

## Numerično simuliranje premočrtnih stičnih problemov z elastohidrodinamičnim mazanjem

### Numerical Simulation of Elastohydrodynamic Lubricated Line Contact Problems

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V prispevku je prikazan vpliv elastohidrodinamičnega (EHD) mazanja na napetostno polje v dotikalnem področju mehanskih elementov pri kotalnem in drsnem gibanju. Raziskava obravnava premočrte dotike, ki jih pogosto srečujemo pri dotikalnih elementih, npr.: zobjniki, valjčni ležaji, kolesa itn. V prispevku uporabljene računske analize so zasnovane na nadomestnem modelu dveh valjev, ki imata enak polmer, kakor sta krivinska radija dotikalnih mehanskih elementov na mestu realnega dotika. Nadomestna valja sta obremenjena z normalno in tangencialno silo, ki ju poprej določimo s Hertzovo kontaktno teorijo. Pogoji EHD mazanja so upoštevani posredno prek dodatnih zunanjih obremenitev. Za določitev napetostnega polja za različne pogoje mazanja v dotikalnem področju je pri numeričnih analizah uporabljeni metoda končnih elementov.

Na temelju rezultatov tovrstnih izračunov lahko popišemo vpliv tipičnih veličin mazanja (viskoznost maziva, srednja hitrost stičnih površin) na napetostno polje v dotikalnem področju. Na ta način je mogoče poiskati optimalne mazalne parametre glede na napetostno polje v dotikalnem področju.

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(Ključne besede: mehanika dotika, mazanje elastohidrodinamično, Hertzova teorija, simuliranje numerično)

This paper presents the influence of EHD-lubrication on stress fields in the contact area of rolling and sliding mechanical elements. The research is related to the line contacts which usually appear with contacting elements such as gears, roller bearings, wheels etc. The computational analysis was performed using the equivalent model of two contacting cylinders that have the same radii as are the curvature radii of contacting mechanical elements at any point of a real contact. The equivalent cylinders are subjected to normal and tangential traction, determined by the Hertz contact theory with consideration of additional external contact forces arising from the EHD-lubrication conditions. The finite element method is used for numerical computations of the stress fields for different lubrication conditions.

The results from such computations provide the basis for determination of the influence of typical lubrication parameters, such as viscosity of lubricant and mean surface velocity of the contacting cylinders on the stress field in contact area. Consequently it is possible to find the optimal lubrication parameters relating to the stress field in the contact area.

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(Keywords: contact mechanics, elastohydrodynamic lubrication, Hertz theory, numerical simulation)

#### 0 UVOD

Mehanika dotika obravnava napetosti in deformacije kot posledico dotika dveh trdih teles. V primeru dotika neskladnih površin (zobjniki, ležaji, tekalna kolesa itn.) je dotikalna površina praviloma majhna v primerjavi z dimenrijami obravnavanih teles. V tem primeru so napetosti koncentrirane na ozko področje blizu dotikalnih površin in so v splošnem neodvisne od oblike obravnavanih teles v večji oddaljenosti od dotikalnih površin [1]. Za preprečitev poškodb na dotikalnih površinah mora biti največja dotikalna napetost vedno manjša od lokalne meje plastičnosti materiala [2]. Mejo plastičnosti materiala določimo v splošnem z enosnim nateznim ali strižnim preskusom. V praksi se pojavi plastičnost materiala v splošnem kot posledica večosnega napetostnega stanja pri napetosti, ki jo določimo po eni izmed porušnih hipotez. Za večino kovinskih materialov je zelo uporaben Misesov kriterij:

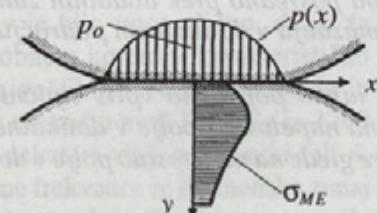
#### 0 INTRODUCTION

Contact mechanics provides the means for determining the stresses and deformations that result from the contact between two solid bodies. In non-conforming contact (gears, bearings, wheels, etc.), the contact area is generally small, if compared with the body dimensions. In this case the stresses are concentrated in the region close to the contact zone, and they are usually not influenced by the body shape at some distance from the contact area [1]. In order to prevent any fatal surface damage occurring on contact surfaces, the maximum stress due to contact loading should always be lower than the local yield stress of the material [2]. The yield stress is usually determined by the simple one-dimensional tension or shear experimental test. The load at which plastic yielding occurs in realistic structures is related to the uni-axial yield stress through the appropriate yield criterion. For most metallic materials the Mises yield criterion is the best choice, and is postulated as:

$$\text{Do določenih vrednosti} \quad \sigma_{ME} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \sigma_y = \sqrt{3} \cdot \tau_y \quad (1),$$

kjer so  $\sigma_1$ ,  $\sigma_2$  in  $\sigma_3$  glavne normalne napetosti,  $\tau_y$  in  $\sigma_y$  meji plastičnosti materiala pri enoosnem strigu in enoosnem nategu ali tlaku [3]. Razmere v dotiku dveh mehanskih elementov so v splošnem zelo zapletene, tako da vpliva na primerjalno napetost v dotikalnem področju  $\sigma_{ME}$  več dejavnikov (sl. 1 [2]). V tem prispevku je obravnavan le vpliv EHD mazanja.

Hertzov kontakt (Hertz contact)  
 $\mu=0; R_f=0; h_o=0; \sigma_r=0; \Delta\vartheta=0$

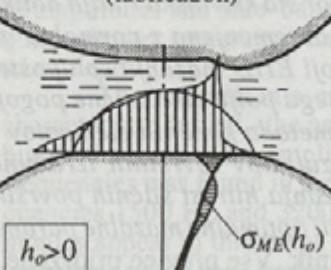


trenje (friction)  
 $\mu>0$

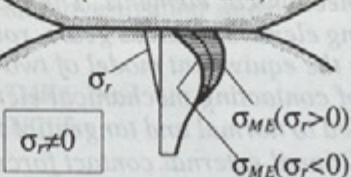
površinska hrapavost (surface roughness)  
 $R_f>0$

where  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  are the principal stresses, and  $\tau_y$  and  $\sigma_y$  are the yield stress of the material in simple shear and simple tension or compression, respectively [3]. The conditions between contacting mechanical elements are complex, and many parameters influence the equivalent stress  $\sigma_{ME}$  appearing in the contact area (Fig. 1 [2]). In this paper the study is restricted to the influence of EHD-lubrication.

