

Exponential Tracking Control of an Electro-Pneumatic Servo Motor

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According to the fundamental importance of the tracking theory on technical systems, the main goal of this paper is a further development of the theory and the application of the tracking, especially to the practical tracking concept.

The new definition of the practical exponential tracking is shown. The practical tracking is defined by the output vector, which is different from the former definitions, given by the output error vector. The defined exponential tracking is elementwise. The new criterion of the practical exponential tracking is shown. Based of the new criteria, the control algorithm of practical exponential tracking is determined by using the self-adaptive principle. The structural characteristic of such a control system is the existence of two feedback sources: the global negative of the output value and the local positive of the control value. Such a structure ensures the synthesis of the control without the internal dynamics knowledge and without measurements of disturbance values.

The plant under consideration is an electro-pneumatic servo motor. This system is often applied as the final control element of a controller in automatic control systems. The correction device for the mentioned plant will be a digital computer. The mentioned control forces the observed plant output to track the desired output value with prespecified accuracy. In this paper the simulation results produced by practical tracking control algorithm on an electro-pneumatic servosystem will be presented.

The results show a high quality of the practical exponential tracking automatic control. The type of the control ensures the change of the output error value according to a prespecified exponential law.

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0 INTRODUCTION

The practical tracking concept is very important from the technical viewpoint. The consideration of the dynamics behavior of technical plants during a limited time interval, with a prespecified quality of that behavior, imposes a practical request and necessities, which can be placed to any technical plant. For many plants the most adequate tracking concept is the practical tracking concept. The concept most completely satisfies practical technical requirements on the dynamics behavior as well as the quality of the dynamics behavior. The practical tracking concept includes physically possible and realizable initial deviations of the output value, maximum deviations of the output value permitted in relation to the desired output value (according to the desired accuracy), a set of expected and unexpected

disturbances during such a time interval, which is of a technical interest. The elementwise exponential tracking has been defined. Each element of vector y should exponentially approach the appropriate element of vector y_d . Elementwise exponential tracking was introduced by Grujić and Mounfield [1] to [6]. In those papers the Lyapunov approach to the exponential tracking study is used. The approach assumes the existence of the bound (envelope of the output error vector) which limits the exponential evolution of the output error vector, but that bound is not predefined. In this framework the bounds are predefined and determined with the function set $I_A(\cdot)$ and scalar β .

The nonuniform practical exponential tracking is introduced by Lazić [7], where definitions, criteria and algorithms for such tracking are presented for a certain class of technical objects.

1 PROBLEM STATEMENT AND SIGNIFICANCE

The object considered can be described by a mathematical model expressed by the state and the output equations:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{f}[\mathbf{x}(t), \mathbf{d}(t)] + \mathbf{B}\mathbf{u}(t) \\ \mathbf{y}(t) &= \mathbf{g}[\mathbf{x}(t)]\end{aligned}\quad (1).$$

The admitted bounds of the vector \mathbf{y} of the object real dynamic behavior are determined by the vector of desired dynamic behavior \mathbf{y}_d and sets E_i and E_A , as follows:

$$I_i(\mathbf{y}_{d0}; E_i) = \{\mathbf{y}_0 : \mathbf{y}_0 = \mathbf{y}_{d0} - \mathbf{e}_0, \mathbf{e}_0 \in E_i\} \quad (2),$$

$$I_A(t; \mathbf{y}_d(\cdot); E_A) = \{\mathbf{y} : \mathbf{y} = \mathbf{y}_d(t) - \mathbf{e}, \mathbf{e} \in E_A\} \quad (3).$$

2 DEFINITION

The system Eq. (1) controlled by $\mathbf{u}(\cdot) \in S_u$ exhibits the practical exponential tracking with respect to $\{\tau, \Lambda, \beta, I_i(\cdot), I_A(\cdot), S_d, S_z\}$, (Fig. 1), if $[\mathbf{y}_0, \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in I_i(\mathbf{y}_{d0}) \times S_d \times S_z$ implies:

$$\mathbf{y}[t; \mathbf{y}_0, \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in I_A(t), \forall t \in R_\tau \quad (4),$$

and for $\forall i \in \{1, 2, \dots, n\}$ and $\forall t \in R_\tau$ holds

$$\begin{aligned}y_i[t; \mathbf{y}_{i0}, \mathbf{u}(\cdot), \mathbf{y}_{di}(\cdot), \mathbf{d}(\cdot)] &\geq \\ y_{di}(t) - \alpha_i(y_{d0} - y_{i0})e^{-\beta t}, y_{i0} &\leq y_{d0}\end{aligned}\quad (5),$$

and

$$\begin{aligned}y_i[t; \mathbf{y}_{i0}, \mathbf{u}(\cdot), \mathbf{y}_{di}(\cdot), \mathbf{d}(\cdot)] &\leq \\ y_{di}(t) - \alpha_i(y_{d0} - y_{i0})e^{-\beta t}, y_{i0} &\geq y_{d0}\end{aligned}\quad (6).$$

