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Gear Vibrations in Supercritical Mesh-Frequency Range Caused by Teeth Impacts

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After gear teeth impact, natural free vibrations arise, attenuating in a short period of time. Teeth impacts repeat with the frequency of teeth entering the mesh, vibrations become restorable, and restore with teeth mesh frequency. In the range of sub-critical teeth mesh frequency range these natural free vibrations are covered by forced vibrations caused by the fluctuation of teeth deformations. In the super-critical mesh frequency range, restorable free vibrations dominate in the frequency spectrum of gear system vibrations. These restorable free vibrations effectuate the increase of total vibration level with the speed of rotation increase. Also, in this frequency range the modal structure (natural frequency) of the gear system is not stable and effectuates super-critical resonances arising. Gear vibration measurements and frequency analysis (FFT-Analysis) are performed in very high speeds of gear rotations as high as 40,000 rpm. A mathematical model for experimental results synthesis is established. For this purpose, the theory of singular systems is used. Gear teeth mesh is treated as a singular system, with a continual process of load transmission with singularities caused by teeth impacts. Damping coefficients and energy attenuation is determined using the developed mathematical model.

Keywords: gears, vibration, singular system, frequency spectrum

0 INTRODUCTION

Gear vibrations have been the subject of studies for a long time. Research was oriented, both theoretically and experimentally, toward identifying the teeth mesh process or toward analysing gear drive system behaviour. Different models were used, starting from a single-degree model for teeth mesh analysis to complex multi-degree models for identification of effects in a complete gear transmission system. Excitation arises from the teeth mesh. Basically, during the process of gear meshing two excitation processes are performed: fluctuation of gear teeth deformation and teeth impacting. Both of them enable the inclusion of the influence of teeth transmission errors, teeth stiffness non-linearity, friction forces, etc. Calculations of gear vibrations are predominantly performed in the form of forced vibrations caused by the fluctuation of gear teeth deformation. Compared with experimental results, this approach produces satisfactory results in sub-critical and in resonant mesh frequency range. However, in super-critical mesh frequency range (extremely high rotation speeds) the difference between the calculated and measured vibrations is significant. In this frequency range, the calculated level of vibrations decreases with increase of rotation speed, however, measured gear vibration level slightly increases (Fig.1).

In the Gear research centre, TU Munich (FZG) managed very intensive investigations of gear vibrations 40 years ago. This research was analytical and experimental [1] and [2]. Knabel [2] performed detailed calculations of forced vibration caused by fluctuation of teeth deformations. Calculations were carried out using the model of three-degree freedom of meshed gear pair and analogy calculation system (computer). In Fig. 1, line 2 presents one of these results. The same vibrations were measured (line 1 in Fig. 1), and the difference between the obtained vibration levels in the supercritical teeth mesh-frequency range is evident. In [1] and [2] the basic line 3 (Fig. 1) is defined, which presents basic and general increase of gear vibrations. Using the results of the FZG, the gear calculation procedure was standardized [3]. Internal dynamic forces are involved in calculations by dynamic factor K_v (Fig. 2). In supercritical teeth mesh-frequency range is defined as independent of the teeth mesh-frequency, however, with the value close to maximal values in sub-critical range if teeth mesh frequency f is not higher than natural f_n more than 2.5 times.

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Further research was performed mainly in sub-critical and critical gear rotation speeds. Vibrations in those calculations are excited by time function of fluctuation of teeth in mesh deformation [4] and [5]. This function is suitable for inclusion of transmission errors and their effects analysis [6], inclusion of sliding friction effects [7] and [8], dynamic loads calculations [6] and [9]. All of these and similar calculations or analyses were processed in sub-critical and critical teeth mesh frequency ranges. Supercritical teeth mesh frequency range was not processed in the mentioned analysis. Research in the field of gear vibration was also performed in respect of non-linearity [10] and [11], and it was found that the effects of non-linearity are not significant. In some research works on gears, analyses of balance of vibration energy [12] and [13] were also performed. Calculations of gear vibrations using a discrete systems approach in some of the research works were coupled with FEM calculations of elastic systems [13] and [14].

In order to define the nature of gear vibrations in supercritical teeth mesh frequencies and to define a mathematical model, so as to make a synthesis of the measured results, a new approach to gear vibrations treatment is taken. Gear vibrations are defined as restorable free (natural) vibrations caused by teeth impact at the moment of contact start (addendum impact). Every impact disturbs natural free damped vibrations which are restored with a new teeth impact. The theory of singular systems is used to develop a mathematical model.

1 PROBLEM FORMULATION

1.1 Comparing Measured and Calculated Gear Vibrations

The measured results of vibration for one gear pair in a very wide teeth mesh-frequency range [2] show fluctuations, which can be explained by comparing them to those calculated (Fig. 1). Measurements were carried out to 20,000 rpm and the main resonance was at 5000 rpm (Fig. 1, line 1). The range of other 15,000 rpm was supercritical, where the level of vibration was approximately between 50*g* and 100*g* (*g* - earth acceleration). After the main resonance acceleration decreased to the level of 40*g*, it then slightly increased with the speed of rotation increase. Using the model presented in Fig. 1, vibrations were calculated and are presented by Fig. 1, line 2. Excitation was performed by the function of stiffness fluctuation in the gear teeth mesh. In the supercritical mesh-frequency range, the calculated level of vibration decreased to minimal values. This is a phenomenon which needs a new approach to the process of description and modelling.

