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Prediction of Unavoidable Distortions in Transformation-Free Cooling by a Newly Developed Dimensionless Model

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Former investigations in transformation-free cooling processes of long cylindrical workpieces in a gas nozzle field showed a typical behaviour of the dimensional changes, which correlated to only few dimensionless numbers. These numbers are created by the following parameters: shape and dimensions of component, its initial temperature, temperature of the quenching media, heat transfer coefficient, heat conductivity, heat capacity, density, thermal expansion coefficient, Young's modulus, Poisson's ratio, yield strength and strain hardening behaviour. The representative group of 28 austenitic stainless steels was selected from literature. Their properties were statistically analyzed in order to carry out a systematic investigation of the most significant material properties and process parameters of dimension and shape changes during transformation-free cooling. The characteristic values of statistically significant interval of considered austenitic steel properties – average and standard deviation - as well as the range of usual process parameters for their heat treatment are used for simulations execution by the commercial FEM program SYSWELD. The relative changes of the component dimensions obtained from numerical simulations, have been analyzed in dependence of six autonomous dimensionless numbers with their interactions in order to find the proper equations for prediction of unavoidable distortion in a transformation-free cooling. To define these equations, the method of nonlinear regression analysis was used.

Keywords: heat treatment, austenitic steel, thermal strain, cooling, distortion

0 INTRODUCTION

Predicting distortion after steel heat treatment by computer simulation is a complex task that involves a number of process parameters and nonlinear variation of steel properties. The most important process in this respect is a martensitic hardening. Cooling of austenitized parts has always been accomplished with inhomogeneous temperature distributions, which have thermal stresses and thermal strains as a consequence. If material cannot elastically accommodate these stresses, the thermal strains cause plastic deformation. These deformations occur before reaching the first phase transformation when steel microstructure is in the supercooled austenitic phase. Therefore, investigating the problem of unavoidable distortions after transformation free cooling provides knowledge about influential factors on the behaviour of workpieces and presents us with some practical measures to reduce these distortions. Consequently, the behaviour of austenitic steels, which shows no phase transformations during cooling, should be investigated [4] and [5].

The analysis of transformation-free cooling leads to a set of coupled differential equations, which describe the development of thermal stresses with strain hardening caused by the processes of heat conduction and transfer [1], [3] and [11]. This analysis represents a comparable simple case of the distortion phenomena simulation because complex processes connected to phase transformations were considered. However, considering this most simple case, it can be mentioned that an analytic solution for the differential equations set of that thermo-mechanical problem is not available even for the simple part geometries such as finite length cylinders, discs or plates [11]. On the other hand, numerical solutions with finite element programs such as SYSWELD give an insight into a specific simulated case without general trends and validity of workpieces behaviour. To get a general insight into the distortion behaviour after transformation-free cooling, the dimensionless analysis was proposed. The proposed

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dimensionless numbers contain process parameters, material properties, and geometrical dimensions [6]. A nonlinear regression analysis was used to determine the correlation between proposed dimensionless numbers and relative change in cylinders length.

1 DIMENSIONLESS NUMBERS

Former investigations of transformation-free cooling of long cylindrical workpieces in a gas nozzle field showed a typical behaviour of the dimensional changes depending on few dimensionless numbers [5] and [6]. As the changes of length and diameter are functions of position, their mean values must be calculated, as shown in Fig. 1a.

The first of the dimensionless numbers, which indicates heat transfer at workpieces with a very good heat conductivity, is the Biot number.

$$F_1 = Bi = \frac{(V/S)\cdot\alpha}{\lambda} \tag{1}$$

The volume to surface ratio (*V*/*S*) at Eq. (1) indicates a characteristic linear dimension of a part, α means the average heat transfer coefficient and λ is the average heat conductivity in the considered temperature interval.

Fig. 1b shows the dimensional changes of cylinders with the length of 50 up to 200 mm and diameter between 10 and 50 mm. For a different length and diameter of a cylinder, the plot of relative changes in length against Biot number offers uniform curves. From Fig. 1 it is obvious that thermal strains will appear only when the Biot number is more than 0.1. It is in correspondence with the results of a number of investigations that showed it is possible to neglect temperature gradients and thermal stresses and strains for small Biot numbers (Bi < 0.1).

