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Fatigue Life Estimation of Notched Structural Components

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This work considers the analytical/numerical methods and procedures for obtaining the stress intensity factors and for predicting the fatigue crack growth life of notched structural components. Many efforts have been made during the past two decades to evaluate the stress intensity factor for corner cracks and for through cracks emanating from fastener holes. A variety of methods have been used to estimate the stress intensity factor (SIF), values, such as approximate analytical methods, finite element method (FEM), finite element alternating, weight function, photo elasticity and fatigue tests. In this paper the analytic/numerical methods and procedures were used to obtain SIF, and predict the fatigue crack growth life for cracks at attachment lugs. Single through crack in the attachment lug analysis is considered. For this purpose analytic expressions are evaluated for SIF of cracked lug structures. For validation of the analytic stress intensity factors of cracked lugs, FEM with singular finite elements is used. Good agreement between computation and experimental results for fatigue life of aircraft cracked lugs was obtained. To determine crack trajectory of cracked structural components under mixed modes,, conventional singular finite elements and X-FEM are used.

Keywords: Notched structural components, lugs, analytic stress intensity factor of lugs, finite elements, X-FEM, fatigue life estimation

0 INTRODUCTION

Surface and through-thickness cracks frequently initiate and grow at notches, holes in structural components. Such cracks are present during a large percentage of the useful life of these components. Hence, understanding the severity of cracks is important in the development of life prediction methodologies [1]. Current methodologies use the stress intensity factor (SIF) to quantify the severity of cracks and the development of SIF solutions for notched structural components using analytical, numerical and semi-analytical methods has continued for the last three decades.

The adoption of the damage tolerance design concept [2] and [3] along with an increased demand for accurate residual structure and notched component life predictions have provided a growing demand for the study of fatigue crack growth in aircraft mechanical components. The damage tolerance approach assumes that the structure contains an initial crack or defect that will grow under service usage. The crack propagation is investigated to ensure that the time for crack growth to a critical size takes much longer than the required service life of notched structural components. For damage tolerance program to be effective it is essential that fracture data can be evaluated in a quantitative manner. Since the establishment of this requirement not only the understanding of fracture mechanics has greatly improved, but also a variety of numerical tools have become available to the analyst. These tools include Computer Aided Design (CAD), Finite Element Modelling (FEM) and Computation Fluid Dynamics (CFD). Fracture mechanics software provides the engineering community with this capability.

Computer codes can be used to predict fatigue crack growth and residual strength in aircraft structures.

They can also be useful to determine in-service inspection intervals, time-to-onset of widespread fatigue damage and to design and certify structural repairs. Used in conjunction with

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damage tolerance programs fracture analysis codes can play an important role in extending the life of "high-time" aircraft. Traditional applications of fracture mechanics have been concerned on cracks growing under an opening or mode I mechanism. However, many service failures occur from cracks subjected to mixed mode loadings. A characteristic of mixed mode fatigue cracks is that they usually propagate in a non-self similar manner. Therefore, under mixed mode loading conditions, not only the fatigue crack growth rate is of importance, but also the crack growth direction. Several criteria have been proposed regarding the crack growth direction under mixed mode loadings. In this work, maximum strain energy density criterion [4] and [5] is used. This S-criterion allows stable and unstable crack growth in mixed mode. The application of this criterion can be found from the works by several authors [6] and [7]. The aim of this work is to investigate the strength behaviour of an important aircraft notched structural elements such as cracked lugs and riveted skin.

1 NUMERICAL SIMULATION OF CRACK GROWTH

Numerical simulations of crack growth provide a powerful predictive tool to be used during the design phase as well as for evaluating the behaviour of the existing crack. These simulations can be used to compliment experimental results and allow engineers to economically evaluate a large number of damage scenarios. Numerical methods are the most efficient way to simulate fatigue crack growth because crack growth is an incremental process where stress intensity factors (SIF) values are needed at each increment as input to crack growth equations.

In order to simulate mixed-mode crack growth an incremental type analysis is used, where knowledge of both the direction and size of the crack increment extension are necessary.

For each increment of crack extension, a stress analysis is performed using the quarter-point singular elements (Q-E) [8] and SIF are evaluated. The incremental direction and size along the crack front for the next extension are determined by fracture mechanics criteria involving SIF as the prime parameters. The crack front is re-meshed and the next stress analysis is carried out for the new configuration.

