

# Optimum Design of a Hybrid-Driven Mechanical Press Based on Inverse Kinematics

Hui Li\* - Lihui Fu - Yuping Zhang

Department of Electromechanical Engineering, Shijiazhuang Institute of Railway Technology, P.R. China

*A novel hybrid-driven mechanical press for deep drawing is presented. The main feature of hybrid-driven linkage is to combine the motion of a large constant velocity motor with a small servomotor via a two degree of freedom mechanism. The former provides the main power and motion required, while the latter acts as a motion modulation device. Therefore, the hybrid-driven linkage can provide for programmable motion output. In this paper, a mechanical press driven by hybrid mechanism is developed. It considers the displacement and velocity requirement of the drawing process. Inverse kinematic analysis and dimension synthesis optimization of this hybrid-driven mechanism is presented. It points out a new way for press research and provides a new method for press application in flexible manufacture.*

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**Keywords:** hybrid-driven mechanism, press, deep drawing, optimum design, servomotor

## 0 INTRODUCTION

Mechanical presses are widely used in the metal industry. The crank press consisting of a slider-crank linkage is unsuitable for precision drawing because of its lack of the characteristics of constant working velocity [1]. Therefore, in order to obtain the constant working velocity, mechanical press manufacturers have developed the multi-links presses. These attempts have focused on increasing the approach and return velocity. To slow down the slider velocity in the working stroke, it is important to obtain the constant working speeds or make the load-stroke characteristics suitable for special application. The multi-links presses, however, do not have the flexibility for different drawing technology [2]. In order to provide for the more suitable load-stroke characteristics, several researchers have developed the flexible press which uses a servomotor as the prime mover [3] to [6]. However, the force capabilities of servomotor are usually limited. Recently, many researchers have paid more attention to the hybrid-driven mechanism. The main idea of the hybrid-driven linkage is to combine the motion of a large constant velocity motor with a small servomotor via a two degree of freedom mechanism. The constant velocity motor provides the main power and motion requirement, while the servomotor acts as a low power motion modulation device.

Therefore, the hybrid-driven linkage can provide for programmable motion output. The idea of hybrid-driven linkage was initially presented by Tokuz and Jones [7] to [10]. They used a differential gearbox as the hybrid mechanism to drive a slider-crank linkage. Greenough and Brashaw [11] took a seven bar mechanism as the hybrid linkage to generate a dwell motion. Conner [12] synthesized a hybrid mechanism using a genetic algorithm. Kirecci and Dulger [13] to [14] proposed a hybrid actuator. In the study, the arrangement comprised of a planar two degree of freedom, seven-link mechanism is driven by a constant speed motor and a servomotor. The dynamic behavior of hybrid actuator was studied by applying numerical simulation on the whole system. Lagrangian mechanics was applied to derive equations of motion. Simulation results were presented to demonstrate the ability of a model developed with PID controller action. Herman [15] presented a hybrid cam mechanism. The hybrid cam mechanism adds much flexibility to the system requiring only small peak power and peak torque from the servo motor. Seth and Vaddi [16] to [17] introduced the concept of programmable function generator mechanism. Vohar [18] optimized a link-drive mechanism for deep drawing mechanical press, Li and Zhang [19] to [20] investigated the feasibility of the hybrid-driven mechanical presses. The scheme of hybrid-driven

\*Corr. Author's Address: Department of Electromechanical Engineering, Shijiazhuang Institute of Railway Technology, Shijiazhuang 050041, P.R. China, HuiLi68@163.com

mechanical press is put forward. A hybrid-driven nine-bar mechanical press for precision drawing is presented. The hybrid-driven mechanical press for precision drawing is optimized by two step's optimization. The dimension of the linkage and the motion rules of servomotor are obtained. By properly optimizing the displacement trajectory of the servomotor, the output motion of the slider can pass through the desired trajectories. Therefore, the hybrid-driven mechanical presses have flexible output motion characteristics suitable for precision drawing. The advantages of a hybrid-driven mechanical press are analyzed.

