# Iskanje okvar ležajev z uporabo Meyerjevih algoritmov

# Bearing-Fault Detection Using the Meyer-Wavelet-Packets Algorithm

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Pri mnogih uporabah je bila uspešno upoštevana omejitvena lastnost valčni paket (VP) pri časovnofrekvenčni analizi. V tem prispevku raziskujemo novo metodo iskanja okvar ležajev na osnovi VP z uporabo
Meyerjevega filtra, tako dobimo Meyerjev algoritem (MVP). Predlagani MVP algoritem smo ocenili za
simulirane signale in signale v dejanskem času. Z upoštevanjem tega smo uporabili učinkovito metodo za
skrajšanje preračunov algoritma. Tako je algoritem primernejši za sprotno odkrivanje okvar ležajev in zato
učinkovit način za obdelavo signalov nihanj in drugih mehanskih sistemov.

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(Ključne besede: napake na ležajih, odkrivanje napak, algoritmi, vibracijski signali)

The localization property of wavelet packets in a time-frequency analysis has been successfully considered in many applications. In this paper, a new method of bearing-fault detection is investigated using a WP basis with the Meyer filter leading to the Meyer wavelet packets (MWPs) algorithm. The proposed MWP algorithm is evaluated for simulated and real-time signals. In this respect, an efficient method is used to greatly reduce the algorithm's computations. This makes the algorithm more suitable for the online detection of failures in bearings, and also an effective candidate for the processing of vibration signals in other mechanical systems.

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(Keywords: bearing fault detection, MWP, algorithms, vibration signals)

#### **0 INTRODUCTION**

A ball bearing consists of an inner race, an outer race and a number of rolling balls, as shown in Fig. 1 [1]. Normally, the metal fatigue produced between the above elements yields some mechanical vibrations, which in time lead to bearing damage and increases in the machine's noise level. The contamination, corrosion, improper installation, and lubrication of bearings can effectively speed up the damage rate [2].

Bearing failures may be detected by analyzing vibration signals, which contain the machine's dynamic information [3]. This is performed by inspecting the characteristic frequencies of the defect computed from the bearing dimensions and shaft's rotating speed as [2]:

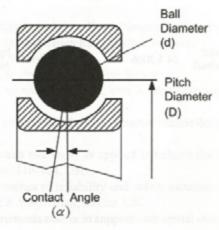


Fig. 1. A typical bearing structure

$$f_i = \frac{N}{2} \left[ 1 + \frac{d}{D} \cos(\alpha) \right] f_r \tag{1}$$

$$f_0 = \frac{N}{2} \left[ 1 - \frac{d}{D} \cos(\alpha) \right] f_r \tag{2}$$

$$f_b = \frac{D}{d} \left\{ 1 - \left[ \frac{d}{D} \cos(\alpha) \right]^2 \right\} f_r \tag{3},$$

where N shows the number of balls,  $f_r$  is the shaft's rotating speed,  $\alpha$  denotes the contact angle of the balls and races,  $f_b$ ,  $f_r$ ,  $f_{\sigma}$  and express, respectively, the frequencies of the defective ball, the inner race, and the outer race, and d and D are the balls and pitch diameters, as illustrated in Fig. 1.

Vibration signals measured on the machine's surface are normally embedded in background noise. and therefore, high-precision techniques should be established for detecting and/or diagnosing machine failures. Consistent with other findings in the literature, to resolve the frequency content of a signal using the short-time Fourier transform, a sufficient data record is required. It is well known, however, that when the latter technique is applied to a large number of samples, the time localization is lost. This becomes more significant for non-steady signals when the detection of transients and movements (containing drift, trends, abrupt changes, etc.) is required. The ineffectiveness of this algorithm in such cases may lead to poor results and wrong conclusions.