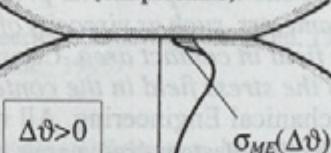
mazanje (lubrication)



zaostale napetosti (residual stresses)



temperatura (temperature)



Sl.1. Vpliv nekaterih veličin na porazdelitev primerjalne napetosti po Misesu  $\sigma_{ME}$  pri dotikalni obremenitvi

Fig.1. The influence of various factors on the distribution of the equivalent Mises stress  $\sigma_{ME}$  due to contact loading

## 1 HERTZOVI DOTIKALNI POGOJI

Po Hertzovi teoriji je porazdelitev tlaka v dotikalnem področju dveh valjev podana z analitično enačbo [3] slika 2:

$$p(x) = \frac{2F_N}{\pi b^2} \sqrt{b^2 - x^2} \quad (2),$$

kjer sta  $F_N$  normalna sila na enoto dolžine nadomestnih valjev in  $b$  polovična širina dotikalne ploskve:

$$b = \sqrt{\frac{8F_N R}{\pi E}} \quad (3),$$

## 1 HERTZ CONTACT CONDITIONS

According to the Hertz contact theory, the distribution of contact pressure in the contact area can be analytically determined by [3] figure 2:

in which  $F_N$  is the normal force per unit length of equivalent cylinders and  $b$  is the half-width of the contact area, which is given as:

kjer sta  $R^*$  nadomestni polmer in  $E^*$  nadomestni modul elastičnosti, definirana kot:

$$R^* = \frac{R_1 R_2}{R_1 + R_2} \quad (4)$$

$$E^* = \frac{2 E_1 E_2}{E_2 (1 - v_1^2) + E_1 (1 - v_2^2)} \quad (5)$$

Z upoštevanjem Coulombovega zakona lahko tangencialno obremenitev  $q(x)$  zaradi drsenja dotikalnih površin določimo kot zmnožek normalne obremenitve  $p(x)$  in koeficiente trenja  $\mu$ :

$$q(x) = \mu \cdot p(x) \quad (6)$$

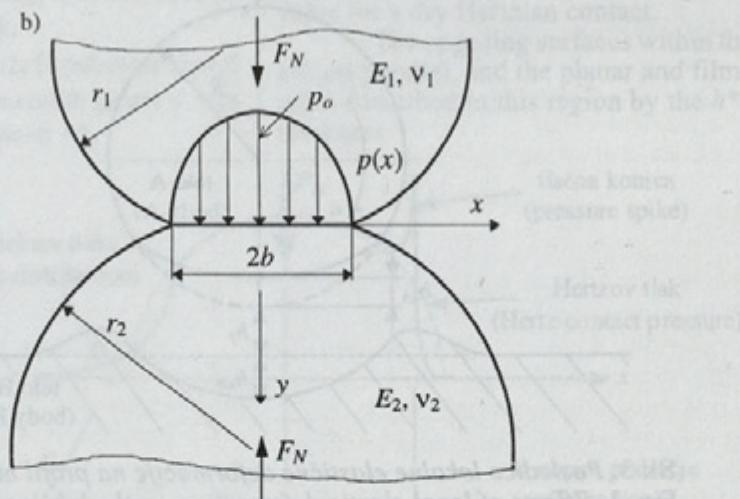
Zaradi lažjega upoštevanja dotikalnih parametrov pri numeričnih preračunih je v splošnem primerno nadomestiti zapleteno geometrijsko obliko mehanskih elementov z nadomestnim modelom dveh valjev (sl. 2). Nadomestna valja imata enaka krivinska polmera kakor obravnavana mehanska elementa v izbrani točki realnega dotika. Omenjena nadomestitev omogoča uporabo Hertzove teorije za določitev dotikalnih obremenitev. Vpliv EHD mazanja na razmere v dotiku je mogoče popisati prek dodatnih zunanjih obremenitev, opisanih v poglavju 2.

where  $R^*$  is the equivalent radius and  $E^*$  is the equivalent Young's modulus, defined as:

Utilising the Coulomb friction law the tangential loading  $q(x)$  - due to sliding of contacting surfaces - can be easily determined from the normal loading  $p(x)$  and coefficient of friction  $\mu$  by:

$$q(x) = \mu \cdot p(x) \quad (6)$$

In order to survey more closely the contact parameters in numerical computations, one should replace the complex geometry of contacting mechanical elements by an equivalent model of two cylinders (Figure 2). The equivalent cylinders have the same curvature radii as the contacting mechanical elements at any chosen point of real contact. This enables the use of the Hertz contact theory for determination of contact conditions as presented above. The influence of EHD-lubrication can also be considered in relation to additional external contact loading, described in chapter 2.



Sl. 2. Nadomestni model dveh valjev  
Fig. 2. Equivalent model of two cylinders

## 2 TEORIJA ELASTOHIDRODINAMIČNEGA MAZANJA

### 2.1 Elastohidrodinamična mazalna plast

Izraz elastohidrodinamična mazalna plast se nanaša na plast mazalnega olja, ki lahko loči dve nasproti ležeči površini pri zgoščenem dotiku. Pri oblikovanju mazalnih plasti pri elastohidrodinamičnem mazanju imajo pomembno vlogo naslednji trije dejavniki:

## 2 ELASTOHYDRODYNAMIC LUBRICATION THEORY

### 2.1 Elastohydrodynamic lubricating film

The term elastohydrodynamic lubricating film refers to the lubricating oil which separates the opposing surfaces of a concentrated contact. The three following effects play a major role in the formation of lubrication films in elastohydrodynamic lubrication:

- oblikovanje hidrodinamične plasti,
- modifikacija geometrije plasti zaradi elastične deformacije,
- sprememba viskoznosti in reologije mazalnega olja v odvisnosti od tlaka.