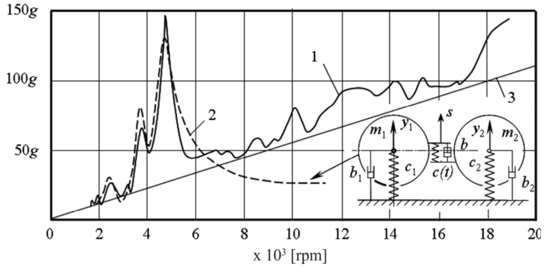

Fig. 1. *Comparing measured and calculated gear vibration level* [2]

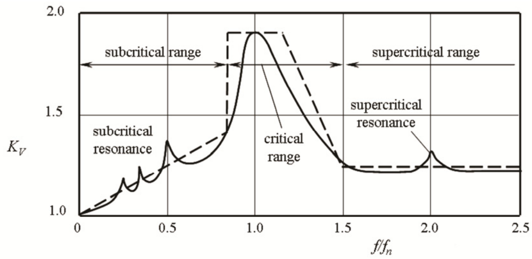

Fig. 2. *Dynamic factor* K_v [3] *and approximation of gear dynamic forces and vibration*

For practical use and load capacity calculation of gear drives, dynamic factor K_v is defined and standardized by ISO 6336. For this purpose, measured dynamic forces and vibrations were used, and values of K_v were defined separately for sub-critical, critical and supercritical teeth mesh-frequency range (Fig. 2). The levels of dynamic forces for a practical use are approximated by interrupted lines. Resonant teeth mesh-frequency is defined based on middle gear teeth stiffness and equivalent mass of connected gears and other rotating masses. In the supercritical teeth mesh-frequency range, the level of dynamic forces is approximated by a horizontal line, i.e. the value of K_v is independent of teeth mesh-frequency increase (if $f < 2.5\ f_n$). However, it is important that the level of K_v in this range is significant, and it is necessary to investigate this phenomenon.

1.2 Addendum Gear Teeth Impact

There are a few kinds of teeth impacts during gear meshing. Much stronger than others is addendum impact, especially in spur gears without teeth flank corrections. Teeth deformations are proportional to teeth load and teeth stiffness. Deformations replace the first point of contact from the right position A, to position A' which is ahead of point A. Contact of teeth pair starts with intensive addendum impact (Fig. 3a). Collision speed v_c is proportional to teeth deformation, speed of rotation n and gear design parameters. By analyzing teeth geometry, deformations and speeds, collision speed at the first point of teeth contact is defined as:

$$v_c = r_{b1}\omega_1\left(1+\frac{1}{u}\right)\left(1-\frac{\cos(\alpha'+\varphi)}{\cos\alpha_w}\right). \tag{1}$$

Angular speed $\omega_1 = 2\pi n_1$, n_1 - revolution per minute (rpm) of pinion, transmission ratio $u = z_2/z_1$, teeth number of connected gears are z_1 and z_2, and other parameters are presented in Fig. 3a. In Fig. 3b relative collision speed v_c/n for the chosen parameters for spur gear pair is presented ($z_1 = z_2 = 25$, module $m = 5$ mm, offset factors $x_1 = x_2 = 0$). For the other spur gear pairs with the radius of basic circle of pinion r_{b1} in mm, using ratio (v_c/n) from diagram in Fig. 3b, collision speed is

$$v_c = \left(\frac{v_c}{n}\right)\left(1+\frac{1}{u}\right)\frac{r_{b1}}{117.5}n_1 . \tag{2}$$

Collision speed v_c is defined in the direction of teeth contact line. For a further application of this speed, it is necessary to transform the model of rotating masses into a harmonic oscillator. Inertia moments of rotating masses (of gears and all others connected to them) J_1 and J_2 should be transformed in concentrated masses in the teeth contact line direction,

$$m_{t1} = \frac{J_1}{r_{b1}^2}; \qquad m_{t2} = \frac{J_2}{r_{b2}^2}. \tag{3}$$

The radii of the basic circle of gear pair are $r_{b1} = (mz_1/2)\cos\alpha$ and $r_{b2} = (mz_2/2)\cos\alpha$, where $\alpha = 20^0$. The collision force of concentrated masses is:

$$F_c = v_c\sqrt{c' m_e}\ ; \quad m_e = \frac{m_{t1}m_{t2}}{m_{t1}+m_{t2}}, \tag{4}$$

where c' is teeth stiffness at the moment of collision and m_e equivalent mass.