In this paper, the hybrid-driven system is initially applied to a mechanical press for deep drawing. The purpose of this paper is to present a design method for dimensional synthesis of a hybrid-driven mechanical press that is suitable for deep drawing and optimization of the displacement trajectory of the servomotor based on inverse kinematics.

### 1 INVERSE KINEMATICS ANALYSIS OF HYBRID-DRIVEN MECHANICAL PRESS

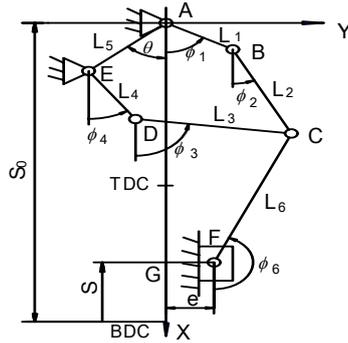


Fig. 1. Sketch of hybrid-driven press

A hybrid-driven mechanical press is shown in Fig. 1. It is composed of a planar five-bar mechanism ABCDE and a dyad of CF. The length of links are denoted by  $L_1, L_2, L_3, L_4, L_5$  and  $L_6$ . The crank  $L_1$  is driven by a constant speed motor. The crank  $L_4$  is driven by a servomotor. Link  $L_5$  is the fixed link. The X-axis of the coordinate system in Fig. 1 is set along the vertical direction. The coordinate axes are fixed at point A. In order to improve the load characteristics of the slider, the point in which  $L_1, L_2$  and  $L_6$  are lined up in a straight line, as the BDC (Bottom Dead Center) of the slider. The

starting point of the displacement is taken as BDC. Top Dead Center of the slider is TDC.

According to Fig. 1 the loop vector equations are given as follows:

$$\vec{L}_1 + \vec{L}_2 = \vec{AG} + \vec{GF} + \vec{L}_6 \quad (1)$$

$$\vec{L}_1 + \vec{L}_2 = \vec{L}_5 + \vec{L}_4 + \vec{L}_3 \quad (2)$$

From Eqs. (1) and (2), the following equations are obtained:

$$\begin{aligned} L_1 \cos \phi_1 + L_2 \cos \phi_2 &= S_0 - S + L_6 \cos \phi_6 \\ L_1 \sin \phi_1 + L_2 \sin \phi_2 &= e + L_6 \sin \phi_6 \end{aligned} \quad (3)$$

$$L_1 \cos \phi_1 + L_2 \cos \phi_2 = x_E + L_4 \cos \phi_4 + L_3 \cos \phi_3$$

$$L_1 \sin \phi_1 + L_2 \sin \phi_2 = y_E + L_4 \sin \phi_4 + L_3 \sin \phi_3,$$

where  $L_1, L_2, L_3, L_4, L_5, L_6, e, \theta$  are the dimension parameters of the mechanism as shown in Fig. 1.  $S$  stands for the displacement function of the slider required by deep drawing process;  $S_0$  is given as follows:

$$S_0 = \sqrt{(L_1 + L_2 + L_6)^2 - e^2} \quad (4)$$

From Eq. (3) we can obtain the function of the displacement of the crank  $L_4$  with respect to the rotation angle  $\phi_1$  and slider displacement  $S$ :

$$\phi_4 = f(\phi_1, S) \quad (5)$$

By differentiating Eq. (3) with respect to time, the following equation can be obtained:

$$\begin{aligned} L_2 \sin \phi_2 \omega_2 - L_6 \sin \phi_6 \omega_6 &= \dot{S} - L_1 \sin \phi_1 \omega_1 \\ L_2 \cos \phi_2 \omega_2 - L_6 \cos \phi_6 \omega_6 &= -L_1 \cos \phi_1 \omega_1 \end{aligned} \quad (6)$$

$$L_4 \sin \phi_4 \omega_4 + L_3 \sin \phi_3 \omega_3 = L_1 \sin \phi_1 \omega_1 + L_2 \sin \phi_2 \omega_2$$

$$L_4 \cos \phi_4 \omega_4 + L_3 \cos \phi_3 \omega_3 = L_1 \cos \phi_1 \omega_1 + L_2 \cos \phi_2 \omega_2$$

