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Economic Design of Control Charts

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Control charts are widely used in industry for monitoring and controlling manufacturing processes. They should be designed economically in order to achieve minimum quality control costs. In this paper, an economic design of Shewhart control charts for process mean is proposed that takes into account various parameters. Standards for sample size within statistical process control do not exist due to high diversity of modern production. In the proposed economic model process-mean shift is assumed as random variable. This is a better approximation of the real world, than the models that assume process-mean shift as a constant value. Probability density function is used for description of process-mean shift. The optimum sample size is computed on base of loss function, regarding to constraints of particular production process. The comparison of optimum sample sizes assuming process-mean shift as a constant value versus random variable is presented.

Keywords: control charts, quality control, economic design, statistical process control

0 INTRODUCTION

A quality of a product is usually understood as a capability of fulfilling needs of a customer. A manufacturer desires to produce such products that match the desired specification and also fulfil customer needs. Hence, quality control is an essential part of the manufacturing process. In modern production the product's specifications are not controlled directly. The quality control is carried out by controlling the manufacturing process. A process quality is usually deducted from the manufactured products. In this case, not all of the products can be inspected directly, but only statistically. The quality control is therefore called Statistical Process Control (SPC). A control chart is a primary tool for SPC. A purpose of the control chart is to detect possible changes of the process, and inform the process operator about it. Many types of control charts have been developed in the past. Shewhart control charts, Cumulative sum (*Cusum*) control chart, Exponential weighted moving average (*EWMA*) control chart, and others are presented elsewhere for instance in [1]. Control chart is a quality control technique. Some quality improvement techniques are discussed in [2].

Shewhart control charts are widely used in industry. The design of the control charts has economic consequences since the cost of sampling and testing, the investigating out-of-control signals and possibly correcting probable causes, as well as costs of allowing nonconforming products to reach the customer are all affected by choice of the control charts parameters [3]. The most important parameter that affects the cost of quality control and indirectly the cost of production is sample size. In literature, description of standards that deal with sample size for acceptance sampling can be found in [3]. Due to their generality of use, these standards are very robust. The standards for SPC, which deal with sample size, do not exist. The reason is a standard's uselessness due to its generality and high diversity of modern production. It is not the same if low- or high-priced products are manufactured. Due to diversity of quality control costs, optimum sample size should be calculated for every production process respectively. In industrial environment, no statistical expertise is available. Normally, a sample size is defined by a rule of thumb, lacking the optimality in economical sense. Due to fierce competition on the global market, an economic control charts design can be a critical issue for a manufacturing enterprise.

Some approaches to the economic control charts design are given in literature [4] to [7]. In all of these approaches, the process-mean shift (*PMS*) is assumed constant. In [4], Duncan's loss-cost function is used, which is originally proposed in [8]. In [5], simplified theoretical backgrounds and directions of economic design of control charts are proposed. In [6], economic design of

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control charts of cumulative count of conforming products is presented. In [7], an economic design of control charts is presented for variable sampling size and sampling interval. In approaches [4] to [7], a *PMS* parameter is supposed to occupy an exact constant value. *PMS* is actually not known in advance. If constant value for *PMS* is assumed, the economic model is valid only for that *PMS*. *PMS* directly affects the optimum sample size calculation.

In this paper, an economic design of Shewhart control chart for the mean value of the process ($\overline{x}$ chart) is discussed. The approach does not assume an exact value for *PMS*. We show that by finding its probability density function of *PMS*, we can compute the optimum sample size (n^*).

In Section 1, some theoretical backgrounds and parameters of the Shewhart control chart are discussed. In Section 2, the sampling policy and the economic model are proposed. In Section 3, the optimisation is presented in form of optimum sample size as a function of the *PMS* parameter. These curves are created for different economic parameters (costs). In Section 4, discussion and conclusions are presented.

1 BACKGROUNDS OF SHEWHART CONTROL CHART

Shewhart control chart method is about 80 years old. It is a method for statistical process control, in sense of process-mean and process variability supervision. The statistical parameters are deduced from a statistical sample. The accuracy depends on the sample size (n). In this paper, we are concentrating on $\overline{x}$ control chart for *PMS* detection. The control chart is a kind of hypothesis testing hence the process is in a state of statistical control. The hypothesis testing is done with sample statistics. The sample statistics is assumed to follow a normal distribution with mean (m) and sample standard deviation ($\sigma_{<x>}$). A sample standard deviation depends on standard deviation of population (σ) and sample size (n). The null hypothesis states that the mean of a measured process variable has a desired value. The alternative hypothesis is that this mean value may be changed by $k\sigma$. That means that the new mean value is $\mu+k\sigma$.