The mutual connections between thermal stresses and strains in the range of elastic deformation are defined by the Hooke´s law [3]. Based on the equations of Hooke´s law, the following three dimensionless numbers are derived:

$$F_2 = \alpha_{\text{th}}\left(T_0 - T_\infty\right) \tag{2}$$

$$F_3 = \nu \tag{3}$$

$$F_4 = \frac{E}{\sigma_0} \tag{4}$$

The term T_0 corresponds to initial temperature and T_∞ is the temperature of cooling media. α_{th} is the mean coefficient of thermal dilatation, and ν is Poisson's coefficient. E is Young’s modulus, and the σ_0 is materials yield strength.

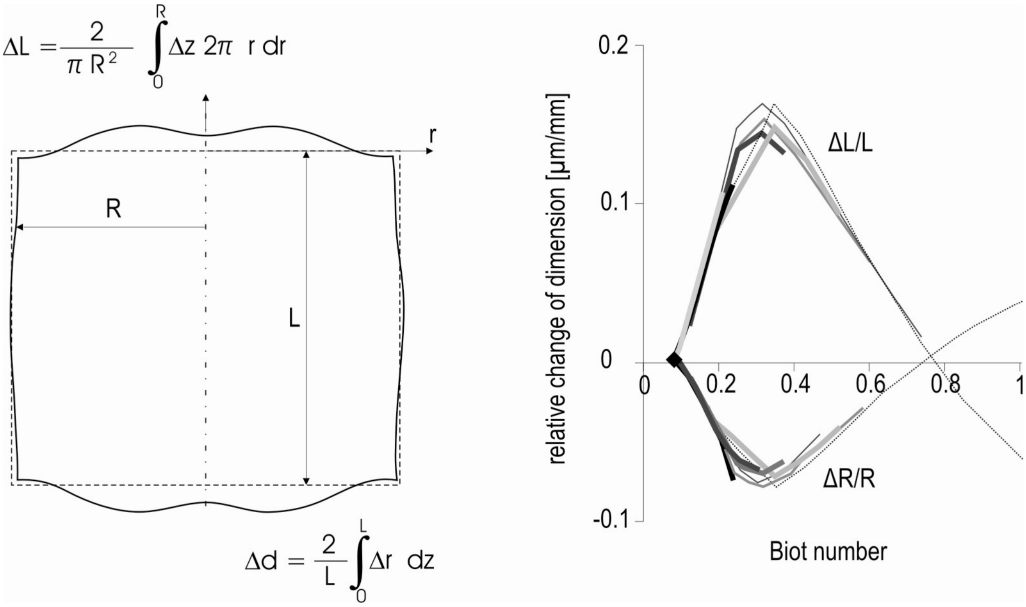


Fig. 1. *a)Determination of relative changes in length and in diameter [5]*
b) Relative change of length and diameter depending on Biot number [6].

At the field of plastic deformation the total strain (ε) consists of elastic (ε_e) and plastic strain (ε_p). To describe the stress-strain dependence in the case of materials hardening by plastic deformation, the modified Ramberg-Osgood model [9] is applied. This model was shown as the most applicable by previous experiments [5]. It describes the stress-strain correlation in the field of plastic deformation by Eq. (5) [10] and [12]:

$$\varepsilon = \varepsilon_{\mathrm{e}} + \varepsilon_{\mathrm{p}} \tag{5a}$$

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{\frac{1}{\mathrm{n}}} \tag{5b}$$

A parameter K signifies the absolute stress level, and n is a strain-hardening exponent. It is a measure of the strength, which is called the "modulus of plasticity" in reference [10]. The parameter K is not equal to the yield or tensile strength, but to the strength extrapolated to a total strain ε equal 1 [10].