Then, the angular velocity of each link can be derived as follows:

$$\begin{aligned} \omega_2 &= \frac{L_1 \sin(\phi_1 - \phi_6) \omega_1 - \cos \phi_6 \dot{S}}{L_2 \sin(\phi_6 - \phi_2)} \\ \omega_6 &= \frac{L_1 \sin(\phi_1 - \phi_2) \omega_1 - \cos \phi_2 \dot{S}}{L_6 \sin(\phi_6 - \phi_2)} \end{aligned} \quad (7)$$

$$\omega_4 = \frac{L_1 \sin(\phi_1 - \phi_3) \omega_1 + L_2 \sin(\phi_2 - \phi_3) \omega_2}{L_4 \sin(\phi_4 - \phi_3)}$$

$$\omega_3 = \frac{L_1 \sin(\phi_4 - \phi_1) \omega_1 + L_2 \sin(\phi_4 - \phi_2) \omega_2}{L_3 \sin(\phi_4 - \phi_3)}$$

By differentiating Eq. (6) with respect to time, yields

$$\begin{aligned} &L_2 \sin \phi_2 \varepsilon_2 - L_6 \sin \phi_6 \varepsilon_6 = \\ &= \ddot{S} - L_1 \cos \phi_1 \omega_1^2 - L_2 \cos \phi_2 \omega_2^2 + L_4 \cos \phi_4 \omega_4^2 \\ &L_2 \cos \phi_2 \varepsilon_2 - L_6 \cos \phi_6 \varepsilon_6 = \\ &= L_1 \sin \phi_1 \omega_1^2 + L_2 \sin \phi_2 \omega_2^2 - L_6 \sin \phi_6 \omega_6^2 \\ &L_4 \sin \phi_4 \varepsilon_4 + L_3 \sin \phi_3 \varepsilon_3 = \\ &= L_1 \cos \phi_1 \omega_1^2 + L_2 \cos \phi_2 \omega_2^2 - L_4 \cos \phi_4 \omega_4^2 - L_3 \cos \phi_3 \omega_3^2 + L_2 \sin \phi_2 \varepsilon_2 \\ &L_4 \cos \phi_4 \varepsilon_4 + L_3 \cos \phi_3 \varepsilon_3 = \\ &= -L_1 \sin \phi_1 \omega_1^2 - L_2 \sin \phi_2 \omega_2^2 + L_4 \sin \phi_4 \omega_4^2 + L_3 \sin \phi_3 \omega_3^2 + L_2 \cos \phi_2 \varepsilon_2 \end{aligned} \quad (8)$$

Therefore, the angular acceleration of each link can be derived as follows

$$\begin{aligned} \varepsilon_2 &= \frac{L_1 \cos(\phi_1 - \phi_6) \omega_1^2 + L_2 \cos(\phi_6 - \phi_2) \omega_2^2 - L_6 \omega_6^2 - \cos \phi_6 \ddot{S}}{L_2 \sin(\phi_6 - \phi_2)} \\ \varepsilon_6 &= \frac{L_1 \cos(\phi_1 - \phi_2) \omega_1^2 + L_2 \omega_2^2 - L_6 \cos(\phi_6 - \phi_2) \omega_6^2 - \cos \phi_6 \ddot{S}}{L_6 \sin(\phi_6 - \phi_2)} \\ \varepsilon_4 &= \frac{L_1 \cos(\phi_1 - \phi_3) \omega_1^2 + L_2 \cos(\phi_2 - \phi_3) \omega_2^2 - L_4 \cos(\phi_4 - \phi_3) \omega_4^2 - L_3 \omega_3^2 + L_2 \sin(\phi_2 - \phi_3) \varepsilon_2}{L_4 \sin(\phi_4 - \phi_3)} \\ \varepsilon_3 &= \frac{-L_1 \cos(\phi_1 - \phi_4) \omega_1^2 - L_2 \cos(\phi_2 - \phi_4) \omega_2^2 + L_4 \omega_4^2 + L_3 \cos(\phi_3 - \phi_4) \omega_3^2 + L_2 \sin(\phi_4 - \phi_2) \varepsilon_2}{L_3 \sin(\phi_4 - \phi_3)} \end{aligned} \quad (9)$$