The centre line of the control chart is kept at the mean of the process (μ). The upper control limit (UCL) and the lower control limit (LCL) are:

$$UCL = \mu + L\sigma/\sqrt{n} \qquad (1)$$

$$LCL = \mu - L\sigma/\sqrt{n} \qquad (2)$$

where L is the control limit coefficient (usually 3 is used).

The interval of normal distribution between $\mu-3\sigma$ and $\mu+3\sigma$ covers 99.73% of the entire population. Statistically, this means that 99.73% of sample values are supposed to fall between the control limits. In spite of no change in a process, 0.27% of the sample values are to exceed the control limits. This is treated as type I error (false-positive). The probability of type I error (α) is defined as:

$$\alpha = 2\Phi(-L) \qquad (3)$$

where $\Phi(.)$ is a cumulative distribution function of a standard normal random variable. Type I error, which means a false alarm, has a geometric probability distribution. Its expected value is ARL_0. A type I error, therefore, appears after every ARL_0 samples on average. ARL_0 is calculated as follows:

$$ARL_0 = \frac{1}{\alpha} \ . \qquad (4)$$

When $L = 3$, a false alarm appears about every 370 samples on average.

The type II error denotes a false-negative detection (process change was not detected). The probability of type II error (β) is defined as:

$$\beta(k,n,L) = \\ = \Phi(L - k/\sqrt{n}) - \Phi(-L - k/\sqrt{n}) \ . \qquad (5)$$

The probability of type II error (β) depends on the shift of the process mean value, denoted by $k\sigma$. It also depends on L and n. When the process-mean changes, it is not detected immediately. The shift is detected on average after a number of samples, which is denoted by ARL_1 (Average Run Length). ARL_1 is the mean value of geometric distribution of shift detection, when the process is out of control and it is calculated as follows:

$$ARL_I(k,n,L)=\frac{1}{1-\beta(k,n,L)}\ . \qquad (6)$$

The chart of β and ARL_I can be found elsewhere, for instance in [9]. From these charts one can conclude that sample size (n) has a significant impact on the efficiency of *PMS* detection. Greater n enables faster detection of *PMS*. Process capability ratio (C_p) is defined elsewhere [3] as:

$$C_p=\frac{USL-LSL}{6\sigma}\ . \qquad (7)$$

USL and *LSL* are specification limits. The control limits are allowed values for quality characteristic. C_p will be used in the following sections.

2 PRESENTATION OF SAMPLING POLICY AND ECONOMIC MODEL

2.1 Background of Economic Model

Economic model is a framework for the economic design of control charts. It is closely linked to a sampling policy, which determines a method of sampling. A sampling policy depends on the production type. Economic design of control charts is carried out on the basis of a loss-cost function optimisation. When the sampling policy changes, the results of optimisation are not valid anymore.

The issue of quality control prevents to produce nonconforming products. A fraction of nonconforming products, which are manufactured on a particular work system, has a very important impact on design. The average fraction of nonconforming products is described with probability of nonconforming product (P_{nc}), which depends on specification limits (*USL*, *LSL*), μ, σ and *PMS*. *PMS* is expressed by the multiplier of standard deviation (k). A normal probability density function for a process nature is assumed. The average fraction of nonconforming products can be expressed as follows:

$$P_{nc}(USL,LSL,\mu,\sigma,k)=\frac{1}{2}+ +\Phi\left[\frac{LSL-(\mu+k)}{\sigma}\right]+\frac{1}{2}-\Phi\left[\frac{USL-(\mu+k)}{\sigma}\right]. \qquad (8)$$

The value of Eq.(8) defines also the probability of product's nonconformity. The same value can be used for the fraction of nonconforming products in population. The value of this expression is always greater than 0. That follows from the assumed normal nature of the observed production process. The expression is minimum, when the process mean is in the middle of the tolerance interval. When the process mean changes, P_{nc} increases.

2.2 Sampling Policy

For the proposed economic model, the following sampling policy is assumed.

Suppose we have a discrete production process, which is not self-correcting. The production process is assumed as a series of independent cycles over time. Each cycle begins with the production process in the in-control state and continues until the process changes and its change is detected and identified. A statistical process control is carried out periodically lot-by-lot, where N_l is a lot size. From every lot a sample of size n is taken, so that the last n products of the lot form a sample (Fig. 1). When a false alarm appears, the production is halted until the alarm identification is done. After a false alarm identification, the production continues. When an alarm is identified as a real alarm, which means that the process has been changed, a process adjustment is needed. Meanwhile, the process is out of control and an increased fraction of nonconforming products is manufactured. After the adjustment, during which the process is returned to the in-control state, the new cycle begins. One cycle of the production run in sense of the assumed sampling policy is presented in Fig. 1.

The sampling policy is determined by its lot size, sample size and inspection strategy. The lot size indirectly defines the sampling frequency (v_s), which means how often a statistical sample is taken.