Eq. (5a-b) can also be noted in a dimensionless form, using the maximum elastic strain ε_0 as a reference value [12]. At the first step, the left and the right side of Eq. (5a) are divided by ε_0, and also the equality $\varepsilon_0=\sigma_0/E$ is introduced on the right side of this new expression:

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0 E} + \frac{1}{\varepsilon_0}\left(\frac{\sigma}{K}\right)^{\frac{1}{\mathrm{n}}} \equiv \frac{\sigma}{\sigma_0} + \frac{E}{\sigma_0}\left(\frac{\sigma}{K}\right)^{\frac{1}{\mathrm{n}}} \tag{6a}$$

$$\frac{E}{\sigma_0}\left(\frac{\sigma}{K}\right)^{\frac{1}{\mathrm{n}}} \equiv \left(\frac{E}{\sigma_0}\frac{\sigma^{\frac{1}{\mathrm{n}}}}{K^{\frac{1}{n}}}\right)\frac{\sigma_0^{\frac{1}{\mathrm{n}}}}{\sigma_0^{\frac{1}{\mathrm{n}}}} \equiv \frac{E\sigma_0^{\frac{1}{\mathrm{n}}-1}}{K^{\frac{1}{\mathrm{n}}}}\left(\frac{\sigma}{\sigma_0}\right)^{\frac{1}{\mathrm{n}}}. \tag{6b}$$

By rearranging the terms in Eq. (6.b) and after introducing them in Eq. (6.a) the final Eq. (7) has been derived. It is used for definition of the next two dimensionless numbers F_5 and F_6.

$$\frac{\varepsilon}{\varepsilon_0} = \frac{\sigma}{\sigma_0} + \frac{E}{\sigma_0}\left(\frac{\sigma_0}{K}\right)^{\frac{1}{\mathrm{n}}}\left(\frac{\sigma}{\sigma_0}\right)^{\frac{1}{\mathrm{n}}} \equiv \frac{\sigma}{\sigma_0} + F_4\left(F_5\right)^{F_6}\left(\frac{\sigma}{\sigma_0}\right)^{F_6} \tag{7}$$

$$F_5 = \frac{\sigma_0}{K} \tag{8}$$

$$F_6 = \frac{1}{n}. \tag{9}$$

The proposed six dimensionless numbers (Table 1) are not the only possible set of numbers.

Table 1. *Definition of dimensionless numbers.*

	Dimensionless number					
	$Bi=F_1$	F_2	F_3	F_4	F_5	F_6
Term	$\frac{(V/S)\cdot\alpha}{\lambda}$	$\alpha_{\mathrm{th}}\cdot(T_0-T_\infty)$	ν	$\frac{E}{\sigma_0}$	$\frac{\sigma_0}{K}$	$\frac{1}{n}$
Length L [m]						
Diameter D [m]						
Initial temp. of material T_0 [°C]						
Temp. of quenching media T_∞ [°C]						
Heat transfer coeff. α [W/(m²K)]						
Heat conductivity λ [W/(mK)]						
Coeff. of therm. expan. α_{th} [1/K]						
Poisson's ratio ν [1]						
Young's modulus E [MPa]						
Yield strength σ_0 [MPa]						
Plasticity modulus K [MPa]						
Strain hardening exponent n [1]						

Nevertheless, their influence on thermal strains is confirmed by numerical simulations and by the conducted experiments [4], [5] and [6]. All products and quotients of these numbers are also dimensionless (e.g. F_4F_5=E/K), but the six terms are independent from each other. That means that it is not possible to create one of them with mathematical operation of two or more other terms. From the proposed set of dimensionless numbers in Table 1, their effect on thermal distortion was systematically investigated for the following four numbers (F_1, F_2, F_4 and F_5). Their influence on thermal distortions and a relative change in length of cylinders made of an austenitic steel was shown as the most significant [6].

2 NUMERICAL INVESTIGATIONS

Numerical simulations with the computer program SYSWELD calculated distortions of austenitic cylinders after transformation-free cooling. During these simulations, the heat transfer problem was solved simultaneously with the problem of mechanical stresses and strains. A cylinder of 20 mm diameter and 200 mm length was chosen as a geometrical domain for all simulations. The choice of this geometry can be explained as follows: the ratio of length and diameter is significantly larger than 3 and the most experimental results are sampled for those dimensions. All simulations were carried out with 2D models using kinematic strain hardening [4]. Its selection is justified by former simulations and experiments performed on austenitic stainless steel SAE30300 (German grade X8CrNiS18.9) [5] and [6].