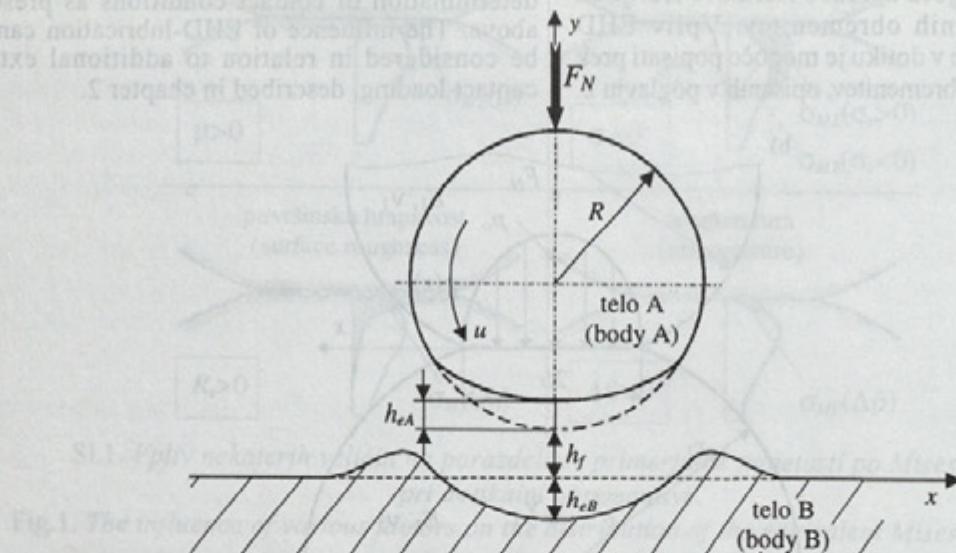
Geometrija površin v Hertzovem dotiku ima na vstopu konvergentno in na izstopu divergentno obliko klina, kar omogoča oblikovanje hidrodinamičnega mazanja. V primerjavi z običajno hidrodinamiko sta tako geometrijska oblika dotika kakor tudi viskoznost maziva funkcija porazdelitve in velikosti hidrodinamičnega tlaka. Zaradi tega je za natančno določevanje geometrijske oblike plasti in viskoznosti najprej treba rešiti Reynoldsovo enačbo.

Za vse materiale, ne glede na njihov modul elastičnosti, se površini v Hertzovem dotiku, pod dovolj veliko obremenitvijo, elastično deformirata. Glavna posledica elastične deformacije na profil mazalne plasti je oblikovanje navidezno vzpoprednega dotika med vhodnim in izhodnim klinastim področjem. Elastičen dotik dveh teles (ravnova površina in valj) prikazuje slika 3. Dotik je prikazan v eni ravnini s dotikalnim radijem za ravno površino neskončno in za valj s polmerom  $R$  [4].

- hydrodynamic film formation,
- modification of the film geometry by elastic deformation,
- transformation of the lubricant's viscosity and rheology under pressure.

The geometry of the interacting surfaces in Hertzian contacts contains converging and diverging wedges, so that some form of hydrodynamic lubrication occurs. Unlike classical hydrodynamics, both the contact geometry and lubricant viscosity are a function of hydrodynamic pressure. It is therefore impossible to specify precisely the film geometry and viscosity before resolving the Reynold's equation.

For all materials, whatever their modulus of elasticity, the surfaces in a Hertzian contact deform elastically (according to loading). The principal effect of elastic deformation on the lubricant film profile is to interpose a central region of quasi-parallel surfaces between the inlet and outlet wedges. This geometric effect is shown in Figure 3, where two bodies - i.e. a flat surface and a roller, in elastic contact - are illustrated. The contact is shown in one plane, and the contact radii are infinite for the flat surface and  $R$  for the roller [4].



Sl. 3. Posledice lokalne elastične deformacije na profil mazalne plasti  
Fig. 3. Effects of local elastic deformation on the lubricant film profile

Profil mazalne plasti v smeri  $y$  je podan po [5]:

The film profile in the  $y$  direction is given by [5]:

$$h = h_f + h_{eA} + h_{eB} \quad (7)$$

kjer so  $h_f$  debelina mazalne plasti in  $h_{eA}$  elastična deformacija telesa A in  $h_{eB}$  elastična deformacija telesa B (sl. 3).

Pri skoraj vseh Hertzovih dotikih geometrijska oblika neskladnih površin v dotiku povzroča močno koncentracijo obremenitve na majhni površini. Ko tekočina loči dve taki površini, se v njej pojavi tlak, nekajkrat večji od tistega, ki

where  $h_f$  is the thickness of the lubricating film, and  $h_{eA}$  and  $h_{eB}$ , are the elastic deformations of the bodies A and B (Fig. 3).

The non-conformal geometry of the contacting surfaces causes an intense concentration of load over a very small area for almost all Hertzian contacts of practical use. When a liquid separates the two surfaces, extreme pressures - many times higher than those encountered in hydrodynamic lubrication - are inevitable. The viscosity of the oil

nastane pri hidrodinamičnem mazanju. Viskoznost olj in večine drugih mazalnih sredstev se s tlakom močno poveča. Odvisnost viskoznosti od tlaka je popisana z enačbo, znano tudi kot Barusov zakon [6] in [7]:

$$\eta_p = \eta_o e^{\alpha p} \quad (8)$$

kjer pomenijo:  $\eta_p$  viskoznost mazalnega sredstva pri trenutnem tlaku in temperaturi,  $\eta_o$  viskoznost pri atmosferskem tlaku in trenutni temperaturi in  $\alpha$  tlačni koeficient viskoznosti.

## 2.2 Porazdelitev tlaka v elastohidrodinamični plasti

Skladno s Hertzovo teorijo je pri statičnih dotikih profil porazdelitve tlaka polkrogelen ali elipsoiden. Tlačno polje se spremeni, ko se ob mazalnem sredstvu (olje), začneta površini ena proti drugi relativno gibati. Relativno gibanje dveh površin v dotiku povzroči ob še nekaterih dodatnih pogojih (oljni klin na vstopu, dovolj velika obremenitev) oblikovanje hidrodinamičnega mazalne plasti, ki nekoliko spremeni porazdelitev tlaka (sl. 4). Največje spremembe se zgodijo na vstopnem in izstopnem področju dotika. V vstopnem področju je hidrodinamični tlak celo nižji od vrednosti, izračunane za suh Hertzov dotik.

V dotiku sta nasproti ležeči površini skoraj vzporedni in ravni. Debelina mazalne plasti v tem področju je osrednja debelina plasti  $h^*$ .