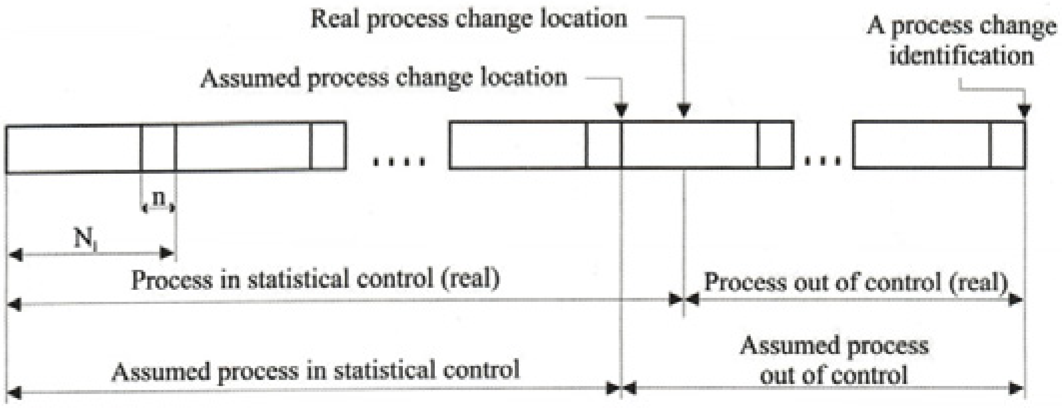

Fig. 1. *Diagram of production run in sense of the assumed sampling policy*

A sampling frequency is calculated using expression:

$$\nu_s = \frac{1}{N_1} \quad . \tag{9}$$

The economic model of control chart is related to the sampling policy. It represents a framework for the mathematical formulation of total quality costs. Using our economic model, an optimum sampling size and sampling frequency for a specific production can be found. When a process changes, this change is located after ARL_1 lots of manufactured products. Due to different batch sizes, variable processing times and appearance of different technical-organisational malfunctions in an industrial environment, economic design of control charts is not based on time, i.e., on the number of manufactured products.

From every lot, a sample is formed that is located at the end of the lot. In our economic model the most inconvenient case of a process change is assumed. We assume that the process changes immediately after the statistical sample has been formed. This would produce the maximum possible fraction of nonconforming products.

The economic designs reported in literature assume *PMS* to be a constant value. In our approach, *PMS* is assumed to be a random value, described by probability density function (PDF_k). The shape of probability function PDF_k depends on the nature of *PMS*. *PMS* is expressed by k in units of σ.

If we assume PDF_k to be a symmetric bimodal probability density function with mode at ($\pm M_k$) and standard deviation (σ_k), PDF_k is described with the following function:

$$\begin{aligned} PDF_k &= PDF_k(k \mid M_k, \sigma_k) = \\ &= \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi}\sigma_k} \cdot e^{-\frac{(k-M_k)^2}{2\sigma_k^2}} + \\ &+ \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi}\sigma_k} \cdot e^{-\frac{(k+M_k)^2}{2\sigma_k^2}} \quad . \end{aligned} \tag{10}$$

The states of the process, where $k = 0$ (process in statistical control), are not assumed as *PMS*s.

2.3 Economic Model

The goal of economic design of control charts is to obtain the minimum costs of the production process. With quality control we want to achieve that only conforming products reach customers. Within the proposed model, the costs related to quality control should be minimized. In the proposed economic model, the following costs are included: (1) costs of quality inspection, (2) costs of false alarm, (3) costs of manufacturing nonconforming products and (4) costs of location and repairing an assignable cause of the process. The economic model is the

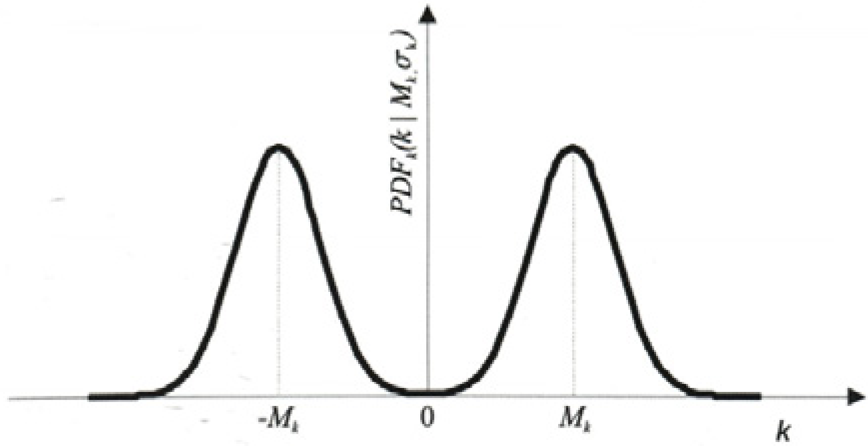

Fig. 2. *Probability density function of PMSs in units of process standard deviation*