Most of the material properties in Table 1 are dependent on temperature and are given in [?] for the above mentioned steel. However, in order to provide a better insight into the influences of individual dimensionless number on distortion behaviour of cylinders, the numerical simulations were conducted with mean values of steel properties. The chosen steel properties are mean values averaged over considered temperature range, which is typical of the analysed set of austenitic steels. The values of the heat transfer coefficient on all cylinder surfaces were taken as equal and constant during the complete cooling cycle.

The aim of the research was to define an adequate expression (10) which enables the connection of the changes of dimensionless numbers shown in table 1 with the corresponding relative changes in length ΔL/L.

$$\frac{\Delta L}{L} = (F_1, F_2, F_3, F_4, F_5, F_6). \tag{10}$$

In searching for a form and coefficients of expression (10), the nonlinear regression analysis was applied. During this analysis, the nonlinear least squares functions from Matlab Statistics Toolbox fit a polynomial model that has a known form but unknown parameter values.

The Eq. (10) could have a very complicated form, but it can be simplified by derivation of some typical combination of the above mentioned numbers (Table 1). By introducing Eq. (5a) and the equality $\varepsilon_0=\sigma_0/E\equiv 1/F_4$ into Eq. (7) and rearranging the terms in a way that the plastic deformation term remains on the left side and all other terms are placed at the right side, the following form is derived:

$$\frac{\varepsilon_p+\varepsilon_0}{\varepsilon_0}=\frac{\sigma}{\sigma_0}+F_4\left(F_5\right)^{F_6}\left(\frac{\sigma}{\sigma_0}\right)^{F_6} \tag{11.a}$$

$$\varepsilon_p=\frac{\sigma}{F_4\sigma_0}+\left(F_5\right)^{F_6}\left(\frac{\sigma}{\sigma_0}\right)^{F_6}-\frac{1}{F_4}. \tag{11.b}$$

From the analytical equations of thermal stresses and strains for cooling cylinders with an axially symmetrical distribution of temperature gradients [1] and [12] it can be seen that the ratio σ/σ_0 is dependent from value of thermal stresses caused by temperature gradients and Poisson's number with the following equality:

$$\frac{\sigma}{\sigma_0}=C_0^{'}\frac{\alpha_{th}\Delta T}{1-\nu}\equiv C_0^{'}\frac{F_2}{1-F_3}. \tag{12}$$

A dimensionless constant $C_0^{'}$ has introduced as constant of proportionality. After the Eq. (12) has inserted into Eq.(11.b) and by rearranging the dimensionless constants the final expression for a new "dimensionless deformation parameter" p_ε was achieved:

$$p_\varepsilon=\frac{C_0F_2}{\left(1-F_3\right)F_4}+\left(\frac{F_2F_5}{1-F_3}\right)^{F_6}+\frac{C_1}{F_4}. \tag{13}$$

The terms of Eq. (13) represent the influences on distortion and plastic deformations caused by thermal stresses (the first term), materials hardening (the second term) and mechanical properties of material (the third term). With the proposed form of dimensionless deformation parameter p_{ε}, a very good regression model can be derived, and the correlation (10) was simplified, because it consists of only two, instead of six variables.

$$\Delta L/L = f(Bi, p_{\varepsilon}). \tag{14}$$

3 MATERIAL DATA

The correlation (14) for predicting of relative change in cylinders length caused by thermal stresses and deformation hardening was proved for a whole group of twenty-eight representative austenitic stainless steels (Table 2). For this group of steels the mean values of all properties, which are relevant to the numerical simulation, were determined at room temperature by statistical analysis (Table 3) according to the database of Cambridge Engineering Selector programme [2]. Afterwards, all considered properties were extrapolated from room temperature to 548 °C, which is an average temperature of cooling process (Table 3). Extrapolation equations were derived from the experimentally determined changes of the considered properties dependent on temperature and stresses for stainless steel grade SAE30300 (DIN X8CrNiS18.9) whose properties are very close to the mean values of properties of analysed austenitic stainless steels set [4].