2 STRESS INTENSITY FACTOR SOLUTIONS OF CRACKED LUGS

In general geometry of notched structural components and loading it is too complex for the stress intensity factor (SIF) to be solved analytically. The SIF calculation is further complicated because it is a function of the position along the crack front, crack size and shape, type loading and geometry of the structure. In this work analytic [3] and FEM [2] and [17] were used to perform linear fracture mechanics analysis of the pin-lug assembly. Analytic results are obtained using relations derived in this paper. Good agreement between the finite element and analytic results is obtained. This is very important because analytic derived expressions can be used as a useful approach in crack growth analyses. Lugs are essential components of an aircraft for which proof of damage tolerance has to be undertaken.

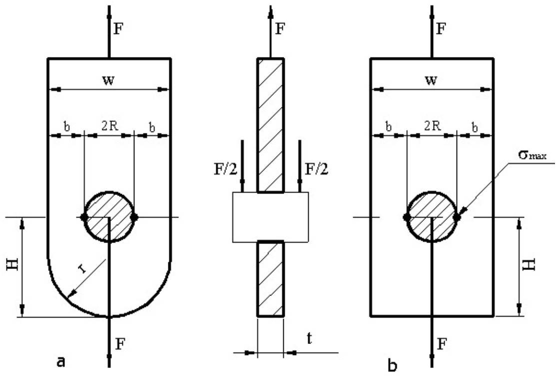

Fig.1. *Geometry and loading of lugs*

Since the literature does not contain the stress intensity solution for lugs which are required for proof of damage tolerance, the problems posed in the following investigation are: selection of a suitable method of determining other SIF, determination of SIF as a function of crack length for various form of lug and setting up a complete formula for calculation of the SIF for lug, allowing essential parameters. The stress intensity factors are the key parameters to estimate the characteristic of the cracked structure. Based on the stress intensity factors, fatigue crack growth and structural life predictions have been investigated. The lug dimensions are defined in Fig. 1.

To obtain the stress intensity factor for the lugs it is possible to start with a general expression for the SIF in the next form:

$$K = Y_{SUM}\,\sigma\sqrt{\pi a}\,, \tag{1}$$

where Y is the correction function, a is the crack length. This function is essential in determining the stress intensity factor. Primary, this function depends on stress concentration factor, k_t, and geometric ratio a/b. The correction function is defined using experimental and numerical investigations. This function can be defined in the next form [9] and [10]:

$$Y_{SUM} = \frac{1.12 k_t A}{A + \frac{a}{b}} z\ Q\,, \tag{2}$$

$$z = e^{r\sqrt{a/b}}\,, \tag{3}$$

$$b = \frac{w - 2\cdot R}{2}\,, \tag{4}$$

$$r = -3.22 + 10.39\cdot\left[\frac{2\cdot R}{w}\right] - 7.67\cdot\left[\frac{2\cdot R}{w}\right]^2, \tag{5}$$

$$Q = \frac{U\cdot\frac{a}{b} + 10^{-3}}{\frac{a}{b} + 10^{-3}}\,, \tag{6}$$

$$U = 0.72 + 0.52\cdot\left[\frac{2\cdot R}{H}\right] - 0.23\cdot\left[\frac{2\cdot R}{H}\right]^2. \tag{7}$$

The stress concentration factor k_t is very important in calculation of correction function, Eq. (2). In this investigation a contact finite element stress analysis was used to analyse the load transfer between the pin and lug.

3 CRACK GROWTH ANALYSES OF DAMAGED STRUCTURAL ELEMENTS UNDER MIXED MODES

To determine crack growth trajectory for structural components under mixed modes here conventional singular finite elements and X-FEM are used. The finite element method is widely used in industrial design applications and many different software packages based on FEM techniques have been developed. It has proved to be very well suited for the study of crack initiation and crack growth [1]. Over the past few decades, several approaches have been proposed to model crack problems: method based on quarter-point finite element [8]. To avoid the re-meshing step in crack modelling, drives techniques were proposed: the incorporation of a discontinuous mode on the element level [11], a moving mesh technique [4], and an enrichment technique based on a partition-of unity X-FEM. The essential idea in the extended finite element method is to add discontinuous enrichment functions to the finite element approximation using the partition of unity. An overview of the developments of the X-FEM method has been given by Rashid [12].