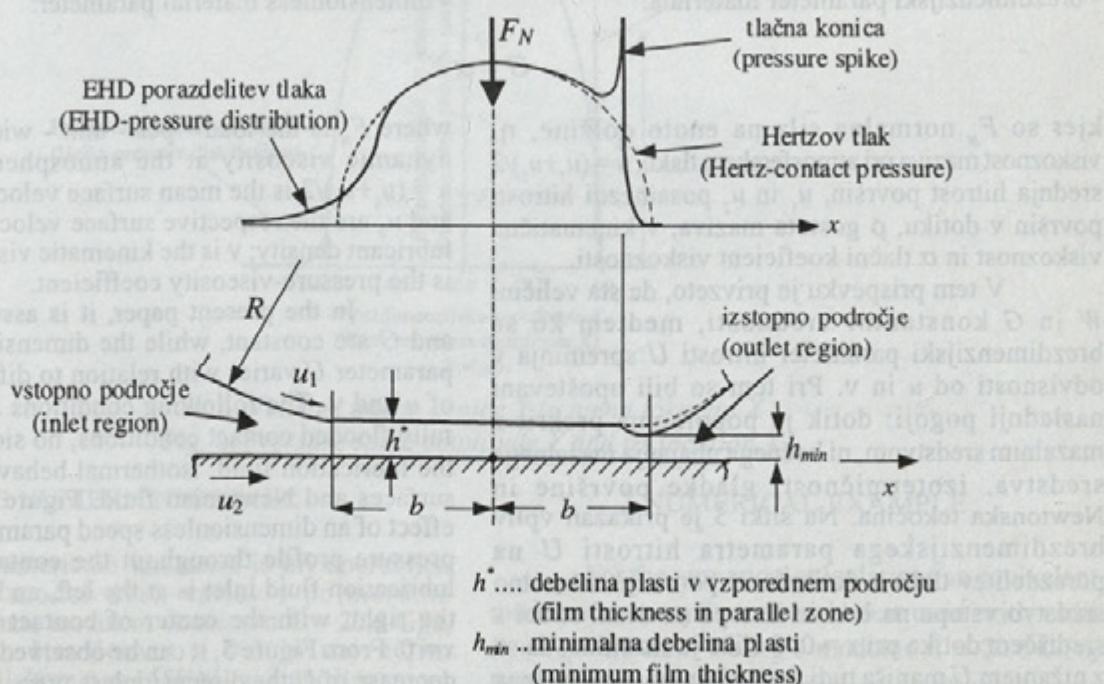
increases dramatically with pressure. The viscosity-pressure relationship is usually described by a mathematically convenient but approximate equation, known as the Barus law [6] and [7]:

where  $\eta_p$  is the lubricant viscosity at current pressure and temperature,  $\eta_o$  is the viscosity at atmospheric pressure, and the current temperature and  $\alpha$  is the pressure-viscosity coefficient.

## 2.2 Pressure distribution in elastohydrodynamic lubricating film

In a static contact, the pressure distribution is hemispherical or ellipsoidal in profile according to the classical Hertzian theory. However, the pressure field will change when the surfaces start moving relative to each other in the presence of a lubricant such as oil. Relative motion between two surfaces causes a hydrodynamic lubricating film to be generated, which to a certain extent modifies the pressure distribution (Fig. 4). The greatest changes to the pressure profile occur at the entry and exit regions of the contact. The combined effect of rolling and of the lubricating film results in a slightly enlarged contact area. Consequently at the entry region, the hydrodynamic pressure is lower than the value for a dry Hertzian contact.

The opposing surfaces within the contact are almost parallel, and the planar and film thickness is often described in this region by the  $h^*$  central film thickness.



Sl. 4. Pogoji pri EHD mazanem premočrtnem dotiku  
Fig. 4. Conditions within an EHD-lubricated line contact

Hitremu dvigu viskoznosti maziva ob vstopu v dotik sledi na izstopu iz dotika strm padec (posledica Hertzove porazdelitve tlaka) do vrednosti, ki jo ima viskoznost pri razmerah v okolini. Da se zagotovi stalen pretok in se izravna nena den padec viskoznosti maziva pri izstopu iz dotika, se oblikuje blizu izstopa zožitev. Najmanjša debelina plasti  $h_{min}$  je pomemben parameter, saj je od njene velikosti odvisna verjetnost dotika vršičkov dveh površin v dotiku. Gradient viskoznosti je na izstopu večji kakor na vstopu v dotik. Malo pred zožitvijo (v delu, ko se mazalno sredstvo bliža najožjemu delu) se v mazivu oblikuje tlačna konica. Takoj, ko mazivo zapusti zožitev, tlak v mazivu hitro pada na raven, ki je nižja kakor pri suhem Hertzovem dotiku. Tlačna konica je navadno višja od največjega Hertzovega tlaka v dotiku. Lega in velikost tlačne konice je močno odvisna od tlačno-viskoznih karakteristik maziva. Navzočnost tlačne konice v izstopnem področju mazanega dotika EHD ima za posledico nastanek velikih strižnih napetosti tik pod površino. Parametri, ki se navadno uporabljajo v mazanih premočrtnih dotikalnih EHD analizah, so [6] do [9]:

- brezdimenzijski parameter obremenitve:

V tem prispevku je privzeto, da sta veličini  $W$  in  $G$  konstantni vrednosti, medtem ko se brezdimenzijski parameter hitrosti  $U$  spreminja v odvisnosti od  $u$  in  $v$ . Pri tem so bili upoštevani naslednji pogoji: dotik je popolnoma prekrit z mazalnim sredstvom, ni bočnega uhajanja mazalnega sredstva, izotermičnost, gladke površine in Newtonska tekočina. Na sliki 5 je prikazan vpliv brezdimenzijskega parametra hitrosti  $U$  na porazdelitev tlaka v dotikalnem področju. Mazalno sredstvo vstopa na levi in izstopa na desni strani s središčem dotika pri  $x = 0$ . Slike je razvidno, da se z nižanjem  $U$  manjša tudi tlačna konica in njena lega se pomika proti izstopu.

