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Variation of Normal Anisotropy Ratio "*r*" during Plastic Forming

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Cold rolled thin sheet metals are typical anisotropic materials as a consequence of the production process of rolling. Anisotropy is usually described from two aspects: as plane anisotropy and as a normal one. Most frequently used anisotropy parameter is the normal anisotropy ratio or the r value, which represents strains ratio in sheet metal plane and along its thickness. It is determined experimentally, usually as mean constant numeric valu, without testing the possible variation in dependence on realized plastic strain.

Accurate defining of the r value has great significanc, since it is not only the formability parameter, but also the crucial anisotropy parameter in software for simulation of the plastic forming process.

The paper presents the results of experimental investigation of three typical materials: low carbon Al killed steel DC04 sheet metal, alloy AlMg4.5Mn0.7 sheet metal and stainless steel X5CrNi18-10 sheet metal. Particular attention is given to defining the dependence of the r value on plastic strain during the forming process. The experiment requires sophisticated devices and accurate calculations. In the actual case, the computerized measuring system Zwick/Roell Z 100 was used. In addition, the paper contains the proposal on how to take into account variations of the r values during the plastic forming process.

Keywords: sheet metal, anisotropy, r value

0 INTRODUCTION

Cold rolled sheet metals are very significant in modern industry. Registered export of cold rolled steel sheet metals (including coated ones) in 2006 was 71.9 million tons on the global level [1]. Total production of primary aluminum in the world in 2007 was 38.7 million tons [2]. Approximately one quarter of that quantity is processed into thin sheets, foils excluded [2].

Due to complex metallurgical process of rolling and its structural specifics, thin sheet metals have prominent anisotropy, i.e., inequality of different characteristics depending on the considered directions in volume. Since sheet metal has a specific geometry, two dimensions in plane which are significantly larger than the third dimension – thickness, anisotropy is considered from two aspects: in sheet metal plane and perpendicular to that plane, along thickness.

In the first case, one referent direction, most often the rolling direction and two other corresponding directions are defined with respect to the rolling direction. Those most often are at 45 and 90° with respect to the rolling direction. For each of these directions, the necessary material characteristics are determined, like the elasticity modulus, yield strength, tensile strength, strain at fracture, the mean r value, strain hardening exponent, etc. The major differences in their differences along different directions indicate the significant influence of plane anisotropy.

The second case is related to the so-called normal anisotropy, i.e., the difference in sheet metal strain in plane and along its thickness. The quantitative indicator of that difference is the normal anisotropy ratio or the r value, which is very important for sheet metals. In anisotropic plasticity theory, starting with the Hill's approach and including all the other [3] and [4], it is used in almost all important relations. In practice, it is an obligatory parameter of the sheet metal formability by deep drawing. Apart from theory, the r value is very important for the functioning of all the software for computer simulation of plastic forming process.

It is determined only experimentally, by uniaxial tension process, according to proper standard procedures, likes ASTM E 517, EN 10130-B etc. Usually, the result is a constant numeric value, which represents the mean value during the measurement or the value that corresponds to relations at the end of the forming process. The following question is reasonable: what should be done if there are materials for which the changes of the r values during the

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process are prominent? These variations can have the greatest significance in software for computer simulation. According to [3], not a single modern commercial software contains implemented r values that vary with strain, for different materials. The aim of this paper is to point to the nature of those variations of the r value in typical cold-rolled sheet metals and to propose how those variations should be taken into account.

1 PROCEDURE FOR DEFINING THE r VALUE

The r value is defined as the ratio of true plastic strain of width (φ_b) and true plastic strain of thickness (φ_s) of specimen at uniaxial tensile test:

$$r = \frac{\varphi_b}{\varphi_s} = \frac{\int_{b_0}^{b} \frac{db}{b}}{\int_{s_0}^{s} \frac{ds}{s}} = \frac{\ln \frac{b}{b_0}}{\ln \frac{s}{s_0}} = \frac{\ln \frac{b_0}{b}}{\ln \frac{s_0}{s}} = \frac{\log \frac{b_0}{b}}{\log \frac{s_0}{s}} , \qquad (1)$$

where b_0 and s_0 are the initial values of specimen width and thickness, respectively, and b and s are true values of width and thickness, respectively, which contain only plastic strain.