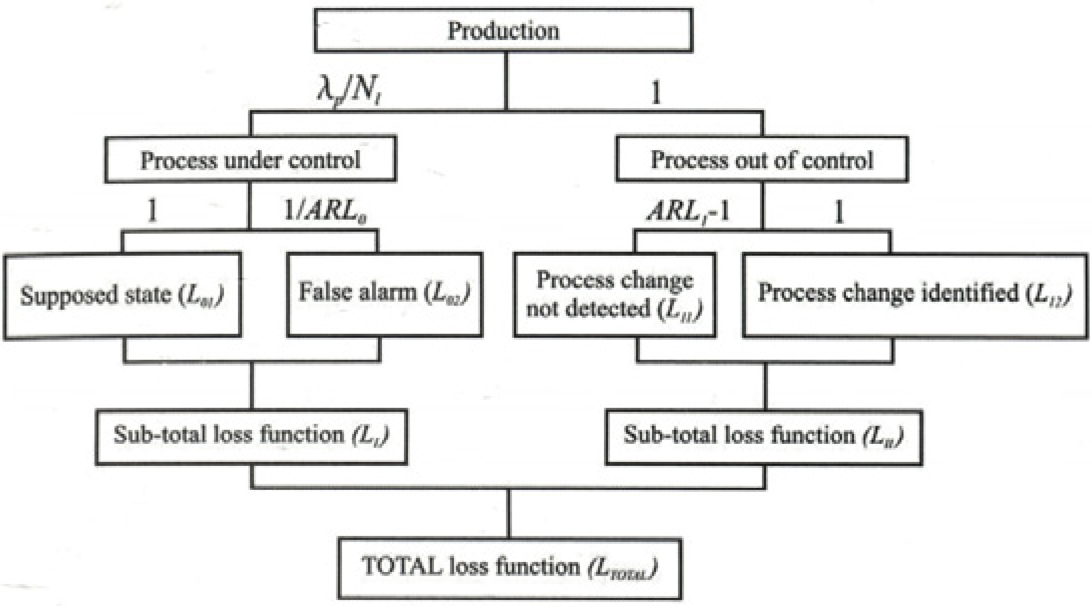

Fig. 3. *Interpretation of the loss functions*

framework for a loss-cost function derivation. Loss-cost function is a powerful tool, due to its simplicity of problem representation in the engineering area. Loss-cost functions are presented in [10]. For the case of simplicity of notation the loss-cost function will be named *loss function*.

In the statistical process control, the following states are possible (Fig. 3). These states are linked with partial loss functions (L_{01}, L_{02}, L_{11}, L_{12}), which are grouped in 2 of separated sub-total loss functions (L_I and L_{II}). The sub-total loss functions define 2 states: process is under control and process is out of control.

For the loss function derivation, the following cost constants are needed:

C_{cl} cost per unit (product) controlled,
C_{nc} cost per nonconforming unit,
C_{fa} cost per false alarm,
C_{rac} cost to locate and repair an assignable cause of the process malfunction.

Loss function design is presented in Fig. 3. Loss function is designed regarding 1 cycle of production run, presented in Fig. 1. In 1 cycle of production run the losses of two states are considered. The states are named Process under control and Process out of control. Their contributions are weighted with λ_p/N_l and 1 respectively. The sub-total and partial loss functions are carried out analogously.

Partial loss functions are as follows:

1. If the process is under control (k=0) and a sample mean value is between the control limits, we have

$$L_{01}(n,k,USL,LSL,\mu,\sigma,N_l,L)=nC_{cl}+N_lP_{nc}(USL,LSL,\mu,\sigma,k)C_{nc}\,. \tag{11}$$

The partial loss function L_{01} assumes the state of the process under control and a sample mean between the control limits. That means that PMS is 0. L_{01} defines a loss of production and the cost of control per lot. The first summation term defines a loss due to control costs and the second one defines a loss due to the fraction of nonconforming parts.

2. If the process is under control (k=0) and a sample mean value exceeds the control limits (Type I error - false alarm), we have

$$L_{02}(n,k,USL,LSL,\mu,\sigma,N_l,L)=nC_{cl}+N_lP_{nc}(USL,LSL,\mu,\sigma,k)C_{nc}+C_{fa}\,. \tag{12}$$

The partial loss function L_{02} assumes the state of the process that is under control and a sample mean falls beyond the control limits. In this case, a false alarm appears. C_{fa} defines costs of false alarm identification.

