

# The Application of a New Formula of Nakaoka Coefficient in HF Inductive Welding

Miroslav S. Milićević\*  
High Technical School Beograd, Serbia

The high-frequency welding procedure represents the complex theory which is possible to be used as an approximate calculation of important data such as current, voltage and the power of inductor and the welding object itself as well as the degree of usage. By applying so far known procedures, long and complex calculations are done through the use of the terms as well as many tables and graphic dependences. In this paper, a new analytical dependence is shown by the use of which the value of Nakaoka coefficient is being calculated through specific approximation, and by which important parameters and the range of high-frequency inductive welding of steel pipes are also calculated. By applying the results of this paper, the calculating procedure is shortened and made easier which makes way for optimizing the choice of thermal regime.

© 2010 Journal of Mechanical Engineering. All rights reserved.

**Keywords:** numerical approximation, electromagnetic field, HF welding, equivalence scheme, Nakaoka coefficient, practical application

## 0 INTRODUCTION

The case of high-frequency inductive welding of steel pipes can be reduced to the induction case on the semi-infinite area according [1] to [19]. Thus, the semi-infinite area in Fig. 1. is exposed to the electromagnetic wave influence under the assumption that this is electro-conducting board. The wave influence occurs from the dielectric area into the conducting area where displacement currents are being neglected, so that Maxwell equations can be written according [1] to [4].

$$-\frac{\partial H_z}{\partial x} = \gamma E_y, \quad (1)$$

$$-\frac{\partial E_y}{\partial x} = \mu \frac{\partial H_z}{\partial t}, \quad (2)$$

where  $H_z$  represents magnetic field,  $E_y$  electric field,  $\mu$  magnetic permeability and  $\gamma$  specific electric conductivity.

Since to the magnetic field  $H$  and electric field  $E$  are sine wave function of the time  $t$ , we have

$$\dot{H} = H_m e^{j(\omega t + \theta_H)} = H_m e^{j\theta_H} e^{j\omega t} = \dot{H}_m e^{j\omega t}, \quad (3)$$

$$\dot{E} = E_m e^{j(\omega t + \theta_E)} = E_m e^{j\theta_E} e^{j\omega t} = \dot{E}_m e^{j\omega t}, \quad (4)$$

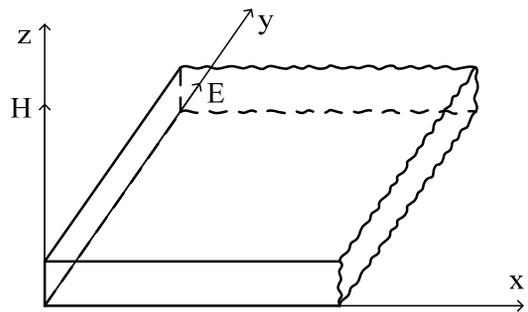


Fig.1. The influence of electro-magnetic wave on semi-infinite area

where  $\dot{H}_m$  and  $\dot{E}_m$  are complex amplitude value of working fields  $H_m$  and  $E_m$ ,  $\theta_H$  and  $\theta_E$  are relevant phases. Exchanging Eqs. (3) and (4) in (1) and (2), and excluding indexes "y" and "z", after transformations we get

$$\frac{d^2 \dot{H}_m}{dx^2} - j\gamma\omega\mu \dot{H}_m = 0. \quad (5)$$

Solving Eq. (5) we have a solution

$$\dot{H}_m = A e^{-\sqrt{j\omega\mu\gamma} \cdot x} + B e^{\sqrt{j\omega\mu\gamma} \cdot x}, \quad (6)$$

which after getting constants from boundary conditions gives (index "0" value for  $x = 0$ )

$$\dot{H}_m = \dot{H}_{m0} e^{-\sqrt{\frac{\omega\mu\gamma}{2}} \cdot x} \cdot e^{-j\sqrt{\frac{\omega\mu\gamma}{2}} \cdot x}. \quad (7)$$

\*Corr. Author's Address: High Technical School Belgrade, Bul. Nemanjića 33/39, 18000 Niš; Serbia; Lavmiro@eunet.rs

From the Eq. (7) we conclude that while penetrating the wave becomes suppressed in the conductible area according to the exponential law. Variable  $\Delta = \sqrt{2/\omega\mu\gamma}$  is the penetration depth within which the value of surface magnetic field falls down to  $1/e$  value of the field.