Due to high sensitivity of the r value to small measurement errors and small measurement bases of sheet metal thickness, substitution of the s/s_0 ratio is usually performed according to the well-known postulate of plasticity theory on volume invariability:

$$l_0 \cdot b_0 \cdot s_0 = l \cdot b \cdot s = const., \qquad (2)$$

where l_0 is the initial value of the specimen gauge length, and l is the true value of gauge length, which contains only the plastic strain.

Finally, the expression for calculating the r value has the following form:

$$r = \frac{\ln \frac{b}{b_0}}{\ln \frac{l_0 b_0}{l b}} = \frac{\ln \frac{b_0}{b}}{\ln \frac{l b}{l_0 b_0}} . \qquad (3)$$

In the classic procedure for determining the *r* value, the specimen is deformed up to maximally allowed strain, and after that unloading is performed, followed by the removal of specimen off the testing device and accurate measurement of length and width according to standard recommendations. Therefore, the obtained value is related to the end of the forming process, and it corresponds to the maximal strain.

The situation is more complex when there is a need to measure plastic strains of length and width simultaneously during the specimen forming process [5]. Since the specimen is constantly under load, the measuring system must constantly perform corrections in the sense of eliminating the elastic portion of strain. One method for solving that problem is given in [6].

The following expressions represent a simple possibility for correcting the width and length of gauge part of the specimen due to influence of elastic strain. Due to that influence, the width is decreased and it needs to be increased, while the length is increased and it needs to be decreased.

$$b = b_{pl} = b_{tot} + \Delta b_{el} , \qquad (4)$$

$$b = b_{pl} = b_0 - \Delta b_{tot} + b_0 \mu_P \frac{\sigma}{E} , \qquad (5)$$

where b_{tot} is the total width (it contains both plastic and elastic part); Δb_{el} represents elastic change of width which is lost during unloading; Δb_{tot} is the total change of width, which is usually measured during the experiment; μ_P is Poisson's ratio, σ is the nominal stress, and E is the elasticity modulus.

$$l = l_{pl} = l_{tot} - \Delta l_{el} , \qquad (6)$$

$$l = l_{pl} = l_0 + \Delta l_{tot} - l_0 \frac{\sigma}{E} ; \qquad (7)$$

where l_{tot} is the total gauge length, which contains both plastic and elastic part; Δl_{el} represents the elastic change of gauge length, which is lost during unloading; Δl_{tot} is total change of gauge length, which is usually measured during the experiment.

2 THE EXPERIMENT

The main characteristics of this experiment the results of which are presented in this paper, is an attempt to define, as accurately as possible, the dependence of the *r* value on realized plastic strain during uniaxial tension process for three cold rolled sheet metals intended for deep drawing. The necessary condition for such an experiment is a sophisticated equipment of high accuracy and reliability. Here, this is the measuring system for testing materials - Zwick/Roell Z 100 with pneumatic jaws. For determining the *r* value, two transducers are used: extensometer for measuring elongation type TC-EXMACRO.H01, Zwick/Roell, with gauge length 10 to 100 mm, accuracy above 1 μm, according to EN 10002-4 grade 1 and extensometer for measuring the change of width type TC-EXICLWD.001, Zwick/Roell, measurement range 1.5 to 11.5 mm with accuracy above 1μm, according to EN 10002-4 grade 0.5 [7].

The specimens were prepared according to the standard for uniaxial tensile test EN 10002-1 with initial gauge length $L_0 = l_0 = 80$ mm, parallel sides length $L_c = 120$ mm and initial width $b_0 = 20$ mm. The test was performed in line with recommendations of standard EN 10002-1 and EN 10130-B, whose regulations are implemented in software TestXpert ver. 10, which assists the specified measuring system. Fig. 1 shows the work space of testing machine with the specimen and both extensometers in working position.