Several criteria have been proposed to describe the direction of crack propagation for mixed mode crack growth. Only the minimum strain energy density criterion [4] and [5] is discussed in this work. The strain energy density criterion is based on the postulate that the direction of crack propagation at any point along the crack front toward the region where the strain energy density factor is minimum. The strain energy density factor, *S*, is given as:

$$S(\theta) = a_{11}K_I^2 + a_{12}K_I K_{II} + a_{33}K_{III}^2\,, \tag{9}$$

where the factors a_{ij} are functions of the angle θ, and are defined as:

$$\begin{aligned} a_{11} &= \frac{1}{16G\pi}\left[(1+\cos\theta)(k-\cos\theta)\right], \\ a_{12} &= \frac{1}{16G\pi}\sin\theta\left[2\cos\theta-(k-1)\right], \\ a_{22} &= \frac{1}{16g\pi}\left[(k+1)(1-\cos\theta)+(1+\cos\theta)(3\cos\theta-1)\right], \end{aligned} \tag{10}$$

where G is the shear modulus and *k* is a constant depending upon stress state, and is defined as: $k = (3-\nu)/(1+\nu)$ for plane stress. The direction of crack of crack growth is determined by minimizing this equation with respect to the angle theta (θ). In mathematical form, the strain energy density criterion can be stated as:

$$\begin{aligned} &\left[2(1+k)\mu\right]\tan^4\frac{\theta}{2} + \left[2k(1-\mu^2)-2\mu^2+10\right]\tan^3\frac{\theta}{2} - \\ &24\mu\tan^2\frac{\theta}{2} + \left[2k(1-\mu^2)+6\mu^2-14\right]\tan\frac{\theta}{2} + \\ &2(3-k)\mu = 0\,, \end{aligned} \tag{11}$$

$$\left[2(k-1)\mu\right]\sin\theta - 8\mu\sin 2\theta + \left[(k-1)(1-\mu^2)\right] \cos\theta + \left[2(\mu^2-3)\right]\cos 2\theta \rangle 0 , \tag{12}$$

where $\mu = K_I / K_{II}$. Once S is established, crack initiation will take place in a radial direction *r*, from the crack tip, along which the strain energy density is minimum.

The main advantage of this criterion is its ease and simplicity, and its ability to handle various combined loading situations. The crack growth direction angle in the local coordinate plane perpendicular to the crack front can then be determined for each point along the crack front. In this work, the crack inclination angle is taken into account in the calculations by means of the values of the SIF, K_I and K_{II}, because their values are a function of the orientation of the crack plane.

4 NUMERICAL EXAMPLES

In order to demonstrate the accuracy and efficiency of the methodology discussed in the preceding sections, two crack growth applications are described. The first applications describe crack growth in aircraft wing lug and the second illustrates the use of the finite element methodology to simulate crack trajectory under mixed-mode.

4.1 Fatigue Crack Growth in an Aircraft Wing Lug

This example describes the analytical and numerical methods for obtaining the stress intensity factors and for predicting the fatigue crack growth life for cracks at attachment lugs. Straight-shank male lug is considered in the analysis, Fig. 2. Three different head heights of lug are considered in the analysis. The straight attachment lugs are subjected to axial pin loading only. Material properties of lugs are (Al 7075 T7351) [10]: R_m = 432 MPa $\Leftrightarrow$ Ultimate tensile strength, $R_{P0,2}$ = 334 MPa, C_F= 3.10^{-7}, n_F = 2.39, $K_{IC} = 70.36 MPa\sqrt{m}$.

The stress intensity factors of cracked lugs are calculated under stress level: $\sigma_g = \sigma_{max}$ = 98.1 MPa, or corresponding axial force, F_{max} = $\sigma_g \cdot (w-2R) \cdot t$ = 63716 N. In the present work finite element analysis of cracked lug is modelled with special singular quarter-point six-node finite elements around crack tip, Fig. 3. The load of the model, i.e. a concentrated force, F_{max}, was applied at the centre of the pin and reacted at the other and of the lug. Spring elements were used to connect the pin and lug at each pair of nodes with identical nodal coordinates all around the periphery. The area of contact was determined iteratively by assigning a very high stiffness to spring elements which were in compression and very low stiffness (essentially zero) to spring elements which were in tension. The stress intensity factors of lugs, analytic and finite elements, for through-the-thickness cracks are shown in Table 2. Analytic results are obtained using relations from previous sections, Eq. (1).