The lubricant experiences a precipitous rise in viscosity as it enters contact, followed by an equally sharp decline to the ambient viscosity levels at the exit of the contact (as a result of the Hertz contact pressure). To maintain continuity of flow and compensate for the loss of lubricant viscosity at the contact exit, a constriction is formed close to the exit. The minimum film thickness  $h_{min}$  is an important parameter since it controls the likelihood of asperity interaction between the two surfaces. Viscosity declines even more sharply at the exit than at the entry to the contact. A large pressure peak is generated next to the constriction on the upstream side, and downstream the pressure rapidly declines to less than dry Hertzian values. The peak pressure is usually larger than the maximum Hertzian contact pressure and diminishes as the severity of lubricant starvation increases and dry conditions are approached. The site and the steeples of the pressure peak depend vitally on the lubricant's pressure-viscosity characteristic. The presence of a pressure spike in the outlet region of the EHD-lubricated contact produces large shear stresses, which are localised very close to the surface. The operating parameters normally used in EHD-lubricated line contact analysis are [6] to [9]

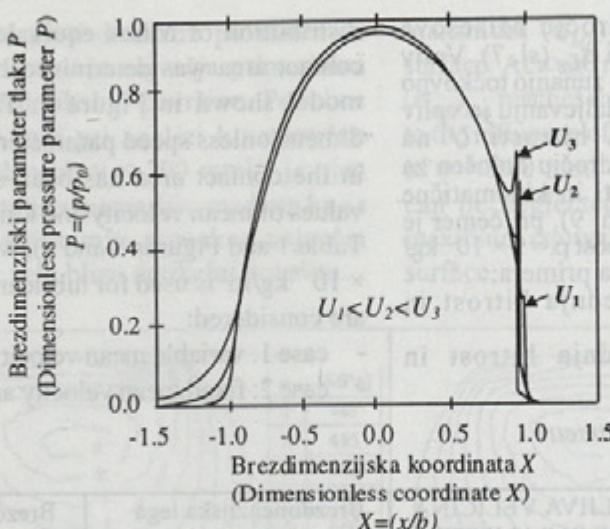
$$W = \frac{F_N}{E^* r^*} \quad (9),$$

$$U = \frac{\eta_o u}{E^* r^*} ; \quad \eta_o = \rho \cdot v \quad (10),$$

$$G = \alpha E^* \quad (11),$$

where  $F_N$  is the load - per - unit - width;  $\eta_o$  is the dynamic viscosity at the atmospheric pressure;  $u = (u_1 + u_2)/2$  is the mean surface velocity; where  $u_1$  and  $u_2$  are the respective surface velocities;  $\rho$  is the lubricant density;  $v$  is the kinematic viscosity; and  $\alpha$  is the pressure-viscosity coefficient.

In the present paper, it is assumed that  $W$  and  $G$  are constant, while the dimensionless speed parameter  $U$  varies with relation to different values of  $u$  and  $v$ . The following conditions are assumed: fully flooded contact conditions, no side leakage of the lubrication fluid, isothermal behaviour, smooth surfaces and Newtonian fluid. Figure 5 shows the effect of an dimensionless speed parameter  $U$  on the pressure profile throughout the contact area. The lubrication fluid inlet is at the left, and the outlet at the right, with the center of contact occurring at  $x = 0$ . From Figure 5, it can be observed that with the decrease of  $U$  the dimensionless pressure spike also decreases, and its location moves towards the outlet.

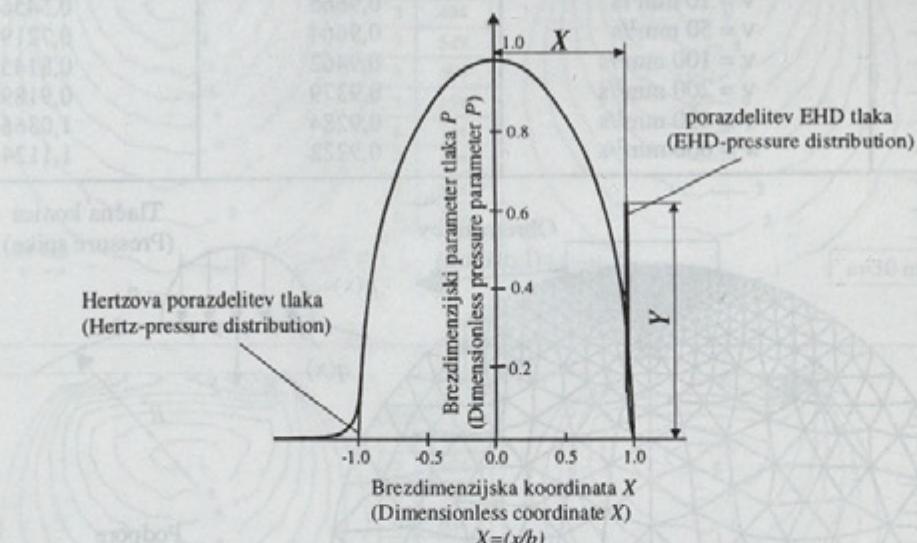


Sl. 5. Porazdelitev brezdimenzijskega parametra tlaka  $P$   
Fig. 5. Dimensionless pressure distribution  $P$

Hamrock je predlagal naslednje enačbe za izračun brezdimenzijske amplitudne tlačne konice  $Y$  in brezdimenzijske lege tlačne konice  $X$  (sl. 6 [10]):

$$Y = 0.267 W^{-0.375} U^{0.174} G^{0.219} \quad (12),$$

$$X = 1 - 2.469 W^{-0.941} U^{0.206} G^{-0.848} \quad (13).$$



Sl. 6. Velikost tlačne konice  $Y$  in njena lokacija  $X$   
Fig. 6. Pressure spike amplitude  $Y$  and its location  $X$

### 3 NUMERIČNI PRIMER

Pri numeričnih izračunih je bil uporabljen nadomestni model dveh valjev s polmeroma  $R_1 = R_2 = 20$  mm, modulom elastičnosti  $E = 20,6$  GPa, Poissonovim številom  $\nu = 0,3$  in največjim površinskim tlakom  $p_o = 1550$  MPa. Z upoštevanjem priporočene vrednosti koeficienteja trenja pri kontaktnih problemih z dobrim mazanjem  $\mu = 0,04$  [11] je bila z uporabo metode končnih elementov (linearna numerična analiza, izotropen material),

Hamrock proposed the following equations for the dimensionless spike amplitude  $Y$  and dimensionless spike location  $X$  (see Figure 6 [10]):

For the numerical calculations an equivalent model of two cylinders has been used with radii  $R_1 = R_2 = 20$  mm, Young's modulus  $E = 20.6$  GPa, Poisson's ratio  $\nu = 0.3$  and maximum contact pressure  $p_o = 1550$  MPa. Taking into account the recommended value for the coefficient of friction in contact problems with good lubrication  $\mu = 0.04$  [11], the