Therefore, according to Eqs. (7) and (9), the angular velocity  $\omega_4$  and angular acceleration of  $\varepsilon_4$  of the crank  $L_4$  can briefly be written as:

$$\omega_4 = f_1(\phi_1, \omega_1, S, \dot{S}) \quad (10)$$

$$\varepsilon_4 = f_2(\phi_1, \omega_1, \varepsilon_1, S, \dot{S}, \ddot{S}), \quad (11)$$

where  $\omega_1, \varepsilon_1$  are the angular velocity and angular acceleration of link  $L_1$  and  $S, \dot{S}, \ddot{S}$  are the displacement, velocity and acceleration of the slider, respectively.

## 2 OPTIMUM DESIGN OF A HYBRID-DRIVEN MECHANICAL PRESS FOR DEEP DRAWING

The optimization of the hybrid-driven mechanism includes determining the objective function, the design variables, the constraints conditions and the optimization method. Taking the widely used mechanical drawing press as an example, we will demonstrate the optimization of the hybrid-driven mechanical press.

### 2.1 The Parameters of Optimized Hybrid-Driven Mechanical Press

(1) The geometric parameters of the mechanism are:  $L_1, L_2, L_3, L_4, L_5, L_6, \theta, e$ .

(2) The angular velocity  $\omega_1$  of the input link  $L_1$  which is driven by the constant velocity motor.

(3) The ideal displacement function  $S$  of the slider required by the deep drawing process

$$S=S(t)=S(\phi_1, \phi_4). \quad (12)$$

Based on these parameters we can obtain the displacement trajectory of input link  $L_4$  by optimization.

### 2.2 Objective Function

When a servomotor is used to drive the mechanism, the tracing capability of the

servomotor should be taken into account because it is not unlimited. Furthermore, the more complex the motion of punch pin is, the larger the fluctuation of the motion of the servomotor, which required the higher tracing capability of a servomotor. In view of the limitation of the tracing capability of the servomotor, the objective function of the optimization is given as follows:

$$f(x) = \min \sum \varepsilon_4^2. \quad (13)$$

### 2.3 Design Variables

The design variables of the optimization are the geometric parameters of the linkage:

$$[x]=[L_1, L_2, L_3, L_4, L_5, L_6, \theta, e]^T.$$

### 2.4 Constraints Conditions

- In order to satisfy different motion trajectory of the slider, the input link  $L_1$  and  $L_4$  must make complete rotations. Li [20] finished the classifications of hybrid five-bar linkages. In this paper, the unrestraint double-crank type is used. The input link  $L_1$  and  $L_4$  are unrestraint cranks. Therefore, the inequality constraints are defined as follows:

$$\begin{aligned} L_5 + L_1 + L_4 &< L_2 + L_3 \\ L_2 + L_1 + L_4 &< L_3 + L_5 \\ L_3 + L_1 + L_4 &< L_2 + L_5 \end{aligned} \quad (14)$$

- In order to minimize the power required of the servomotor, the power distribution conditions should be satisfied [20]:

$$L_1 > \lambda \cdot L_4, \quad (15)$$

where  $\lambda$  is the power distribution coefficient, and  $\lambda > 1$ .

- Pressure angle condition: Pressure angle is significant as an indicator of good force and motion transmission. Therefore, it is reasonable to attempt to minimize the maximum pressure angle in a linkage design.

The pressure angle inequality constraints is given as follows:

$$|180^\circ - |\phi_6|| \leq [\alpha], \quad (16)$$

where  $[\alpha]$  is the allowable pressure angle.