3. If the process is out of control (k≠0) and a sample mean value falls between the control limits (Type II error) then

$$L_{11}(n,k,USL,LSL,\mu,\sigma,N_l,L)==nC_{cl}+N_lP_{nc}(USL,LSL,\mu,\sigma,k)C_{nc}\,. \tag{13}$$

The partial loss function L_{11} assumes the state of the process, which is out of control and a sample mean lies between the control limits. *PMS*, which is greater than 0, has not been detected yet. In this state, the fraction of nonconforming parts is increased. This situation contributes the most to the overall loss.

4. If the process is out of control (k≠0) and a sample mean value is beyond the control limits, we have

$$L_{12}(n,k,USL,LSL,\mu,\sigma,N_l,L)==nC_{cl}+C_{rac}+N_lP_{nc}(USL,LSL,\mu,\sigma,k)C_{nc}\,. \tag{14}$$

The partial loss function L_{12} assumes the state of the process that is out of control and a sample mean beyond the control limits. A real alarm is detected. C_{rac} defines costs due to *PMS* identification of the assignable cause of *PMS* and its removal.

From two aforementioned process states and belonging partial loss functions, two sub-total loss functions are formed. First of them is used for lots when a process is under control. The second is used for lots when a process is assumed out of control. From ARL_1 it is possible to calculate how many samples have to be inspected until a change in the process is identified (if k≠0).

The first sub-total loss function is:

$$L_I(n,k,USL,LSL,\mu,\sigma,N_l,L)==nC_{cl}+\frac{C_{fa}}{ARL_0}+N_lP_{nc}(USL,LSL,\mu,\sigma,k)C_{nc}\,. \tag{15}$$

The second sub-total loss function is:

$$L_{II}(n,k,USL,LSL,\mu,\sigma,N_l,L)==ARL_1(k,n,L)[nC_{cl}+N_lP_{nc}(USL,LSL,\mu,\sigma,k)C_{nc}]+C_{rac}\,. \tag{16}$$

The sub-total loss functions correspond to the assumed process states, described in Fig. 1. The first sub-total loss function expresses costs per inspected lot. The second sub-total loss function expresses cost per out-of control process state, considering quality inspection and a fraction of nonconforming products that are manufactured in this state. The total loss function can be deducted from Fig. 3. For *PMS* the probability density function PDF_k is used. The total loss function is computed per manufactured product. This offers a possibility to compare losses of different lot sizes. There is additional information that is needed for the economic design of control charts. It is the process failure rate (λ_p) that denotes after how many products, manufactured by the observed work system, the process changes on the average. λ_p should be found by a long-term observation of the process.

A total loss function Eq.(17) consists of numerator and denominator. A numerator is a total loss during one cycle of the production run (Fig. 1). A denominator is a number of manufactured products in one cycle of the production run.

$$L_T(\lambda_p, n, USL, LSL, \mu, \sigma, N_l, L, \mu_k, \sigma_k) = \\ = \frac{\frac{\lambda_p}{N_l}\cdot L_I(.) + \int_{-\infty}^{0} L_{II}(.)\cdot PDF_k(.)\cdot dk + \int_{0}^{\infty} L_{II}(.)\cdot PDF_k(.)\cdot dk}{\left(\frac{\lambda_p}{N_l} + \int_{-\infty}^{0} ARL_1(.)\cdot PDF_k(.)\cdot dk + \int_{0}^{\infty} ARL_1(.)\cdot PDF_k(.)\cdot dk\right)\cdot N_l} \quad (17)$$

Eq. (17) does not have k among its arguments. *PMS*-multiplier (k) is handled by PDF_k. Since the zero case (k=0) is not considered as *PMS*, there are two integrals needed. Due to symmetry, (17) can be simplified to

$$L_T(\lambda_p, n, USL, LSL, \mu, \sigma, N_l, L, \mu_k, \sigma_k) = \\ = \frac{\frac{\lambda_p}{N_l}\cdot L_I(.) + 2\cdot\int_{0}^{\infty} L_{II}(.)\cdot PDF_k(k, M_k, \sigma_k)\cdot dk}{\left(\frac{\lambda_p}{N_l} + 2\cdot\int_{0}^{\infty} ARL_1(.)\cdot PDF_k(k, \mu_k, \sigma_k)\cdot dk\right)\cdot N_l} \quad (18)$$

Since the loss is expressed per manufactured product, it is possible to compare losses of different lot sizes and for each of them it is possible to find the optimum sample size (n^*). By changing the lot-size, within the proposed sampling policy, the sampling frequency is changed as well.

3 SIMULATION STUDY

The optimum sample size is the sample size that minimizes L_T. All of the discussed parameters have impact on the optimum sample size. In Fig. 4 we can see loss function (L_T) as a function of the sample size for different modes (M_k). The following constants were assumed: C_{cl}=10, C_{nc}=100, C_{fa}=C_{rac}=200, λ_p=500, C_p=1 (process capability), N_l=100, σ=0.2.

