Strojniški vestnik - Journal of Mechanical Engineering 53(2007)10, 696-705
UDK - UDC 620.172.21
Izvirni znanstveni članek - Original scientific paper (1.01)

Časovno odvisno vedenje pogonskih jermenov pod vplivom periodične mehanske obremenitve - analiza lokacije enojne spektralne črte

Time-Dependent Behaviour of Drive Belts under Periodic Mechanical Loading - An Analysis of the Location of a Single Line Spectrum

Barbara Zupančič - Anatolij Nikonov - Urška Florjančič - Igor Emri
(Fakulteta za strojništvo, Ljubljana)

Pri zagotavljanju vzdržljivosti motorjev je kritična trajnost pogonskih jermenov. Zato je razumevanje mehanizmov, ki na trajnost jermenov vplivajo, velikega pomena. S tem prispevkom nadaljujemo raziskavo o časovno odvisnem obnašanju dinamično obremenjenih elastomernih izdelkov. V ta namen je bil razvit teoretični model, ki omogoča analizo nabiranja napetostno-deformacijskega stanja [1]. Razvoj modela temelji na konstitutivnem modeliranju prenosnih jermenov, ki so izpostavljeni zobati periodični obremenitvi med obratovanjem motorja. V vsakem obremenitvenem ciklu je elastomerni material izpostavljen kombinaciji pojavov lezenja in kasnitve. V določenih razmerah, ki jih definirata geometrijska oblika jermena in kotna hitrost obratovanja jermenic, pojav kasnitve med dvema zaporednima cikloma ni končan. Tako material vstopi v naslednjo obremenitveno fazo s preostalo deformacijo. Posledično se deformacijsko stanje prične nabirati, kar vodi do utrjevanja materiala, nastajanja razpok in končno lahko tudi do odpovedi jermena.

Dosedanji izsledki raziskave so pokazali, da obstajajo kritične razmere obremenjevanja, pri katerih se pojavi postopek nabiranja deformacije. Predvidevamo lahko, da bo izdelek med obremenjevanjem v kritičnih razmerah (oz. v bližini) skoraj zagotovo odpovedal! Kritične razmere obremenjevanja so odvisne od zakasnilnega časa materiala (opredeljuje ga lokacija spektra), medtem ko je velikost nabrane deformacije odvisna od jakosti (intenzivnosti) pripadajoče spektralne diskretne linije. Iz tega lahko sklepamo, da je mehanski spekter polimernega materiala, iz katerega je izdelek narejen, odločilna materialna funkcija za napoved trajnosti dinamično obremenjenih elastomernih izdelkov.

V tem prispevku analiziramo vpliv lokacije enojne spektralne linije na kritične obratovalne razmere. Rezultati analize kažejo, da daljši odzivni časi materiala pomenijo nižje kritične obratovalne kotne hitrosti.

(Ključne besede: časovno odvisno modeliranje, prenosni jermeni, viskoelastične analize, mehanski spektri)

A drive belt's durability is a critical factor for the sustainable operation of engines. Thus, understanding the mechanisms that affect their durability is extremely important. In this paper we present the continuation of our research on the time-dependent behavior of dynamically loaded elastomeric products. For this purpose the constitutve model has been developed, which made possible the analyses of the stress-strain state [1]. The development of the costitutive model is based on the modelling of the transmission belts when exposed to tooth-like periodic mechanical loading during the operation of the engine. Within each loading cycle the elastomeric material undergoes a combination of creep and the retardation process. Under certain conditions, defined by the drive belt's geometry and the pulleys' angular velocity, the retardation process between two loadings cannot be fully completed to a strain-free state. Consequently, the strain state starts to accumulate, which leads to hardening of the material, crack formation, and ultimately to the failure of the belt.

The results have shown that there exists some critical loading conditions for the strain-accumulation process to occur. We can predict that the product will almost certainly fail if it operates under critical conditions, which depend on the retardation time (location of the spectrum). The magnitude of the accumulated strain is dictated by the intensity of the spectrum line. Thus, the mechanical spectrum of the *polymeric material, from which the product is made, is the most important material function for predicting the durability of the polymeric product.*

In this paper we analyze the effect of the location of a single spectrum line on the critical operating conditions. We found that longer response times for a material mean lower critical operating angular velocities.

(Keywords: time-dependent constitutive modeling, transmission belts, viscoelastic analysis, mechanical spectrum)

1 OBREMENITVENI POGOJI POGONSKEGA JERMENA

Tip obremenitve pogonskega jermena je bil določen s tržnim MKE paketom ANSYS, pri čemer smo privzeli elastično obnašanje vseh komponent jermena. Jermen je bil prednapet s silo F in obremenjen z želenim momentom M, tako da je bila sila prednapetja razdeljena na silo F_1 v natezni veji jermena in silo F_2 v razbremenitveni veji jermena, kakor je shematično prikazano na sliki 1. Na sliki je hkrati označena tudi medosna razdalja med jermenicama l, polmer jermenic R, ter časa t_1 in t_2, ki označujeta začetek in konec stika jermena z jermenico.