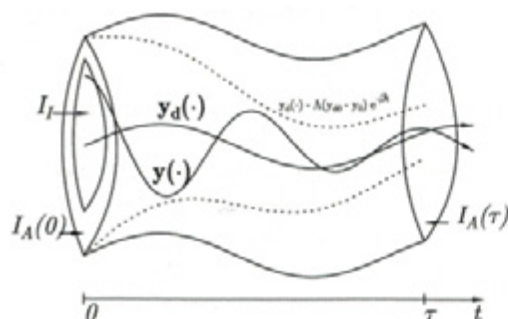


Fig. 1. Practical exponential tracking

3 CRITERIA

In order for the system Eq. (1) controlled by $\mathbf{u}(\cdot)$ to exhibit practical exponential tracking with respect to $\{\tau, \Lambda, \beta, I_i(\cdot), I_A(\cdot), S_d, S_z\}$ it is sufficient that control $\mathbf{u}(\cdot)$ guaranties:

$$\begin{aligned}e[t; \mathbf{e}_0, \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] &= \\ -\gamma e, \forall [t; \mathbf{e}_0, \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] &\in R_\tau \times E_i \times S_d \times S_z\end{aligned}\quad (7),$$

where $\gamma \in [\beta, +\infty[$.

Arbitrary $\mathbf{y}_d(\cdot) \in S_d$, $\mathbf{d}(\cdot) \in S_z$, $\mathbf{e}_{i0} \in E_{i0}$ and $i \in \{1, 2, \dots, n\}$ is considered. From Eq. (7) it follows that:

$$e(t; \mathbf{e}_0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)) = \mathbf{e}_0 e^{-\gamma t}, \forall t \in R_\tau,$$

or in scalar form:

$$e_i[t; \mathbf{e}_{i0}; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] = e_{i0} e^{-\gamma t}, \forall t \in R_\tau. \quad (8).$$

This equation expresses exponential decrease of the function $e_i(\cdot)$, from starting value e_{i0} towards zero. Since $e_{i0} \in E_{i0} \subseteq E_{Ai}$ the value of the error in any time is less than $e_{i, \max}$, and greater than $e_{i, \min}$. It means that the object exhibits practical tracking with respect to $\{\tau, I_i(\cdot), I_A(\cdot), S_d, S_z\}$, Lazić [7]:

$$\mathbf{y}[t; \mathbf{y}_0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in I_A(t)$$

$$\forall [t; \mathbf{y}_0, \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in R_\tau \times I_i(\mathbf{y}_{d0}) \times S_d \times S_z \quad (9).$$

Based on Eq. (8) and the condition of theorem $\gamma \in [\beta, +\infty[$, it follows that

$$e_i[t; \mathbf{e}_{i0}; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \leq e_{i0} e^{-\beta t}, \forall t \in R_\tau, e_{i0} > 0 \quad (10),$$

$$e_i[t; \mathbf{e}_{i0}; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \geq e_{i0} e^{-\beta t}, \forall t \in R_\tau, e_{i0} < 0 \quad (11),$$

and

$$e_i[t; 0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] = 0, \forall t \in R_\tau, e_{i0} = 0 \quad (12).$$