The use of wavelet packets has recently been considered for analyzing bearing defects, using coif4 wavelets [4]. The capability of this technique in concentrating on a desired portion of the frequency content of a signal in the time-frequency domain has received increasing attention in different applications. In this paper, a Meyer-packet-wavelets algorithm is

proposed for bearing-fault detection. Accordingly, an effective technique is designed based on the relevant WP tree to reduce the number of computations. The outline of the paper is as follows. In Section 2, a brief review of wavelet packets and the Meyer filter is presented. In Section 3, the proposed wavelet-packet algorithm is described for bearing fault detection and is evaluated using the simulated and real-time data. In Section 4, a method is considered for reducing the required computations, and the conclusions are presented in Section 5.

### 1 WAVELET PACKETS

Assume that the quadrature mirror lowpass and highpass filters of an orthogonal wavelet are, respectively, given by h(n) and g(n). The wavelet-packet coefficients are then defined by subsampling the convolutions of  $d_j^p(n)$  with h(-2n) and g(-2n) as [5]:

$$d_{j+1}^{2p}(n) = d_j^p(n) * h(-2n)$$
  
 $d_{j+1}^{2p+1}(n) = d_j^p(n) * g(-2n)$  (4),

where 0<p<2)-1. The corresponding structure of the above wavelet-packet filter-band is depicted in Fig. 2 for two levels of decomposition. As it can be seen, the output of each branch passes separately through the quadrature mirror lowpass and highpass filters. Appropriate selection of the branches results in the desired sub-band frequency in the time-frequency domain.

To analyze precisely the frequency content of a signal, wavelets with high-frequency localization are desirable. Orthogonal Shannon wavelets have the highest frequency resolution among the various

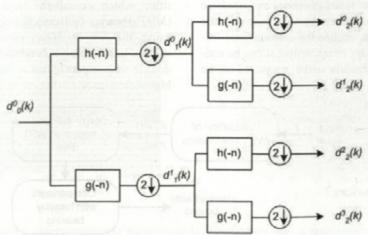


Fig. 2. Wavele- packet filter-bank decomposition with successive filtering and subsampling

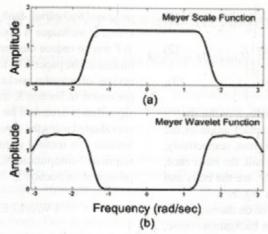


Fig. 3. Meyer filters, a) lowpass, b) highpass

wavelets. The lowpass and highpass filters corresponding to these wavelets divide the frequency domain in rectangular forms within  $[-\pi/2, \pi/2]$  and  $[-\pi, -\pi/2] \cup [\pi/2, \pi]$  [5]. The sharp edges of these filters, however, lead to non-causal wavelets in the time domain. In practice, an approximation filter may be used. Such an approximation in the time domain is the Meyer wavelet, whose frequency band is shown in Fig. 3. The smoothed edges of this filter correspond to much faster decaying wavelets than the Shannon wavelets in the time domain. Due to the unlimited length of the Meyer wavelet, its approximation, the discrete Meyer wavelet, is considered.

## 3 THE PROPOSED MAYER-WAVELET-PACKET BEARING-FAULT-DETECTION ALGORITHM

Consider a hole inside the inner race or outer race of a bearing. According to Section 1, a rotating shaft with a constant speed generates an impulsive vibration on the spot where the balls pass over the defect. This, in turn, excites the characteristic frequencies of a bearing component that can be estimated from the vibration signal measured on the machine surface.

To detect a bearing fault, the signal energy can be measured within a sub-band around the characteristic frequency of the bearing component. When this energy increases compared to that of the healthy one, we can decide if the inner race (or the outer race) is defective. This procedure is illustrated in Fig. 4.