Gibanje, ko se izbrana točka na jermenu pri vrtenju pogonske in gnane jermenice vrne v začetno lego, imenujemo zaključen obremenitveni cikel jermena. Za viskoelastično analizo smo izbrali točko pod zobom jermena, v kateri ima časovni potek strižne napetosti znotraj vsakega obremenitvenega cikla obliko sunka. Izbrana lokacija je prikazana na sliki 2a in označena z A, medtem ko slika 2b prikazuje pripadajoči časovno odvisni razvoj strižne napetosti.

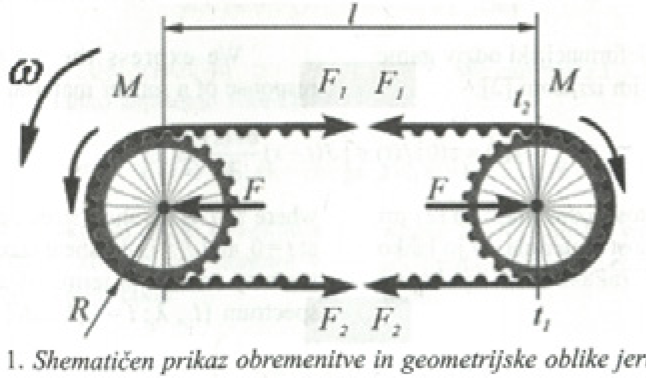
V prvem približku lahko potek strižne napetosti modeliramo z razliko dveh koračnih

1 THE LOADING CONDITIONS OF A DRIVE BELT

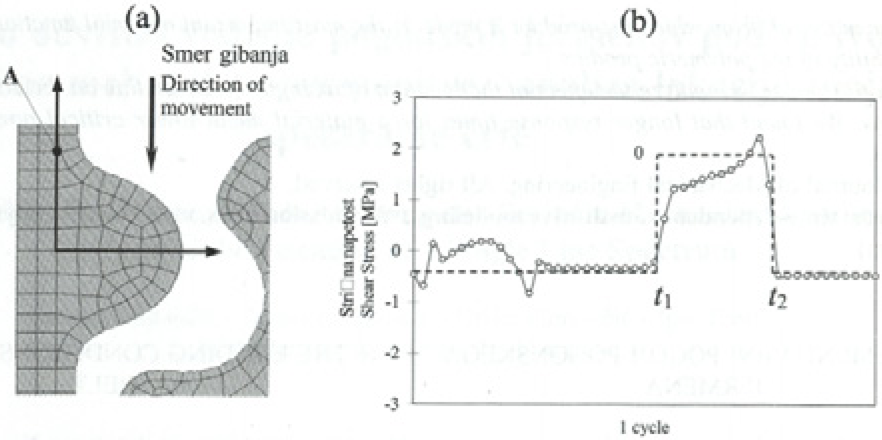
The loading conditions of a synchronous belt were determined with the commercial FEM program ANSYS, assuming the elastic behaviour of all the belt components. The belt was pre-stressed with a force F and loaded with the desired torque M, so that the pre-stressing force was appropriately divided into the strand on the tension side, F_1, and on the slack side, F_2, as schematically shown in Fig. 1. The figure also indicates the distance between two pulleys, l, their radius, R, and the times, t_1 and t_2, which indicate when the belt enters and leaves the driving pulley.

The driving pulley and the driven pulley were then rotated in such way that a selected point on the belt would return to the initial position. The described movement we designated as a complete loading cycle of the belt. For the present viscoelastic analysis we selected the location on the belt tooth where the shear stress state within each loading cycle has the form of an impulse function. The selected location is shown in Fig. 2a and is indicated as A, while Fig. 2b shows the corresponding calculated time-dependent evolution of the stress state.

In the first approximation the shear stress can be modelled as the difference between two step



Sl. 1. *Shematičen prikaz obremenitve in geometrijske oblike jermena*
Fig. 1. *Schematics of the belt's loading conditions and its geometry*



Sl. 2. *Časovno odvisni razvoj strižne napetosti v točki A v zaključenem obremenitvenem ciklu*

Fig. 2. *The time-evolution of the shear stresses at point A within a complete loading cycle*

funkcij z jakostjo striga τ_0, kakor je prikazano na sliki 2b. Torej:

functions with the shear intensity, τ_0, as shown in Fig. 2b. Hence:

$$\tau(t)=\tau(0)h(t)+\tau_0\sum_{n=1}^{N}\left\{h\left[t-t_1-(n-1)\xi\right]-h\left[t-t_2-(n-1)\xi\right]\right\} \quad (1),$$

za $t \le N\xi$. Časa t_1, t_2 in trajanje enega obremenitvenega cikla ξ, lahko izrazimo kot funkcije geometrijske oblike jermena in kotne hitrosti obratovanja jermenic ω. Če l označuje medosno razdaljo med jermenicama, R je polmer jermenic, N pa število obremenitvenih ciklov, ki jim je bil jermen izpostavljen, tedaj lahko pišemo: $t_1=(l+\pi R)/(\omega R)$, $t_2=(l+2\pi R)/(\omega R)$, $\xi=(2l+2\pi R)/(\omega R)$. Model za popis strižne napetosti lahko zapišemo v naslednji obliki:

