Entropy Generation and Exergy Efficiency in Adiabatic Mixing of Nitrogen and Oxygen Streams of Different Temperatures and Environmental Pressures

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The paper presents a non-dimensional model of entropy generation and exergy efficiency in the adiabatic mixing of a nitrogen stream and oxygen stream with different temperatures and environmental pressure. The resulting mixture has the same, i.e. environmental, pressure. Non-dimensional variables in the model represent the ratio of thermodynamic stream temperatures u, the ratio between the thermodynamic temperature of one stream and the environmental temperature u_1, and the mole fraction y_1 of one of the streams. The model comprises irreversibility due to different temperatures of streams because of the mixing of different gases, while exergies of streams also comprise their mole fractions in the ambient air (atmosphere). In this respect, the model takes the standard values of mole fractions of oxygen and nitrogen of $y_{O2} = 0.21$ and $y_{N2} = 0.79$. The values of stream mole fractions at which maximum values of entropy generation and minimum values of exergy efficiencies occur are given. Calculation results are given in respective diagrams.
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0 INTRODUCTION

Adiabatic mixing of ideal gas streams often occurs in thermal engineering. Usually, it is required for obtaining a particular temperature of the stream at the mixing chamber inlet. For given conditions of inlet streams, the mass flow ratio of streams, gives the required temperature at the outlet of the mixing chamber. Such processes belong to a group of characteristic irreversible processes in which the irreversibility is caused by two factors: a change in stream temperatures and the mixing of different ideal gases. Both of these factors, and consequently the total irreversibility and the exergy efficiency, are directly affected by mole fractions of particular streams in the mixture. As expressions for exergy streams in this paper also contain their partial pressures (or concentrations) in the environment, the analytic model also gives expressions for the streams of two-atom gases, i.e. of oxygen and nitrogen, whose mole fractions in the surrounding air (atmosphere) are known. These gases have the same isentropic exponent of $\kappa = 1.4$ [1].

1 ANALYTIC MODEL

1.1 Non-Dimensional Representation of Entropy Generation

In the observed case, two streams of gases enter the isolated system, relevant for the entropy increase; therefore, according to [2], the entropy increase of the isolated system is expressed by the Eq. (1):

$$\left(\Delta \dot{S} \right)_{\text{is.syst}} = q_{\text{n1}} \left(C_{\text{m}p} \ln \frac{T_{\text{m}}}{T_1} - R_{\text{m}} \ln \frac{p_1^{'}}{p} \right) + \\ + q_{\text{n2}} \left(C_{\text{m}p} \ln \frac{T_{\text{m}}}{T_2} - R_{\text{m}} \ln \frac{p_2^{'}}{p} \right). \tag{1}$$

One can easily obtain the temperature T_m at the outlet of the adiabatic mixing chamber from the First Law of thermodynamics:

$$T_{\text{m}} = y_1 T_1 + y_2 T_2 . \tag{2}$$

Using known Eqs., according to [2]:

$$C_{\text{m}p} = \frac{\kappa R_{\text{m}}}{\kappa - 1}; \ y_1 = \frac{p_1^{'}}{p} = \frac{q_{\text{n1}}}{q_{\text{n}}}; \ y_2 = 1 - y_1$$

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and introducing the non-dimensional ratio of the temperatures of inlet streams,

$$u = \frac{T_2}{T_1} . \qquad (3)$$

Eq. (1) can be easily transformed into a non-dimensional form of:

$$\frac{\left(\Delta \dot{S}\right)_{is.syst}}{q_n R_m} = (y_1 - 1)\ln(1 - y_1) - y_1 \ln y_1 +$$
$$+ \frac{\kappa}{\kappa - 1}\left[y_1 \ln u + \ln \frac{y_1(1-u)+u}{u} \right]. \qquad (4)$$

In Eq. (4), the first two members on the right side of the equation represent the entropy increase caused by the mixing of two different gases, while the last addend represents the entropy increase caused by different inlet temperatures of gases. The value of isentropic exponent κ is 5/3, 7/5 and 4/3 for one-atom, two-atom and three-atom gases, respectively [1]. From the same Eq., a physically justified fact that for y_1 = 0.0 and y_1 = 1.0, the value of entropy increase of the isolated system is equal to 0 can easily be obtained. If $u = 1.0$ is inserted into Eq. (4), then only the first two members remain, representing only the entropy increase caused by the mixing of two streams of different ideal gases. A detailed analysis of such a case can be found in [3]. If Eq. (4) is represented as a function of the mole fraction y_1, for given values of u and κ, a diagram given in Fig. 1 is obtained.