The energy of the wavelet-packet coefficients is computed at each node, (*j*,*p*), of the tree as [4].

$$E = \left(\sum_{k=1}^{M} (d_{j}^{p}(k))^{2} / M\right)^{\frac{1}{2}}$$
 (5),

where  $d_j^p(k)$  are the wavelet-packet coefficients and M shows the number of coefficients. Note that the amount of computed energy shows the severity of a defect [2]. These energies are next compared to the same values measured from the associated healthy bearing. This procedure has already been applied to coif4 wavelets [4]. In this paper, the proposed algorithm, which we call the Meyer wavelet packets (MWPs) bearing-fault detection algorithm is designed using the discrete Meyer Meyer Meyer wavelet. This wavelet has a narrower bandwidth in the frequency domain with respect to that of the coif4, leading to a higher frequency resolution in the time-frequency

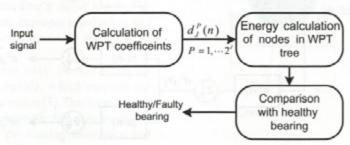


Fig. 4. Bearing-fault detection algorithm using wavelet packets

domain. To present this, Fig. 5 shows the amplitudes of the corresponding wavelet-packet coefficients for an input signal composed of two sinusoids at 10 and 30 Hz as.

$$x(t) = \sin(2\pi 10t) + \sin(2\pi 30t)$$
 (6).

This signal is sampled at 64 Hz and decomposed to the level of J=5 by utilizing the dmey and coif4 wavelets. Observe that the frequency bands of the sinusoids, shown by darker lines, are clearly resolved by the dmey wavelet compared to those of the coif4, indicating its higher frequency resolution.

# 3.1. Performance of MWP for Simulated Signals

The capability of the MWP algorithm is evaluated for simulated bearing signals assuming that the outer race is damaged. The bearing specifications are assumed to be pd = 65 mm, bd = 15 mm, n = 8 and  $\beta = 0$ , for which the characteristic frequencies are obtained based on (1 to 3) as  $f_r = 33.3$ ,  $f_{th} = 20.5$  and  $f_{th} = 68.31$  Hz. The healthy bearing-vibration signal is assumed to be the sum of three low-amplitude sinusoids given by:

$$x_k(t) = 0.5\cos(2\pi \cdot f_r t) + 0.1\cos(2\pi \cdot f_{sk}t) + 0.1\cos(2\pi \cdot f_{sk}t)$$
  
+0.1cos(2\pi \cdot f\_{sk}t) (7),

where the subscript h denotes the healthy state. To define the outer-race defect, another sinusoidal signal is also added to (7) in three individual experiments representing different levels of severity as

$$x_{d_i}(t) = x_k(t) + 0.5\cos(2\pi . f_o t)$$
  
 $x_{d_i}(t) = x_k(t) + \cos(2\pi . f_o t)$  (8),  
 $x_{d_i}(t) = x_k(t) + 2\cos(2\pi . f_o t)$ 

where the subscript *d* indicates the signal of the defective bearing. A white Gaussian noise with zeromean and a variance of 0.01 is also added to the signal as the background noise. The signals are sampled at 1024 Hz, leading to 16384 samples. The *demy* wavelet-packet coefficients are computed for 8 levels (j=8). In this way, the frequency region between 0 and 512 Hz is divided into 256 two-Hz bandwidths. Fig. 6 shows the energies of the 70 nodes of the wavelet-packet tree for bearing signals defined in (8). From the figure it is clear that the measured energies between nodes 43 to 47 have significantly increased as a result of the defect.

#### 3.2. Performance of MWP for Real Data

In this experiment the performance of the MWP algorithm is inspected for real-time bearing signals. Accordingly, Figs. 7-b and c show the recorded signals of a bearing with a hole in the inner or outer race. The real data are provided for a shaft speed of  $2000 \, rpm$  with a  $40 \, kHz$  sampling rate filtered out from a low-pass filter with a cut-off frequency of  $15 \, kHz$  [3]. The number of input samples is 4096. The test specifications are similar to those used in Section 3.1. From (1) and (2), the theoretical frequencies of the defective inner and outer race bearings are, respectively, obtained as  $f_i = 164$  and  $f_o = 102.5 \, Hz$ .