Now, using Eq. (10), Eq. (11), Eq. (12), $e_i(t) = y_{di}(t) - y_i(t)$, $\forall t \in R_\tau$, the function set definitions $I_i(\cdot)$ and $I_A(\cdot)$, Eq. (2) and Eq. (3), and $\alpha_i = 1$, $\Lambda = 1$, one gets:

$$\begin{aligned}y_i[t; \mathbf{y}_{i0}; \mathbf{u}(\cdot), \mathbf{y}_d, \mathbf{d}(\cdot)] &\geq y_{di}(t) - \alpha_i(y_{d0} - y_{i0})e^{-\beta t} \\ y_{i0} &\leq y_{d0}, \forall t \in R_\tau\end{aligned}\quad (13),$$

$$\begin{aligned}y_i[t; \mathbf{y}_{i0}; \mathbf{u}(\cdot), \mathbf{y}_d, \mathbf{d}(\cdot)] &\leq y_{di}(t) - \alpha_i(y_{d0} - y_{i0})e^{-\beta t} \\ y_{i0} &\geq y_{d0}, \forall t \in R_\tau\end{aligned}\quad (14),$$

which considering the arbitrary chosen $[e_{i0}, i, \mathbf{y}_d, \mathbf{d}(\cdot)] \in E_{i0} \times \{1, 2, \dots, n\} \times S_d \times S_z$, together with Eq. (9) satisfies the definition and proofs the theorem.

4 ALGORITHM

The algorithm is based on the natural tracking control concept introduced by Grujic. The main characteristic of this concept, which follows from the self-adaptive principle, Grujic [8] to [10], is the existence of the local positive feedback in the control u (with possible derivative and/or integrals of u). The local positive feedback compensates for the influences of the disturbances and the internal dynamics of the controlled object, since, during the control construction the information about them is not used.

The main negative feedback loop in the output y (with possible derivatives and/or integrals of y) provides the desired quality of the error evolution.

The values of all vector elements $y(t)$ and $\dot{y}(t)$ from Eq. (1) are measurable in any time instant $t \in R_+$.

Let Assumption 1 hold, let $S_u = \{u(\cdot)\}$ and control $u(\cdot)$:

$$u(t) = u(t^-) + D^T (DD^T)^{-1} [\dot{e}(t) + \gamma e(t)] \quad (15),$$

$$\forall [t, e_0, y_d, d(\cdot)] \in R_t \times E_t \times S_d \times S_z$$

where D is an arbitrary matrix satisfied $\det(DD^T) \neq 0$, and $\gamma \in [\beta, +\infty[$.

System Eq. (1) controlled by $u(\cdot)$, Eq. (15), exhibits The practical exponential tracking with respect to $\{\tau, l, \beta, I_f(\cdot), I_d(\cdot), S_d, S_z\}$.

If there is no delay in the feedback loop than $u(t) = u(t^-)$, Grujic [8] to [10], and following the vector equation Eq. (15) one gets:

$$D^T (DD^T)^{-1} [\dot{e}(t) + \gamma e(t)] = 0 \quad (16),$$

$$\forall [t, e_0, y_d, d(\cdot)] \in R_t \times E_t \times S_d \times S_z$$

By multiplying that equation with matrix D from the left side, the following equation is obtained:

$$\dot{e}(t) = -\gamma e(t), \quad \forall [t, e_0, y_d, d(\cdot)] \in R_t \times E_t \times S_d \times S_z \quad (17),$$

which, based on $\gamma \in [\beta, +\infty[$ and the former theorem proves this theorem.

This algorithm presents a further justification of the approach of the natural tracking control by Grujic and Mounfield. The natural trackability condition is not considered here. A further implementation of this algorithm in the present paper is a simplify attempt and matrix D will be chosen as an arbitrary matrix.

5 APPLICATION

In this case an electro-pneumatic servo motor as a plant, presented in Figure 2, is considered. It consists of:

1. a single acting membrane pneumatic motor,
2. potentiometer (displacement transducer),
3. electro-pneumatic transducer (EPT).

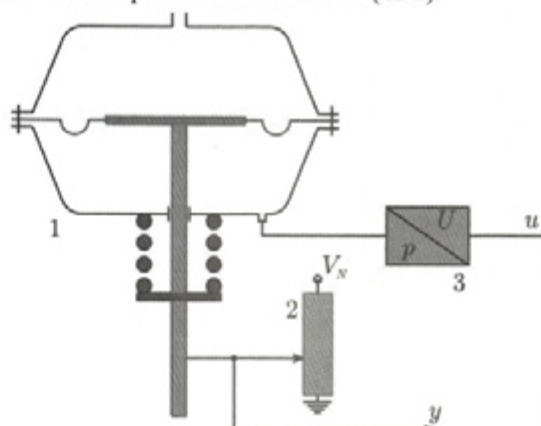


Fig. 2. Electro-pneumatic servo motor

The mathematical model of the mentioned plant is shown Lazic [11]. Here the first order EPT is accepted and verified by experimental results. The electrical part of EPT is very fast and the air volume of pneumatic line and motor chamber are relatively small. For this plant that simplification is very closed to the exact mathematical model of EPT with a pneumatic motor when their air volume determines variable τ_p by a very complex formula.