The diagram shows that each curve, with its respective u, has its maximum for a physically justified local extreme and that each parametric curve u has a determined mole fraction y_{1stat} (abscissa of a stationary point), which follows from a necessary condition for the existence of the function extreme (4). From this condition, the following equation is derived:

$$\left[y_{1stat}(1-u)+u \right]\ln \frac{1-y_{1stat}}{y_{1stat}} +$$
$$+ \frac{\kappa}{\kappa-1}\left[\left(y_{1stat}(1-u)+u \right)\ln u + 1 - u \right] = 0. \qquad (5)$$
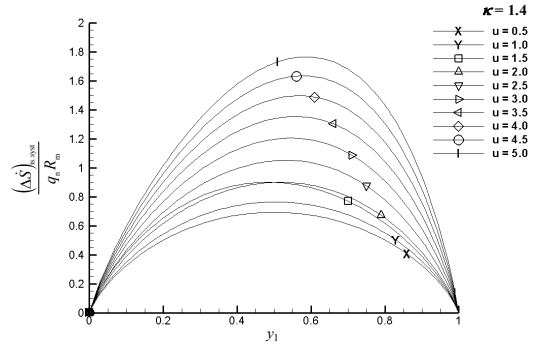

Fig. 1. *Quantitative representation of non-dimensional entropy generation depending on the mole fraction y_1, temperature ratio u in the adiabatic mixing of two streams of two different two-atom gases, $\kappa = 1.4$*

It is obvious that the required value y_{1stat} from Eq. (5) cannot be expressed explicitly; therefore, Newton's method [4] is used to solve the above equation. Table 1 gives the results of a numerical estimation of y_{1stat} together with the respective maximum entropy increases, for $\kappa = 1.4$ and for specified values of u.

From Table 1, it can be observed that y_{1stat} increases with an increase in u. If $u = 1.0$, then the entropy increase is caused only by the mixing of two different gas streams. In that case the value of $y_{1stat} = 0.5$ with the respective entropy increase of 0.693 follows explicitly from Eq. 5 since the addend from this Eq. is dropped. If the mixing of two streams of the same gases, the same pressure and different temperatures is concerned, then y_{1stat} can also be directly (explicitly) expressed from Eq. 5 since the first addend from the above Eq. is dropped.

$$y_{1stat} = \frac{u - 1 - u\ln u}{(1-u)\ln u} . \qquad (6)$$

If the values of y_{1stat} obtained from Eqs. (5) and (6) are shown in a diagram, the result is a quantitative representation given in Fig. 2.
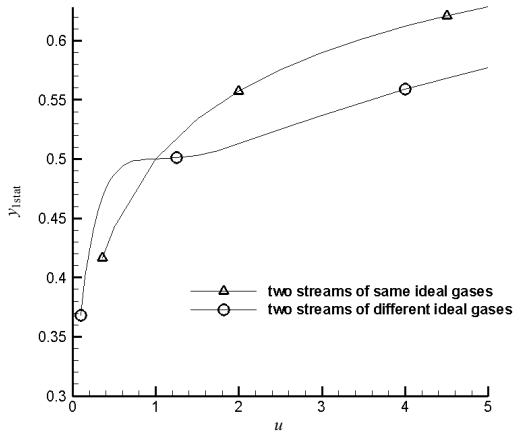
Table 1. *Values of mole fraction y_{1stat} and respective entropy increases depending on the value u*

u	0.1	0.25	0.50	0.75	1.0	2.0	3.0	4.0	5.0
y_{1stat}	0.36	0.432	0.483	0.498	0.50	0.517	0.545	0.569	0.587
$\Delta S_{is.syst}/(q_n R_m)$	2.808	1.497	0.900	0.729	0.693	0.900	1.204	1.497	1.765

Fig. 2. *Effect of the mixing of two streams of different ideal gases on the displacement of y_{1stat} depending on the temperature ratio u*

The diagram shows the effect of the mixing of two different ideal gases on the displacement of y_{1stat} at which maximum entropy increases occur. For $u < 1.0$, the mixing of different ideal gases displaces the stationary point to higher values, while for $u > 1.0$ the displacement is in the direction of lower values. For $u = 1.0$, $y_{1stat} = 0.5$ and at that point, the function under consideration has the inflection point with the horizontal tangent.