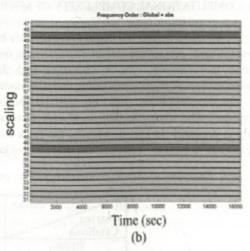


Fig. 5. Time-frequency representation of wavelet-packet coefficients for a sinusoidal signal composed of two harmonics at 10 and 30 Hz for a) coif4 wavelets, b) dmey wavelets

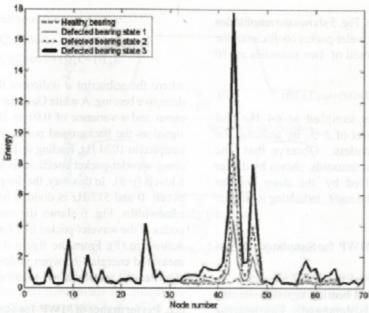


Fig. 6. Energies of wavelet-packet nodes for the three simulated signals of a presumably defective bearing based on (8)

The wavelet-packet coefficients of these signals are computed using the *dmey* wavelet to the level of j=12. As a result, the frequency band between 0 and 20 kHz is divided into 4096 sub-bands of 4.88 Hz. The corresponding defect frequencies are then given by the nodes between 12 and 50 or 12 and 31 for the inner or outer races, respectively. Table 1 presents the measured energies of the healthy and defective signals for the mentioned sub-bands. The respective increase of the energy in each case clearly shows the presence of a fault.

#### 4 COMPUTATIONAL COMPLEXITY OF MWP

To find a defect, our inspection may only be concentrated around the defect characteristic frequency that is known, a priori. This starts with the determination of the corresponding path and nodes of the wavelet-packet tree towards the defect frequency. The schematic diagram of such a procedure is depicted in Fig. 8, wherein the thick line represents a typical path of the wavelet-packet

tree towards an assumed defect frequency with 3 levels of decomposition (j=3). In this way there is no need to compute all the wavelet packet coefficients, but only the coefficients of the specified path. Next, the total energy is computed based on these coefficients, which accordingly leads to an extensive reduction in the algorithm computations.

In general, for a signal of length N, the number of real additions,  $A_{mal}$ , and multiplications,  $A_{mul}$ , of a wavelet-packet tree for the decomposition level of J with the quadrature-mirror filter of order M is given by [7]:

$$A_{add} = A_{mol} = M \times J \times N$$
 (9).

Using the specified path of the waveletpacket tree mentioned above, the number of computations reduces to [7]:

$$A_{add} = A_{mal} = \frac{1}{2} M \times N \times \left(1 - \frac{1}{2^{J}}\right) \qquad (10),$$

which is 1/2J of that given by (9).

Table 1. Energies of the dmey wavelet-packet coefficients for the bearing signals

Nodes Bearing Condition	12 to 50	12 to 31
Healthy	0.0439	0.0987
Inner-race defected	0.0779	0.0380
Outer-race defected	0.0261	0.1056

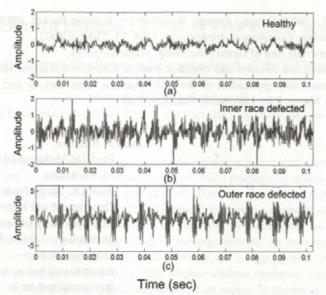


Fig. 7. Real-data bearing signals for a) healthy, b) defective inner race, and c) defective outer race

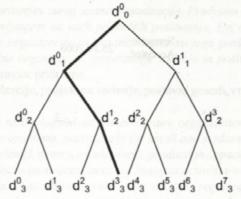


Fig. 8. A wavelet-packet tree with a typical specified path (thick line) towards the defect frequency

#### 5 CONCLUSIONS

In this paper a new bearing-fault detection algorithm, which we called the MWP algorithm, was proposed, based on the *dmey* wavelet-packet coefficients. This algorithm gives a higher resolution when detecting bearing defects compared to that of the *coif4* wavelet; however, it requires more compu-

tations. To resolve the latter problem, an efficient technique was designed by considering a specific path of the wavelet-packet tree towards the defect frequency. The number of computations was then reduced by a factor of twice the number of decomposition levels. This, as a consequence, made the proposed MWP algorithm more appropriate for the online processing of bearing-fault detection.

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Prejeto: Received:

21.4.2004

Sprejeto: Accepted:

25.10.2006

Odprto za diskusijo: 1 leto Open for discussion: 1 year