for $t \le N\xi$. We can express the times t_1, t_2 and the duration of one loading cycle, ξ, as functions of the belt geometry and the angular velocity of the belt drive, ω. If l is the distance between the axes of the pulleys, R is the radius of the pulleys, and N is the number of loading cycles that the belt has been exposed to, we obtain the following expressions, $t_1=(l+\pi R)/(\omega R)$, $t_2=(l+2\pi R)/(\omega R)$, $\xi=(2l+2\pi R)/(\omega R)$. Hence, the shear-stress model can be rewritten in the form:

$$\tau(t)=\tau(0)h(t)+\tau_0\sum_{n=1}^{N}\left\{h\left[t-(2n-1)\frac{l+\pi R}{\omega R}\right]-h\left[t-\frac{2n(l+\pi R)-l}{\omega R}\right]\right\} \quad (2),$$

za $t \le N\xi$.

for $t \le N\xi$.

2 ANALIZA NABIRANJA DEFORMACIJSKEGA STANJA

2 ANALYSIS OF THE STRAIN ACCUMULATION

Časovno odvisni deformacijski odziv gume lako popišemo z naslednjim izrazom [2]:

We express the time-dependent strain response of a rubber material as [2]:

$$\gamma(t)=\tau(0)J(t)+\int_0^t J(t-s)\frac{\partial\tau(s)}{\partial s}ds \quad (3),$$

kjer je $\tau(0)$ strižna napetost, izražena z en.(2) pri $t=0$, $J(t)$ pa strižna voljnost materiala, ki jo lahko popišemo z diskretnim zakasnilnim spektrom $\{L_i, \lambda_i; i=1,2,...,K\}$ kot:

where $\tau(0)$ is the shear stress, expressed with Eq.(2) at $t=0$, and $J(t)$ is the shear creep compliance, which we may written in terms of a discrete retardation spectrum $\{L_i, \lambda_i; i=1,2,...,K\}$ as :

$$J(t)=J_g+\sum_{i=1}^{K}L_i\left(1-e^{-\frac{t}{\lambda_i}}\right) \quad (4).$$

J_g je strižna voljnost materiala v steklastem stanju, λ_i pa označuje retardacijske čase, kjer so locirane ustrezne spektralne črte $L_i = L(\lambda_i)$, K označuje število spektralnih črt, s katerimi modeliramo $J(t)$.

Če izraz en.(2) za strižno napetost in izraz za strižno voljnost en.(4) vstavimo v en.(3), dobimo:

J_g stays for glassy compliance, when λ_i denotes the retardation times, where the corresponding spectrum lines $L_i = L(\lambda_i)$ are located. K stays for a number of spectrum lines for modelling $J(t)$.

Introducing Eq.(2) for the shear stress and Eq.(4) for the shear creep compliance into Eq.(3), we obtain:

$$\gamma(t, t \leq N\xi) = \tau(0)J(t) + \tau_0 \sum_{n=1}^{N} \sum_{i=1}^{K} L_i \left[e^{-\frac{t}{\lambda_i} + \frac{2n(l+\pi R)-l}{\omega R \lambda_i}} - e^{-\frac{t}{\lambda_i} + (2n-1)\frac{l+\pi R}{\omega R \lambda_i}} \right] \quad (5).$$

En.(5) popisuje časovno odvisni razvoj (postopek zbiranja) deformacije v materialu kot funkcijo kotne hitrosti ω, geometrijskih parametrov l in R ter števila obremenitvenih ciklov N, ki jim je jermen izpostavljen. Nabrano deformacijo lahko izrazimo kot:

Eq.(5) describes the time-dependent evolution (accumulation process) of the strain in a material as a function of the angular velocity ω, the geometry parameters l and R, and the number of loading cycles, N, to which the belt is exposed. Let the accumulated strain in N cycles be expressed as $\Gamma_N(t)$, so:

$$\gamma(t, t \leq N\xi) = \tau(0)J(t) + \Gamma_N(t) \quad (6),$$

kjer je:

where:

$$\Gamma_N(t) = \tau_0 \sum_{n=1}^{N} \sum_{i=1}^{K} L_i \left[e^{-\frac{t}{\lambda_i} + \frac{2n(l+\pi R)-l}{\omega R \lambda_i}} - e^{-\frac{t}{\lambda_i} + (2n-1)\frac{l+\pi R}{\omega R \lambda_i}} \right]$$

En.(5) uporabimo za vrednotenje vpliva posameznega parametra na obnašanje jermenov v daljšem časovnem obdobju (trajnost).

We can use Eq.(5) to analyze how each of the listed parameters affects the long term behaviour (durability) of the belt.