2 EXERGY EFFICIENCY AND AN ANALYSIS OF THE MIXING PROCESS

Exergy destruction of the observed process can be adequately described by means of the so-called exergy efficiency, which can, according to [5], be defined in the following way:

$$\varepsilon_{ex} = 1 - \frac{T_{env}\Delta\dot{S}_{is.syst}}{\dot{E}_1 + \dot{E}_2}, \qquad (7)$$

where $\dot{E}_1$ and $\dot{E}_2$ represent the exergies of streams of oxygen and of nitrogen at the mixing chamber inlet, which can be determined, according to [6], by the following Eq.:

$$\dot{E}_1 = q_{n1}\left(C_{mp}(T_1 - T_{env}) - T_{env}\left(C_{mp}\ln\frac{T_1}{T_{env}} + R_m\ln\frac{1}{y_{1env}}\right)\right), \qquad (8)$$

$$\dot{E}_2 = q_{n1}\left(C_{mp}(T_2 - T_{env}) - T_{env}\left(C_{mp}\ln\frac{T_2}{T_{env}} + R_m\ln\frac{1}{y_{2env}}\right)\right). \qquad (9)$$

The mole fraction of oxygen in the air (atmosphere) is $y_{O2} = y_{1env} = 0.21$, and of nitrogen $y_{N2} = y_{2env} = 0.79$, so that these members in the above Eqs. give an additional contribution to the exergy value of the streams with respect to their concentration in the environment.

By using Eqs. (1) to (3) and Eqs. (7) and (8), it is possible to transform Eq. (7) into the following form:

$$\varepsilon_{ex} = 1 - \frac{u_1\left[y_1\left(\ln u + \frac{\kappa-1}{\kappa}\ln\frac{1-y_1}{y_1}\right) + \ln\frac{y_1(1-u)+u}{u} - \frac{\kappa-1}{\kappa}\ln(1-y_1)\right]}{y_1\left(1 - u + u_1\ln u + u_1\frac{\kappa-1}{\kappa}\ln\frac{1}{y_{1env}}\right) + u - u_1 - u_1\ln\frac{u}{u_1} + u_1\frac{\kappa-1}{\kappa}\ln\frac{1}{y_{2env}}}, \qquad (10)$$

where

$$u_1 = \frac{T_{ok}}{T_1}. \qquad (11)$$

It can be easily proved that from Eq. (11) it follows that the value of exergy efficiency is equal to one for $y_1 = 0.0$ and for $y_1 = 1.0$ since in these cases, either stream 2 or stream 1 is involved. Consequently, there is no mixing process and as a result no exergy destruction.

The results of exergy efficiency calculation for $u_1 = 0.5$, 1.0 and 2.0, depending on the mole fraction and parametric values of $u = 0.5$; 1.0; 2.0; 3.0; 4.0 and 5.0, are shown in diagrams in Figs. 3 to 8.
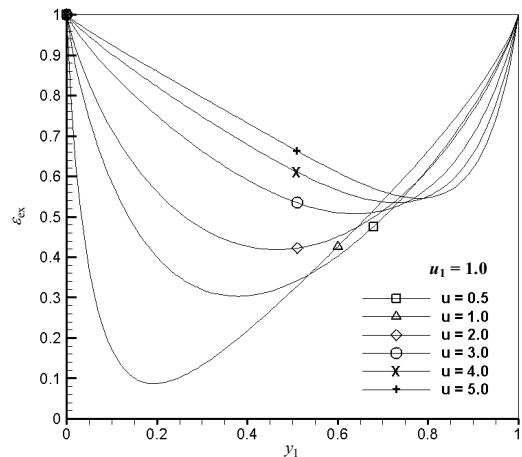

Fig. 3. *Dependence of exergy efficiency on the mole fraction y_1 and the temperature ratio u for $u_1 = 1.0$*