There is a term for magnetic field now

$$\dot{H}_m = \dot{H}_{m0} e^{-\frac{x}{\Delta}} \cdot e^{-j\frac{x}{\Delta}} \quad (8)$$

In a similar way we get the electric field power along axis x:

$$\dot{E}_m = \frac{\sqrt{2}}{\gamma\Delta} \dot{H}_{m0} e^{-\frac{x}{\Delta}} \cdot e^{j\left(\frac{\pi}{4} - \frac{x}{\Delta}\right)}, \quad (9)$$

which enables us to see that it is being moved in phases and that it is being exponentially suppressed along penetration axis. Penetrating, both fields are more and more late compared to inward field, and electric field is always ahead of magnetic field for  $\pi/4$ .

Current density is:

$$\dot{J}_m = \gamma \cdot \dot{E}_m, \quad (10)$$

that is after the shift from Eqs. (9) and (10):

$$\dot{J}_m = \frac{\sqrt{2}}{\Delta} \dot{H}_{m0} e^{-\frac{x}{\Delta}} \cdot e^{j\left(\frac{\pi}{4} - \frac{x}{\Delta}\right)}. \quad (11)$$

Surface power density is derived from Poynting vector:

$$\underline{S} = \frac{1}{2} [\underline{E}_m \cdot \underline{H}_m], \quad (12)$$

where  $\underline{S}$  represents flow energy vector,  $\underline{E}_m$  vector complex amplitude of the electric field and  $\underline{H}_m$  conjugated complex amplitude of the magnetic field, which transfer vector to scalar value, after being changed and fixed, gives:

$$\dot{S} = \frac{\sqrt{2}}{2\gamma\Delta} H_{m0}^2 e^{-\frac{2x}{\Delta}} \left( \cos \frac{\pi}{4} + j \sin \frac{\pi}{4} \right). \quad (13)$$

Only active power part performs heating and it is:

$$P = \frac{H_{m0}^2}{2\gamma\Delta} e^{-\frac{2x}{\Delta}}. \quad (14)$$

At the entrance into semi-infinite area,  $x=0$  and substitute value of  $\Delta$ , area power is:

$$P_0 = 0.5 \sqrt{\frac{\omega\mu}{2\gamma}} H_{m0}^2. \quad (15)$$

Within the penetration depth  $\Delta$ , 86% of overall heat is lost.

Previously described case of semi-infinite area expands in the [1] to [4] to the real case, that is to say, to welding of the rims of steel band of V loop which is in the zone of inductor and concentration field influence.

Starting from the basic equations of electro-magnetic circuit, an equivalent scheme can be established with concentrated parameters and it is shown in Fig. 2.

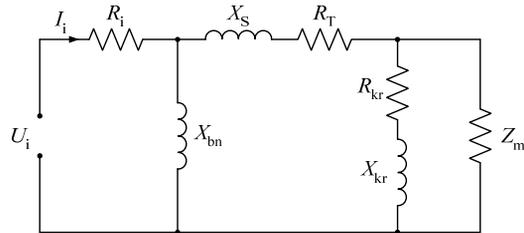


Fig. 2. Changeable scheme with concentrated parameters

Fig. 2 is valid for the case of encircling inductor which is most applied in practice, and the signs are:  $R_i$  is active resistance of single coil inductor,  $R_t$  is active resistance of steel pipe,  $X_s$  is inductive resistance of dispersion between inductor and pipes,  $X_{bn}$  is external resistance of dispersion,  $R_{kr}$  is active resistance of the steel band rims,  $X_{kr}$  is inductive resistance of the steel band rims and  $Z_m$  is impedance conditioned by the resistance of internal pipe opening and internal magnetic concentrator.

External inductive resistance of inductor dispersion is calculated by [1] to [4]:

$$X_{bn} = \frac{X_s k_m}{1 - k_m}, \quad (16)$$

where  $k_m$  represents Nakaoka coefficient whose values are in the Figure 3. depending on the relationships  $D_i/D$  and  $a_i/D$ , where  $D_i$  represents internal inductor diameter,  $D$  is external pipe diameter and  $a_i$  inductor length.

Nakaoka coefficient  $k_m$  is a function of many variables and its dependability is given in the Fig. 3 within the function  $D_i/D$  and the range  $a_i/D$  is taken as a parameter.

If known interpolation formulas are to be used we will get complex analytical terms which are inadequate for use in engineering practice.

In this paper with adequate substitution, before approximation, a simple link  $k_m$  is reached in the function of dependable variables which in the end gives simple analytical dependability.

The precision of Nakaoka coefficient calculation according to this approximation is illustrated by a practical example where other ranges which characterize the model of high-frequency welding of one solid steel pipe are calculated as well.