2.1 Deformacijsko stanje po določenem številu zaključenih obremenitvenih ciklov

2.1 The strain state during completed cycles

Recimo, da nas zanima zbrano deformacijsko stanje po N zaključenih ciklih, t. j. pri času $t = t_N$:

Let us assume that we are interested in the accumulated strain at the end of N completed cycles, i.e., at $t = t_N$:

$$t_N = N\xi = 2N\frac{l+\pi R}{\omega R} = 2N\frac{\kappa+\pi}{\omega} \quad (7),$$

kjer je $\kappa = l/R$. V tem primeru izraz en.(6) zavzame obliko:

where $\kappa = l/R$. In this case Eq.(6) takes the following form:

$$\gamma(N) = \tau(0)J(2N\frac{\kappa+\pi}{\omega}) + \Gamma_N(N) \quad (8).$$

Izraz za zbrano deformacijo po N zaključenih ciklih $\Gamma_N(N)$ lahko zapišemo v obliki:

We can write $\Gamma_N(N)$ as a recursive formula:

$$\Gamma_N(n) = \Gamma_N(n-1) + \Delta\Gamma_n(n), n = 1, 2, 3, ..., N \quad (9),$$

pri čemer $\Gamma_N(N)$ lahko v skrčeni obliki izrazimo s končno vsoto prirastkov zbrane deformacije v posameznem obremenitvenem ciklu:

where $\Gamma_N(N)$ can be written as the sum of the accumulated strain in each consecutive cycle in the form:

$$\Gamma_N(N) = \sum_{n=1}^{N} \Delta\Gamma_n(n) \quad (10),$$

kjer je $\Delta\Gamma_n(n)$ oblike:

where $\Delta\Gamma_n(n)$ is of the form:

$$\Delta\Gamma_n(n)=\tau_0\sum_{i=1}^{K}L_i\exp\left(-\frac{(\kappa+\pi)(2n-1)-\pi)}{\omega\lambda_i}\right)\left[1-\exp\left(-\frac{\pi}{\omega\lambda_i}\right)\right].$$

Iz zadnjega je razvidno, da je tudi prirastek zbrane deformacije $\Delta\Gamma_n(n;\ n=1,2,3,\ldots,N)$ za posamezen cikel odvisen od geometrijske oblike jermena $\kappa=l/R$, kotne hitrosti jermenic ω, števila obremenitvenih ciklov N in zakasnilnega spektra materiala $\{L_i,\lambda_i;\ i=1,2,\ldots,K\}$.

Lokacijo kritične kotne hitrosti obratovanja ω_{CR}, pri kateri je zbiranje deformacije $\Delta\Gamma_n(n;\ n=1,2,3,\ldots,N)$ najintenzivnejše, matematično določimo z naslednjo enačbo:

We can also see that the strain accumulated in each consecutive cycle, $\Delta\Gamma_n(n;\ n=1,2,3,\ldots,N)$ depends on the geometry of the belt, $\kappa=l/R$, the angular velocity of the pulleys, ω, the number of completed cycles, N, and the retardation spectrum of the material, $\{L_i,\lambda_i;\ i=1,2,\ldots,K\}$.

The location of the critical angular velocity, ω_{CR}, of the belt-drive, at which $\Delta\Gamma_n(n;\ n=1,2,3,\ldots,N)$ has an extremum, can be determined from:

$$\frac{\partial}{\partial\omega}\{\Delta\Gamma_n(n)\}_{\omega=\omega_{CR}}=0 \quad (11).$$

Z upoštevanjem dejstva, da je $\Delta\Gamma_n(n)$ funkcija geometrijskih parametrov jermena, števila obremenitvenih ciklov in časovno odvisnih materialnih lastnosti, podobno velja tudi za kritično kotno hitrost ω_{CR}. Na podlagi teoretičnega ozadja, predstavljenega v [1], smo v sklopu nadaljnjih raziskav sistematično analizirali vpliv geometrijske oblike jermena, števila obremenitvenih ciklov in lokacije enojne spektralne linije na lokacijo kritične kotne hitrosti ω_{CR} in postopek zbiranja deformacije.

Taking into consideration that $\Delta\Gamma_n(n)$ is a function of the drive belt's geometrical parameters, the number of consecutive cycles, and the material time-dependent property, the same holds also for the location of ω_{CR}. Considering the theoretical background in [1] we aimed our further research at a systematic analysis of the effect of belt-drive geometry, the number of loading cycles and the single spectrum line location at the location of ω_{CR} and the strain-accumulation process.

2.2 Vpliv geometrijske oblike jermena

Za popis vpliva geometrijske oblike jermena, ki jo izrazimo s razmerjem $\kappa=l/R$, analiziramo deformacijo, zbrano v prvem ciklu , $n=N=1$, pri čemer časovno odvisno materialno lastnost popišemo z najpreprostejšim spektrom, t. j. z enojno spektralno linijo. V tem primeru se izraz za $\Delta\Gamma_n(n)$ poenostavi do oblike:

2.2 The effect of the belt-drive geometry

To grasp the influence of the belt-drive geometry, expressed by the ratio $\kappa=l/R$, we shall analyze the strain accumulated during the first cycle, $n=N=1$, and keep the spectrum as simple as possible, *i.e.*, assume that the time-dependency of the material can be modelled with a single spectral line. In this case the expression for $\Delta\Gamma_n(n)$ becomes:

$$\Delta\Gamma_n(n=1)=\tau_0 L_1\,e^{-\frac{\kappa}{\omega\lambda_1}}\left[1-e^{-\frac{\pi}{\omega\lambda_1}}\right] \quad (12).$$