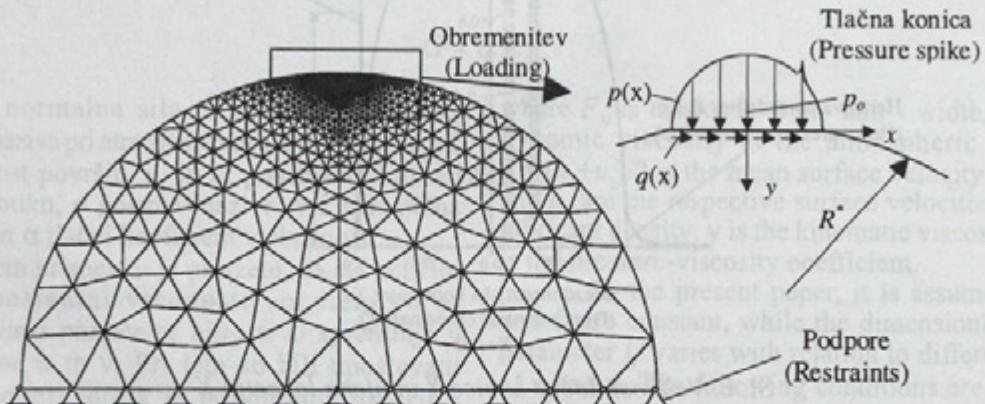
izračunana v dotikalnem področju Misesova primerjalna porazdelitev napetosti  $\sigma_{ME}$  (sl. 7). Vpliv EHD mazanja je popisan z dodatno zunanjim točkovno silo velikosti tlačne konice. V nadaljevanju je vpliv brezdimenzijskega parametra hitrosti  $U$  na napetostno polje v dotikalnem področju določen za različne vrednosti srednje hitrosti in kinematicne viskoznosti (preglednica 1, sl. 8 in 9), pri čemer je za gostoto maziva uporabljenja vrednost  $\rho = 9 \times 10^{-7} \text{ kg/m}^3$ . Pri preračunih upoštevamo dva primera:

- primer 1: spremenljiva srednja hitrost in konstantna viskoznost,
- primer 2: konstantna srednja hitrost in spremenljiva viskoznost.

Preglednica 1. Obremenitveni parametri

Table 1. The loading parameters

OBREMEMENITVENI PRIMER LOAD CASE	SPREMENLJIVA VELIČINA VARIABLE PARAMETER	Brezdimenzijska lega tlačne konice Dimensionless pressure spike location $X$	Brezdimenzijska velikost tlačne konice Dimensionless pressure spike amplitude $Y$
PRIMER 1 CASE 1 $v = 220 \text{ mm}^2/\text{s}$	$u = 1 \text{ m/s}$	0,9610	0,6156
	$u = 5 \text{ m/s}$	0,9462	0,8146
	$u = 10 \text{ m/s}$	0,9379	0,9190
	$u = 20 \text{ m/s}$	0,9284	1,0368
	$u = 30 \text{ m/s}$	0,9215	1,1126
	$u = 50 \text{ m/s}$	0,9135	1,2160
PRIMER 2 CASE 2 $u = 10 \text{ m/s}$	$v = 10 \text{ mm}^2/\text{s}$	0,9665	0,5456
	$v = 50 \text{ mm}^2/\text{s}$	0,9664	0,7219
	$v = 100 \text{ mm}^2/\text{s}$	0,9462	0,8145
	$v = 200 \text{ mm}^2/\text{s}$	0,9379	0,9189
	$v = 400 \text{ mm}^2/\text{s}$	0,9284	1,0366
	$v = 600 \text{ mm}^2/\text{s}$	0,9222	1,1124



Sl. 7. Numerični model za določitev  $\sigma_{ME}$   
Fig. 7. Numerical model for determination of  $\sigma_{ME}$

Rezultati s slik 8 in 9 kažejo, da je porazdelitev primerjalne napetosti  $\sigma_{ME}$  v dotikalnem področju odvisna od srednje hitrosti dotikalnih površin in kinematicne viskoznosti maziva. Pri manjših vrednostih srednje hitrosti ( $u < 10 \text{ m/s}$ ) je njen vpliv razmeroma majhen in ga lahko zanemarimo. S povečevanjem srednje hitrosti se veča tudi največja primerjalna napetost, hkrati pa se njen