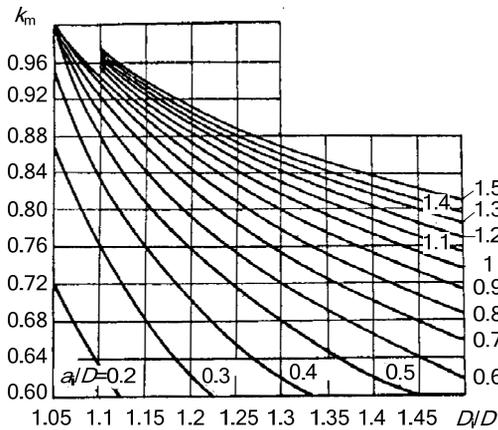


Fig. 3. Nakaoka coefficient values

1 APPROXIMATIVE CALCULATION BY THE USE OF NUMERIC METHOD

In order to calculate Nakaoka coefficient, whose graphic dependences are given in the Fig. 3, numeric method known from [7] as the method of the smallest squares will be used. Having in mind what has been presented in [7] as well as the experience in application and further research of the approximation of the author of this paper, adequate form for its use in programming consists of the following.

If there is an experimental or other cluster ( $x_i, y_i$ ) of specific values where ( $i = 1, 2, \dots, n$ ), then we can find the polynomial in the form of

$$y = \sum_{i=0}^m a_i x^i, \tag{17}$$

which approximates the function given by the cluster of points, where coefficients  $a_i$  are being determined by the system of

$$\sum_{j=0}^m w_{j+i} a_j = z_i, \quad (i = 0, 1, \dots, m). \tag{18}$$

The coefficient  $w_k$  from the system of Eqs. (18) is calculated by the formula:

$$w_k = \sum_{i=1}^n x_i^k, \quad (k = 0, 1, \dots, 2m), \tag{19}$$

unknown  $z_l$  from the relation:

$$z_l = \sum_{i=1}^n y_i x_i^l, \quad (l = 0, 1, \dots, m). \tag{20}$$

The precision of approximated function is achieved by an adequate choice of the  $m$  degree in the polynomial (17). The way this approximation is laid out in this paper offers the possibility of doing the programming in C language. The system of Eqs. (18) is solved by Gauss's elimination by means of which we do the calculation of coefficients of  $a_i$  polynomial (17) which approximates a specific functional dependence which is required.

Nakaoka coefficient, whose Graphic dependences are given in Fig. 3, is a function of 2 variables  $D_i/D$  and  $a_i/D$  and it is obvious that those are non-linear functions. Due to this, calculations at HF welding have been done in such a way that the value  $k_m$  has been determined graphically.

To make it simple, we introduce shift:

$$k_m' = \frac{1}{k_m}, \tag{21}$$

and thus by applying cited approximation and C programming, which is not being mentioned in this paper due to its size, linear dependences good at approximation are achieved, with a small bug, coefficient Nakaoka. During the approximation proportion  $a_i/D$  is used as the parameter and thus how many linear analytical dependences we will get depends on how many approximations we do. Let us list some of the achieved approximations:

$$\frac{a_i}{D} = 0.5, \quad k_m' = 0.9712 + 1.6210 \left( \frac{D_i}{D} - 1 \right), \tag{22}$$

$$\frac{a_i}{D} = 0.8, \quad k_m' = 0.9735 + 0.9987 \left( \frac{D_i}{D} - 1 \right), \tag{23}$$

$$\frac{a_i}{D} = 1.1, \quad k_m' = 0.9772 + 0.7298 \left( \frac{D_i}{D} - 1 \right). \tag{24}$$

From the cited relations the change of the coefficient in front of the bracket in the function  $a_i/D$  can be seen since that function is non-linear, and having in mind other approximations, finding reciprocal values algorithm for determining coefficients is done in a similar way. Based on the approximation bug, on many practical examples,

we get adequate linear approximation for engineering practice.

Since we introduced shifts in two steps and applied algorithm for approximate calculation of polynomial coefficient twice, by replacing the known and ordering we get:

$$k_m' = 1 + 0.73171 \left( \frac{D_i}{D} - 1 \right) \frac{a_i}{D} \quad (25)$$

and

$$k_m = \frac{1}{1 + 0.73171 \left( \frac{D_i}{D} - 1 \right) \frac{a_i}{D}} \quad (26)$$

In the Table 1 there are examples of calculation with the help of new analytical dependence of Nakaoka coefficient (26) which were compared to the values from Fig. 3 and the percentage of relative error discrepancies were calculated.

