dot{\vartheta}_i+$$
$$+K_{u,i-1}(\dot{\vartheta}_i-\dot{\vartheta}_{i-1})+K_{u,i}(\dot{\vartheta}_i-\dot{\vartheta}_{i+1})+ \qquad (2).$$
$$+c_{i-1}(\vartheta_i-\vartheta_{i-1})+c_i(\vartheta_i-\vartheta_{i+1})=\mathfrak{M}^*_{g,i}(\omega t)-\omega^2\frac{d\mathcal{J}_i^*(\omega t)}{d(\omega t)}$$

Based on the results presented in [9], which demonstrate that the variability of the mass moment of inertia of the crank gears and the angles of oscillation in vehicle diesel engines have relatively small influence, we disregard the second and third terms on the left-hand side of (2), and the actual value of the moment of inertia ($\mathcal{J}_i^*$) we replaced with the mean value of the moment of inertia ($\mathcal{J}_i$). The last term on the right-hand side of (2) is actually the “excitation” of the inertia forces of the oscillating masses of the crankshaft rod.

Under these conditions, the equation has the following form:

$$\mathcal{J}_i \cdot \ddot{\vartheta}_i + K_{s,i} \cdot \dot{\vartheta}_i + K_{u,i-1}(\dot{\vartheta}_i - \dot{\vartheta}_{i-1}) + K_{u,i}(\dot{\vartheta}_i - \dot{\vartheta}_{i+1}) +$$
$$+ c_{i-1}(\vartheta_i - \vartheta_{i-1}) + c_i(\vartheta_i - \vartheta_{i+1}) = \mathfrak{M}^*_{g,i}\left(\omega t\right) + \mathfrak{M}^*_{in,i}\left(\omega t\right) \qquad (3).$$

Equation (3) presents the usual form of describing the oscillations of the i-th disc. The equation system (3) for *n* discs is a system of linear, non-homogenous differential equations of the second order. These equations are solved either by a numerical method for solving differential equations of the second order (e.g., a central differences method), where the solution is $\vartheta_i = f(\omega t)$, or by dividing the excitation into a group of harmonic functions, and thereby effectively transforming (3) into an ordinary algebraic equation, which defines the resonance modes of the system operations and amplitudes (A_{ij}) of the twisting angles for certain excitation orders (j).

By considering the arrangement of the vectors of the excitation moments for individual excitation orders (j) in multi-cylinder engines, the orders of the resulting moments can be viewed as either main, strong or weak. The "main" excitation orders of a 6-cylinder four-stroke engine are: $j = 3$; 6; 9; 12 and so on, the "strong" excitation orders are $j = 4.5$, 7.5, 10.5, and the remaining excitation orders are considered weak. Since engineering practice is concerned only with the main and strong orders, we only consider these when solving differential Equations (3). For example, the "interesting" excitation orders for 6-cylinder engines are only 4.5, 6, 7.5 and 9. Consequently, the number of algebraic equations to be solved is considerably reduced. For the correct analysis of the torsional oscillations, it is very important to define as precisely as possible the parameters of the torsional-oscillatory system, which are the moments of inertia of the discs ($\mathcal{J}_i$), the reduced torsional stiffness (c_i), the internal ($K_{u,i}$) and external ($K_{s,i}$) damping coefficients, as well as the excitation $\mathfrak{M}_i = \mathfrak{M}_{g,i} + \mathfrak{M}_{in,i}$..

We now continue with the discussion of the system parameters.

2 PARAMETERS OF THE TORSIONAL-OSCILLATORY SYSTEM

2.1 Mass Moments of Inertia

The first step in defining mass moments of inertia is the grouping of the individual subsystems and replacing the mass moments of inertia with their counterparts on the axis of rotation of the equivalent system. Certain engine elements have simple forms, thereby making the calculation of their mass moments of inertia easy. The most difficult form for a calculation of the moment of inertia is the crank gear shown in Figure 2, where all the construction characteristics of the mechanism are indicated. Using the principle of substitution of the connecting rod by two real masses (m_A) and (m_B), as a standard approach in engine dynamics, the moment of inertia of the crankshaft rod can be expressed as:

$$\mathcal{J}^*_i\left(\alpha_i\right) = \left[k^2 \cdot m + r^2\left(1 - \frac{s'}{l}\right) \cdot m'\right] + \qquad (4).$$
$$+ r^2\left(\frac{s'}{l} \cdot m' + m''\right) \cdot \left(\frac{\dot{x}}{r\omega}\right)^2 - \lambda^2\left(\frac{s'}{l - s'}\right) \cdot m' \cdot \left(\frac{\dot{\beta}}{\lambda\omega}\right)^2 r$$