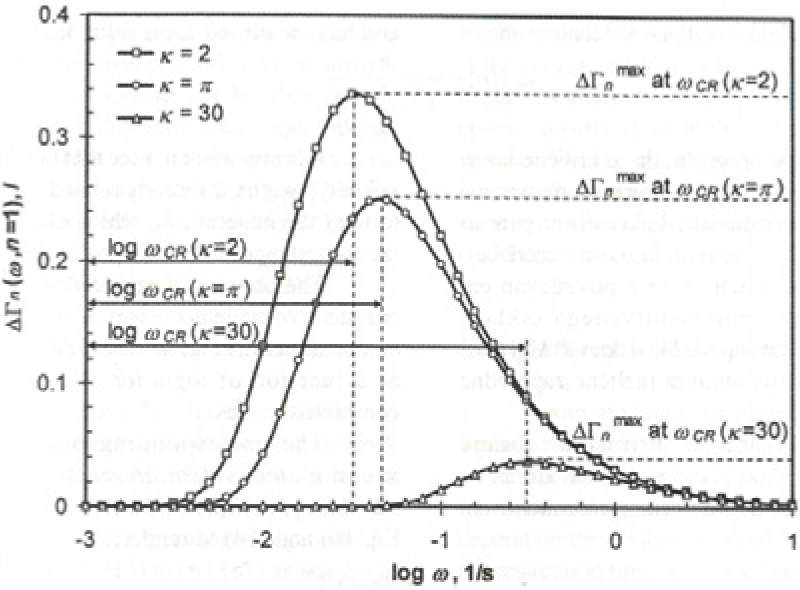
Zbrano deformacijsko stanje po prvem ciklu obratovanja je kot funkcija kotne hitrosti, $\Gamma(\omega,N=1)=\Delta\Gamma_n(\omega,n=1)$, prikazano na sliki 3. Rezultati se nanašanjo na tri različne vrednosti parametra κ in retradacijski čas $\lambda_1=100$ s [3].

Za izbrane vrednosti $N=n=1$ (en obremenitveni cikel) in $K=1$ (enojna spektralna linija), lahko lokacijo kritične kotne hitrosti $\omega_{CR}(\kappa)$ določimo analitično iz en.(11) v sklenjeni obliki:

The strain accumulated during the first cycle as a function of frequency, $\Gamma(\omega,N=1)=\Delta\Gamma_n(\omega,n=1)$, is shown in Fig. 3. The results are also displayed for three different values of κ and for $\lambda_1=100$ s [3].

When $N=n=1$ (one loading cycle) and $K=1$ (single spectral line), the location of the critical angular velocity, $\omega_{CR}(\kappa)$, can be determined as the closed form solution of Eq.(11). In this case we express $\omega_{CR}(\kappa)$ as:

$$\omega_{CR}(\kappa)=\frac{\pi}{\lambda_1\ln\frac{\kappa+\pi}{\kappa}} \quad (13).$$



Sl. 3. *Vpliv geometrijske oblike jermena $\kappa = l/R$, na postopek zbiranja deformacije v prvem obremenitvenem ciklu pri $\lambda_1 = 100$ s*

Fig. 3. *The effect of belt-drive geometry, $\kappa = l/R$, on the strain accumulated during the first cycle, for $\lambda_1 = 100$ s*

Kritična kotna hitrost $\omega_{CR}(\kappa)$ podaja informacijo o pogojih obratovanja jermenic in jermena, pri katerih je zbiranje deformacije največje. Seveda pa se postopek zbiranja deformacije odvija tudi pri obratovalnih kotnih hitrostih, ki so večje ali manjše od kritične, $\omega_{CR}(\kappa)$.

Iz zgornje analize lahko povzamemo, da mora biti razmerje $\kappa = l/R$ čim večje, seveda v mejah uporabnosti jermena.

The critical angular velocity, $\omega_{CR}(\kappa)$, indicates the operational conditions for the belt drive at which the magnitude of the accumulated strain achieves its maximum. Of course, strain accumulation also occurs at angular velocities that are larger or smaller than $\omega_{CR}(\kappa)$.

From the analysis presented in this section we can summarize that $\kappa = l/R$ should be chosen to be as large as the drive-belt application allows.

2.3 Vpliv števila obremenitvenih ciklov

Pri analizi vpliva števila obremenitvenih ciklov še vedno ohranimo predpostavko o najpreprostejšem modeliranju časovno odvisne materialne lastnosti z enojno spektralno črto, locirano pri odzivnem času $\lambda_1 = 100$ s. Izberemo vrednost geometrijskega parametra $\kappa = \pi$. V tem primeru se izraz za prirastek zbrane deformacije poenostavi do oblike:

2.3 The effect of the number of completed cycles

To analyze the effect of the number of loading cycles we shall again keep the material time-dependence simple by using the same single spectral line model as before, with $\lambda_1 = 100$ s. We shall let $\kappa = \pi$. The expression for the strain accumulation in each consecutive cycle then becomes:

$$\Delta\Gamma_n(n) = \tau_0 L_1 \, e^{-\frac{\pi(4n-3)}{\omega\lambda_1}} \left[1 - e^{-\frac{\pi}{\omega\lambda_1}}\right]; \; n = 1, 2, 3, \cdots, N \tag{14}$$