For determining the *r* value, the most important condition is to have the process to occur within the limits of homogenous forming. For low carbon steel sheet metals it was accepted that homogenous forming limit would be the strain of 20%. For the Al alloy and stainless sheet metals that can be different, so it is necessary to carry out additional testing.

The *r* value is very sensitive to measurement accuracy. Small errors in the measured strains cause a large error in the determined *r* value. According to [8], if the coefficient of width measurement variation is about 0.1% the estimated coefficient of variation of *r* is 40 to 70 times larger, i.e., 4 to 7%! This is the main reason for using the highly accurate, sophisticated and reliable testing device.

Prior to measurement of the *r* value dependence on the value of strain, it was necessary to determine all the important mechanical properties of sheet metals: elasticity modulus, yield strength (R_p), tensile strength (R_M), strain at maximal force (A_g), strain at fracture (A) and strain hardening exponent (n). The influence of plane anisotropy was observed by defining the three directions in sheet metal plane: the rolling direction, 0°, and directions at 45 and 90° with respect to the rolling direction. Specimens were prepared and proper characteristics were determined for each direction.

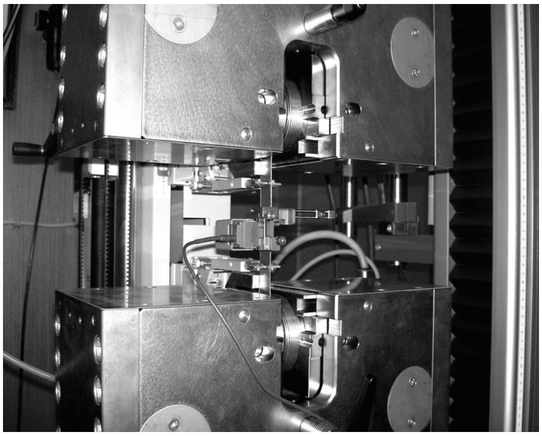

Fig. 1. *Work space of testing machine with the specimen and both extensometers*

3 RESULTS AND DISCUSSION

Basic mechanical properties of all the tested materials are given in Tables 1, 2 and 3. The first material is low carbon aluminum killed steel sheet DC04, thickness s_0 = 0.8 mm, the second is aluminum alloy sheet series 5000 AlMg4.5 Mn0.7 in annealed tempered state, thickness s_0 = 1.0 mm and the third is the stainless austenitic steel sheet X5CrNi 18-10, thickness s_0 = 0.7 mm.

The influence of plane anisotropy on the *r* value should be pointed out. In classic low-carbon sheet metals, such as investigated DC04, the smallest *r* value is for direction, which is at 45° with respect to the rolling direction. The influence this on formability by deep drawing is well known, particularly in rectangular geometries. If the position of non-deformed sheet metal, i.e., blank, is adjusted so that the specified direction of 45° goes along the middle of lateral sides, and much more favorable directions of 0 and 90° with higher *r* values over the corner zones, better results are achieved: larger drawing ratio, fewer defects, etc.

Table 1. *Properties of DC04 sheet metal*

DC04 (Mat. N° 1.0338, DIN EN 10130) s_0 = 0.8 mm										
Angle [°]	L_0 [mm]	b_0 [mm]	S_0 [mm^2]	E [GPa]	$R_{P0,2}$ [MPa]	R_m [MPa]	A_g [%]	A [%]	n	r
0	80	20.33	16.26	173.7	171.11	305.76	20.24	35.44	0.222	1.845
45	80	20.32	16.26	191.3	160.87	297.18	22.92	39.82	0.234	1.626
90	80	20.34	16.27	183.2	169.21	294.10	20.52	40.15	0.223	2.32
Mean value				184.88	165.52	298.56	21.65	38.808	0.228	1.854