The mean value of the moment of inertia of the crank gear is then computed as:

$$\mathcal{J}_i = k^2 \cdot m + r^2\left[\left(1 - \frac{s'}{2l}\right) \cdot m' + \frac{m''}{2}\right] \qquad (5).$$

Expression (5) contains only one parameter, which is the crank-arm radius of inertia (k). This parameter is fairly difficult to determine due to the complex form of the crankshaft. Either experimental or analytical method can be used to determine it. The experimental method is based on the rolling pendulum principle, i.e., the torsional oscillator and "small" oscillations theory. The calculation method is based on a 3D model representing the crankshaft divided into a large number of finite elements, for each of which the expressions of the mass moment of inertia are known.

Both methods are acceptable and provide satisfactory results. Regarding the first method, where errors during measuring "small" oscillations can be accrued, and the fact that one crankshaft has to be destroyed, the second method has an advantage in practice.

2.2 Torsional Stiffness in a Component of the Equivalent System

Since the torsional stiffness (c) of a component of the equivalent torsional-oscillatory system has the most influence on the calculations of torsional oscillations, it is necessary to determine it precisely. An actual part in the torsional-oscillatory system, whether on the crankshaft rotational axis or on another axis, is converted into an equivalent part, based on equating the potential energies of the real and the equivalent part. When determining the stiffness, the cranks are the most sensitive part of the IC engine crankshaft oscillation system, due to their complex construction. The methods used for determining the engine crank stiffness can be either analytical, which range from using classical semi-empirical sources to finite-element methods, or experimental, or a combined analytical-experimental approach. We have used all three methods in our research and we adopt the combined analytical-experimental method as the most efficient one, especially taking into account the very advanced contactless torsional oscillations measurement methods available today [8].

2.3 Damping of Torsional Oscillations in a Crankshaft

The dampings are caused by the relative movement of adjoining elements and the molecules in the material. They can be either external (when caused by surrounding media such as air, oil, water, or an adjoining element), or internal (when caused by the friction between the molecules in the material resulting from the elastic deformations of the material).

The total external and internal dampings in the engine, usually presented under complex working conditions, are very difficult to determine in practice. Available literature on this topic is mostly concerned with addressing the total loss caused by damping, usually by using indirect methods such as the external drive method and the method of disconnecting cylinders of the engine. However, these methods do not address the real dynamic working conditions of the engine, and neither do they specifically address the dampings caused by torsional oscillations. Therefore, the results found in the bibliography need to be treated with caution. Considering the aforementioned, it is obvious that the damping caused by torsional oscillations around the state of constant rotation speed is even more complex. We now present a new way to define the external and internal damping in IC engines, and we compare it to the existing literature.

2.3.1 External Damping as a Result of Torsional Oscillations

The most important locations in which external dampings in the crank gear of an IC engine occur are the piston assembly, the link between the piston and the connecting rod, and the crank pin and crank journal bearings. By considering the working conditions in these locations, including the mechanical and thermal loads, the manner of lubrication and oil quality, the tolerance, and so on, it becomes clear that the external damping caused by torsional oscillations cannot be measured by any direct methods. It is, however, possible to use the so-called indirect experimental-calculation methods. Unfortunately, these are fairly complex, and any error made can result in utterly incorrect values of the external damping coefficients. Most of the bibliographical sources express the external damping coefficient (K_s) through the specific external damping coefficient (μ_s) per piston surface area ($D_k^2 \pi/4$) and per squared crank diameter (r^2).

The coefficient (μ_s) and/or the coefficient (K_s) are given in the bibliographical sources as follows:

- $\mu_s = (4 \text{ to } 5)\ \ 10^4\ Ns/(m^3 rad)$, for larger diesel engines ([3] to [5]),
- $\mu_s = 10^5\ \ Ns/(m^3 rad)$, for diesel engines with dampers of torsional oscillations,
- $\mu_s = 1.5\ \ 10^5\ Ns/(m^3 rad)$, for diesel engines without dampers of torsional oscillations [3],
- $K_s = 0.7 \text{ to } 1.3\ \ Nms/rad$, for mono-cylinder diesel engines [13],
- $K_s = 600\ Nms/rad$, for ship diesel engines, low-speed engines [20], etc.

This data suggests that there is a discordance in the treatment of external damping coefficients in various engines. Some of the mentioned bibliographical sources include the total damping (both external and internal) in the coefficient, and some sources differentiate between engines with and without dampers of torsional oscillations.