Lokacijo kritične kotne hitrosti $\omega_{CR}(n)$ ponovno določimo iz en.(11), ki se v primeru enojne spektralne črte pri $\lambda_1 = 100$ s in $\kappa = \pi$ preoblikuje v:

The location of $\omega_{CR}(n)$ can be obtained from Eq.(11), which for a single spectral line $\lambda_1 = 100$ s and $\kappa = \pi$ reads:

$$\frac{\partial \Delta\Gamma_n(n)}{\partial \omega} = \frac{\partial}{\partial \omega}\left\{\tau_0 L_1 \, e^{-\frac{\pi(4n-3)}{\omega\lambda_1}} \left[1 - e^{-\frac{\pi}{\omega\lambda_1}}\right]\right\} = 0 \tag{15}$$

rešitev te enačbe pa lahko podamo v sklenjeni obliki:

and has the closed-form solution:

$$\omega_{CR}(n) = \frac{\pi}{\lambda_1 \ln \dfrac{4n-2}{4n-3}} \quad (16).$$

Pomembno je opozoriti, da je kritična kotna hitrost v zgornjem izrazu, $\omega_{CR}(n)$, neposredno povezana z zakasnilnim časom materiala, λ_1, ki nazorno poudari pomembnost časovne odvisnosti lastnosti materiala.

Zbiranje deformacije se s povečevanjem zaporedne številke obremenitvenega cikla n zmanjšuje, kar je prikazano na sliki 4, kjer je $\Delta\Gamma_n(\omega, n)$ prikazana kot funkcija $\log\omega$ za različne zaporedne obremenitvene cikle.

Največjo vrednost prirastka zbrane deformacije $\Delta\Gamma_n^{max}(\omega_{CR}, \kappa = \pi, n)$, za določen obremenitveni cikel določimo s kombinacijo en. (16) in en. (14):

It is important to note that the critical angular velocity, $\omega_{CR}(n)$, is directly related to the retardation time of the material, λ_1, which clearly emphasizes the importance of the material's time-dependent.

The amount of accumulated strain in each consecutive cycle decreases as n increases. This is demonstrated in Fig. 4, where $\Delta\Gamma_n(\omega, n)$ is shown as a function of $\log\omega$ for different numbers of completed cycles.

The corresponding peak value of the accumulated strain in each loading cycle, $\Delta\Gamma_n^{max}(\omega_{CR}, \kappa = \pi, n)$, is obtained by combining Eq.(16) and (14) to render:

$$\Delta\Gamma_n^{max}(\omega_{CR}, \kappa = \pi, n) = \tau_0 L_1 \frac{1}{4n-2} \cdot \left(\frac{4n-3}{4n-2}\right)^{(4n-3)} ; \; n = 1, 2, 3, \cdots, N \quad (17).$$

Opozorimo na pogoj:

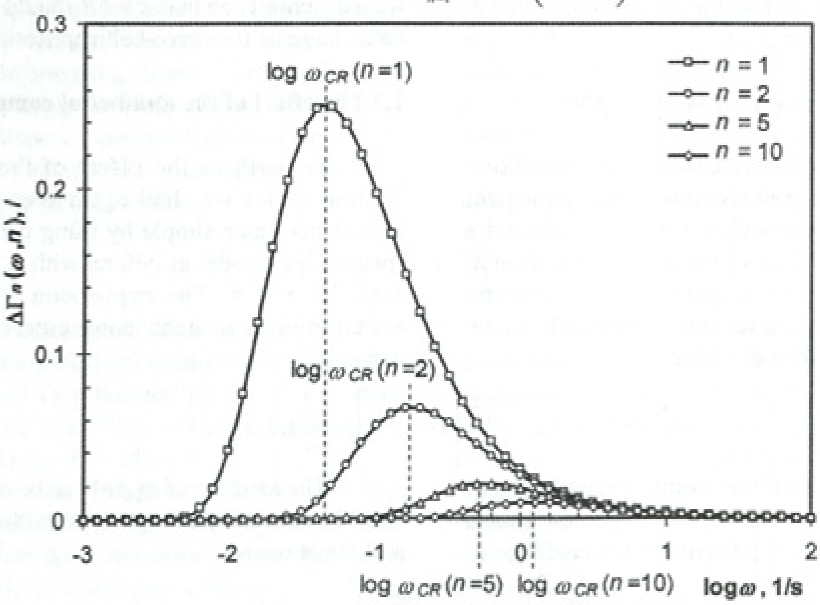
It is important to stress that the condition

$$\lim_{n\to\infty} \Delta\Gamma_n^{max}(\omega_{CR}, \kappa = \pi, n) = \lim_{n\to\infty} \tau_0 L_1 \frac{1}{4n-2} \cdot \left(\frac{4n-3}{4n-2}\right)^{(4n-3)} = 0 \quad (18),$$

ki pomeni, da bo zbiranje deformacije čedalje šibkejše, ko $n\to\infty$. Poleg tega pa za kumulativno zbrano deformacijsko stanje glede na število obremenitvenih ciklov velja:

prevails, which means that the strain accumulated in each cycle will tend towards zero as $n\to\infty$. However, it is also true that the accumulated strain:

$$\lim_{n\to\infty} \Gamma_n^{max}(\omega_{CR}, N) = \lim_{n\to\infty} \tau_0 L_1 \sum_{n=1}^{N} \frac{1}{4n-2} \cdot \left(\frac{4n-3}{4n-2}\right)^{(4n-3)} = \infty \quad (19),$$