Table 2. *Properties of AlMg4.5Mn0.7 sheet metal*

EN AW-5083 (AlMg4.5Mn0.7- 0 ; Mat. N° 3.3547, DIN EN 573-3) s_0 = 1.0 mm										
Angle [°]	L_0 [mm]	b_0 [mm]	S_0 [mm^2]	E [GPa]	$R_{P0,2}$ [MPa]	R_m [MPa]	A_g [%]	A [%]	n	r
0	80	20.23	20.23	69.6	138.17	264.91	15.50	18.79	0.260	0.619
45	80	20.26	20.26	68.2	123.88	251.49	17.48	21.30	0.257	1.005
90	80	20.31	20.31	71.0	130.20	260.84	18.69	20.00	0.248	0.676
Mean value				69.46	129.03	257.18	17.29	20.35	0.256	0.826

Table 3. *Properties of X5CrNi 18-10 sheet metal*

X5CrNi18-10 (Mat. N° 1.4301, DIN EN 10088) s_0 = 0.7 mm										
Angle [°]	L_0 [mm]	b_0 [mm)	S_0 [mm^2]	E [GPa]	$R_{P0,2}$ [MPa]	R_m [MPa]	A_g [%]	A [%]	n	r
0	80	20.2	14.14	182.4	263.43	602.34	39.37	40.37	0.366	0.815
45	80	20.14	14.10	184.1	248.63	561,62	48.03	55.64	0.355	1.240
90	80	20.00	14.00	189.8	265.70	565.61	35.70	36.42	0.338	0.963
Mean value				185.1	256.6	572.8	42.78	47.02	0.3535	1.065

For Al alloys, as well as for stainless steels, in the direction of 45° with respect to the rolling direction the *r* value is maximal. For Al alloys from series 5000, to which the alloy used here belongs, *r* values are small: 0.62 to 1. In such conditions, sheet metal is very prone to thinning, which is unfavorable for forming by deep drawing from the technological aspect [9]. The tested stainless austenite steel sheet metal has somewhat higher values: 0.82 to 1.15, but its formability is relatively bad as well, especially when compared to low-carbon steel sheet metals [10].

In certain cases, like the deep drawing of the axially symmetric pieces, it is convenient to use the mean values according to formula $X=(X_0+2X_{45}+X_{90})/4$, where X refers to the values of the corresponding characteristics in directions at 0, 45 and 90° with respect to the rolling direction. In that way, one takes into account the planar anisotropy [11], but the effect of the fictituous isotropy are also obtained. The mean values defined in this way, should be applied cautiously, after an analysis of each concrete froming process. For instance, in deep drawing of the rectangular shapes, it is more adequate to use values of characteritics corresponding to each direction. The mean values are presented in the last rows in Tables 1 to 3.

The Figs. 2 to 10, show the main experimental results. Dependencies of the *r* value and natural, true, width plastic strain on true plastic length strain during the specimen forming process are given.

Figs. 2 to 4 show very good linearity of natural width strain, φ_b and relatively small changes of *r* value during the process. Important divergence occurs only in the zone of very small strains, up to 2% approximately. It should be emphasized that according to the definition of the *r* value, expression (3), its value is unknown at the beginning of the process ($b=b_0$ and $l=l_0$, therefore $r=0/0$). Consequently, at low-carbon steel sheet metals, adoption of the mean value as constant during the entire process does not lead to major errors.

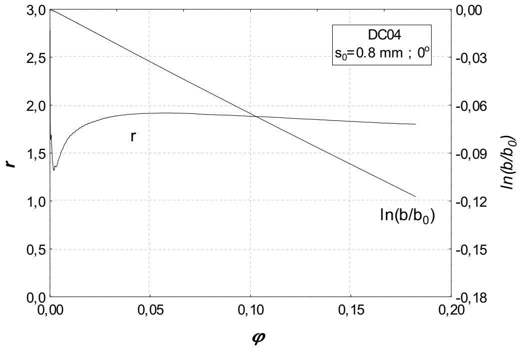

Fig. 2. *Dependence of the r value and true width strain on true strain for sheet metal DC04, 0° with respect to the rolling direction (RD)*