Table 2. *A representative group of analysed austenitic stainless steels*

Steel grades
27Cr-9Ni (as cast); 19Cr-10Ni-2,5Mo; 19Cr-9Ni-0,2C; 19Cr-10Ni; 19Cr-11Ni-2,5Mo; 19Cr-9Ni; 19Cr-11Ni-3,5Mo; 24Cr-13Ni; 25Cr-20Ni; X8CrNiS18.9; AISI 201; AISI 202; AISI 205; AISI 216; AISI 301; AISI 302; AISI 304; AISI 305; AISI 308; AISI 309; AISI 310; AISI 314; AISI 315; AISI 316; AISI 317; AISI 321; AISI 329; AISI 330

Table 3. *Typical range of all relevant properties in the observed set of austenitic steels denoted by Table 2*

Material property		Average value	Standard deviation
Density, ρ [kg/m^3]	at 20 °C	7859	± 109
	at 548 °C	7616	± 217
	at 1084 °C	7378	± 219
Heat conduction, λ [W/mK]	at 20 °C	15.38	± 1.56
	at 548 °C	21.64	± 3.12
	at 1084 °C	27.75	± 3.13
Heat capacity, c [J/kgK]	at 20 °C	499.5	± 16.2
	at 548 °C	619	± 32
	at 1084 °C	735	± 33
Coefficient of thermal expansion, α_{th} [1/K]	at 20 °C	$16.41\cdot10^{-6}$	$\pm 1.420\cdot10^{-6}$
	at 548 °C	$19.96\cdot10^{-6}$	$\pm 2.850\cdot10^{-6}$
	at 1084 °C	$23.441\cdot10^{-6}$	$\pm 2.854\cdot10^{-6}$
Young's modulus,·E [MPa]	at 20 °C	196 000	± 6450
	at 548 °C	150 000	± 13000
	at 1084 °C	105 000	± 13000
Poisson´s ratio, ν at 20, 548 and 1084 °C		0.27	± 0.05
Yield strength, σ_0, [MPa]	at 20 °C	306	± 99
	at 548 °C	242	± 198
	at 1084 °C	179	± 99
Plasticity modulus, K = 1100.6 MPa (at 548 °C)			
Strain hardening exponent, n = 0.5178 (at 548 °C)			

Table 4. *The plan for selection of dimensionless numbers for numerical simulations of transformation free cooling of cylinders made of austenitic steels*

Levels of Biot number F_1 and corresponding heat transfer coefficients α [W/m^{2}K]															
Bi	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
α	454	909	1363	1817	2272	2726	3180	3635	4089	4543	4998	5452	5906	6361	6815

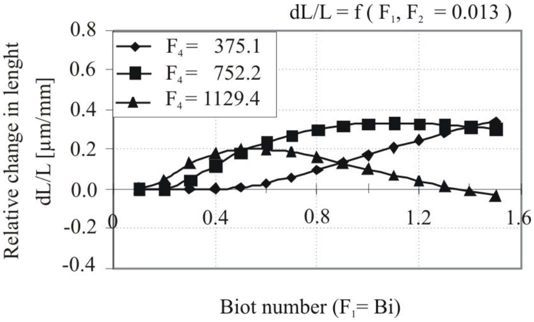
Levels of other dimensionless numbers for every Biot number selected above						
F_2			F_4			F_3
0.013	0.0171	0.0213	375.1	752.2	1129.4	0.27
Initial temperature T_0 [^{o}C]			F_5			F_6
778	1020	1084	0.36455	0.18182	0.12091	1.9313

According to Table 3, every property has its own mean value and an estimated standard deviation. This kind of determination of steel properties enables the selection of property values at three levels, which is typical of the whole group of steels (the statistically lower value, the mean value and the statistically upper value). This means that it is also necessary to consider the dimensionless numbers F_2, F_4 and F_5 with three typical levels of values (min., average and max.). Contrary to them, the Biot number was changed in a broader range of values between 0.1 and 1.5 with the same average heat transfer coefficient for all surfaces of the cylinders. The plan for the selection of dimensionless numbers combination, F_1, F_2, F_3, F_4 and F_5 and for numerical simulations is shown in Table 4.