By examining our own experimental results for torsional oscillations in 6-cylinder, in-line, water-cooled diesel engines for transport vehicles, with powers between 140 kW and 190 kW, and indirect calculation methods, we estimated the following range for the external damping coefficient for our research: $\mu_s = 1.45 \cdot 10^5 \text{ to } 1.75 \cdot 10^5\ Ns/(m^3 rad)$.

2.3.2 Internal Damping in the Material as a Result of Torsional Oscillations

When analyzing the internal damping in the material caused by elastic deformation due to torsional oscillations, some researchers disregard this damping because of the small amount of resulting deformation, or compensate its influence by increasing the outer damping values. In such cases researchers only address more prominent damping, such as in oscillation dampers, couplings, propeller shafts, etc. Another approach is to observe the internal damping in all the elements, regardless of its intensity. This principle has been used in this paper as well. When determining internal damping caused by torsional oscillations, two approaches are generally adopted:

- A model with constitutional equations of continuum mechanics, and the knowledge of the physical and chemical characteristics of the materials,
- A practical semi-empirical model, which can be found in the existing literature.

On account of the complicated and costly experiments needed to determine the physical and chemical characteristics of the materials for different temperatures and load conditions, as well as a highly complicated equation system, we adopt the latter method for determining the internal damping (K_u).

One of the most common approaches in defining internal damping is to use the damping ratio (ψ), defined as the ratio of the dissipation energy caused by internal friction per oscillation cycle (W_d) and potential energy (W_p), as $\psi = W_d / W_p$. Under the assumption that the hysteresis loop, which accompanies internal friction, can be approximated by an ellipsoid (which can be accepted as satisfactory if the oscillations in question are harmonic), the internal damping coefficient can be expressed as:

$$K_u = \frac{c \cdot \psi}{2 \cdot \pi \cdot \omega}. \qquad (6).$$

Several authors have dealt with the issue of internal damping in materials in IC engines as a function of its cause. For example, the crankshaft material ([1] and [3]), the viscous damper [3], the viscoelastic damper ([1], [3], [10] and [12]), and the combined viscoelastic-viscous damper [7], are all considered as significant sources of internal damping.

We now focus on defining the internal damping in the crankshaft material and the viscoelastic damping in the torsional oscillation damper (TOD), which is of interest for the vehicle diesel engines analyzed in this paper.

2.3.2.1 Internal Damping in IC Engine Crankshaft Material as a Result of Torsional Oscillations

When determining the coefficient of internal damping in a crankshaft material, the empirical expression $W_d = H\omega^m A$ for dissipation energy per oscillation cycle is frequently used. The recommended values of the constants are $m = 0$ and $n = 3$, and the constant H depends on the type of material [1].

Using expression (6) and the expression for Ψ, as well as the fact that the potential energy per oscillation cycle can be written as $E_p = c\,A^2/2$, where A is the amplitude of oscillation, we express the internal damping coefficient as:

$$K_u = \frac{c}{H_1 \cdot \omega} \cdot \left(\frac{\mathfrak{M}_j^*}{W_0} \right)^{1/2} \qquad (7),$$

where:

- $\mathfrak{M}_j^* = \left[\left(a_j^g \right)^2 + \left(b_j^g + b_j^{in} \right)^2 \right]^{1/2}$ is the amplitude of the excitation moment [Nm] of the j-th order for which the damping is being calculated and whose frequency is ω [rad/s],
- W_0 [m^3] is the polar resistance moment of the section for which the damping is being calculated
- H_1 is a constant (it contains the constant H).

The value of the constant H_1 is given in [1] for the crankshaft materials most frequently used in IC engines.

2.3.2.2 Internal Damping in the Visco Elastic Damper of Torsional Oscillations

Because of its simplicity, cost and efficiency, the viscoelastic torsional oscillations damper (TOD) is used almost exclusively in medium- and high-speed diesel engines. The design of such a TOD is shown in Figure 3.a., and the equivalent scheme of the damper in series with other parts of the torsional-oscillatory mechanism is shown in Figure 3.b. The characteristic parameters of the torsional-oscillatory damper, shown in Figure 3, are the stiffness (c_v) and the internal damping coefficient ($K_{u,v}$). They depend on the material, the dimensions and the working conditions. Since these parameters are defined using measurements during oscillations, they are usually referred to with the prefix "dynamic". The viscoelastic damper of torsional oscillations has a fairly non-linear value of torsional stiffness, c_v. This stiffness can generally be viewed as a function $c_v = f(t_v, A_{rv}, \omega,$ *type and dimensions of viscoelastic material*) . With our