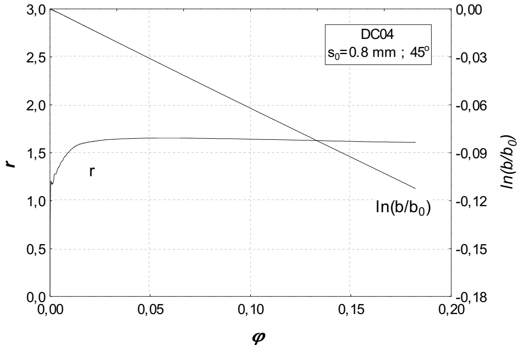

Fig. 3. *Dependence of the r value and true width strain on true strain for sheet DC04, 45° to RD*

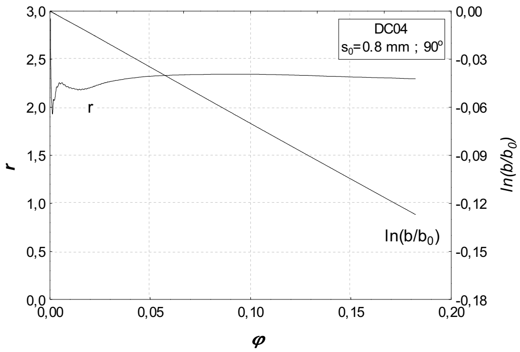

Fig. 4. *Dependence of the r value and true width strain on true strain for sheet DC04, 90° to RD*

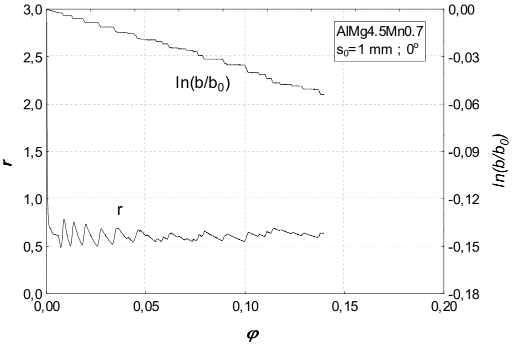

Fig. 5. *Dependence of the r value and true width strain on true strain for sheet AlMg4.5 Mn0.7, 0° to the rolling direction*

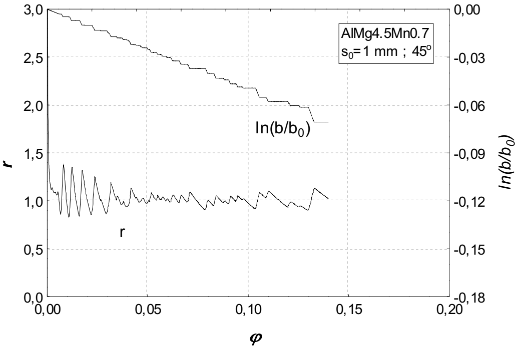

Fig. 6. *Dependence of the r value and true width strain on true strain for sheet AlMg4.5 Mn0.7, 45° to the rolling direction*

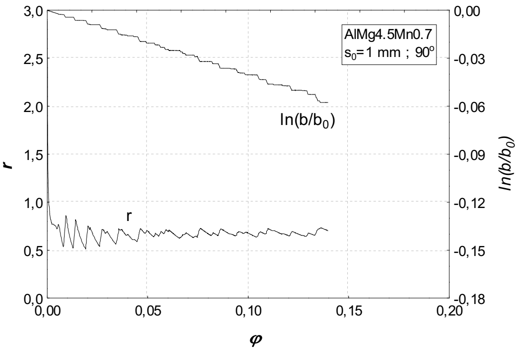

Fig. 7. *Dependence of the r value and true width strain on true strain for sheet AlMg4.5 Mn0.7, 90° to the rolling direction*