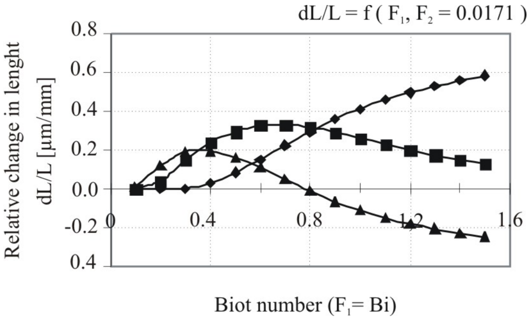
4 RESULTS AND DISCUSSION

4.1 Results of Numerical Simulations

Afterwards, numerical simulations were performed by computer programme SYSWELD, Fig. 2 shows the results: the calculated relative change in length dependent on the combinations of dimensionless numbers according to Table 2.



a)



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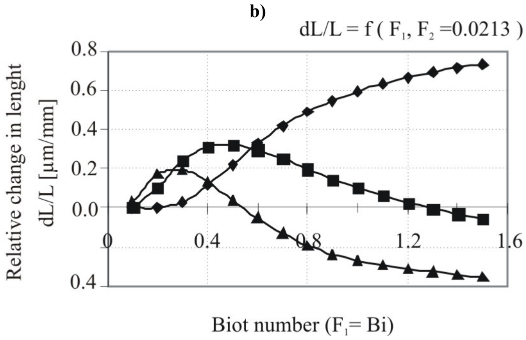

c)

Fig. 2. *The relative change in length after transformation free cooling of cylinders made of austenitic steels dependent on F_1 and different combinations of F_2, F_4 and F_5*

a) $\Delta L/L = f(F_1)$ with $F_2 = 0.013$, $F_4 = (375.1; 752.2; 1129.4)$, $F_5 = (0.36455, 0.18182, 0.12091)$
b) $\Delta L/L = f(F_1)$ with $F_2 = 0.0171$, $F_4 = (375.1; 752.2; 1129.4)$, $F_5 = (0.36455, 0.18182, 0.12091)$
c) $\Delta L/L = f(F_1)$ with $F_2 = 0.0213$, $F_4 = (375.1; 752.2; 1129.4)$, $F_5 = (0.36455, 0.18182, 0.12091)$

Based on Fig. 2, it can be concluded that all the curves which presented dependence of relative change in length from Biot number $\Delta L/L = f(F_1)$ have the same behaviour towards mathematical definitions of continuity, position of null points as well as maximum and minimum. This means that these curves may come from the same function class and can be interpolated by similar regression functions.

4.2 Interpolation of Numerical Simulation Sesults by Newly Proposed "FL3" Model

For the mathematical description of the relative length change of cylinders as function of dimensionless numbers (Fig. 2), a new parameter model is proposed. This model can give a unique analytical expression for prediction of relative change in length after transformation free cooling of cylinders made of any austenitic steel mentioned in Table 2. The model was named after the three levels of selection of materials' properties used for their derivation, and the first letters of the names of its authors (as the **F**rerichs - **L**andek - **L**isjak - **L**übben model). The relative change in length after transformation-free cooling of cylinders made of austenitic steel (see Fig. 3) is analysed with the method of nonlinear least squares using the computer program Matlab equipped by the Statistics Toolbox module. The aim of nonlinear modelling was to analyse the influence of relative change in length $\Delta L/L$ originating from the individual change of Biot number F_1, the dimensionless deformation parameter p_ε, as well as common influence both of them. After the nonlinear fitting and prediction of Eq. (12), a multi dimensional polynomial of the degree three was chosen as the most adequate approach to the numerical results of Fig. 2. Accordingly, the obtained R-square value of the approximation expressed by Eq. (15 a and b) is equal to 0.9316 with 95% confidence bounds.