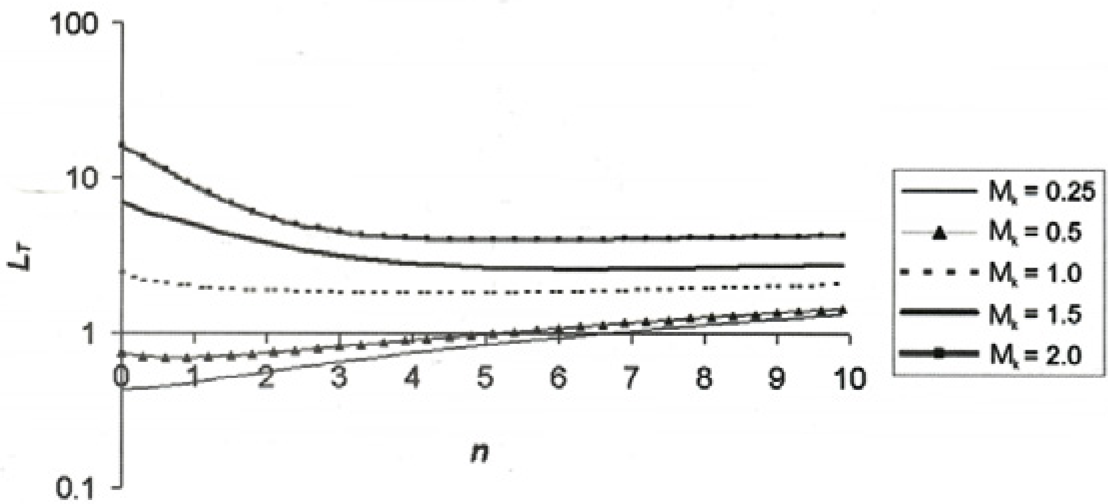

Fig. 4. *Loss function (L_T) vs. the sample size (n), where C_{cl}=10, C_{nc}=100, C_{fa}=C_{rac}=200, λ_p=500, C_p=1, N_l=100, σ_k=0.2; the ordinate is in a logarithmic scale*

From these curves the optimum sample size for a chosen mode M_k can be found. The curve (M_k=0.25) is monotonically increasing. In this case, we can conclude that control of the process is economically unacceptable since the optimum sample size is 0. The curve (M_k=0.5) has the minimum at approximately $n = 1$, but it is not explicitly significant. If greater sample size is chosen, error gets smaller. Curves, with larger M_k, have a significantly expressed optimum sample size. Using expression L_T, the curves for the optimum sample size in dependency of M_k were plotted for various parameters. The optimum sample size and lot size can be calculated numerically, using L_T.

The lot size defines the sampling frequency. In the presented case, the sampling frequency is 1 sample per 100 manufactured products, which means v_s=0.01. The optimum sample size, where lot size is 100, can seen from diagrams, presented in Figs. 5a, b and 5c. By changing C_{fa} and C_{rac}, the optimum sample size curves change. When C_{fa} and C_{rac} increase, the optimum sample sizes lowers and peaks of the optimum sample size curves move to the right. C_{fa} and C_{rac} have impact on the optimum sample size, where M_k is small. For large values of M_k the cost constants C_{fa} and C_{rac} have no significant impact.

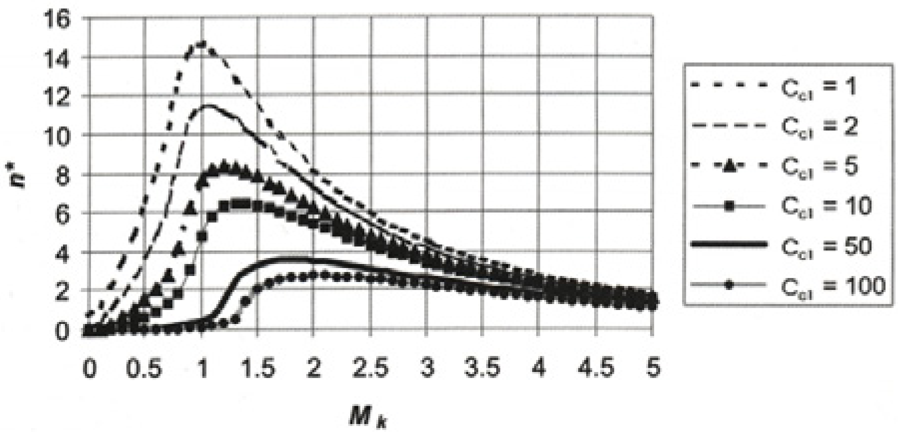

Fig. 5a. *Optimum sample size (n*) vs. mode M_k for different C_{cl}, where, C_{nc}=100, C_{fa}=C_{rac}=100, λ_p=500, C_p=1, N_l=100, σ_k=0.2*