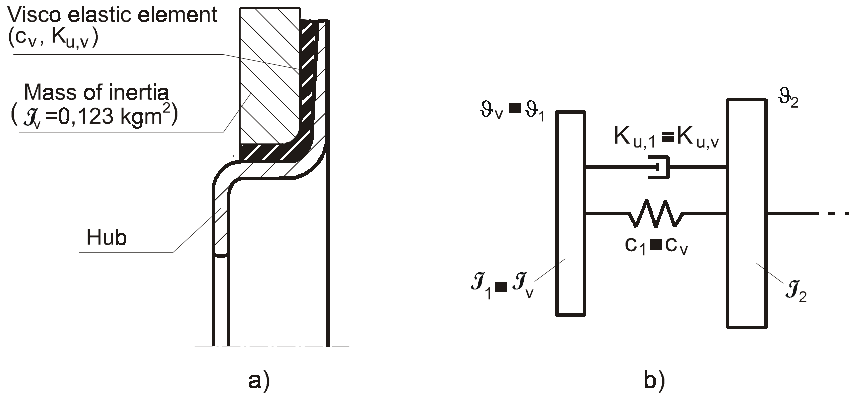

Fig. 3. *Design of viscoelastic damper with equivalent scheme*

experimental measurements, the viscoelastic material surface temperature t_v was in the range $t_v = 60$ to 65 °C, so the average value of the viscoelastic element surface temperature t_v = 62ºC is adopted. Since the focus of our research is only on one damper, the type and dimensions of the material can be excluded as well. Therefore, only the relative amplitude of the oscillation of the viscoelastic element $A_{rv} = A_v - A_2$ and the oscillation frequency are used in the analysis.

As a starting point for defining the dynamic torsional stiffness (c_v) for viscoelastic dampers, the dependence of the natural frequency (f_{vo}) on the relative oscillating amplitude (A_{rv}) is presented. The dependence is based on measurements performed on an assembly specifically made for examining torsional oscillations. The measurements $f_{vo}=f(A_{rv}\, t_v)$ are shown in Figure 4.

Based on the results shown in Figure 4, for the temperature t_v = 62 ºC, we express c_v as, $c_v = \left(2\pi f_{vo}\right)^2 \mathcal{J}_v$. The resulting values of c_v are shown in Figure 5.

To define the internal damping coefficient $K_{u,v}$ we use expression (6), where the potential energy in one cycle is defined as $W_p = \int c_v \cdot \vartheta_{rv} \cdot d\vartheta$. According to [1], for dampers with a viscoelastic element the constants $m = 0$ and $n = 2$ are recommended.

Using the above assumptions, the results shown in Figure 5, as well as the measurements of torsional oscillations in an engine with a torsional oscillations damper, we determine the constants to be $H = 1.6 \cdot 10^4$, $m = 0$, and $n = 1.6$. The coefficient $K_{u,v}$ of the internal damping of the viscoelastic element of the torsional oscillations damper is calculated as:

$$K_{u,v} = \frac{1,6 \cdot 10^4 \cdot A_{r,v}^{1,6} \cdot \left(c_v \cdot \mathcal{J}_v\right)^{1/2}}{2 \cdot \pi \cdot \int_0^{A_{r,v}} c_v \cdot \vartheta_{rv} d\vartheta} \qquad (8).$$

The plot of the coefficient of the internal damping of viscoelastic elements based on (8) is shown in Figure 6. The resulting values of c_v and $K_{u,v}$ are used as the input parameters in (3).

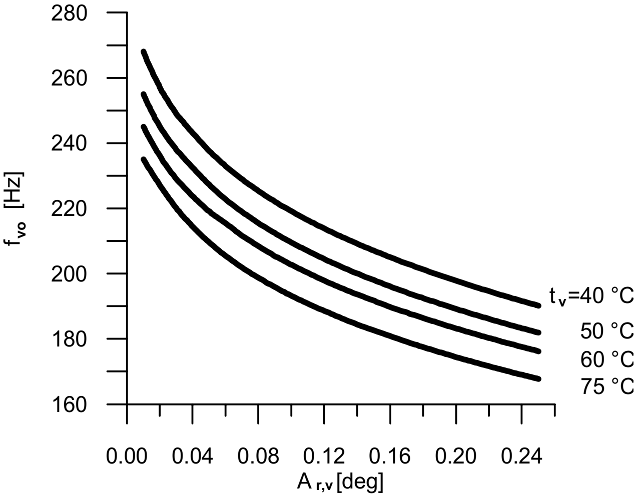

Fig. 4. *Change of natural frequency f_{vo} as a function of the relative amplitude A_{rv} and temperature t_v*