$$\tau_p \frac{dp(t)}{dt} + p(t) = K_p u(t) \quad (18),$$

$$B_v \frac{dy(t)}{dt} + K_o y(t) + c_i \operatorname{sgn}[\dot{y}(t)] = A_m p(t)$$

where:

y - motor spindle displacement,

u - voltage control signal,

p - EPT output pressure.

A block diagram of the considered plant is shown in Figure 3.

The technical characteristics of the plant are:

$\tau_p = 0.45$ s - EPT time constant,

$K_p = 0.229 \cdot 10^5$ Pa/V - EPT gain,

$B_v = 63050$ Ns/m - damping factor,

$c_i = 93.5$ N - Coulomb friction coefficient,

$R_L = 175$ Ω - EPT coil resistance,

$K_o = 150857.14$ N/m - motor spring stiffness,

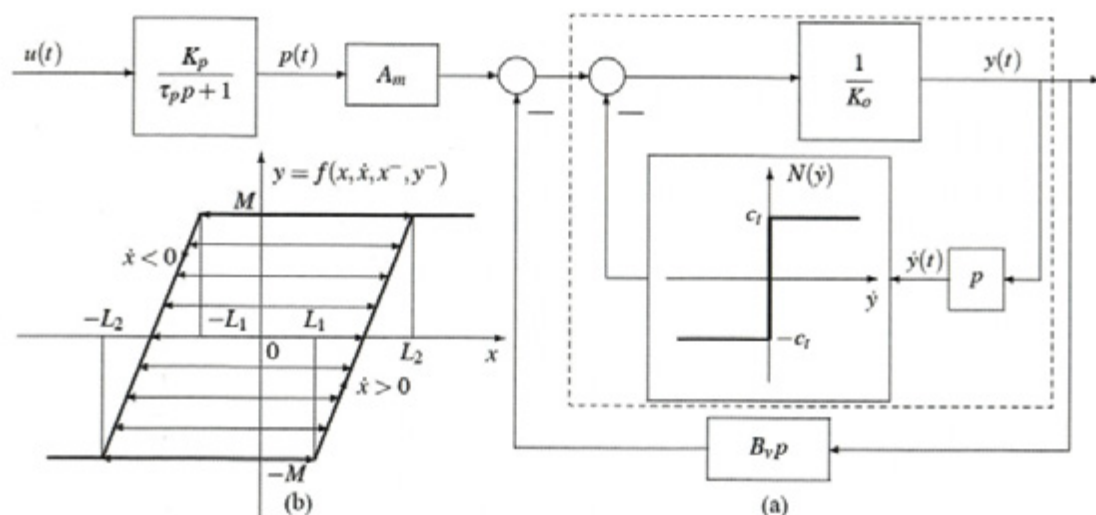


Fig.3. (a) The plant block diagram, (b) the equivalent nonlinearity of the plant part shown in the dashed box of (a)

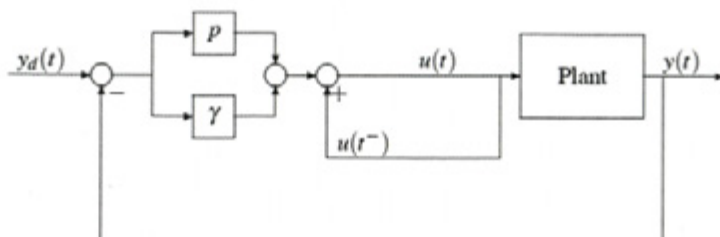


Fig. 4. Control system block diagram

$A_m = 330 \cdot 10^{-3} \text{ m}^2$ - membrane area,
 $y_{max} = 17.5 \text{ mm}$ - maximum motor spindle displacement.

A symbolic block diagram of the control system is presented in Figure 4.

The digital computer simulation of the practical exponential tracking control algorithm, based on the self-adaptive principle, in the form:

$$u(t) = u(t^-) + D[\dot{e}(t) + \gamma e(t)] \quad (19),$$

for a prespecified $\beta = 1$, chosen $\gamma = 1.5$ ($\gamma \in [\beta, +\infty]$), and $D = 0.1$ is done. The illustration of the results achieved by the practical exponential tracking algorithm simulation can be seen in Figures 5 to 8.