Table 1. *Geometric parameters of lugs [10]*

Lug No.	Dimensions [mm]				
	2R	W	H	L	t
2	40	83.3	44.4	160	15
6	40	83.3	57.1	160	15
7	40	83.3	33.3	160	15

Table 2. *Comparisons analytic and FE results for SIF, K_I*

Lug No.	a [mm]	$K_{I\max}^{MKE}$ [daN/mm^2]	$K_{I\max}^{ANAL.}$ [daN/mm^2]
2	5.00	68.78	65.62
6	5.33	68.12	70.24
7	4.16	94.72	93.64

Fig. 4 shows a comparison between the experimentally determined crack propagation curves and the load cycles calculates, and Walker law [11] for several crack lengths. Experimental results of crack growth behaviour of lugs were carried out on the servo-hydraulic MTS system. A detailed description of experimental fatigue behaviour of cracked lugs is described in reference [3]. A relatively close agreement between the test and the presented computation results is obtained. The analytic computation methods presented in this work can be a reliable method for damage tolerance analyses of notched structural components such as lugs-type joints.

4.2 Crack Growth from Riveted Holes

In this section, the modelling of crack propagation in a plate (Al 2024 T351) with cracks emanating from one hole subjected to a far-field tension is considered, σ Fig. 5. In the initial configuration the left crack has 2.54 mm and is oriented at angle $\theta = 33.6°$ to the left hole. The change in crack length for each iteration is taken to be a constant, Δa = 2.54 mm, and the cracks are grown in eight steps. In this analysis the strain energy density criterion (S-criterion) is used to determine the crack trajectory or angle of crack propagation.

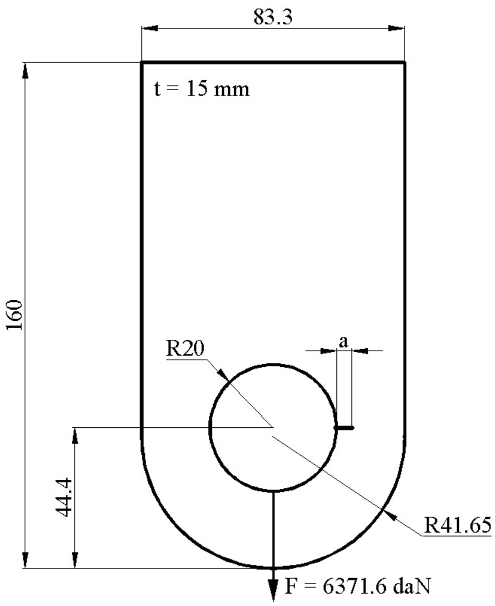

Fig. 2. *Geometry of cracked lug 2*

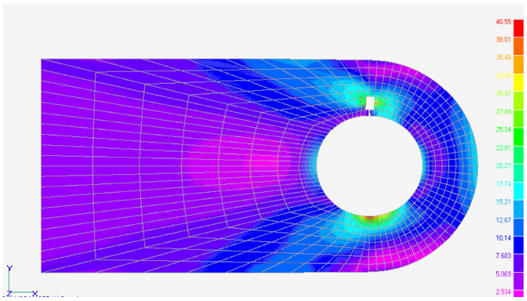

Fig. 3. *Finite element model of cracked lug with stress distribution*

In this work, the crack inclination angle is taken into account in the calculations by means of the values of the SIF, K_I and K_{II}, because their values are a function of the orientation of the crack plane. These parameters were calculated numerically with the finite element method.

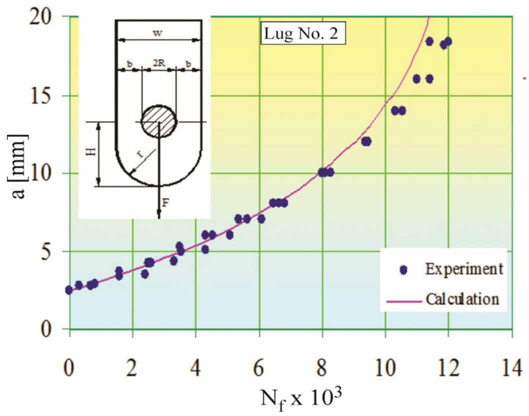

Fig. 4. *Crack propagation at lug –Comparisons analytic results with tests (H = 44.4 mm)*