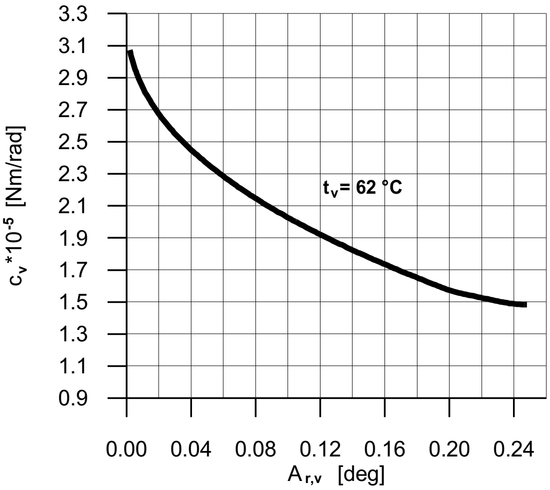

Fig. 5. *Change of torsional stiffness c_v of the viscoelastic element as a function of the relative amplitude $A_{rv.}$*

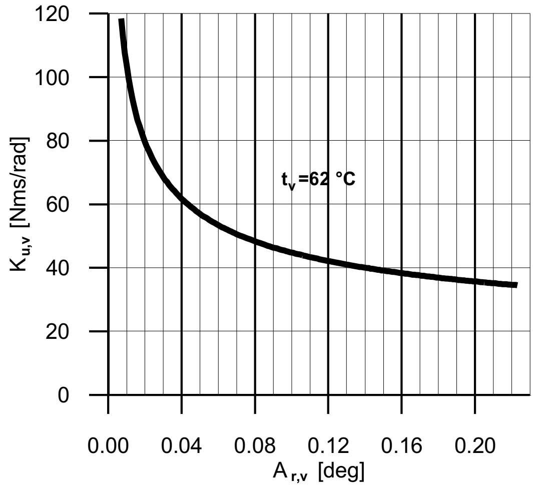

Fig. 6. *Change of the internal damping coefficient $K_{u,v}$ of the viscoelastic element as a function of the relative amplitude $A_{rv.}$*

Other parameters in a torsional oscillations system, such as stiffness and damping of the transmission shaft of the hydraulic test bench, etc., have not been separately addressed here, although they are inevitable in the equivalent system of torsional oscillations when being measured on the engine test bench. The reason for this is the negligible influence of the hydraulic brake on engine oscillations.

3 EXCITATION IN TORSIONAL-OSCILLATORY SYSTEM

According to Equation (3), the excitation in a torsional-oscillatory system consists of the excitation moments of gas forces ($\mathcal{M}_{gi}$) and the moments-of-inertia forces of the oscillating masses ($\mathcal{M}_{in}$) of the crank gear. The moment of the gas forces can be expressed as:

$$\mathcal{M}_g = \left[p(\alpha) - p_0\right] \cdot \frac{D_k^2 \cdot \pi}{4} \cdot \left(\cos\alpha - \frac{\lambda \cdot \sin\alpha \cdot \cos\alpha}{\sqrt{1-\lambda^2 \cdot \sin^2\alpha}}\right) = = \mathcal{M}_{g0} + \mathcal{M}_g^*(\alpha) = \mathcal{M}_{g0} + \sum_{j=0,5}^{\infty}\left[a_j^g \cdot \cos(j\alpha) + b_j^g \cdot \sin(j\alpha)\right] \tag{9}$$

where:

- p_0 is the gas pressure in the crank case,
- $p(\alpha)$ is the gas pressure in the cylinder,
- a_j^g, b_j^g are the coefficients of the Fourier series expansion of the gas-forces excitation.

The excitation moment of the inertial forces of the oscillating masses of the crank gear can be described as:

$$\mathcal{M}_{in}^*(\alpha) \approx \left(m'' + \frac{s'}{l} \cdot m'\right) \cdot r^2 \cdot \omega^2 \sum_{j=1}^{6} d_j^{in} \cdot \sin(j\alpha) = \sum_{j=1}^{6} b_j^{in} \cdot \sin(j\alpha) \tag{10}$$

where only the first six excitation terms are of interest, and the remaining terms are disregarded due to their small values. Coefficient d_j is a function of the parameter $\lambda = r/l$ (Figure 2) and can be found in bibliographical sources [3] and [15].