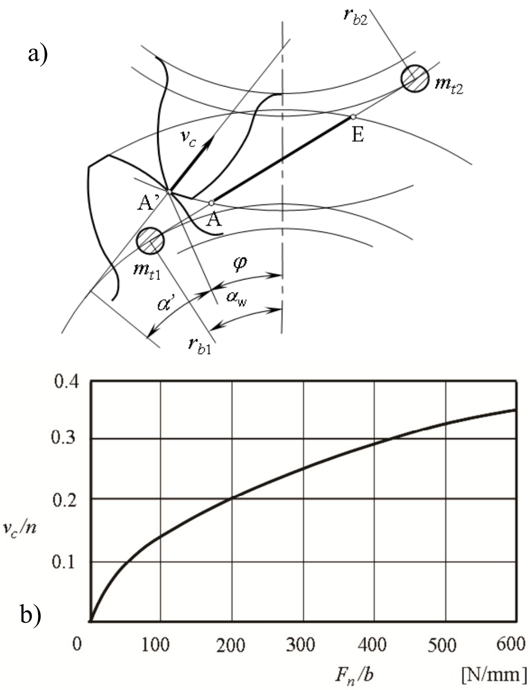

Fig. 3. *Teeth collision: a) addendum collision, b) relative speed of addendum collision for chosen gear parameters*

1.3 Testing Rig for Vibrations in Supercritical Mesh Frequency Range

For the purpose of gear vibration in the supercritical teeth mesh-frequency range, a specific testing rig is designed and realised (Fig. 4). In order to perform the extreme high speed of rotation, the masses of rotating components are of relatively small dimensions, with specific type of lubrication and sealing. The speed of rotation can vary from zero to 40,000 rpm measured at the shaft of pinion z_1. The torque which applies load in gear teeth is made by middle coupling using the principle of back-to-back system. The application of torque takes place before the rotation and vibration measurement. For the experiments presented in this work, the torque in the coupling (gear $z_2 = 47$) was $T_2 = 30$ Nm, and in the pinion z_1 = 32 it was T_1 = 20.4 Nm. The speed of rotation $n = n_1$ in rpm was measured in the shaft of the pinion z_1 = 32 and teeth mesh-frequency was calculated $f = nz_1/60$. The accelerometer for vibration measurement is fixed by the screw on the left gear housing. The position of the accelerometer is presented in Fig. 4b. The direction of the main accelerometer sensitivity is covered with gear contact line direction. The dynamic forces from the teeth mesh area are transmitted to the housing walls through bearings. Vibrations in the teeth contact area and in the housing walls differ in intensity but the structure of vibration spectrums is very similar. This relation is provided by a specific design of housing which eliminates additional natural frequencies with significant effects. The difference in the vibration level is proportional to vibration transmissibility factor between the teeth mesh area and the position point of the accelerometer.

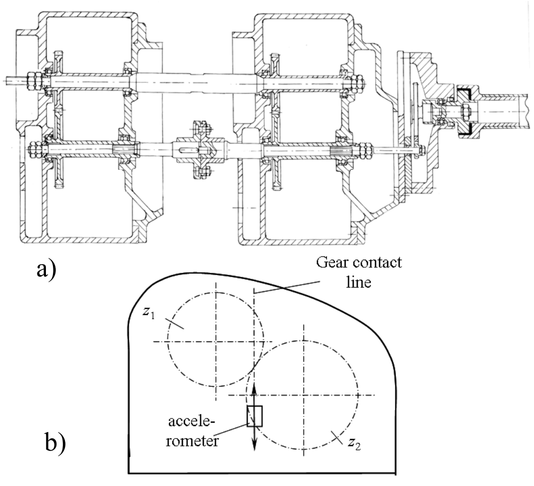

Fig. 4. *Back-to-back test rig: a) gears centre distance surface, b) right side view (centre distance a* = 85 mm, z_1 = 32, z_2 = 47, *m* = 2 mm, *b* = 8.5 mm*)*

Measurement and spectrum analysis were performed using software for this purpose for real time and FFT analysis. Prior to measurement, the modal testing of back-to-back system was performed. The system was excited by the impact in the gear flank. One of those results is presented in Fig. 5. The three natural frequencies were detected, f_{n1} and f_{n2} caused by corresponding shafts and shaft supports elasticity and f_n caused by elasticity of gear teeth in mesh. Before measuring the vibrations with a high speed of rotation, measurements with very slow speed of rotation were done. The aim of those measurements was to detect vibrations caused by separated teeth impacts. The results of these measurements are presented in Fig. 6. After the impact, gears vibrate with natural frequency f_n. This vibration was measured in a tangent direction in the gears. After a short time, the vibration is damped. The next impact excites a new damped vibration, again and again. Small speed of rotation produces vibration, as presented in Fig. 6.

The measured results presented in Fig.6 show that gear vibrations contain restorable free component. Teeth collisions repeat with teeth mesh frequency *f* and restore a new cycle of free damped vibrations. This effect in the gear vibration analysis first used by Umezawa and thereafter Cai, reference [4], is referred to as the Umezawa's effect. Using this effect and doing calculations, Cai [4] obtained an increase in vibrations in the supercritical teeth mesh frequency range, however, the range was very small. If the gears are damaged [16] these effects become dominant in the frequency spectrum.