Table 1. Evaluating the approximation accuracy

$\frac{D_i}{D}$	$\frac{a_i}{D}$	$k_m$ Eq. (26)	$k_m$ Fig. 3	$\varepsilon$ [%]
1.4	1.2	0.804	0.800	0.50
1.1	0.5	0.8723	0.88	0.87
1.25	1.1	0.8574	0.855	0.28
1.325	0.85	0.78138	0.775	0.80
1.50	1.50	0.8039	0.815	1.36

By analyzing the movement of relative flaw from the Table 1 it can be seen that it does not exceed one percentage which is a very good approximation for engineering practice. That is why the dependence (26) for Nakaoka coefficient with the HF welding will be used further by making calculation process easier.

The inductive resistance of dissipation, between the inductor and steel pipes, from the scheme in the Fig. 2 and (16) is [1] to [7] given in the form of:

$$X_s = 0.25 \omega \mu_0 \pi \frac{D_i^2 - D^2}{a_i} \quad (27)$$

which after the shift in (6) gives

$$X_{bn} = 0.25 \omega \mu_0 \pi \frac{D_i^2 - D^2}{a_i} \cdot \frac{k_m}{1 - k_m} \quad (28)$$

By substituting the value  $k_m$  according to the new analytical dependence (26) into (28) we get:

$$X_{bn} = 0.342 \omega \mu_0 \pi (D_i + D), \quad (29)$$

through which we come to the new conclusion which was hidden until now, that the exterior induction resistance of inductor dissipation, which is supplied by HF current, does not depend on the length of the inductor. Such a conclusion can be used when optimizing parameters in the process of HF welding which was presented here by the substitution Fig. 2.

In order to check the results of the paper for calculating according to the new dependence and due to the presentation of the special values of the substitution scheme in the Fig. 2, a practical solving example will be given.

*Example:* Design and calculate the parameters of HF welding of the steel pipe of 76.1 mm in diameter and of 4 mm wall thickness in such a way that production velocity is 60 m/min. The interior diameter of the inductor is 85 mm, generator frequency is 440 kHz, welding temperature is 1500 °C, mean distance of the inductor from the joint point of the tape rim 111 mm, 5.36 mm thick conduction pellet in the space of 600 mm. The gap between ferrite and interior rim of the steel tape is  $b_l = 5$  mm and the inductor length is  $a_i = 72$  mm. Heating characteristics are adopted in the usual way like in [1] to [6].

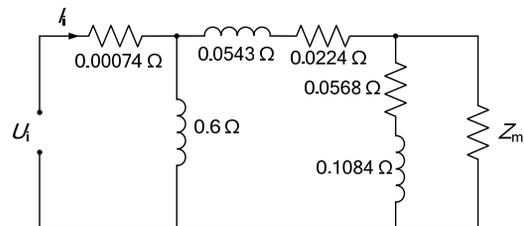


Fig. 4. The substitutional scheme with calculated values for the pipe of 76.1 mm in diameter and 4 mm of wall thickness

After the application of the procedure from [1] to [19] and this paper we get special values of the equivalent scheme.

In order for this welding to take place, the power  $P_{tr} = 163.8$  kW which is released on the rims of the steel tape is needed; thus the power flowing through the tape is  $I_{tr} = 1698$  A and voltage of the steel tape noose is  $U_{tr} = 207.8$  V.

The coefficient of the power use is  $\eta_i = 0.717$  and electric coefficient of the useful effect is  $\eta = 0.985$  and with this the calculation of

the necessary power for the inductor has been enabled:

$$P_i = \frac{P_w}{\eta_i \cdot \eta} = 232 \text{ kW} \quad (30)$$

By elementary calculation we get the inductor current

$$I_i = 2160 \text{ A} \quad (31)$$

and the voltage on the inductor

$$U_i = 297 \text{ V} \quad (32)$$

It has been considered that impedance  $Z_m$ , which is the function of the interior opening of the pipe and the characteristic of ferrite as the concentration field, has an infinite value. The depth of the heat penetration in the steel tape is  $\Delta_k = 0.824 \times 10^{-3} \text{ m}$ .

Having in mind the approximate losses in the oscillator and other generator appliances as well as done calculation it is proven that it is really possible to make a quality welding of this steel pipe by the chosen velocity of the welding.

## 2 CONCLUSION

The general theory of HF welding has been described in this paper starting with the case of electromagnetic wave influence on half-infinite environment.

A real model has been formed, for a so-called V noose formed by the rims of steel tape during the pipe welding and equivalent substitutional scheme has been given. The problem of calculating unknown values is pointed out when using complex analytical expressions and graphic dependences dependent on many parameters.