When defining the amplitudes of gas-excitation harmonics (a_j^g, b_j^g) and inertia forces (b_j^{in}), it is important to accurately determine the pressure flow in the engine cylinder $p(\alpha)$. It is possible to simulate one engine cycle using a simplified model (e.g., with a zero-dimensional model in the engine cylinders, and a one-dimensional model in the suction and exhaust part of the engine [17]) and still obtain satisfactory results for the cylinder pressure $p(\alpha)$. The results for $p(\alpha)$ obtained from a simulation are in turn used to determine the coefficients a_j^g, b_j^g using Expression (9). We simulated one cycle on Engine 1 and simultaneously measured the pressure in the cylinder of the engine.

The parameters of Engine 1, on which the simulation was performed, are:

- Suction, six-cylinder, in-line diesel engine with ignition order *1-5-3-6-2-4*,
- Piston diameter/stroke *123 mm/140 mm*, compression ratio $\varepsilon = 16.5$,
- Maximum power/rotation speed *143 kW/ 2200 rpm*;

The amplitudes of the *j*-th order of the excitation moments of the gas forces $R_j^g = [(a_j^g)^2 + (b_j^g)^2]^{1/2}$ are calculated using the instant pressures values

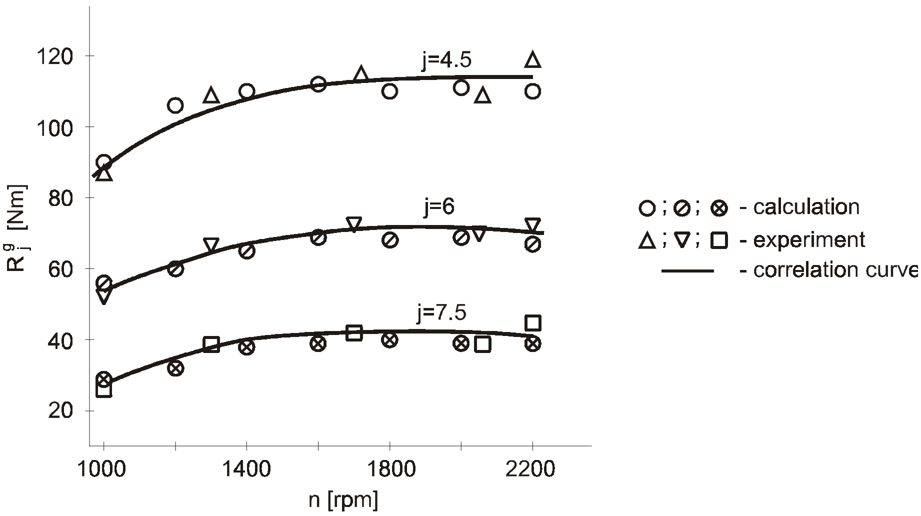

Fig. 7. *Relationship between the excitation amplitude R_j^g and engine revolution n*

measured in the engine cylinder, using piezoelectric sensors, as well as the pressure observed during the simulation of one engine cycle [17], which are shown in Figure 7 for j = 4.5, 6 and 7.5.

Based on the calculation and the experimental results (plotted as points in Figure 7), the resulting correlation curve (continuous line) to be used in the calculations of torsional oscillations is derived.

4 ANALYSIS OF THE CALCULATION RESULTS

The presented analysis includes the calculation and measurements performed on Engine *1*. According to the discussion presented in Section 2, an equivalent torsional-oscillatory engine system with and without a TOD is formed. Typical values of this equivalent torsional-oscillatory system, with and without a torsional oscillations damper, have been calculated based on the discussion in Section 2, and are as follows:

Engine ***1*** without a TOD (Figure 8)

$\mathcal{J}_1=0.03$ *kg m²*
$\mathcal{J}_2=\mathcal{J}_4=\mathcal{J}_5=\mathcal{J}_7=0.083$ $kg\ m^2$
$\mathcal{J}_3=\mathcal{J}_6=0.062$ $kg\ m^2$
$\mathcal{J}_8=1.542$ $kg\ m^2$; $\mathcal{J}_9=0.096$ $kg\ m^2$
$\mathcal{J}_{10}=0.33$ $kg\ m^2$
$K_{s2}=K_{s3}=\ldots=K_{s7}=8.7$ *Nms/rad*
$c_1=3.074\cdot10^6$ *Nm/rad*
$c_2=c_3=\ldots=c_6=1.996\cdot10^6$ *Nm/rad*
$c_7=2.7\cdot10^6$ *Nm/rad*; $c_8=0.15\cdot10^4$ *Nm/rad*
$c_9=0.923\cdot10^6$ *Nm/rad*

