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Logarithmical Crowning for Spur Gears

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Gear tooth modifications, such as lead crowning, are often recommended to compensate for misalignment (e.g. assembly deviations). Lead crowning means that the tooth centre is slightly thicker than the tooth edges and is usually described as a circular arc profile. The use of crowning shifts the peak load from the tooth flank edges and therefore reduces the risk for high contact pressures at the edges, which can otherwise result in a shortened service life.

In this study a logarithmical lead profile was compared with traditional lead profile modifications for gears. The profiles were applied on a spur gear pair and a numerical method for contact analysis was used to calculate the contact pressure distribution at the pitch diameter. All lead profiles were optimised with respect to low contact pressure at a specific normal load, a specified maximum misalignment in the plane of action and a tooth flank edge contact criteria.

The results show that the logarithmical profile responds differently to misalignments compared to the traditional lead profile modifications. The logarithmical profile resulted in lower maximum contact pressures for small misalignments and is therefore of further interest in terms of achieving a robust gear design.

Keywords: spur gear, lead crowning, misalignment, contact pressure

0 INTRODUCTION

It is widely accepted that the use of tooth modifications enhances the load capacity by decreasing the risk for extreme tooth load deflection interference. Earlier, experiments investigated the influence of tooth modifications and misalignments on tooth surface contact strength and root bending strength. Today, the understanding achieved through the use of theoretical methods has led to further understanding of the need for modifications which make the gear design insensitive to manufacturing errors and misalignments. Li [1] developed a 3-D finite element method to calculate surface contact stress and root bending stress for a pair of spur gears with machining errors, assembly errors and tooth modifications. Li suggest that misalignment error in the plane of action should not be greater than 0.01°; the amount of lead (circular arc profile) crowning should not be greater than 20 µm and the machining error should not be greater than 10 µm to obtain high strengths gears. Guilbault et al. [2] found that contact perturbations (e.g. misalignments) led to very high or even excessive contact pressure, while the corresponding fillet stress remained moderate. Guilbault et al. made use of a combined finite strip model with a pseudo-three-dimensional model to calculate tooth bending deflections and fillet stresses.

According to Harianto and Houser [3] lead modifications to compensate for misalignments are selected with respect to manufacturing accuracy, company based standards or by performing a load distribution analysis based on established peak misalignments.

Lead crowning for gears is traditionally expressed as a circular arc profile or as a flat profile at the centre with modification only near the tooth flank edges. In this study a logarithmical lead profile, expressed mathematically as a special logarithmical function, was compared with the traditional lead profile modifications.

The logarithmical profile was first developed for roller bearings in order to optimise the stress distribution, but there is not much evidence to show that it has been used for lead profile modification for gears. Roller bearings have undergone many technical improvements since they were originally introduced [4] and it was Lundberg's theory [5] that resulted in this logarithmical profile which was found to be superior to conventional profiles.

The results showed that the logarithmical profile gives noticeably lower contact pressures for small misalignments compared to traditional lead profile modifications.

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0.1 Test Setup

The lead profiles were applied on a spur gear pair consisting of two identical gears with equal face width length (40 mm). Contact pressure calculations were carried out for a load case with normal load 11500 N at the pitch diameter, representing a "frozen" moment of the mesh cycle. The applied misalignment, which has similar consequences as helix slope deviation, was applied by rotating gear 2 in the plane of action as Fig. 1 illustrates.

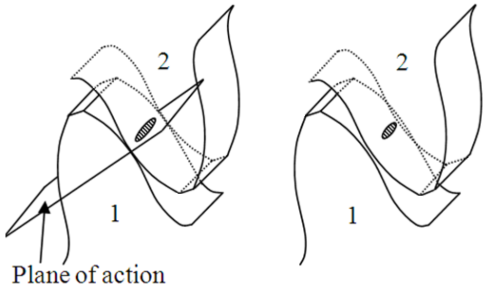

Fig. 1. *Illustration of the gear contact with no misalignment (left) and with angular misalignment of gear 2 in the plane of action (right)*

0.1.1 Contact Calculation Program

The contact pressure was calculated using a numerical method for contact analysis [6], which can be described as a boundary element method applied for infinite elastic planes. This means that the contact area must be small compared to the size of the bodies.