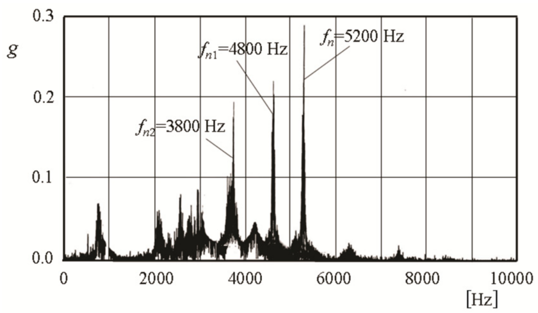

Fig. 5. *Natural frequencies of testing rig*

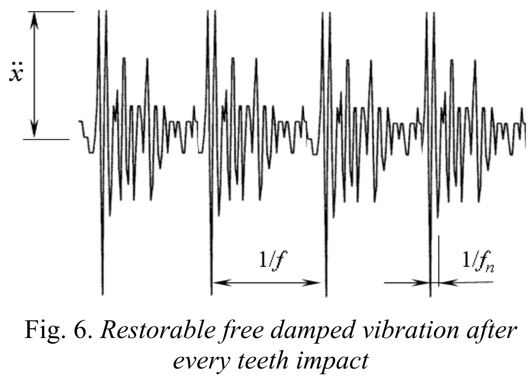

Fig. 6. *Restorable free damped vibration after every teeth impact*

2 ANALYSES OF EXPERIMENTAL RESULTS

Using installation, as presented in Fig. 4, the measurement was performed in the range as high as 40,000 rpm of pinion z_1 = 32. The main objective was to obtain similar results presented in Fig. 1 [2] and identify the phenomenon which increases vibration level in the supercritical mesh-frequency range. In addition, an objective was to identify "supercritical resonances" (Fig. 2) and the phenomenon which creates fluctuation of vibration level in this teeth mesh-frequency range. One of the measured results is presented in Fig. 7. At the teeth mesh-frequency $f = f_n$ = 5200 Hz (Fig. 5) the main resonance increased vibration level to 168*g*. There is a small difference between the natural frequencies f_{n1} = 4800 Hz and f_n = 5200 Hz and it is possible to conclude that both of them were affected at a very high level of resonant vibration. After resonance, the level of vibration was decreased to 40*g* and then fluctuated to 60*g* for 20,000 rpm. In the range of up to this speed, something was changed in the tested system. The level of vibrations decreased to 20*g* for 28,000 rpm and then fluctuated between 20*g* and 45*g*. These results are not identical with those presented in Fig. 1, because the testing rigs are not the same, but the phenomenon is similar.

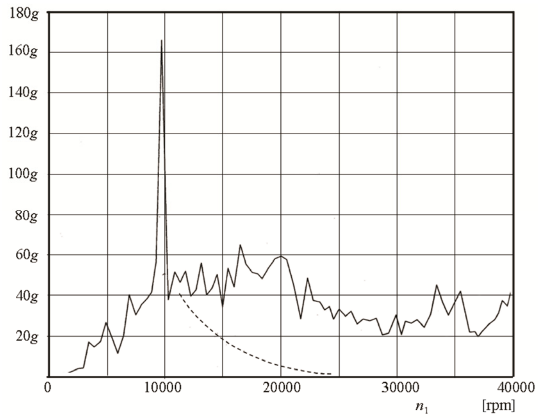

Fig. 7. *Total level of gear vibration measured using installation presented in Fig. 4*

For the purpose of identifying the structure of gear vibrations, a spectral analysis was carried out. Using software for Furrier transformation, the spectrums of frequencies and amplitudes of component time functions were obtained. In the range of sub-critical area frequency, the spectrums consist of time functions with teeth mesh-frequencies f and their higher harmonics $2f$, $3f$, etc. and with natural frequencies f_{n1}, f_{n2} and f_n. The value of amplitudes of free vibrations increases when mesh frequency gets close to some of naturals. In Fig. 8a one of those spectrums is presented for the speed of rotation n_1 = 4050 rpm. In full resonance, teeth mesh-frequency became equal to the main natural one $f = f_n$ = 5200 Hz. It was for the speed of rotation n_1 = 9750 rpm. In frequency spectrum (Fig. 8b) only very high amplitudes dominate with resonant frequency of 5200 Hz.

After resonance, a further increase of the speed of rotation revealed one specific phenomenon. In frequency spectrum, vibration with teeth mesh-frequencies f increasingly decreases until it disappears, as indicated by the interrupted line in Fig. 7. The total level of vibration in this area is the result of natural vibrations with natural frequencies (Fig. 8c). Within the range of speed of rotation n_1=10,000 to 20,000 rpm (Fig. 7), frequency spectrums are similar to the spectrum in Fig. 8c, i.e. vibrations within this range are natural (free) vibrations with frequency f_n. With an increase in the speed of rotation, the intensity of the teeth impact also increases and the level of natural vibrations increases. In the frequency spectrum in Fig. 8c, the vibration amplitude increases with the increase of the speed of rotation. It should be noted that in the $f > f_n$ range the situation is opposite to that in Fig. 6. The time of impact repetition is shorter than the period of natural free vibrations $1/f < 1/f_n$.