In order to get analytical dependences, a numerical coefficient approximation has been approached and, so far, it has had only graphic presentation. A numerical approximation method has been chosen and adjusted and it is adequate for solving through the application of the C programming language.

The procedure of numerical approximation is made easier by introducing adequate scheme prior to algorithm application. As a result, the simple formula for calculating Nakaoka coefficient value has been reached as a function of two variables.

By applying it to many practical examples it can be concluded that approximative formula, beside being simple, has good approximation

because relative flaw of line deviation is equal to one percentage. A new conclusion has been drawn that the inductive resistance of inductor dissipation is independent of the inductor length which opens up a new way of optimizing parameters.

The results of this paper are applied to the complete calculation of HF welding of the steel pipe of 76.1 mm in diameter and 4 mm wall thickness which is of great interest for engineering and production practice and it represents a new way of further research in the field of electromagnetic theory applied in the field of thermia. The application is principal through optimizing parameters with the tendency of achieving greater energy saving and better use coefficient along with improved structure and the quality of welded and thermally treated products.

## 3. REFERENCES

- [1] Slukhotskii, A.E. (1968). *Application HF current for electrothermia*. Mechanical Publ., St. Petersburg. (In Russian)
- [2] Slukhotskii, A.E., Ryskin, S.E. (1974). *Inductors for induction heating*. Mechanical Publ., St. Petersburg. (In Russian)
- [3] Shamov, N.A., Lunin, V.I., Ivanov, N.V. (1977). *High frequency metal welding*. Mechanical Publ., St. Petersburg. (In Russian)
- [4] Milićević, M., Milićević, V. (2002). Impeder for HF inductive welding of steel tubes. *IEE Proceedings, Science, measurement and technology*, vol. 149, no. 3., p. 113-116.
- [5] Milićević, M., (2007). Possible defect of the high frequency inductive welding in the steel tubes by applying a ferrite impeder. *39<sup>th</sup> International October Conference on Mining and Metallurgy*, Soko Banja, p. 318-333
- [6] Mitani, K., Shibuya-Ku, H. (1992). *Impeder: How its innovation and design impacts the welding process. The 8<sup>th</sup> annual world tube congress, Proceedings*, Chicago, p. 25-33.
- [7] Milovanović, G. (2005). *Numerical Analysis*. Faculty of Electronic Engineering, Niš. (In Serbian)
- [8] Nemkov, V.S., Demidovich, V.B. (1988). *Theory of induction heating*, Energy Publ., St. Petersburg. (In Russian)

- [9] Wright, J. (1997). *Principles of high frequency induction tube welding*. Electronic heating equipment, Sumner Inc.
- [10] Rudnev, I.V. (1997). *Induction heat treatment*. Steel heat treatment handbook, New York.
- [11] Ruffini, S.R., Ruffini, T.R., Nemkov, S.V. (1998). *Advanced design of induction*. Industrial Heating, Madison Heights, Michigan.
- [12] Wade, J. (1990). *Effective utilisation of magnetic flux concentrators in induction heating at commercial heat treating plant*. Heat treatments, Cleveland.
- [13] Electronic Heating Equipment, inc. (1998). *1998 Catalog & Applications Guide*, Buckley, p. 1-18.
- [14] *TDK Impeder core* (1999). Technical documentation, TDK Italy.
- [15] Todorov, T.S., Ivanov, P.T., Milićević, M., Madjarov, N., Iliev, D., Aleksiev, D. (2000). *Specialized high frequency power supplies for induction heating applications IHS-2000*, Padua.
- [16] Milićević, M., Milićević, T. (2005). New solution for improvement of parameters and quality of HF inductive welding of steel tubes. *37<sup>th</sup> International october conference on mining and metallurgy*, Bor, p. 466-475.
- [17] Milićević, M., Milićević, V., Milićević, T. (2004). MDM impeder for improvement of parameters and quality of HF inductive welding. *36<sup>th</sup> international october conference on mining and metallurgy*, Bor, p. 531-537.
- [18] Milićević, M., Milićević, V. (2004). *Analysis of the transistor converter of power together with energy dosage for the inductive heating and welding of steel tubes*. ETEP, vol. 14, no. 2, p. 111-118.
- [19] Milićević, M., Radaković, Z. (2006). *Quality improvement of steel pipes produced by seam welding with new magneto-dielectric impeder*. *Materials transactions*, The Japan institute of metals, vol. 47, no. 6, p. 1464-1468.