- Slider mobility condition: From Fig. 1 we know that the following inequality constraints should be satisfied:

$$L_6 > y_C - e, \quad (17)$$

where  $y_C$  is the coordinate of point  $C$  in  $y$ -direction.

- Link length condition: in order to have appropriate link lengths, we define the link lengths inequality constraints as follows:

$$L_{i\min} \leq L_i \leq L_{i\max} \quad i = 1, 2, \dots, 6 \quad (18)$$

### 2.5 Optimization Method

Clearly, the optimization model of the hybrid-driven mechanical press is highly nonlinear. Therefore, the Genetic Algorithm (GA), which is a better method for nonlinear optimization, is selected to optimize the hybrid-driven mechanical press [21].

### 2.6 Optimization Results

The optimization of the hybrid-driven mechanical press is to obtain the geometric parameters of the mechanism and displacement trajectory of the servomotor according to the requirement of deep drawing processes. On the basis of this and by properly optimizing the input motion of the servomotor, various desired output motions are obtained.

This optimal example is aimed at making the slider's motion more suitable for deep drawing process. The stroke of the slider is 670 mm and the stroke per minute is 8. With the requirements mentioned above and the displacement, velocity, acceleration curve of the slider given in Figs. 2 to 4, we can optimize the geometric parameters of the press by using the GA [21]. The optimal original values are given as follows:

$$L_1 = 185.1, L_2 = 800, L_3 = 800, L_4 = 185.1, \\ L_5 = 393.8, L_6 = 1000.0, \theta = -49.624^\circ, e = 5.0.$$

The corresponding input motion characteristics of the crank  $L_4$  are shown in Fig. 5 where the geometric parameters are optimal original values.

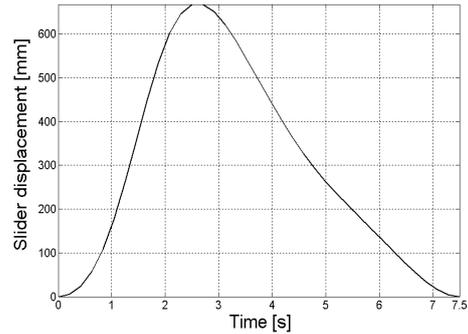


Fig. 2. Slider displacement  $S$

After 70 iterations, the optimum is achieved. The optimal results are obtained as follows:

$$L_1 = 173.2, L_2 = 737.8, L_3 = 635.9, L_4 = 141.2, \\ L_5 = 513.94, L_6 = 840.7, \theta = -25.468^\circ, e = 6.73.$$

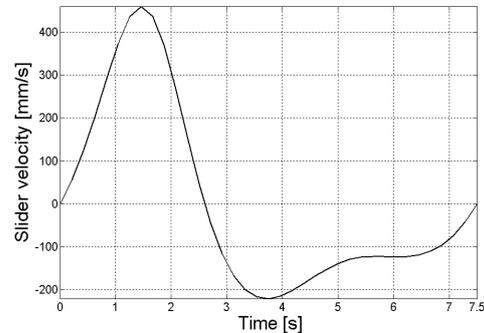


Fig. 3. Slider velocity  $\dot{S}$

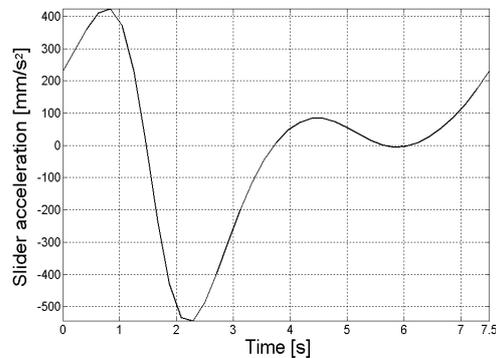


Fig. 4. Slider acceleration  $\ddot{S}$

The corresponding input motion characteristics of the crank  $L_4$  are shown in Fig. 6, 7 and 8, respectively. Hence, by optimizing the slider's displacement trajectory in Fig. 2, the slider's velocity is steadier and suitable for the

need of deep drawing during the working stroke of the slider.