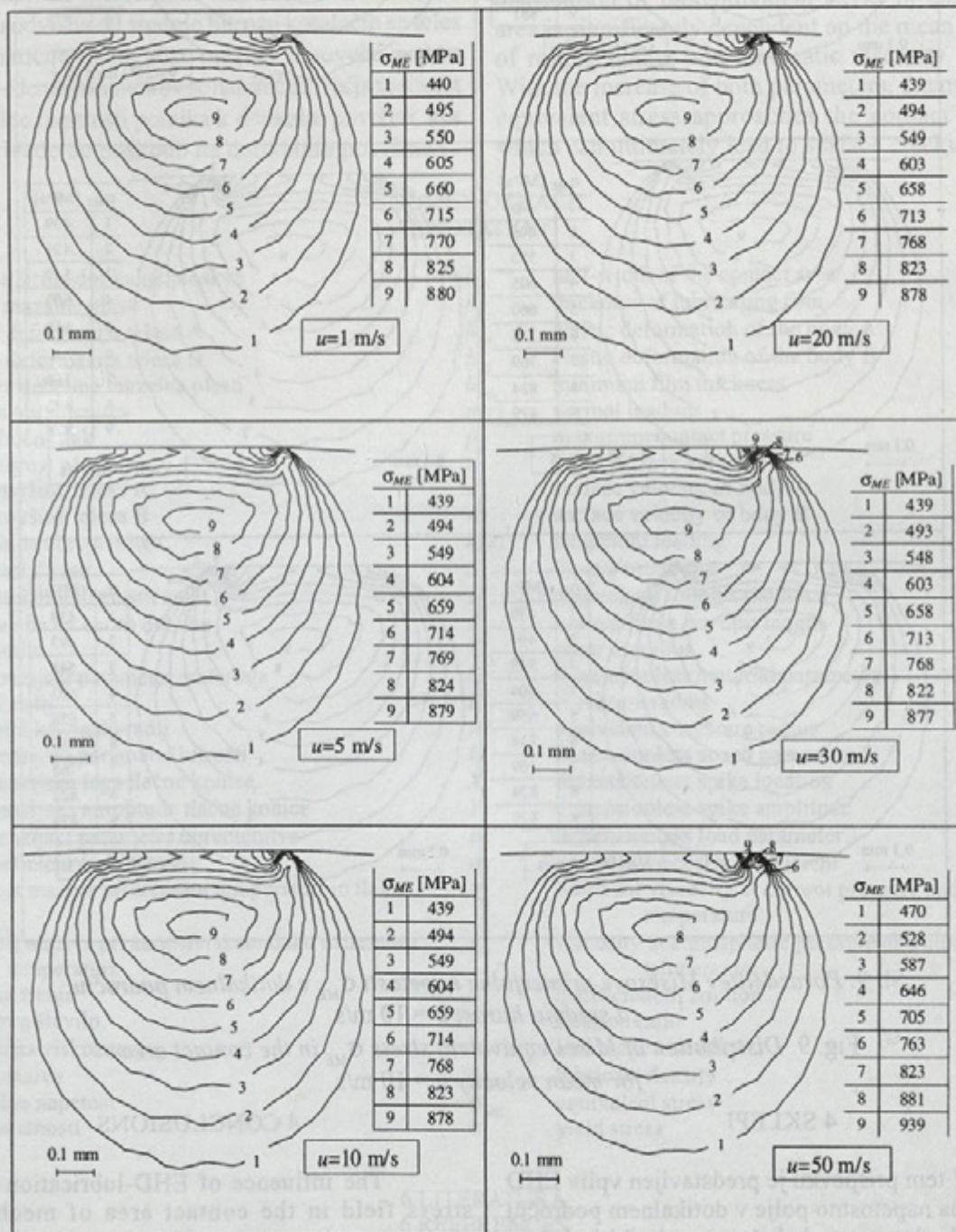
distribution of Mises equivalent stress  $\sigma_{ME}$  in the contact area was determined by the finite element model shown in Figure 7. The influence of the dimensionless speed parameter  $U$  on the stress field in the contact area has been studied for different values of mean velocity and kinematic viscosity (see Table 1 and Figures 8 and 9), where the value  $\rho = 9 \times 10^{-7} \text{ kg/m}^3$  is used for lubricant density. Two cases are considered:

- case 1: variable mean velocity and fixed viscosity,
- case 2: fixed mean velocity and variable viscosity.

The results in Figures 8 and 9 show that the distribution of equivalent stress  $\sigma_{ME}$  in contact area significantly depends on the mean velocity and kinematic viscosity of the lubricant. For the small values of mean velocity ( $u < 10 \text{ m/s}$ ) its influence on the equivalent stress is very small, and can be neglected. With an increase in the mean velocity the maximum equivalent stress also increases and its

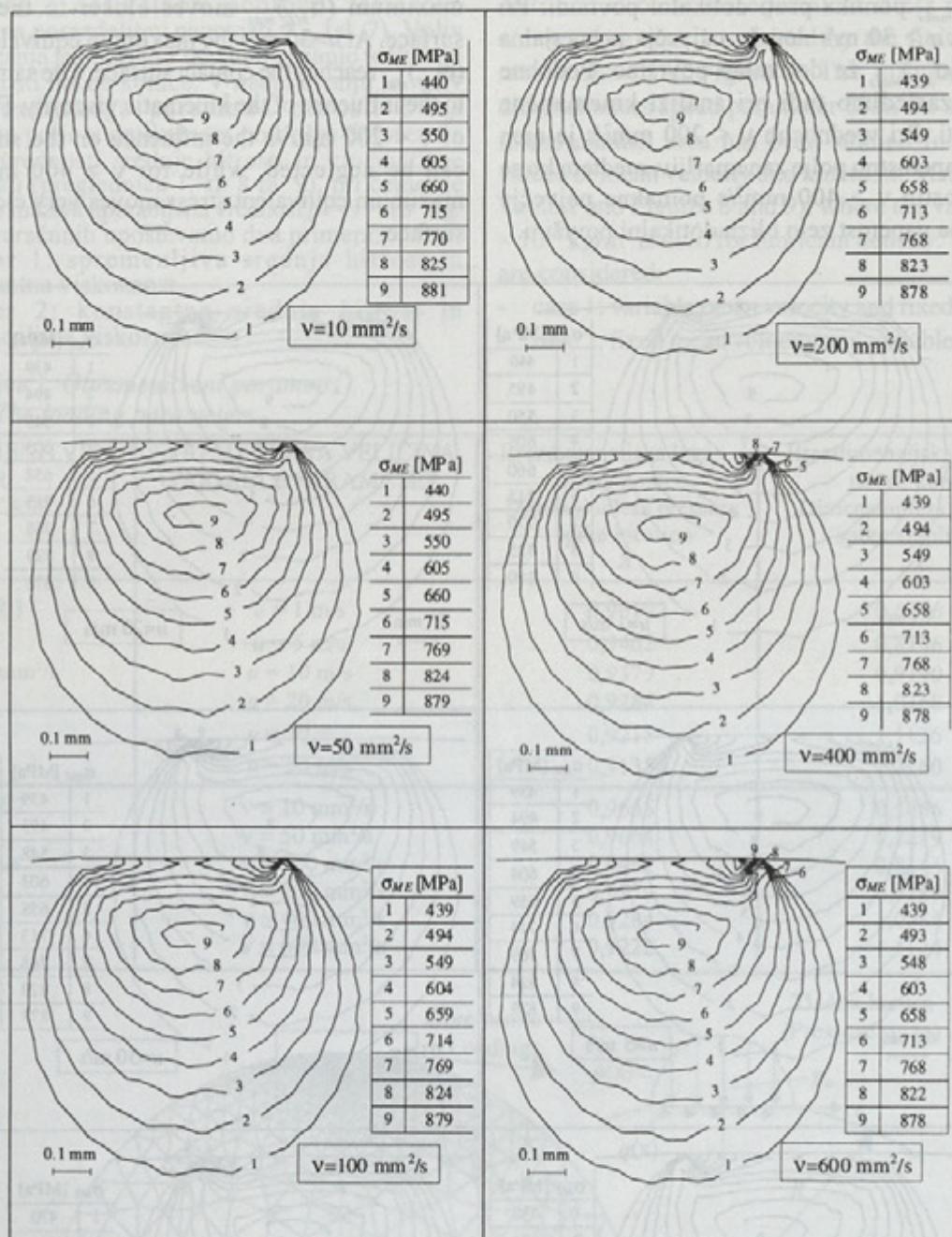
vrh ( $\sigma_{ME,max}$ ) pomika proti dotikalni površini. Pri vrednostih  $u > 30 \text{ m/s}$  doseže največja primerjalna napetost ( $\sigma_{ME,max}$ ) že dotikalno površino. Podobne rezultate zasledimo tudi pri analizi kinematične viskoznosti. Pri vrednostih  $v < 200 \text{ mm}^2/\text{s}$  je njen vpliv na napetostno polje zanemarljiv, medtem ko se pri vrednostih  $v > 400 \text{ mm}^2/\text{s}$  pomakne največja primerjalna napetost zelo blizu dotikalni površini.