The diagram shows that each parametric value of u has the minimum value ε_{min} for the unambiguously determined mole fraction $y_1 = y_{1stat}$. It can be noted that while the value of u is increasing towards one, the value of ε_{min}

decreases, and the values of respective $y_1 = y_{1stat}$ also decrease. For $u > 1.0$, the values of ε_{min} increase with an increase in u, as well as the values of $y_1 = y_{1stat}$, i.e. stationary points move to the right. The values of y_{1stat} follow from the necessary condition of the existence of an extreme in Eq. (10). The equation which follows from this condition has to be solved numerically. In this case, Newton's method is applied again and the calculation results are shown in the diagram in Fig. 4. For the limit values of $y_1 = 0.0$ and $y_1 = 1.0$, regardless of all parametric values of u, the values of exergy efficiency are equal to one.
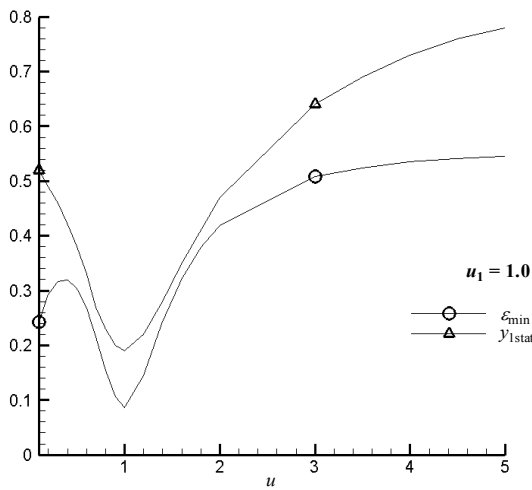

Fig. 4. *Value of mole fractions y_{1stat} and respective minimum exergy efficiencies $\varepsilon_{ex\,min}$ depending on the values of u for $u_1 = 1.0$*

The diagram shows that the value of the function y_{1stat} decreases at first and reaches the minimum for $u = u_1 = 1.0$, and then increases continuously. But the curve representing the function $\varepsilon_{ex\,min}$ shows two local extremes, a maximum and a minimum. The maximum occurs close to $u = 0.5$, and the minimum occurs for $u = 1.0$, where the minimum absolute value of exergy destruction is also obtained. For $u > 1.0$, the function $\varepsilon_{ex\,min}$ rises continuously.

If Eq. (10) is quantified for $u_1 = 0.5$, with the same parametric values of u as in the previous case, the results shown in the diagram in Fig. 5 are obtained.

The diagram shows that in this case the mole fraction y_{1stat} moves to the area of higher values with respect to the previous case. The

values of y_{1stat} with the respective ε_{exmin}, also obtained by a numerical solution of the condition of extreme in Eq. (10), are shown in the diagram in Fig. 6.
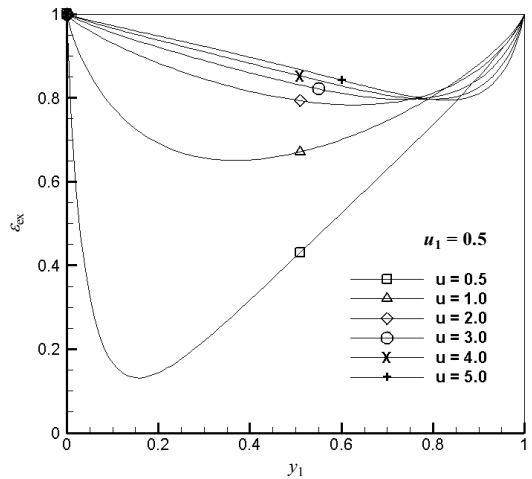

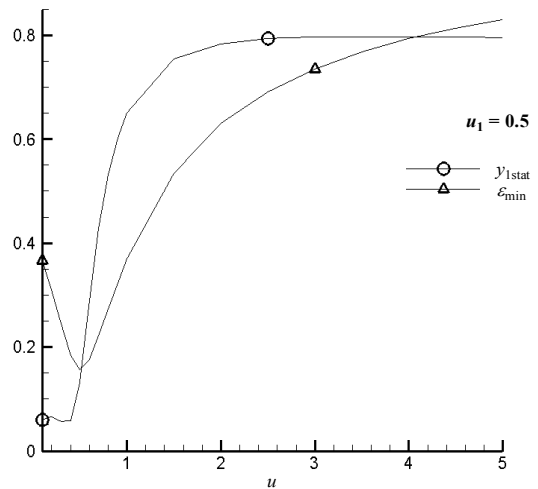Fig. 5. *Exergy efficiency depending on the mole fraction y_1 and the temperature ratio u for $u_1 = 0.5$*