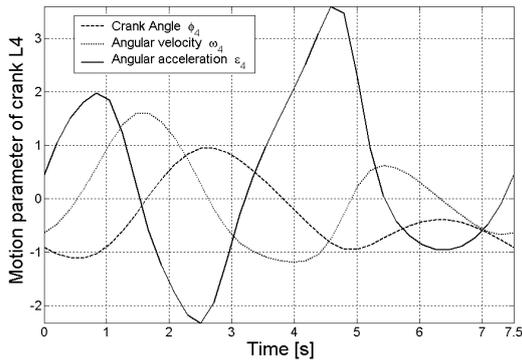


Fig. 5. Motion parameter of crank  $L_4$

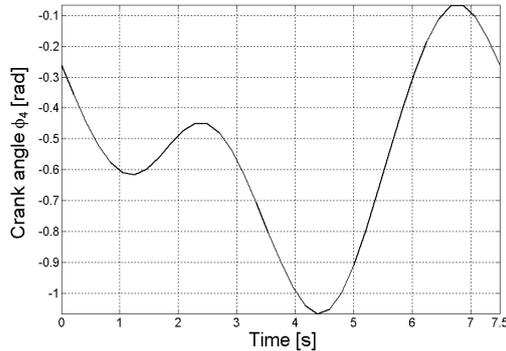


Fig. 6. Crank angular displacement  $\phi_4$

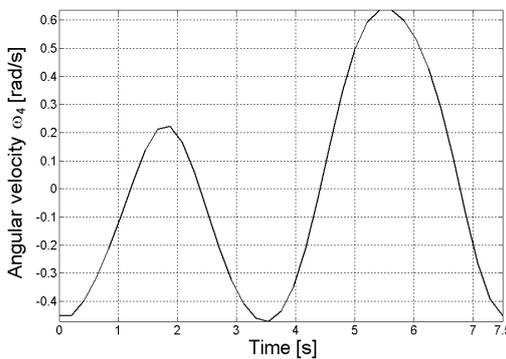


Fig. 7. Crank angular velocity  $\omega_4$

### 3 CONCLUSIONS

This paper presents a novel concept of using a hybrid-driven mechanism for a mechanical press which is suitable for deep drawing. The hybrid-driven mechanism is a new method to apply press in flexible manufacture. This method enables a press to produce several

types of output motion trajectory. The main contribution of this paper may be summarized as follows:

- A hybrid-driven mechanism suitable for a mechanical press is presented. In addition to the advantages of a classical press, the press using this mechanism as driving device is controllable and flexible. By changing the motion trajectory of the servomotor, the press can meet many requirements of the pressing technology.

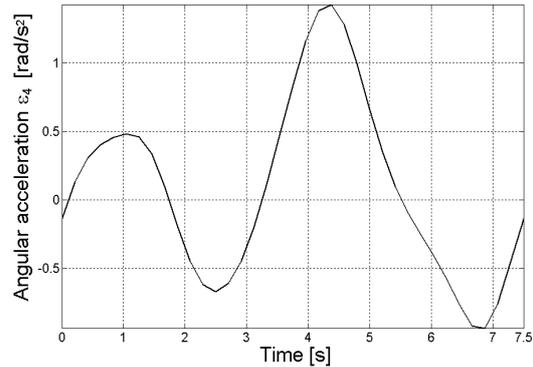


Fig. 8. Crank angular acceleration  $\epsilon_4$

- In order to obtain the motion trajectory of the servomotor, a particular description of inverse kinematic analysis of the hybrid-driven mechanism is presented. The study shows that the more complex the output motion is, the higher the tracing capability required of the servomotor is.
- In view of the limitation of the tracing capability of the servomotor, the parameters of the hybrid-driven mechanism are optimized with minimizing the velocity fluctuation of the servomotor as objective function. The example indicates that after inverse optimization of the mechanism, the fluctuation of velocity and acceleration of the servomotor is reduced markedly, and the value of acceleration is also reduced.

### 4 ACKNOWLEDGEMENT

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