Sl. 4. *$\Delta\Gamma_n(\omega, n)$ kot funkcija* $\log\omega$ *za različne zaporedne številke obremenitvenega cikla pri $\lambda_1 = 100$ s in $\kappa = \pi$*
Fig. 4. *$\Delta\Gamma_n(\omega, n)$ as a function of* $\log\omega$ *for different numbers of loading cycles, for $\lambda_1 = 100$ s and $\kappa = \pi$*

kar pomeni, da bo jermen skoraj zagotovo odpovedal, če bo obratoval pri kritični kotni hitrosti. Rezultati te analize kažejo, da bi obratovanje jermena pri kritični kotni hitrosti ali v njeni bližini zelo verjetno lahko vodilo do porušitve po določenem številu obremenitvenih ciklov. Časovni okvir tega postopka je odvisen od deformacijskega stanja materiala, ki lahko vodi do porušitve, in amplitude uporabljene napetosti τ_0 (sl. 2).

2.4 Vpliv lokacije enojne spektralne črte λ_1

Iz analize vpliva števila obremenitvenih ciklov in vpliva geometrijske oblike na postopek zbiranja deformacije je razvidno, da sta izraza en.(13) in en.(16) za kritično hitrost odvisna od pozicije spektralne črte, t. j. zakasnilnega časa λ_1. Če združimo izraza en.(13) in en.(16) s privzetkom, da je $N = n = 1$ in je $\kappa = \pi$ za spekter z enojno spektralno črto, dobimo naslednji izraz za kritično kotno hitrost:

which means that if a belt was to operate constantly at the critical angular velocity it will always fail! From this analysis we can conclude that belts that operate mostly at, or in the vicinity of, the critical angular velocity will almost certainly fail. The time frame of this process depends on the material failure strain and the amplitude of the applied stress, τ_0 (see Fig. 2).

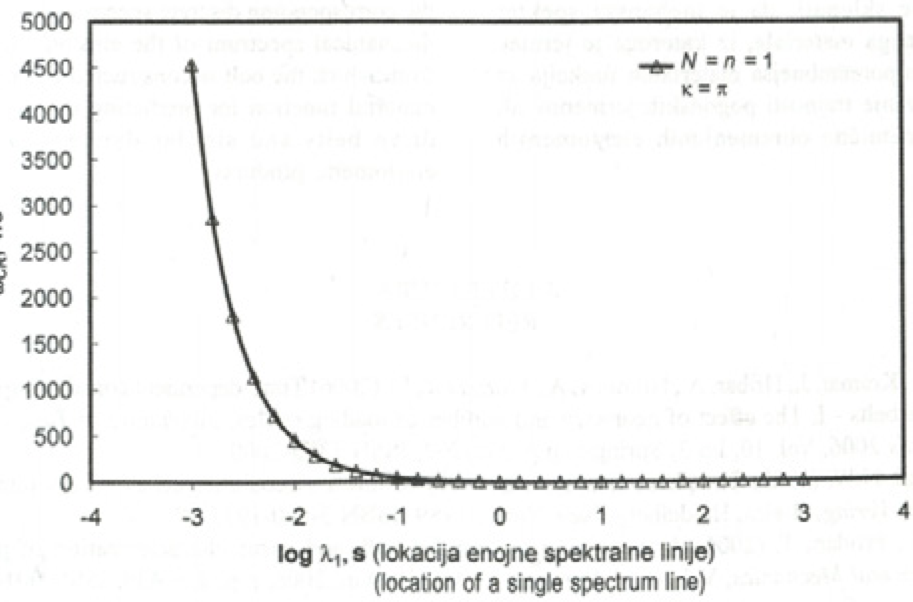
2.4 The effect of a single spectrum line location, λ_1

From studying the effect of the number of loading cycles and the effect of geometry on the strain-accumulation process it is evident that expressions for the critical angular velocity in Eq.(13) and Eq.(16) depend on the position of a single spectrum line, i.e., the retardation time, λ_1. If we bring together Eq.(13) and Eq.(16), assuming that $N = n = 1$ and je $\kappa = \pi$ for a single spectrum line, we obtain the following expression for the critical angular velocity:

$$\omega_{CR}(n) = \frac{\pi}{\lambda_1 \ln 2} \qquad (20).$$

Diagram na sliki 5 kaže, da pri $N = n = 1$ (en obremenitveni cikel) in $K = 1$ (enojna spektralna črta), se lokacija kritične kotne hitrosti pri določeni vrednosti geometrijskega parametra $\kappa = \pi$ pomika k manjšim kotnim hitrostim, ko povečujemo odzivne čase.