0.1.2 Lead Modification

A logarithmical lead profile (Logarithmical profile) was compared with the following two traditional lead profile modifications:

- Flat at the centre and parabolic modification near the face tooth edges (Parabolic modification).
- Flat at the centre with circular modification near the face tooth edges (Circular modification).

This logarithmical profile is a modified version of the profile suggested by Lundberg [5]:

$$L = A \cdot \ln \frac{1}{1 - K\left(\frac{x}{b}\right)^2} \quad (1)$$

where the coordinate x represents the position at the face width (x equal to zero corresponds to the gear tooth centre), b being the half face width length, A being a scale factor and K a profile factor. The scale factor A works as a multiplier for the whole function and the profile factor K is the so called Lundberg's profile factor, which also prevents that infinite dimensioning is given at flank ends. Note that Johns and Gohar [7] have a derivation of the Lundberg's profile factor. Similarly, the parabolic and circular modifications are also expressed using two factors: setting the length of the flat region, and expressing the profile near the face tooth edge. Fig. 2 illustrates each lead profile.

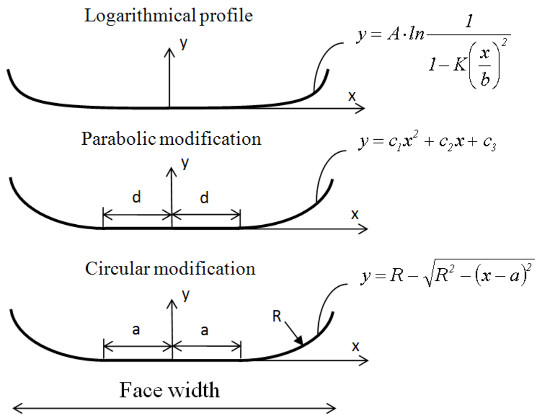

Fig.2 *Schematic illustrations of all lead profiles. The x-axis represents the position at the face width and the y-axis the modified quantity*

0.1.3 Optimisation Criteria

With more crowning, the width of the contact patch becomes smaller, and as a consequence of this the maximum surface pressure becomes higher. On the other hand, too little crowning will cause the gear teeth to make contact too close to the flank edges. Therefore the amount of crowning should be optimised to give as low surface pressure as possible, with the constraint that contact is only allowed at a certain distance from the flank edges.

In this work the three different crowning profiles were optimised to meet the requirement that no contact is allowed closer than 10% of the flank width from the flank edges, at maximum normal load (11500 N) and maximum misalignment (0.04 degrees). The edge contact criteria were roughly chosen with respect to the fact that lead evaluation length is shortened at each end depending on the module and flank width size according to the ISO standard [8]. Each type of lead profile (Fig. 2) is optimised by varying their parameters in a structural manner until the specified contact criteria is achieved. The profile resulting in the lowest maximum contact pressure was set as the optimal profile.

As a reference, an optimised full circular arc profile, with no flat region at the centre, was added to the test profiles.

2 RESULTS

The optimised lead profiles are presented in Fig. 3. where it can be seen that the almost identical parabolic and circular modifications decrease more rapidly from the tooth flank centre than the logarithmical profile. At the edges, the logarithmical profile deviates more rapidly than the parabolic and circular modifications. The amount of modification at the tooth flank edges is 9 μm larger for the logarithmical profile than for the parabolic and circular modification.

Fig. 4. presents the maximum contact pressure for all lead profiles versus the amount of misalignment. Initially, at no misalignment (when the pressure is distributed uniformly axially), the