Fig. 6. *Values of mole fractions y_{1stat} and respective minimum exergy efficiencies $\varepsilon_{ex\,min}$ depending on the value of u for $u_1 = 0.5$*

The diagram shows that the function y_{1stat} has a minimum for y_{1stat}. The function of respective $u = u_1 = 0.5$ also shows a minimum value in the absolute sense for $u = u_1 = 0.5$. After that, the function rises steeply and after $u > 1.5$ it reaches almost a constant value, which is only slightly sensible to the values $1.5 < u < 5.0$. A more detailed analysis shows that for $u = 3.5$, the

function $\varepsilon_{e\ min}$ shows a maximum value of $\varepsilon_{ex\ min} = 0.79726$, while $\varepsilon_{ex\ min}$ ($u = 5.0$) = 0.7942, which provides a numerical confirmation of the previous statement.

Figs. 8 and 9 give a graphical representation of the function analysis (10) for $u_1 = 2.0$.
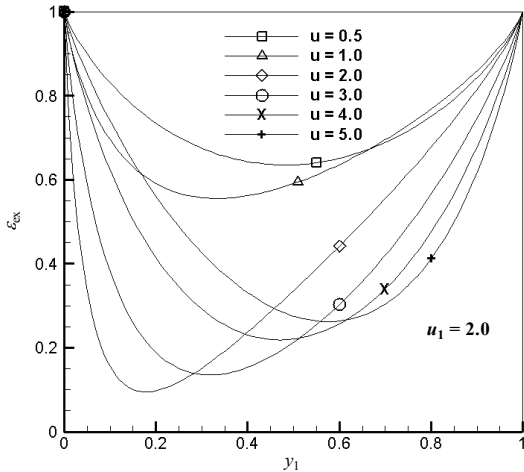

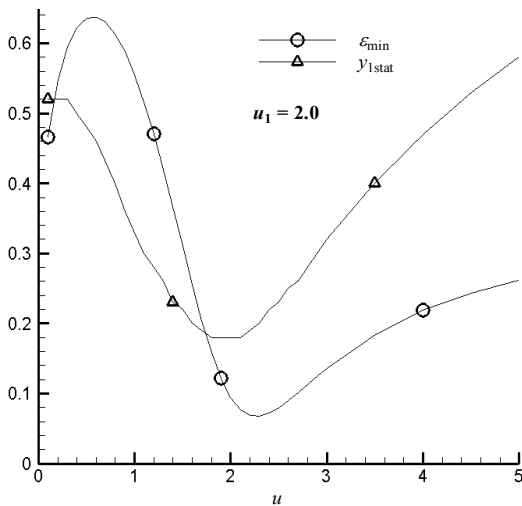Fig. 7. *Exergy efficiency depending on the mole fraction y_1, and temperature ratio u for $u_1 = 2.0$*

Fig. 8. *Values of mole fractions y_{1stat} and respective minimum exergy efficiencies ε_{min} depending on the value of u for $u_1 = 2.0$*

Fig. 7 shows that in the case of a minimum value of $\varepsilon_{ex\ min}$, lower values of mole fractions y_{1stat} are obtained with respect to the case in which $u_1 = 1.0$. In addition, it can be concluded that these values of local extremes are situated in a significantly large area of mole fractions y_{1stat}, with respect to the previous two cases. This is explicitly shown in the diagram in Fig. 8.

The diagram shows that in this case again the function y_{1stat} has a local extreme (minimum) for $u = u_1 = 2.0$, while the maximum of $y_{1stat} = 0.52$ = const. is obtained for a relatively wide range of u, i.e. $0.1 < u < 0.3$. After that, y_{1stat} falls to the absolute minimum of y_{1stat} ($u = 2.0$) = 0.18, and it continuously rises to y_{1stat} ($u = 5.0$) = 0.26197. The function $\varepsilon_{ex\ min}$ shows a local extreme (maximum) for $u = 0.6$ ($\varepsilon_{ex\ min}(u = 0.6) = 0.63753$). After that, the function $\varepsilon_{ex\ min}$ falls, but it does not reach its minimum for $u = 2.0$ but for $u = 2.3$, where the value of the minimum equals 0.06783. Logically, having reached its minimum, the function $\varepsilon_{ex\ min}$ rises continuously and reaches the value of $\varepsilon_{ex\ min}(u = 5.0) = 0.2697$ at the end of the examined interval.