Fig. 5 shows that when $N = n = 1$ (one loading cycle) and $K = 1$ (single spectral line), the location of the critical angular velocity at a defined geometry $\kappa = \pi$ is moving to lower values when the spectral line is located at longer retardation times.



Sl. 5. *Vpliv lokacije zakasnilnega časa λ_1 na kritično obratovalno kotno hitrost*
Fig. 5. *The effect of the location of the retardation time λ_1 on the critical operating angular velocity*

3 SKLEPI

Iz predstavljenega lahko povzamemo, da jermeni izkazujejo zbiranje deformacijskega stanja, če so pri obratovanju izpostavljeni določenim kotnim hitrostim. Postopek zbiranja je najbolj intenziven pri t.i. kritični kotni hitrosti, ki je med drugim odvisna od razmerja med medosno razdaljo jermenic in polmerov jermenic. Pri dani geometrijski obliki se kritična hitrost povečuje s povečevanjem števila obremenitvenih ciklov. Obenem pa se velikost zbrane deformacije zmanjšuje nelinearno, ko se zaporedna številka cikla povečuje. To pomeni, da se postopek zbiranja deformacije upočasnjuje s številom obremenitvenih ciklov in je po določenem številu zanemarljiv. Vendar pa, če bo jermen deloval pri kritični kotni hitrosti ali v njeni bližini, bo skoraj zagotovo prišlo do njegove porušitve.

Analiza je pokazala, da je kritična kotna hitrost odvisna tudi od zakasnilnega časa materiala, ki opredeljuje lokacijo mehanskega spektra. Pri materialih z daljšimi odzivnimi časi (to pomeni, da je spekter v območju daljših zakasnilnih časov) se glede na teoretične izsledke tega prispevka postopek zbiranja deformacijskega stanja najintenzivneje pojavlja pri manjših obratovalnih kotnih hitrostih. Obenem je velikost zbrane deformacije opredeljena z intenziteto diskretne spektralne črte, ki pripada določenemu zakasnilnemu času. Iz izvedene analize je mogoče sklepati, da je mehanski spekter elastomernega materiala, iz katerega je jermen izdelan, najpomembnejša materialna funkcija za napovedovanje trajnosti pogonskih jermenov ali drugih dinamično obremenjenih elastomernih izdelkov.

3 CONCLUSIONS

From the presented analysis we can conclude that drive belts will exhibit an accumulation of strain when exposed to operation at certain angular velocities. The strain accumulation will be most intensive at some critical angular velocity that is proportional to the ratio of the belt length and the diameter of the pulleys. For a given belt geometry the critical angular velocity increases with the number of loading cycles. At the same time the magnitude of the accumulated strain decreases non-linearly as the number of loading cycles increases. Hence, the strain-accumulation process slows down with the increasing number of loading cycles and is negligible after a certain number of loadings. But in any case, if the belt operates at, or in close proximity to, its critical angular velocity it will almost certainly fail.

The analysis has shown that the critical angular velocity also depended on the material retardation time, which defined the location of the mechanical spectrum. According to our theoretical results materials with longer response times (meaning that the spectrum line is located at a higher retardation time) would exhibit the most intensive strain-accumulation process at lower operating angular velocities. At the same time the analysis has also shown that the magnitude of the accumulated strain was dictated by the strength of the corresponding discrete spectrum line. Thus, the mechanical spectrum of the elastomeric material from which the belt is constructed is an important material function for predicting the durability of drive belts and similar dynamically loaded elastomeric products.

4 LITERATURA
4 REFERENCES

[1] Emri, I., Kramar, J., Hribar, A., Nikonov, A., Florjančič, U. (2006) Time-dependent constitutive modeling of drive belts - I. The effect of geometry and number of loading cycles, *Mechanics of Time-dependent Materials* 2006, Vol. 10, Iss 3, Springer, p.p. 245-262, ISSN 1385-2000

[2] Tschoegl, N.W. (1989) The phenomenological theory of linear viscoelastic behavior: An introduction, *Springer-Verlag*, Berlin, Heidelberg, New York , 1989, ISBN 3-540-19173-9

[3] Emri, I., Prodan, T. (2006) A measuring system for bulk and shear characterization of polymers, *Experimental Mechanics*, Vol. 46, 4, Springer-Verlag, Boston, 2006, p.p. 429-439, ISSN 0014-4851

Naslov avtorjev: Barbara Zupančič
dr. Anatolij Nikonov
dr. Urška Florjančič
prof. dr. Igor Emri
Univerza v Ljubljani
Fakulteta za strojništvo
Aškerčeva 6
1000 Ljubljana
barbara.zupancic@fs.uni-lj.si

Authors' Address: Barbara Zupančič
Dr. Anatolij Nikonov
Dr. Urška Florjančič
Prof. Dr. Igor Emri
University of Ljubljana
Faculty of Mechanical Eng.
Aškerčeva 6
1000 Ljubljana
barbara.zupancic@fs.uni-lj.si

Prejeto: 17.7.2007
Received:

Sprejeto: 28.9.2007
Accepted:

Odprto za diskusijo: 1 leto
Open for discussion: 1 year