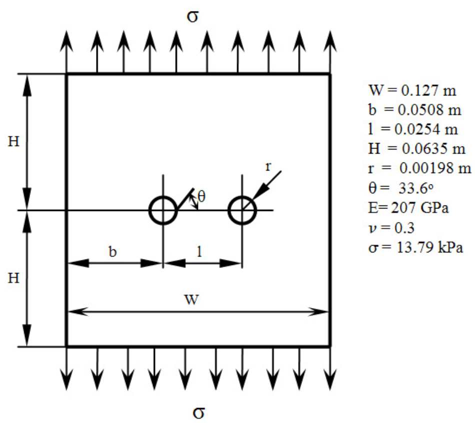

Fig. 5. *Geometry and load of the riveted crack problem*

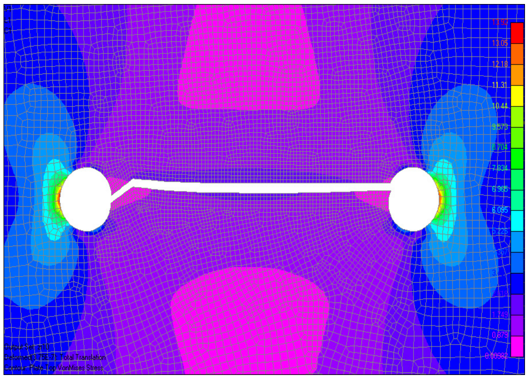

Fig. 6. *The crack trajectory using Q-P elements and S-criterion*

Figure 6 shows the stress contour and crack trajectory for the last configuration. In this crack growth analysis quarter-point (Q-P) singular finite elements are used with S-criterion. These results are compared with an extended finite element method (X-FEM) [13] and [14], Table 3 and Fig. 7.

The predicted crack trajectories using Q-P singular finite elements and X-FEM method are nearly identical. The extended finite element method allows for the modelling of arbitrary geometric features independently of the finite element mesh. This method allows the modelling of crack growth without re-meshing.

Fig. 7 shows good agreement between conventional QP singular finite elements and X-FEM in determining crack growth trajectory.

5 CONCLUSIONS

The finite element method is a robust and efficient technology that can be used to investigate the impact crack on the performance of notched structural components.

Table 3. *Position for left crack tip*

X-FEM [13]		Presented Q-P singular FE solutions	
X_C [mm]	Y_C [mm]	X_C [mm]	Y_C [mm]
54.458	64.618	54.458	64.618
57.404	64.465	56.987	64.389
60.350	64.287	59.525	64.287
63.322	64.287	62.065	64.259
66.294	64.364	64.605	64.247
69.266	64.338	67.145	64.247
72.136	64.262	69.685	64.270
74.168	63.754	72.227	64.315

The aim of this work is to investigate the strength behaviour of the notched structural elements such as the aircraft cracked lugs. In the fatigue crack growth and fracture analysis of lugs, accurate calculation of SIF is essential. Analytic expression for stress intensity factor of cracked lug is derived using the correction function and FEM. The contact finite element analyses for the true distribution of pin contact pressure are used for determining stress concentration factors used in the correction function. Good agreement between the derived analytic SIF of cracked lug with finite elements is obtained.

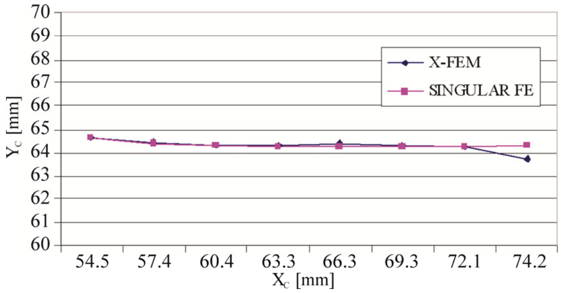

Fig. 7. *Comparison of crack trajectory using present QP singular FE and X-FEM*

Two applications were discussed in this work in order to demonstrate the effectiveness of finite element based on computer codes in evaluating the impact of fatigue crack growth on structural components. Firstly, the predicted crack trajectory is calculated using quarter-point singular finite elements. The applications described a fatigue crack growths analysis of lugs with complex geometry and loading. Good agreements between present computation results with experiments have been obtained. Secondly, the predicted crack trajectory under mixed modes is determined using quarter-point singular finite elements together with the strain energy density criteria. The predicted crack growth trajectory under mixed modes in a plate with cracks emanating from one hole subjected to a far-field tension was nearly identical to the trajectories predicted with X-FEM.

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