Engine ***1*** with a TOD (Figure 9)

$\mathcal{J}_1=0.123$ $kg\ m^2$; $\mathcal{J}_2=0.046$ $kg\ m^2$
$\mathcal{J}_3=\mathcal{J}_5=\mathcal{J}_6=\mathcal{J}_8=0.083$ $kg\ m^2$
$\mathcal{J}_4=\mathcal{J}_7=0.062$ $kg\ m^2$
$\mathcal{J}_9=1.542$ $kg\ m^2$; $\mathcal{J}_{10}=0.096$ $kg\ m^2$
$\mathcal{J}_{11}=0.33$ $kg\ m^2$
$K_{s3}=K_{s4}=\ldots=K_{s8}=8.7$ *Nms/rad*
$c_1\equiv c_v$ *(Figure 5)*; $c_2=3.074\cdot10^6$ *Nm/rad*
$c_3=c_4=\ldots=c_7=1.996\cdot10^6$ *Nm/rad*
$c_8=2.7\cdot10^6$ *Nm/rad*; $c_9=0.15\cdot10^4$ *Nm/rad*
$c_{10}=0.923\cdot10^6$ *Nm/rad*

The internal damping in the crankshaft material and the viscoelastic element of the torsional oscillations damper were calculated based on the discussion given in Section 2.3.2.

Natural oscillating frequencies, as well as the relative oscillation amplitudes, are calculated using the approximate Holtzer method for the 1st and 2nd mode of vibrations. The calculation results for Engine *1* without a TOD are shown in Figure 8, and for Engine *1* with a TOD are shown in Figure 9. These results indicate that in both cases, the 1st mode of vibrations has practically no influence on the oscillations of the engine masses, which is a consequence of the low stiffness of the transmission shaft (c_9 = 0.15 10^4 *Nm/rad*) relative to the stiffness of other components. The calculation results for the “interesting” excitation terms, using the approximate values of the natural oscillation frequencies and the developed software for the calculation of amplitudes of disc oscillations, are shown for Engine *1* without

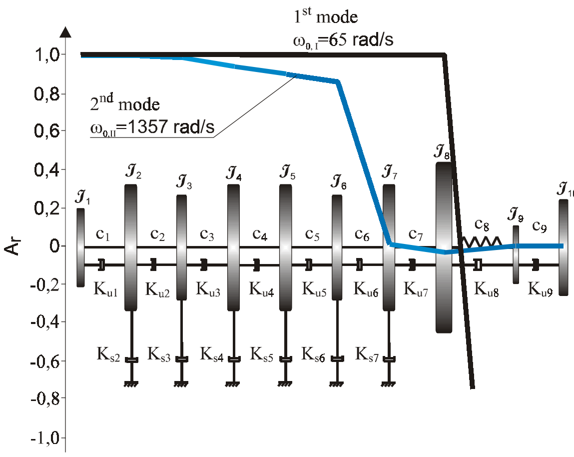

Fig. 8. *Relative oscillation amplitudes of the individual discs for engine 1 without a TOD*

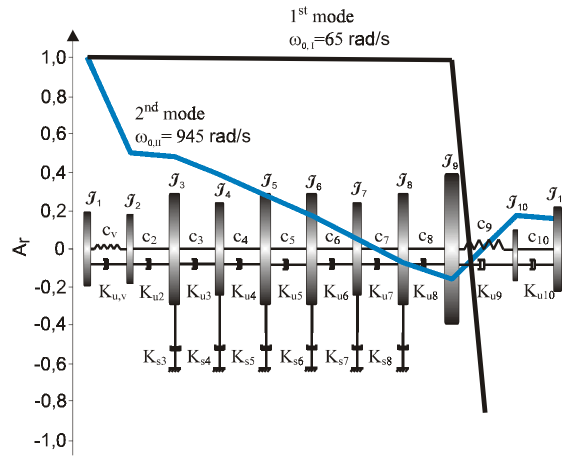

Fig. 9. *Relative oscillation amplitudes of the individual discs for engine 1 with a TOD*