Within the range of the pinion speed of rotation n_1 = 20,000 to 40,000 rpm (Fig. 7) the level of vibrations was first decreased and then, for higher speeds, was increased again. Frequency analyses reveal one additional phenomenon. In the frequency spectrum (Fig. 8d) the amplitude with frequency f disappears completely and all natural frequencies become active. The spectrum becomes crowded with the already detected f_{n1}, f_{n2}, f_n and of many new ones. The amplitudes for these frequencies are relatively small but when combined, using corresponding phase positions, they create a total level of vibrations, which corresponds to the level in Fig. 7. Also, it is noticeable that teeth impact energy is distributed to all of these natural vibrations, and this can be the reason why they have small amplitudes.

Additionally, the modal structure of the tested system at these high speeds of rotation is changed. This phenomenon of modal structure instability at a very high speed of rotation has been identified in other experiments unrelated to gear testing.

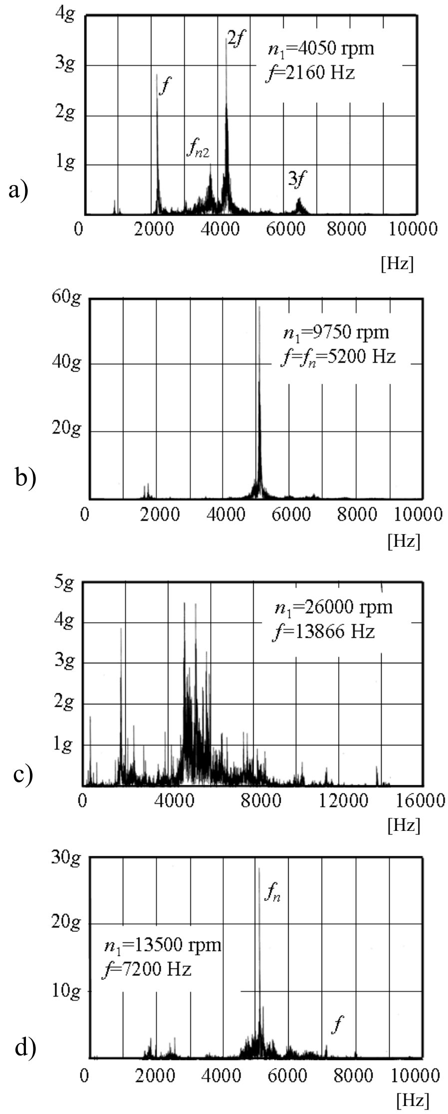

Fig. 8. *Frequency spectrums of gear vibrations presented in Fig.7*

Modal instability and only free vibrations in the range of very high speeds of rotation can explain the fluctuation of vibration level in this range of speeds. When many natural frequencies occur, the level of vibrations decreases. For higher speeds these natural frequencies disappear again and, with one of them or a few of them, the level of vibrations increases. In Fig. 1 this occured for 19,000 rpm, and in Fig. 2 for $f/f_n = 2$, and is marked as supercritical resonance. The possible explanation of supercritical resonances can be found in model instability and in the concentration of all modal frequencies (shapes) in a small group or in one group only.

3 ANALYTIC MODELLING OF MEASURED RESULTS

The gear teeth meshing process presents by itself a singular process [15] which contains two processes: continual and transient one. The continual process is a continual increase of the free vibration level (x_a by frequency f_n) with increase of teeth mesh frequency f. The transient process is the gear response also with a natural frequency f_n after teeth impact, especially in resonance areas.

Collision force (Eq. 4) increases proportionally to the speed of rotation. For a gear pair with a certain tooth load (Fig. 3b), using Eqs. (2) and (4), this force can be presented in direct relation with teeth mesh frequency f and constant K,

$$F_c = K\, f\ ; \qquad f = \frac{n\, z_1}{60}\ ;$$
$$K = \sqrt{c' m_e}\left(\frac{v_c}{n}\right)\left(1 + \frac{1}{u}\right)\frac{r_{b1}}{117.5}\frac{60}{z_1}\ . \tag{5}$$

The impact force increases proportionally to the speed of rotation (teeth mesh frequency f). This force makes deformation in teeth contact direction x_c and produces deformation work

$$W = \frac{F_c\, x_c}{2} = \frac{K\, x_c}{2} f\ . \tag{6}$$

Displacement x_c (deformation) is independent of the force F_c and of the speed of rotation. This deformation depends on the gear load, i.e. consists of teeth deflection and teeth contact deformations. There is a difference between teeth deformations at the moment of teeth impact x_{max} and before impact x_{min} ($x_c = x_{max} - x_{min}$) with amplitude $x_0 = x_c/2$ and frequency f. Deformation work W absorbs elastic system and part of it returns in the form of vibration. These are damped free vibrations with natural frequency f_n. The potential energy of these vibrations is:

$$E_p = \frac{c_\gamma x_a{}^2}{2}; \qquad x_a = Ax_0 f. \tag{7}$$

Displacement x_a presents gear vibration after impact (Fig. 6). This response is much weaker compared to teeth impact. The ratio between the vibration energy and disturbance energy is marked by a constant A and by product Af. The units for the constant A are seconds and product Af is a dimensionless parameter which defines the ratio between amplitude of impact displacement x_0 and amplitude of free vibrations with frequency f_n. This product $(Af)^2$ shows how much energy is produced by vibrations compared to deformation work absorbed by impact elastic deformations.