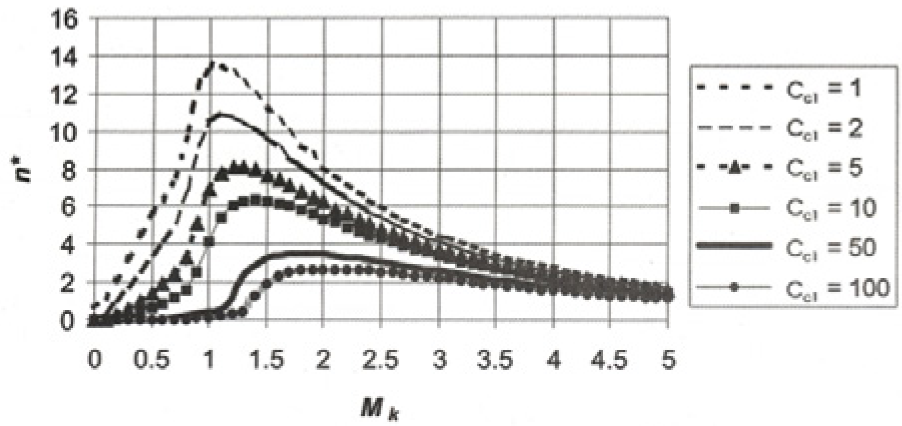

Fig. 5b. *Optimum sample size (n*) vs. mode M_k for different C_{cl}, where, C_{nc}=100,C_{fa}=C_{rac}=200, λ_p=500, C_p=1, N_l=100, σ_k=0.2*

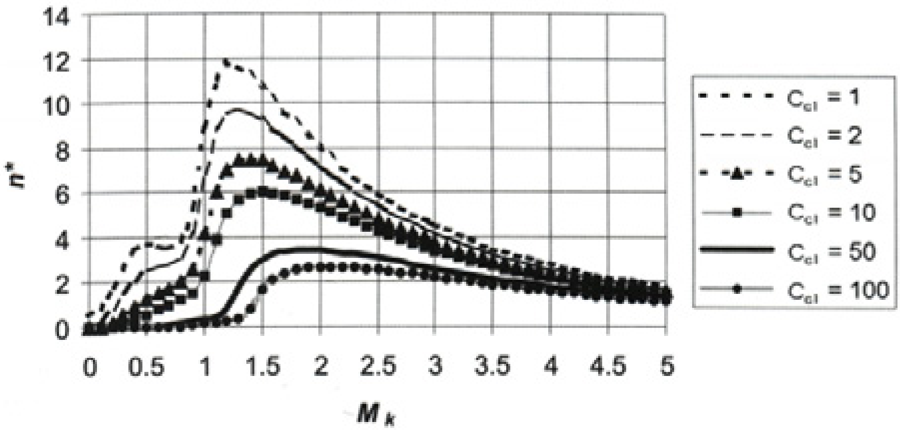

Fig. 5c. *Optimum sample size (n^*) vs. mode M_k for different C_{cl}, where, $C_{nc}=100$, $C_{fa}=C_{rac}=500$, $\lambda_p=500$, $C_p=1$, $N_l=100$, $\sigma_k=0.2$*

In Figs. 5a, b and c the optimum sample size in dependency of the absolute value of mode (M_k) are presented. In some cases when the optimum sample size is 0, that means *SPC* is not recommended, because it is too expensive. In those cases, the total loss function L_T is monotonically increasing, as we can see in Fig. 4 (M_k = 0.25).

4 DISCUSSION AND CONCLUSIONS

In this paper, a new approach for the economic design of $\overline{x}$ charts is given. Optimum intensity of quality inspection, that means sample size and sampling frequency, can be found by the optimisation of the proposed loss function. A sampling frequency is defined by lot size. The majority of the overall loses in quality control are contributed by the state, where the process is out of control and the process-mean shift is not detected. The duration of this state, which is estimated by ARL_1, depends on the sample size, *PMS* and the control limits. Due to different *PMS* values appearing in practice, we describe *PMS* in terms of probability. In our approach *PMS* is not assumed to be a constant value, but it is described by the probability density function PDF_k. The proposed case study is obtained with bimodal probability density function within the proposed economic model. The curves for the optimum sample size for different cost constants are presented. In other approaches, proposed in [4] to [7], a constant value for *PMS* is used, which may be questionable in real production.

The advantage of our approach is that *PMS*, estimated with PDF_k, is closer to reality. In real world, *PMS* varies with time. Our approach takes this fact into account and describes *PMS* as a random variable. For PDF_k an arbitrary probability density function can be used. Zero case (k=0), that means PMS is 0, is excluded from the integral area in Expressions (17) and (18). If there are any constraints for sample size or lot size, given by the observed production process, the optimum sample size (n^*) and the optimum lot size can be found iteratively. For the loss functions in Fig. 4 and optimum sample size in Figs. 5a, b and c the value of C_p is 1. If the process capability ratio is greater (C_p>1), the optimum sample sizes lowers and peaks of their curves move to the right. For greater M_k (M_k>4) no significant difference of optimum sample size for different C_{cl} was observed.