$$\frac{\Delta L}{L} = \begin{cases} \left(0.00841 - 0.011136 p_\varepsilon - 0.009914 p_\varepsilon^2\right) p_\varepsilon F_1^3 + \\ \left(-0.03027 - 0.174116 p_\varepsilon + 0.015132 p_\varepsilon^2\right) p_\varepsilon F_1^2 + \\ \left(-0.040149 - 0.285998 p_\varepsilon + 0.0320051 p_\varepsilon^2\right) p_\varepsilon F_1 + \\ \left(0.006319 + 0.262553 p_\varepsilon + 0.023156 p_\varepsilon^2\right) p_\varepsilon + \\ \left(0.000532 + 0.002315 F_1 + 0.000792 F_1^2\right) F_1 + \\ 0.0001 \end{cases} \tag{15a}$$

$$p_\varepsilon = \frac{0.59284 F_2}{\left(1-F_3\right) F_4} + \left(\frac{F_2 F_5}{1-F_3}\right)^{F_6} - \frac{0.03492}{F_4} \tag{15b}$$

For the above mentioned range of variations of dimensionless numbers F_2 to F_6 the dimensionless parameter p_ε varies at the range from $p_{\varepsilon,\min}$ = -0.064899 to the $p_{\varepsilon,\max}$ = -0.015588. The Biot number F_1 is varied at the range from 0.1 to 1.5. Over the whole temperature range from initial temperature T_0 to the space temperature 20 °C the dimensionless numbers F_1 to F_6 remained constant, as denoted in Table 4.

Within the proposed mathematical model, the relationship between the relative change in length calculated by Eqs. (15a and b) is compared to the results of numerical simulations for the same combination of dimensionless variables F_1 and p_ε (Fig. 3). It is obvious that the proposed mathematical model (15 a and b) might be used for an estimation of relative length change after transformation-free cooling of cylinders made of austenitic steels within the limits of the dimension numbers given in Table 4.

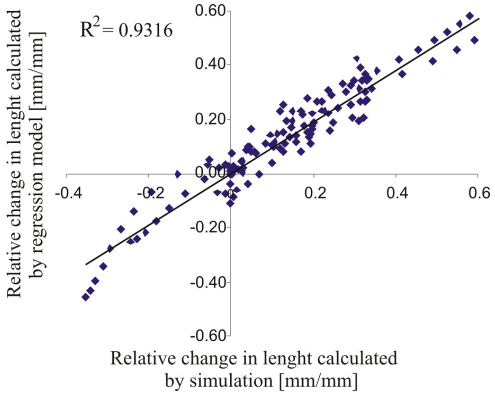

Fig. 3. *Correspondence between relative changes in length after transformation-free cooling of cylinders made of austenitic steels calculated by numerical simulations and predicted by regression model (15 a and b)*

5 CONCLUSION AND OUTLOOK

The knowledge about all the relevant parameters which describe the geometry, material and process, allows the deduction of dimensionless numbers, which govern the distortion behaviour of the system. The analysis with dimensionless numbers brings two main advantages [6, 7, 8]:

- Generally, the quantity of dimensionless numbers is smaller than the total amount of influencing parameters.

- If the values of all the dimensionless numbers of two geometrical similar bodies are equal, the distortion behaviour of these two bodies are equal too.

In this paper it was shown, that due to dimension analysis, the problem of the relative length change prediction of cylinders can be reduced firstly from 14 parameters down to 6 dimensionless numbers, and then with introducing of deformation parameter p_ε it can be further reduced to two dimensionless numbers only:

$$\frac{\Delta L}{L} = f\left(\frac{V\alpha}{S\lambda}, \alpha_{\mathrm{th}}\left(T_0 - T_\infty\right), \nu, \frac{E}{\sigma_0}, \frac{\sigma_0}{K}, \frac{1}{n}\right) = f\left(F_1, F_2, F_3, F_4, F_5, F_6\right) = f\left(F_1, p_\varepsilon\right). \quad (16)$$

The approximation Eq. (13 a and b) for this task was derived by nonlinear regression analysis. The proposed mathematical model named as FL3-model can be used for the optimization of transformation-free cooling parameters, optimization of dimensions and for the selection of optimum steel grades, which get minimum relative changes in length. However, in some cases the differences between the regression model and the simulation by FE method are not negligible. Consequently, in the future the authors will study other combinations of dimensionless numbers in order to receive an improved description of the dimensional changes. With these studies, a simplification of Eq. (15.a-b) might also be achieved.

Furthermore, it should be clarified whether the results can be transferred to other geometries like cylinders with hole, rings and plates.

6 ACKNOWLEDGEMENT

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