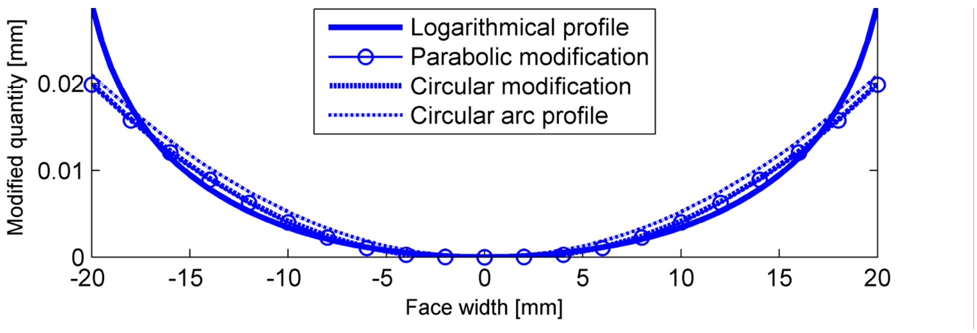

Fig. 3. *Optimised lead profiles*

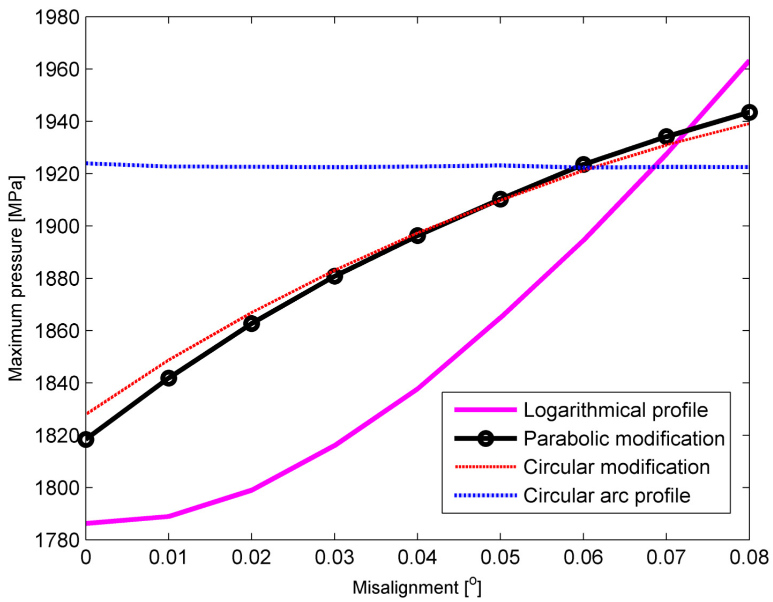

Fig. 4. *The maximum contact pressure for all lead profiles versus the amount of misalignment*

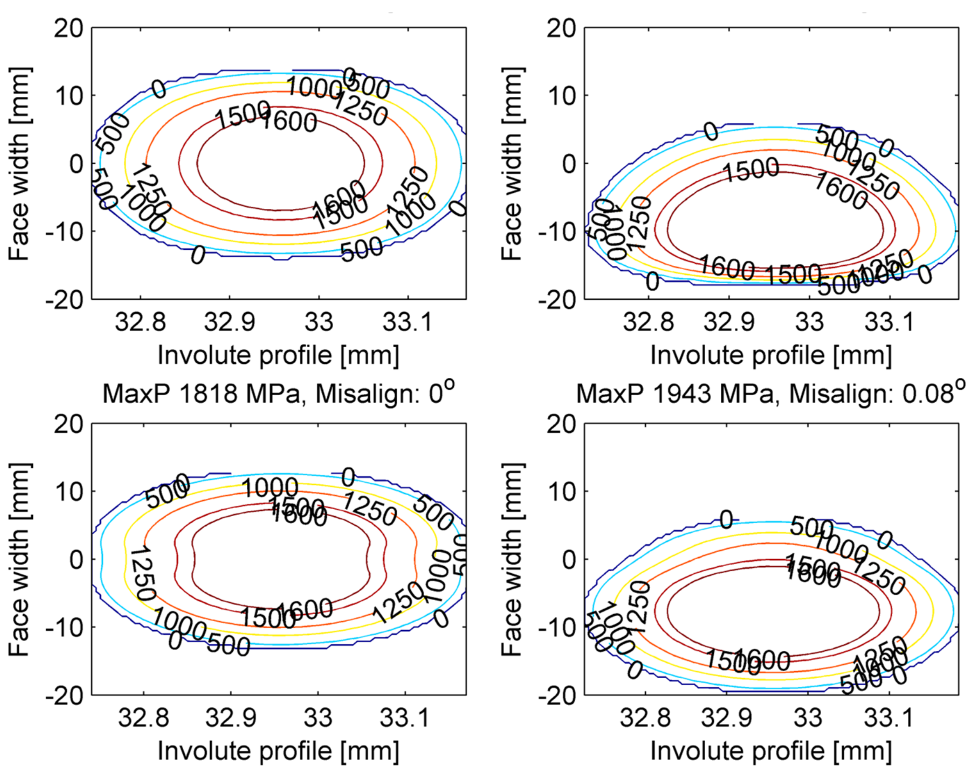

Fig. 5. *Contact pressure distributions for the logarithmical profile (top) and parabolic modification (bottom) using an angular misalignment of 0° and 0.08°*

logarithmical profile has the lowest maximum contact pressure (1786 MPa). The maximum pressure remains relatively constant for small misalignments up to 0.02°, but for misalignments larger than 0.03°, the logarithmical profile increases more quickly than the two traditional lead profile modifications. Further on, for misalignments above 0.07°, the logarithmical profile gives a higher maximum pressure compared to the two traditional lead profile modifications. Naturally the circular arc profile has no variation due to the amount of misalignment.