Figs. 5 to 7 show the prominent oscillatory variation of *r* value which is in line with width change and well-known oscillatory character of tension curves, which are not shown here. It is completely clear that the substitution of such dependence with constant value leads to significant error and neglect of the true material behavior. Approximation of the experimentally obtained dependence can be done by sinusoidal periodic function, in the following form:

$$r = r_{av} + r_a \sin\left(k \cdot 2\pi \cdot \varphi\right) \tag{8}$$

$$r_{av} = \frac{r_{\max} + r_{\min}}{2} \tag{9}$$

$$r_a = \frac{r_{\max} - r_{\min}}{2}, \tag{10}$$

where r_{av}, r_a – are the average and amplitude value of r, respectively; k – the empiric constant which defines number of periods; $r_{\max}$, $r_{\min}$ – maximal and minimal value of r; $\varphi = \varphi_1$ – true plastic strain.

For dependence in Fig. 6 approximation (8) has the form:

$= r$ 1.025 + 0.125 sin(320πφ).

In this case, k = 160, which means that function for r has 16 periods within 0 to 0.1 range of true strain φ, Fig. 8.

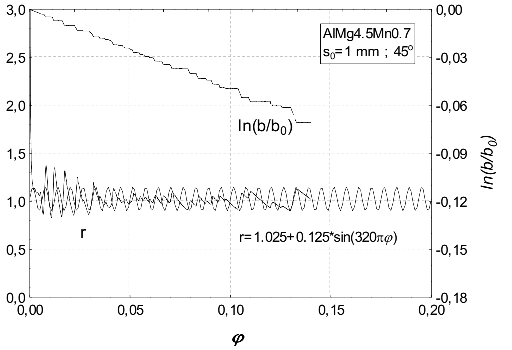

Fig. 8. *Sinusoidal approximation of the r value dependence for sheet AlMg4.5Mn0.7, 45° to the rolling direction*

Figs. 9 to 11 show the dependencies of sheet metal X5CrNi18-10, which corresponds to previous diagrams. For 0° with respect to the rolling direction, the dependence curve of r value increases monotonously, Fig. 9. Fig. 10 shows small variation of the r value, so it is acceptable to adopt the mean constant value. The curve of the r value variation in Fig. 11 has a slightly decreasing character with strain increase on abscissa axis. Adoption of the mean constant value leads to a minor error, which is acceptable.

Dependence of the r in Fig. 9 can be approximated by the function of the following form:

$$r = B\varphi^a, \tag{11}$$

where **B** and **a** are constants which can be approximately determined in the following way:

$$a = \varphi_m;\ B = \frac{r_m}{a^a}, \tag{12}$$

where the subscript m denotes the selected values of r and φ in the diagram.

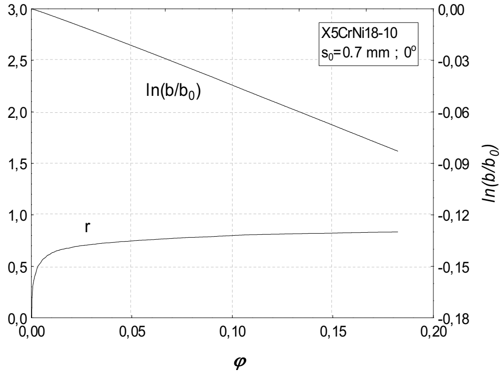

Fig. 9. *Dependence of the r value and true width strain on true strain for sheet X5CrNi 18-10, 0° to the rolling direction*

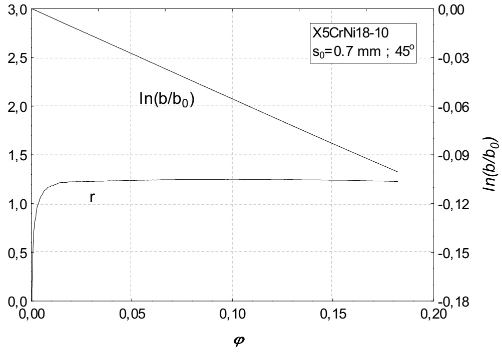

Fig. 10. *Dependence of the r value and true width strain on true strain for sheet X5CrNi 18-10, 45° to the rolling direction*

For the actual case in Fig. 9, the following functions are obtained:

$r = 1.006\varphi^{0.1}$; $\varphi_m = 0.1$ and $r_m = 0.799$,

$r = 1.136\varphi^{0.182}$; $\varphi_m = \varphi_{\max} = 0.182$ and $r_m = r_{\max} = 0.833$,

$r = 1.094\varphi^{0.15}$; $\varphi_m = 0.15$ and $r_m = 0.823$.