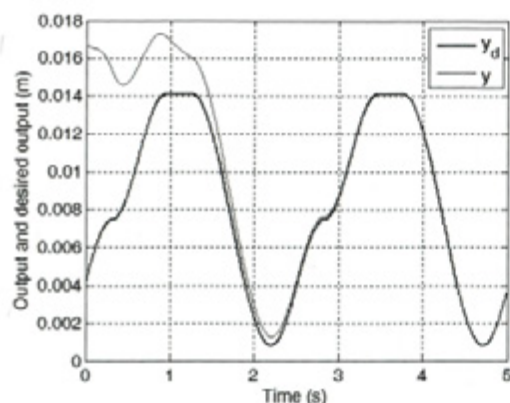


Fig. 5. Output and desired output

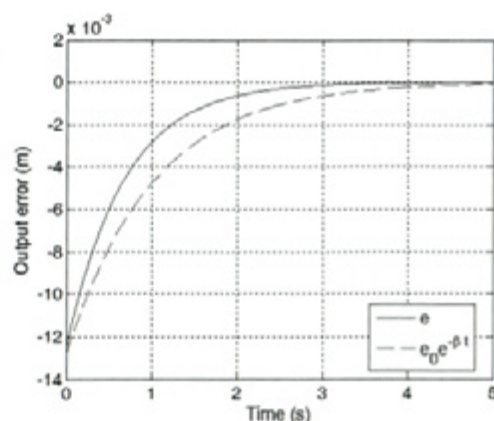


Fig. 6. Output error behavior

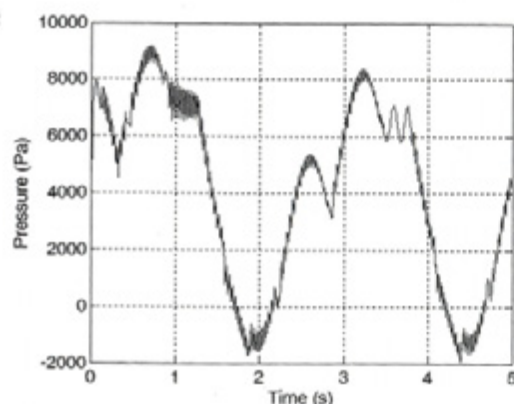


Fig. 7. Electro-pneumatic transducer: output pressure signal

From Figure 5 it can be seen that the real output y is very smooth, and the difference between y and y_d is in permitted boundaries, which can be seen in Figure 6, the exponential error change $e(\cdot)$ is in permitted boundaries $e_0 e^{-\beta(\cdot)}$.

The command pressure (electro-pneumatic transducer pressure) signal behavior is illustrated in Figure 7. The control value u (Fig. 8) is rather rough but lies in the standard signal range -10V to +10V, without saturations. High frequency components in the control signal are a consequence of physical sources - existence of the hysteresis nonlinearity in the plant.

6 CONCLUSIONS

The results show a high quality of the practical exponential tracking automatic control. This type of control ensures a change of output error value according to a prespecified exponential law. For the control design, the internal dynamics of the controlled object need not be known and the measurement of the real output values only is required. The practical tracking control algorithm is based on the self-adjustment principle. The main characteristic of this principle is the existence of the local positive feedback in the control u . The algorithm has been proved based on an assumption that there is no delay in the local positive feedback loop. Over a digital computer simulation, the smaller time step provides the better approximation of the proposed algorithm. Since very small sampling period can be realized by using the up-to-date digital computers, no possible limitations are expected during the implementation on a real system.