In our case study, σ_k = 0.2, but normally it depends on the nature of the process shift. In Fig. 6, the curves of the optimum sample size for various σ_k values are presented. Proposed PDF_k is taken into consideration. When a variance of PDF_k approaches 0 ($\sigma_k \rightarrow 0$), M_k approaches a constant value. Due to this fact, the curve of the optimum sample size taking into account PDF_k differs from the curve of the optimum sample size, where PMS is

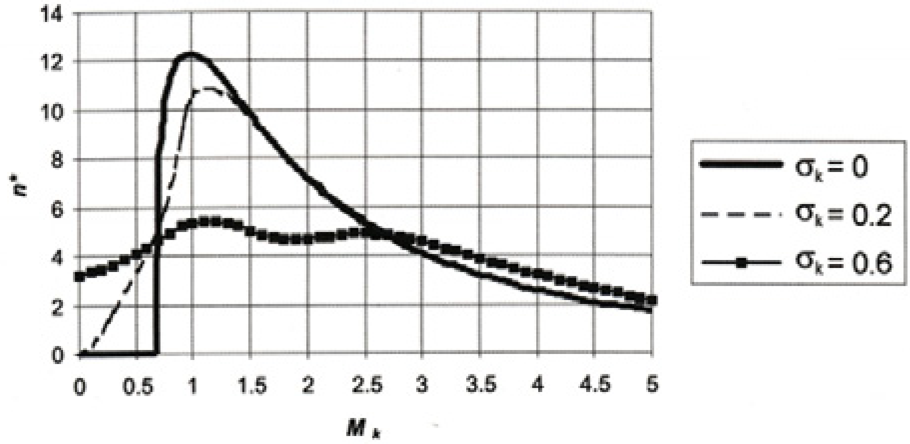

Fig. 6. *Comparison of optimum sample sizes (n*) for different (σ_k), where $C_{cl}=2$, $C_{nc}=100$, $C_{fa}=C_{rac}=200$, $\lambda_p=500$, $C_p=1$, $N_l=100$*

supposed to be a constant value. The difference is significant for small values of M_k, ($M_k<1.5$). If the σ_k increases ($\sigma_k=0.6$), the optimum sample size approaches a constant value. In spite of that, the optimum sample size cannot be generalised. In Fig. 6 comparison of optimum sample size curves for different σ_k is presented. The optimum sample sizes, calculated with the proposed economic model (where $\sigma_k=0.2$ and $\sigma_k=0.6$) are compared with the optimum sample sizes, where PMS is assumed as a constant value.

In this paper, the economic model for Shewhart control charts for process mean design was proposed. An economic design of control charts is a complex operation. In industrial environment, a statistical and quality control expertise is not available. With intention to support this expertise, a web service based operation support will be available soon and proposed in [11]. The web service for this kind of operation support will help operators in the industrial environment not only to control the quality of processes, but also to design the optimum lot sizes and sample sizes. Economic design of control charts is a key operations support for the manufacturing enterprises in order to achieve a competitive position on the global market.

5 ACKNOWLEDGEMENT

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6 GLOSSARY AND NOTATION

ARL_0	Average Run Length of false alarms
ARL_1	Average Run Length of process-out-of-control detection
C_{cl}	Cost per unit (product) controlled
C_{fa}	Cost per false alarm
C_{nc}	Cost per nonconforming unit
C_p	Process capability ratio
C_{rac}	Cost to locate and repair an assignable cause of the process
Cusum	Cumulative sum
EWMA	Exponential weighted moving average
k	Multiplier of a process standard deviation
$k\sigma$	Shift of the process mean value

L	Control limits coefficient
LCL	Lower control limit
LSL	Lower specification limit
L_T	Total loss function
L_{XX}	Loss function
m	Sample statistic mean
M_k	Mode of PDF_k
n	Sample size
n^*	Optimum sample size
N_l	Lot size
PDF_k	Probability density function of k
PMS	Process-mean shift
P_{nc}	Fraction of nonconforming products
SPC	Statistical Process Control
UCL	Upper control limit
USL	Upper specification limit
α	Probability of type I error
β	Probability of type II error
λ_p	Process failure rate
μ	Process mean
ν_s	Sampling frequency
σ	Process standard deviation
σ_k	Standard deviation of PDF_k
$\sigma_{<x>}$	Sample standard deviation
$\Phi(.)$	Cumulative distribution function of a standard normal random variable

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