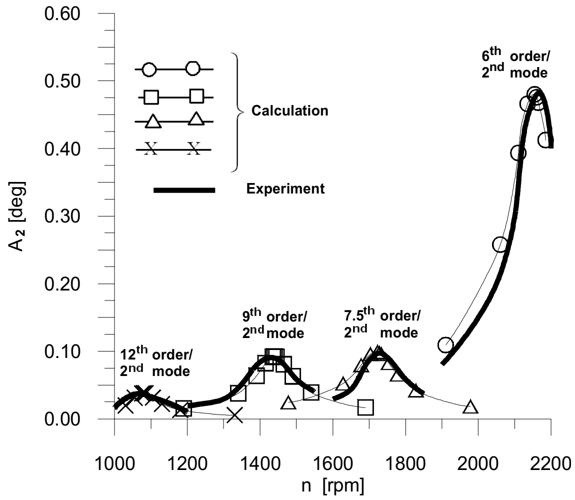

Fig. 10. *Change of twisting amplitude on crankshaft pulley (A_2) in Engine 1 without TOD as a function of engine revolution n, for interesting excitation orders and* 2^{nd} *mode oscillations*

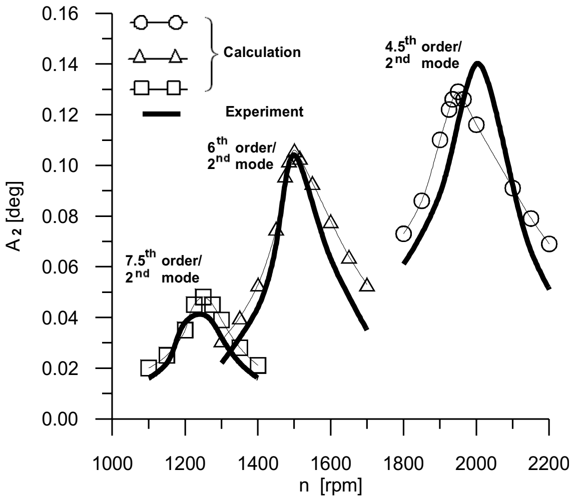

Fig. 11. *Change of twisting amplitude on crankshaft pulley (A_2) in Engine 1 with TOD as a function of engine revolution n, for interesting excitation orders and* 2^{nd} *mode oscillations*

a TOD, and with a TOD in Figures 10 and 11, respectively. These values are taken in the part of the crankshaft pulley where the twisting angles are the greatest.

Both analytical and experimental results are shown in Figures 10 and 11. The equipment for measuring torsional oscillations with a simultaneous harmonic analyses of the mean orders is described in detail in [3]. It is clear that the predicted and measured values agree well for the engine without a torsional oscillations damper, which in turn validates our choice of parameters for the equivalent torsional-oscillatory system.

There is a certain disagreement between the analytical and experimental results for the engine with a torsional oscillations damper (Figure 11). This is due to the choice of the simplified, proposed model, given by (8), used to determine the internal damping coefficient of the TOD. A detailed study of the data indicates that the largest discrepancy occurs during regimes of faster rotational speeds of the engine, which is mostly influenced by the dynamic value of the internal damping coefficient ($K_{u,v}$). An additional analysis of the parameters that determine the characteristics of the viscoelastic elements of the TOD, namely c_v and $K_{u,v}$, has demonstrated better agreement between the analytical predictions and the experimental results. However, the price paid is a more complicated procedure for determining the coefficient $K_{u,v}$, and therefore it is not presented in this paper.

We consider the proposed approach to be acceptable for engineering practice, where the results can be useful in an assessment of the efficiency of a TOD.

Furthermore, the dynamic endurance of the engine crankshaft can be determined by adopting the proposed methodology in calculating the dynamic amplitudes of torsional stress, and its impact on the twisting of the engine crankshaft segments.

5 CONCLUSION

Based on the results of research of the torsional oscillations in an IC engine presented in this paper, the following conclusions can be drawn. The complicated system of torsional oscillations in the IC engine can be replaced with the so-called in-line equivalent torsional-oscillatory system. Typical values in the equivalent system (moments of inertia, stiffness, and damping) can be calculated using proposed simplified methods, which yield satisfactory results. In particular, the proposed method for calculating the characteristics of the viscoelastic element of a torsional oscillations damper with typical non-linear characteristics is new and can be of practical interest. The easiest method for calculating the external excitation of the torsional oscillations system is to use software for an engine-cycle simulation. We demonstrated the agreement between the results for torsional oscillations obtained using the proposed calculation methods and the realistic measured values. Therefore, the proposed procedure for the analysis of torsional oscillations, which consists of the starting physical model, the methodology for determining the input parameters, the adopted mathematical model, and the numerical solution, is satisfactory from a practical standpoint.

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