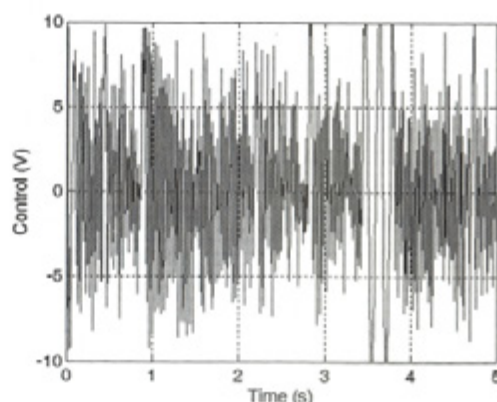


Fig. 8. Control signal

7 NOMENCLATURE

- $B \in R^{q \times r}$ matrix
- $D \in R^{m \times q}$ arbitrary matrix
- $d(\cdot): R \rightarrow R^p$ the disturbance vector function
- $d(t)$ the disturbance vector at time t
- $d \in R^p$ the disturbance vector
- $E_A \subset R^n$ the set of all admitted errors $e(t)$ on R_t ;
- closed connected neighborhood of 0_e
- $E_I \subset R^n$ the set of all admitted initial errors
- $e(0) = e_0$; closed connected neighborhood of 0_e
- $E_{[t,y]} = \{e_i: e_i \in R, e_{im(t)} \leq e_i \leq e_{im(t)}\}, (i)=A,J$
- $e[\cdot; e_0; u(\cdot), y_d(\cdot), z(\cdot)]: R \rightarrow R^p$ the output error response, which at time t represents the output error vector $e(t)$ at the same time
- $e[t; e_0; u(\cdot), y_d(\cdot), z(\cdot)] = e(t)$
- $e \in R^n$ the output error vector, $e = y_d - y$
- $e_{m(t)} = \min \{e: e \in E_{[t,y]}, (i)=A,J, \text{ elementwise minimization}\}$
- $e_{m(t)} = (e_{1m(t)}, e_{2m(t)}, \dots, e_{nm(t)})^T, (i)=A,J$
- $e_{M(t)} = \max \{e: e \in E_{[t,y]}, (i)=A,J, \text{ elementwise maximization}\}$
- $e_{M(t)} = (e_{1M(t)}, e_{2M(t)}, \dots, e_{nM(t)})^T, (i)=A,J$
- $f(\cdot): R^q \times R^p \rightarrow R^q$ the continuous vector function,
- $f(x, z) \in C(R^q \times R^p)$, which describes plant internal dynamics
- $g(\cdot): R^q \rightarrow R^n$ the output function
- $I_A(\cdot): R \times R^n \times 2^{R^n} \rightarrow 2^{R^n}$, the set function of all admitted vector functions $y(\cdot)$ on R_t with respect to $y_d(\cdot)$ and E_A
- $I_A(t) = I_A[t; y_d(\cdot); E_A]$, the set value of the set function $I_A(\cdot)$ at time t , with respect to $y_d(\cdot)$ and E_A
- $I_I(\cdot): R^n \times 2^{R^n} \rightarrow 2^{R^n}$, the set function of all admitted vector functions y_0 with respect to y_{d0} and E_I

$I_f(y_{d0}; E_f)$ the set value of the set function $I_f(\cdot)$ at time t , with respect to y_{d0} and E_f ; if y_{d0} is chosen $\Rightarrow I_f(y_{d0}; E_f) = I_f$
 $R^+ =]0, +\infty[= \{t: t \in R, 0 < t < +\infty\}$
 $R_0 = [t_0, \infty[$
 $R_\tau = [0, \tau[$
 $S_d \subset R^n$ the set of all admitted $y_d(\cdot)$ on R_τ ;
 $y_d(\cdot) \in S_d \Rightarrow y_d(t) \in C(R_\tau, R^n)$
 $S_z \subset R^p$ the set of all admitted $d(\cdot)$ on R_τ
 t time
 $u(\cdot): R \times \dots \rightarrow R^r$ the vector function which describes evolution of the control vector
 $u(t)$ the value of the function $u(\cdot)$ at time t
 $u \in R^r$ the control vector
 $x \in R^q$ the state vector
 $y[\cdot; y_0; u(\cdot), y_d(\cdot), d(\cdot)]$ the real output response, which at time t equals the real output vector at same time,
 $y[t; y_0; u(\cdot), y_d(\cdot), d(\cdot)] = y(t)$
 $y \in R^n$ the real output vector
 $y_d(\cdot): R \rightarrow R^n$ the desired output vector function
 $y_d(t)$ the desired output vector at time t
 $y_d \in R^n$ the desired output vector
 $\beta \in R^+$
 $\gamma \in [\beta, +\infty[$
 $\Lambda \in R^{n \times n}$, $\Lambda = \text{diag}\{e_{1MM}, e_{2MM}, \dots, e_{nMM}\}$
 $\tau \in]0, +\infty[$ the final moment
 $0 = (0 \ 0 \ \dots \ 0)^T \in R^q$

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