3.1 Continuous Process

Impact energy increases continually with teeth mesh frequency f increase. Potential energy E_p is a part of impact energy which also increases continually. Potential energy is released in the form of natural free vibration, such as vibrations presented in Fig. 6. Kinetic energy of gear vibrations is $E_k = m_e \dot{x}_a{}^2/2$. The transformation of potentional into kinetic energy is defined by Lagrange's equations. For a certain mesh frequency f, these equations for natural free vibrations are as follows:

$$\frac{1}{dt}\frac{\partial E_k}{\partial \dot{x}_a} + \frac{\partial E_p}{\partial x_a} = 0; \quad m_e \ddot{x}_a + c_\gamma x_a = 0. \tag{8}$$

This vibration between the two impacts (Fig. 6) is with damping, i.e. with dissipation of vibration energy. The part of Eq. (8) which can present this effect is removed due to the fact that with impact repetition by frequency f, the free natural vibration with frequency $f_n = \left(\sqrt{c_\gamma / m_e}\right)/2\pi$ is restored. Those are restorable free vibrations. In the supercritical mesh frequency range ($f > f_n$) this kind of dissipation is not effective. The damping effect in the system response is included in the transient part of this model. Using Eqs. (7) and (8), the first part of acceleration level in direction of contact line is:

$$\ddot{x}_a = A\frac{c_\gamma x_0}{m_e} f. \tag{9}$$

This is the first (algebraic) part of a singular solution (Fig. 9). Therefore, with the increase in mesh frequency f acceleration increases, and this in turn is proportional to the increase in the absorbed disturbance energy.

3.2 Transient Process

Teeth collision presented in Fig. 3a generates collision force F_c which is repeated with mesh frequency f. This force produces two kinds of teeth deformations (displacements). The total displacement is $x = x_a + x_b$. The first x_a is already included in the continuous process and this part corresponds to the force and torque which meshed gears transmit (corresponds to gear pair load). The second part x_b is additional displacement which is the result of inertia after the teeth impact and corresponds to the gear pair sensitivity.

Gear pair can be presented by an equivalent single mass model with the mass m_e (Eqs. 3 and 4) supported by mean teeth in mesh stiffness c_γ with damping with coefficient b. The action of force F_c in this model produces the additional transmitted force:

$$F_T = b\dot{x}_b + c_\gamma x_b, \tag{10}$$

where x_b is the additional displacement caused by teeth impact. Since forces $b\dot{x}_b$ and $c_\gamma x_b$ are 90° out of phase, the magnitude of the additional transmitted force is:

$$|F_T| = \sqrt{b^2 \dot{x}_b{}^2 + c_\gamma{}^2 x_b{}^2}. \tag{11}$$

The ratio of the additional transmitted force $F_T = m_e \ddot{x}_b$ to the applied $F_{ca} = m_e \ddot{x}_a$ can be expressed in terms of a transmission function, i.e. transmissibility:

$$\frac{F_T}{F_{ca}} = \frac{\ddot{x}_b}{\ddot{x}_a} = \zeta_T \sin(2\pi f t - \varphi),$$

$$\zeta_T = \sqrt{\frac{1 + (2\zeta f / f_n)^2}{\left(1 - f^2/f_n{}^2\right)^2 + (2\zeta f/f_n)^2}}, \tag{12}$$

$$\varphi = \tan^{-1}\frac{2\zeta(f/f_n)}{1 - f^2/f_n{}^2 + 4\zeta^2 f^2/f_n{}^2}.$$

The function ζ_T is well known as a transmission function for a single degree model excited by acceleration. These relations were obtained in the form of response ratio, where $\zeta = 0...1$ is a dimensionless damping parameter. At the moment of impact (t = 0), the responded force is $F_T = F_{ca}\zeta_T \sin\varphi$ and the responded part of vibrations for the single degree mathematical model is:

$$\ddot{x}_b = \ddot{x}_a \zeta_T \sin\varphi . \tag{13}$$

This is the response at the moment of impact after which vibrations continue with natural frequency f_n. This is the transient part of vibration which together with the continuous one produces the total gear vibration level. The maximal response of the system, i.e. ζ_T is for full resonance when $f = f_n$, and $\varphi = \pi/2$.