All the three functions are similar in nature, but the first one is more convenient according to experimental curve, Fig. 12.

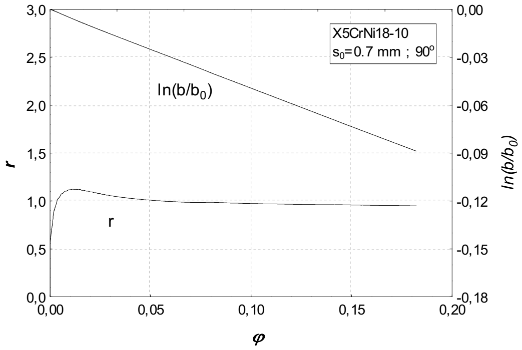

Fig. 11. *Dependence of the r value and true width strain on true strain for sheet X5CrNi 18-10, 90° to the rolling direction*

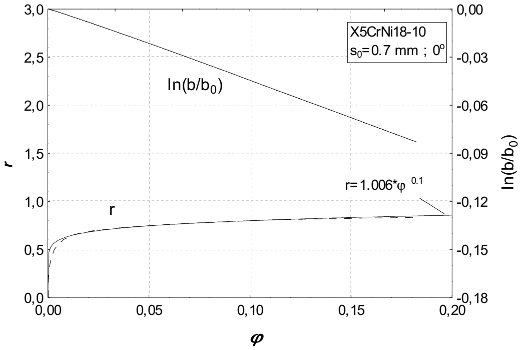

Fig. 12. *Approximation of the r value dependence for sheet X5CrNi 18-10, 0° to the rolling direction*

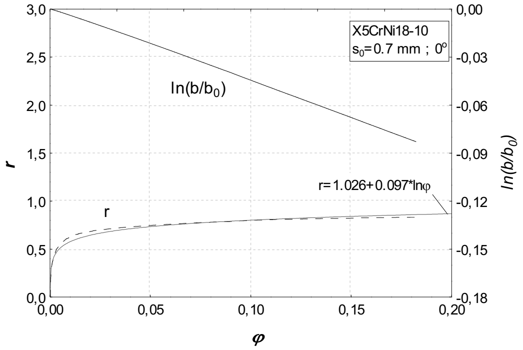

Fig. 13. *Approximation of r value dependence for sheet X5CrNi 18-10, 0° towards rolling direction*

Since all the experimental curves of the *r* value dependence are obtained in digital form like numeric series, during measuring, for very accurate defining of approximation functions it is possible to use appropriate mathematical methods, like the least squares method.

For the dependence of the *r* value from Fig. 9, the logarithmic approximation in software Statistica gives the following functions:

$r = 1.026 + 0.224 \log \varphi$

$r = 1.026 + 0.97 \ln \varphi$,   see Fig. 13.

Log is the sign for the common logarithm, while **ln** is the sign for the natural logarithm.

4 CONCLUSION

The normal anisotropy ratio, the *r* value, represents a very important characteristic of cold rolled thin sheet metals. So far, it has been generally accepted that it is a constant numeric value and as such, it has been used as an indicator of formability of sheet metals by deep drawing and property of materials in software for a numeric simulation of a plastic forming process. This paper has shown, based on a sophisticated experiment, that particular materials, like the Al alloys sheet metals and stainless austenite sheet metals, show significant variations of the *r* value during the forming process. That imposes the need to use adequate approximation functions, some of which were proposed in this paper.

Thus, it is necessary to use a very accurate experiment in order to determine dependency of the *r* value on plastic strain during the forming process, and then to use proper methods in order to define the convenient approximation function, which truly expresses anisotropic behavior of materials.

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