Fig. 5 presents the contact pressure distribution for the logarithmical profile and the parabolic modification at misalignments of 0º and 0.08º respectively. It can be seen that although the maximum pressure is somewhat higher for the logarithmical profile, the contact has not reached the edge in the way that the parabolic modification does. The circular modification had the same behaviour as the parabolic modification shown in Fig. 5.

2 DISCUSSION

For misalignments less than approximately 0.07°, the logarithmical profile resulted in lower maximum contact pressures than the parabolic and circular profile modifications (Fig. 4). Compared to the traditional tooth modifications, the logarithmical profile was found to be less sensitive to misalignments less than 0.02°. This is of interest in terms of achieving a robust design. On the other hand, the contact pressure increased exponentially as the radius of curvature rapidly decreased towards the face tooth edge (Fig. 3).

When comparing the logarithmical profile with the parabolic and circular modifications, the largest difference in maximum contact pressure was approximately 60 MPa at misalignments of 0.02 and 0.03° (Fig. 4). The parabolic and circular modifications behaved in similar manners, which were expected since they are geometrically almost identical (Fig. 3). Additionally, because the reference profile (circular arc profile) has a constant radius of curvature, the maximum pressure will not change for different angular misalignments. For misalignments above 0.07° the reference profile resulted in the lowest maximum pressure (Fig. 4). However, with respect to the risk of edge contact, only the logarithmical profile performed well at the largest misalignment examined (Fig. 5).

It should be pointed out that the suggested lead profiles are only truly optimal for the suggested misalignment, load and contact criteria. There might be other combinations of the parameters expressing each profile that could slightly increase the pressure at the set misalignment (0.04°) but reduce sensitivity. The contact criteria originating from the ISO lead profile evaluation length [8] can be questioned since it is only coupled to evaluation during manufacturing. Additionally, the numerous variables - such as involute profile modification and edge rounding (a surface that separates tooth flank sides and the active profile) - have not been taken into consideration.

When checking the quality of the tooth contact in practice, the gear is mounted in a testing machine and painted with a gear marking compound. The gears are then run under light load and the tooth contact pattern observed. Each gear designer has their own approach to set what is acceptable to avoid edge contact, based on company experience. In this study, the quality of the tooth contact has been roughly estimated using the half plane assumption. In reality, the deflection of the gear teeth is larger at the edges than in the middle of the contact. Moreover, according to Komori et al. [9], the tooth side edge of carburized gear is usually fragile due to hardening. The error made by assuming that the gear teeth are deforming like infinite half planes is probably small and not actually important, since the results are only used for comparing different lead profile modifications. One way to compensate for the potential error would be to use a complementary finite element approach, which is not based on the half plane assumption.

The selection of tooth modification parameters are often set in order to achieve a robust design for a certain performance requirement. Some researchers suggest that lead modifications, together with profile modifications, are important variables in minimizing transmission error. To get the lowest peak-to-peak transmission error, which is considered to be one of the excitations of gear noise, Houser and Harianto [10] found it best to provide parabolic modification near the face tooth edges only, with no modification at the centre. It would be of further interest to investigate whether a logarithmical profile could be set in favour of the parabolic modification with respect to obtaining the lowest peak-to-peak transmission error.

Even with the computer controlled machine tools available today (which can be programmed to cover much more variations than were possible in the past), a logarithmical crowning can be difficult to manufacture due to the challenge that in some cases only negligible amounts of material will need to be removed in order to tighten tolerances.

It is also important for the gear designer to know how the logarithmical profile should be specified in drawings, and how it should be measured in a gear inspection machine [11].

3 CONCLUSIONS

The logarithmical profile is of interest in terms of achieving a robust design.

The logarithmical profile has to be developed so it can attain competitive manufacturing costs.

Further work is needed to use the lead profiles in concert with other tooth modifications.

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