3.3 Total Level of Vibration

By summing the results of continual and transient process the total level of vibrations is:

$$\begin{aligned} \ddot{x} &= \ddot{x}_a + \ddot{x}_b = \ddot{x}_a \left(1+\zeta_T \sin\phi\right) = \\ &= A\frac{c_\gamma x_0}{m_e} f\left(1+\zeta_T \sin\phi\right). \end{aligned} \tag{14}$$

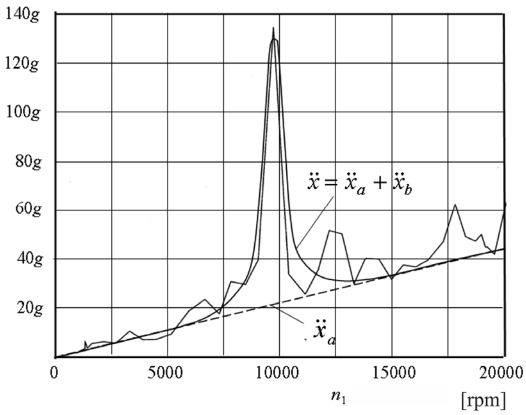

Fig. 9. *Relation between measured and calculated level of gear vibrations*

Using the presented mathematical model and corresponding software, the curve of the total level of gear vibration was calculated (Fig. 9). Calculations were carried out using software developed for equation (14) and parameters of the tested gear pair. The damping parameters A and ζ were adapted to the measured level of vibrations. These parameters offer the opportunity to analyse the relation between the energy of the gear teeth impact and the energy dissipated inside the machine parts. For this purpose, vibrations measured to 20,000 rpm were used, i.e. in the speed range before vibration level starts to fluctuate in a wider range (Fig. 7).

The presented model gives the opportunity to compare the measured and calculated results and to check the hypothesis about the nature of gear vibration as well as to analyze the gear vibration phenomenon. In addition, the model provides the possibility of analyzing impact energy balance of accounts, i.e. energy distribution within the system, and of damping coefficient calculations. For this purpose, vibrations of the testing rig (Fig. 4) were calculated and analyzed. The parameters of the presented testing rig are as follows: inertia moments of rotating masses are $J_1 = 5.351 \cdot 10^{-4}$ kgm^2 and $J_2 = 16.117 \cdot 10^{-4}$ kgm^2 which include mass inertia of the corresponding shaft with both gears and other parts in the shaft. Basic radii of the tested gears are r_{b1} = 0.0323 m and r_{b2} = 0.0475 m (module m = 2 mm and teeth numbers $z_1 = 32$, $z_2 = 47$), transformed masses m_{t1} = 0.5351 kg, m_{t2} = 0.714 kg and equivalent mass of the system m_e = 0.2978 kg. The gears are connected by stiff shafts and one shaft with both gears was treated like one rotating mass. Mean stiffness of the teeth in mesh for both gear pairs, including the shaft effect is $c_\gamma = 3.14 \cdot 10^8$ N/m. Back-to-back system (Fig. 4) was loaded by the torque of 30Nm at the gear z_2, i.e. with the normal force in the flanks of each gear pair F_n = 631 N. This force makes deformations of gear teeth in mesh with amplitude $x_0 = 0.674935 \cdot 10^{-6}$ m. Using Eqs. (9), (13) and (14), and the measured result of gear vibrations, the following gear parameters were calculated: response constant $A = 5.92848 \cdot 10^{-5}$ seconds, and product Af presents vibration response caused by gear teeth impact. In resonant conditions vibration response is increased by $(1+\zeta_T)$ = (1+6.5) = 7.5 times. Dimensionless damping coefficient is ζ= 0.078 and damping coefficient in gear mesh $b = 1.61 \cdot 10^{-5}$ Ns/m.

4 CALCULATED AND MEASURED RESULTS ANALYSIS

By the measurement of the gear vibration in the super-critical mesh frequency range, the phenomenon of gear vibration increase is a proof. Also, in this range natural free vibrations only are identified. After every teeth impact, gears vibrate with natural frequencies and these are restorable free vibrations. With teeth mesh frequency the absorbed disturbance power and the level of natural vibrations are increased. By measured vibration analysis, the nature of gear vibrations in the super-critical mesh frequency range is identified. An additional phenomenon in the form of supercritical resonance is recognised. Following the nature of gear vibrations, a specific mathematical model is established, using the singular system theory. The model of restorable free vibrations consists of two parts; a continuous and transient one. Both of them present the gear vibrations as natural free vibrations with natural frequencies f_{ni} . The teeth mesh frequency f is the parameter which corresponds to disturbance energy absorption by repeatable impacts. The teeth impact force is, in this way, included in both parts of the mathematical model. The results of the continuous part of the mathematical model follow the increasing trend of total vibrations. The results of the transient part follow variations of the total level of gear vibrations caused by resonances.

The main objective of specific mathematical modelling i.e. a synthesis of the measured vibrations is to present the vibrations by following the nature of experimental results. The next objective is to identify the relation between the absorbed disturbance energy and the realised energy by natural free vibration (vibration power). For this purpose, it was necessary to identify damping parameters which include the inside energy dissipation and the outside energy dissipation in contact (elastic deformations, frictions, etc.). Using identified parameters for a calculation of gear vibrations, the calculated results are equal to those measured. The line of the calculated results (Fig. 9) follows the main resonance but not the other smaller resonances. By the presented mathematical model, it is possible to satisfy secondary resonances and cover the measured results much better.