3 CONCLUSION

The non-dimensional analytic model for the calculation of the isolated system entropy increase presented in this paper can generate not only the expressions related to the mixing of two two-atom gases, but also the ones related to the mixing of two streams having the same isentropic exponent κ. Here, the model enables the quantification of the effects of all relevant variables, i.e. of the ratio of stream temperatures, mole fractions and isentropic exponent, on the entropy increase. It has been shown that for every ratio of inlet stream temperatures there is a mole fraction of one of the streams, the so-called "stationary" mole fraction y_{1stat}, for which a maximum entropy increase is obtained, as shown in the diagram in Fig. 1. The effect of the ratio of inlet stream temperatures u on the value of y_{1stat} has been quantified, and the calculation results have shown that the value of y_{1stat} increases with an increase in the ratio u.

Secondly, it has been established that the mixing of two streams of different ideal gases moves y_{1stat} towards higher values for the values of $0 < u < 0.5$, in comparison with the mixing of two streams of the same ideal gases. For $0.5 < u < 1$, the situation is reversed. As expected, for $u = 1.0$, the same value of $y_{1stat} = 0.5$ (Fig. 2) is obtained.

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On the other hand, it has been shown by the model that the exergy efficiency and exergy destruction are affected not only by the factors listed above, but also by the environmental temperature, which has been introduced into the algorithm by the temperature ratio u_1, together with the mole fraction (concentration) of stream gases in the environment (atmosphere). This is the reason why the streams of oxygen and nitrogen, whose mole fractions in the environment (atmosphere) are known (fixed), have been analysed. A quantitative analysis carried out for particular given values of relevant variables shows that for every ratio of inlet temperatures there is a stream mole fraction for which a minimum value of exergy efficiency or a maximum value of exergy destruction is obtained. Figs. 3, 5 and 7 show a quantitative effect of relevant variables on the value of exergy efficiency $\varepsilon_{\text{ex min}}$. If these minimum values for a given u_1 are shown as depending on u, then the minimum value of $\varepsilon_{\text{ex min}}$, also in its absolute value, occurs for $y_{1\text{stat}}(u \approx u_1)$. This means that the highest rate of exergy destruction is generated by processes in which the mole fraction amounts to $y_1 = u = u_1$, as shown quantitatively by Figs. 4, 6 and 8. Also, from these diagrams one can quantitatively determine the effect of the values of u, for a given u_1, on the value of $\varepsilon_{\text{ex min}}$.

4 NOTATIONS

$C_{\text{m}p}$ molar heat capacity $p = $ const [J/(kmol K)]

$\dot{E}_{\text{ex}}$ exergy [W]

ε_{ex} exergy efficiency

κ isentropic exponent

q_{n} quantity flow [kmol/s]

p pressure [Pa]

p' partial pressure [Pa]

R_{m} universal (molar) gas constant [J/(kmol K)]

$\dot{S}$ entropy flow [W/K]

T thermodynamic temperature [K]

u ratio of thermodynamic temperatures of inlet streams

u_1 ratio of thermodynamic stream temperature and the environment

y stream mole fraction in the mixture

Indices

is.syst isolated system

m condition of the stream at the mixing chamber outlet

min minimum

env environmental

stat stationary (in relation to the function local extreme)

1 stream 1 at the mixing chamber inlet (stream of oxygen)

2 stream 2 at the mixing chamber inlet (stream of nitrogen)

5 REFERENCES

[1] Stephan, K., Mayinger, F. (1998). *Thermodynamik, Band 1: Einstoffsysteme, Grundlagen und technische Anwendungen*, 15. Auflage. Springer Verlag, Berlin.

[2] Bošnjaković, F., Knoche, K.F. (1988). *Technische Thermodynamik, Teil I,* 7., vollstaendig neubearbeitete und erweiterte Auflage. Steinkopf Verlag, Darmstadt.

[3] Bejan, A. (1996). *Entropy generation minimization.* CRC Press, New York.

[4] Kreyszig, E. (1993). *Advanced engineering mathematics*, 7. ed. John Wiley & Sons, Inc., New York.

[5] Moran, M.J. (1999). *Engineering thermodynamics.* CRC Press LLC, Ohio.

[6] Fratzscher, W., Brodjanskij, V.M., Michalek, K. (1986). *Exergy, Theirie und Anwendung.* VEB Verlag, Leipzig.