maximum ( $\sigma_{ME,max}$ ) moves closer to the contact surface. At  $u>30 \text{ m/s}$  the maximum equivalent stress ( $\sigma_{ME,max}$ ) reaches the contact surface. The same applies to the influence of the kinematic viscosity. For values of  $v < 200 \text{ mm}^2/\text{s}$  the influence on the stress field can be neglected, while for  $v > 400 \text{ mm}^2/\text{s}$  the maximum equivalent stress moves very close to the surface.



Sl. 8. Porazdelitev Misesove primerjalne napetosti  $\sigma_{ME}$  v dotikalnem področju za kinematično viskoznost  $v = 220 \text{ mm}^2/\text{s}$

Fig. 8. Distribution of Mises equivalent stress  $\sigma_{ME}$  in the contact area for kinematic viscosity  $v = 220 \text{ mm}^2/\text{s}$



Sl. 9. Porazdelitev Misesove primerjalne napetosti  $\sigma_{ME}$  v dotikalnem področju za srednjo hitrost  $u = 10 \text{ m/s}$

Fig. 9. Distribution of Mises equivalent stress  $\sigma_{ME}$  in the contact area for mean velocity  $u = 10 \text{ m/s}$

#### 4 SKLEPI

V tem prispevku je predstavljen vpliv EHD mazanja na napetostno polje v dotikalnem področju mehanskih elementov, kakršni so zobniki in ležaji. Pri numeričnih preračunih je uporabljen nadomestni model dveh valjev, ki sta izpostavljenata normalni in tangencialni obremenitvi zaradi Hertzove teorije in dodatni zunanji obremenitvi kot posledici EHD mazanja.

#### 4 CONCLUSIONS

The influence of EHD-lubrication on the stress field in the contact area of mechanical elements, such as gears and bearings, is presented in the paper. An equivalent contact model of two contacting cylinders is used for computational analyses. The equivalent cylinders are subjected to normal and tangential Hertzian loading conditions with the addition of extra contact loading due to EHD-lubrication.

Za določitev napetostnega polja v dotikalnem področju je bila pri numeričnih analizah uporabljena metoda končnih elementov. Vpliv EHD mazanja na napetostno polje je analiziran za različne vrednosti srednje hitrosti dotikalnih površin in viskoznosti maziva, pri čemer je napetostno polje v dotikalnem področju valja izraženo s primerjalno napetostjo po Misesu. Rezultati numeričnih izračunov kažejo, da je porazdelitev primerjalne napetosti v dotikalnem področju odvisna od srednje hitrosti kotalečih se teles in kinematicke viskoznosti maziva. S povečevanjem obeh navedenih parametrov se namreč največja vrednost primerjalne napetosti pomika k dotikalni površini, kar lahko privede do poškodb na dotikalnih površinah.

The finite element method was used for computational analyses of the resulting stress field in the contact area. The influence of EHD-lubrication on the stress field in the contact area of the cylinder has been studied for different values of mean velocity of the contacting bodies and the viscosity of lubricant, where the stress field in the contact area has been represented by the Mises equivalent stress. The results of the numerical analyses show that the distribution of the equivalent stress in the contact area is significantly dependent on the mean velocity of rolling bodies and kinematic viscosity of fluid. With the increase of both parameters, the maximum equivalent stress approaches the contact surface, which can ultimately lead to surface cracking.

## 5 POMEN OZNAČB 5 NOTATION

polovična širina dotikalne ploskve	$b$	half-width of the contact area
debelina mazalne plasti	$h_f$	thickness of lubricating film
elastična deformacija telesa A	$h_{eA}$	elastic deformation of the body A
elastična deformacija telesa B	$h_{eB}$	elastic deformation of the body B
najmanjša debelina mazalne plasti	$h_{min}$	minimum film thickness
normalna obremenitev	$p(x)$	normal loading
največji bočni tlak	$p_o$	maximum contact pressure
srednja hitrost površin	$u$	mean surface velocity
hitrost površine telesa A	$u_1$	surface velocity of body A
hitrost površine telesa B	$u_2$	surface velocity of body B
tangencialna obremenitev	$q(x)$	tangential loading
modul elastičnosti	$E$	Young's modulus
nadomestni modul elastičnosti	$E^*$	equivalent Young's modulus
normalna sila na enoto dolžine	$F_N$	normal force per unit length
strižni modul	$G$	shear modulus
brezdimenzijski parameter materiala	$G$	dimensionless material parameter
krivinski radij	$R$	curvature radius
nadomestni krivinski radij	$R^*$	equivalent curvature radius
brezdimenzijski parameter hitrosti	$U$	dimensionless speed parameter
brezdimenzijska lega tlačne konice	$X$	dimensionless spike location
brezdimenzijska amplituda tlačne konice	$Y$	dimensionless spike amplitude
brezdimenzijski parameter obremenitve	$W$	dimensionless load parameter
tlačni koeficient viskoznosti	$\alpha$	pressure-viscosity coefficient
viskoznost maziva pri trenutni temperaturi in tlaku	$\eta_p$	lubricant viscosity at current pressure and temperature
viskoznost maziva pri atmosferskem tlaku in trenutni temperaturi	$\eta_o$	viscosity at atmospheric pressure and current temperature
koeficient trenja	$\mu$	coefficient of friction
Poissonovo število	$v$	Poisson ratio
kinematicna viskoznost	$\nu$	kinematic viscosity
gostota maziva	$\rho$	lubricant density
primerjalna napetost	$\sigma_{ME}$	equivalent stress
meja plastičnosti	$\sigma_y$	yield stress

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