5 CONCLUSIONS

By measurement, frequency analysis, mathematical modelling and calculation, the main hypothesis about gear vibrations nature has been confirmed. In the range of supercritical teeth mesh frequency ($f > f_n$), those are restorable free vibrations caused by teeth impact. After every impact the free vibrations with natural frequency are restored. The level of these vibrations increases with an increase in teeth mesh frequency f which increases teeth impact intensity.

A mathematical model is developed to simulate excitation process caused by teeth impacts and to synthesize the measured results of gear vibrations. According to the singular systems theory, gear meshing is presented in the form of a continual and transient process. The continual process is presented by a continual part of the model using algebraic equation. The transient process which includes resonances, is presented by transfer function of the single mass model of the tested gear system.

Using the measured results and the developed mathematical model, a few key facts are analyzed. Restorable free vibrations are proportional to teeth impact intensity and increase with enlargement of the teeth mesh frequency. In the resonant range the system response additionally magnifies free vibration level with natural frequency. The quantity of the impact disturbance energy which is released by gear vibration is defined by the value of constant A. Damping of gear free vibrations is presented by dimensionless coefficient ζ. Numerical values of both of these parameters are calculated.

The presented approach explains gear vibrations in the supercritical teeth mesh frequency range and this explanation is based on the modal structure of the mechanical system. In this frequency range the structure is not stable. For some teeth mesh frequencies those main natural frequencies separate into a number of new ones, while for others they unify. When natural frequencies separate, the level of free vibrations decreases and for the unified it increases. This explains the phenomenon called “supercritical resonance”.

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7 REFERENCES

[1] Winter, H., Gerber, H., Muller, R. (1989). Investigations on the excitation of vibration and noise at spur and helical gears. *Proceedings of the Power Transmission and Gears*, Chicago, vol. 2, p. 765-772.

[2] Knabel, W. (1977). Noise and vibration of gear transmissions. *Doctoral dissertation*, Technical University of München. (in German)

[3] Höhn, B.R. (2002) Modern gear calculation. *Proceedings of the International conference on gears*, München, vol.1, p. 23-43.

[4] Cai, Y. (1995). Simulation on the rotational vibration of helical gears in consideration of the tooth separation phenomenon (A new stiffness function of helical involute tooth pair. *ASME -Journal of Mechanical Design*, vol. 117, p. 460-468.

[5] Huang, K.J., Liu, T.S. (2000). Dynamic analysis of a spur gear by the dynamic stiffness method. *Journal of Sound and Vibration*, vol. 234, p. 311-329.

[6] Tamminana, V.K., Kahraman, A., Vijayakar, S. (2007). A study of the relationship between the dynamic forces and the dynamic transmission error of spur gear pairs. *ASME Journal of Mechanical Design*, vol. 129, p. 75-84.

[7] Vaishia, M., Singh, R. (2003). Strategies for modelling friction in gear dynamics. *ASME Journal of Mechanical Design*, vol. 125, p. 383-393.

[8] He, S., Gunda, R., Singh, R. (2007). Inclusion of sliding friction in contact dynamics model for helical gears. *ASME Journal of Mechanical Design*, vol. 129, p. 48-57.

[9] Verdmar, L., Andersson, A. (2003). A method to determine dynamic loads on spur gear teeth and on bearings. *Journal of Sound and Vibration.* vol. 267, p. 1065-1084.

[10] Parker, R.G., Vijayakar, S.M., Imajo, T.B. (2000). Non-linear dynamic response of a spur gear pair: modelling and experimental comparisons. *Journal of Sound and Vibration,* vol. 237, p. 435-455.

[11] Vaishya, M., Singh, R. (2001). Sliding friction-inducted non-linearity and parametric effects in gear dynamics. *Journal of Sound and Vibration*, vol. 248, p. 671-694.

[12] Dimitrijevic, D., Nikolic-Stanojevic, V. (2007). Eigenfrequency analysis of spur gear pair with moving eccentric masses on the body of one of the gears. *FME Transactuions,* vol. 35, no. 3, p. 157-163.

[13] Pavic, G. (2005). The role of damping on energy and power in vibrating systems. *Journal of Sound and Vibration,* vol. 281, p. 45-71.

[14] Sabot, J. (1999). Integrated vibroacoustic approach to compute and reduce gear transmissions noise. *Proceedings of the International conference on Power transmission*. Pairs, p. 2039-2050.

[15] Debeljkovic, D., Owens, D. (1985). On practical stability of singular systems. *Proceedings of the Melecon Confenence.* Madrid, p. 103-105.

[16] Belšak, A., Flašker, J. (2008). Vibration analysis to determine the condition of gear units. *Strojniški vestnik - Journal of Mechanical Engineering*, vol. 54, no. 1, p. 11-24.