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Some Statistical Aspects of Firecracker Noise

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Since firecracker explosions have the characteristics of an impulse noise, they can cause hearing loss, serious personal injuries, fire hazards, annoyance, and even death. Their peak sound pressure levels at a distance of some meters can exceed the limit value of 140 dB, at which hearing protection is necessary. This article deals with some noise aspects resulting from firecracker explosions, which were measured during New Year's Eve. Apart from principal factors influencing the acoustic power of such an explosion, some new statistical aspects are described. A special emphasis is given to the probability distribution function of peak sound pressure levels, originating from a great number of firecracker explosions. Generally, the probability distribution closely follows the Rayleigh distribution, but when the number of explosions in unit time is high enough, it tends to a Gaussian distribution. Such transition is accelerated when reflections are taken into account, since in this case the number of peaks between two sequential zero crossings of sound pressure increases.

Keywords: firecrackers, explosions, statistics, prediction, noise level

0 INTRODUCTION

Explosions of firecrackers, especially during New Year's Eve, present a rather serious problem worldwide [1] and [2]. It is well known that firecracker explosions can cause loss of hearing and numerous extra-aural effects; they also present a fire hazard and a risk of serious personal injuries (burns of body and face, loss of sight and fingers), and even death [3] and [4].

Although an enormous number of firecrackers is manufactured and detonated each year, very few scientific papers on this subject have been published in serious periodicals. In the available literature we could not find any relevant studies concerning the statistical distribution of a series of random detonations such as those from firecrackers during New Year's Eve and other festivals.

Firecracker explosions belong to a group of intensive impulse noise sources, which are particularly hazardous [5] to [7]. The impulse form of firecracker explosions and their number extensively contributes to an increase of environmental noise; their peak sound pressure level at a distance of some metres can greatly exceed 140 dB: the level adopted for hearing protection. This can further cause annoyance, shift a hearing threshold or even produce deafness. Despite numerous acoustical trauma and extra aural effects due to firecracker explosions, the significance of this problem is mostly neglected and has been ignored by most legislatures around the world. In fact, there are some directives concerning statistics of firearm noise [8] and [9]; however, they do not deal with peak levels whose source locations and sound power change in time, as in the case of firecrackers. Therefore, statistical distribution of peak levels of the impulses at an arbitrary location, lying inside the region of interest, was investigated. The impulses were obtained as a result of the measurements of firecracker explosions taken during New Year's Eve.

The main scope of this paper is to outline statistical distributions of firecracker detonations, which can be used for some practical examples, such as for the prediction of noise during New Year's Eve and other festivals and their impact on environmental noise. The statistical method described can also be extended to working environments, for instance in metalworking shops. In this case, a much greater generalisation must be done, especially due to more complicated boundary conditions.

1 SOME CHARACTERISTIC PROPERTIES OF FIRECRACKER NOISE

Different types of firecrackers are in use. They differ in charge, dimensions, encapsulation and the way in which they are detonated. The rate of particular detonation further depends on granulation. Firecrackers

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with fine granulation possess higher detonation rates with typical values at around 3,500 m/s. The effect of an explosion is usually enhanced by properly constructed encapsulation. This is usually hard paper, sometimes waxed and sometimes consisting of different types of PVC.

Many firecrackers are charged with black powder, usually of fine granulation or with a compound of potassium chlorate and powdered aluminium. The explosion of firecrackers charged with black powder is stepped; their burning rate increases gradually from approx. 400 to 2,000 m/s. In firecrackers charged with a mix of potassium perclorate and powdered aluminium, detonation is faster and the sound of the explosion is more shrilly. As explained below, the black powder has very low mechanical efficiency, which is why it is used for weak firecrackers. Nitro-cotton has approximately twice the efficiency of black powder. The efficiency of various chemical explosives is usually estimated by comparing them with an equivalent weight of TNT. When confined in a firecracker, the powder quickly becomes hot enough to burn very rapidly, releasing its internal energy in a very short time. Pressure rises with the rate of burning. Guncotton releases nearly twice the heat of black powder, and two – thirds that of nitro-glycerine. Black powder is very low, while guncotton and nitro-glycerine are high in brisance. Gunpowder has the least pressure effect of any common explosives. Potassium chlorate ($KClO_3$) is also frequently an ingredient used in pyrotechnics: a powerful oxidizer that gives a low-temperature reaction. Using different additives, such as powdered aluminium, a large amount of energy can be released, although such mixes can be relatively dangerous. The most common firecrackers are filled with approximately 1 g of pyrotechnical ingredients.

Peak overpressure and positive phase duration increase with charge generally in quite a complicated form, but for small charges typical for firecrackers and distances exceeding a few meters, it increases approximately as the cube root of the charge mass [10] and [11]. Encapsulation is also of great importance. The enforced encapsulation increases the initial overpressure. Consequently, the rise time decreases and the intensity of explosion increases.

A firecracker explosion outdoors, in an open space without reflecting obstacles, produces a non-reverberant A -type impulse noise with a single spike -type overpressure, which can be approximated as a Friedlander impulse as shown in Fig. 1a. It is usually described by A duration (see later). On the other hand, a firecracker explosion indoors or in a half-open space has a reverberant effect, usually B (characterised by B duration) - or C -type (characterised by C duration) impulse, Figs. 1b and c.

The main characteristics of these types of impulse noise can be described by using three basic parameters: rise time, impulse duration and peak overpressure. Rise time is the time interval between the start of the impulse and the time when the peak value is achieved. In practice this is taken as the time to rise from 10 to 90% of its maximum absolute value of the sound pressure. Duration is a time interval between the start of the impulse and the time when the impulse decays for a definite amount. Peak value of sound overpressure is the maximum absolute value of the instantaneous sound pressure in Pa. It determines whether the linearized wave equation or some nonlinear form can best describe the propagation of this impulse. These parameters depend on the firecracker used and on its characteristics, especially the containment (mass or charge weight and type of explosive used), encapsulation and its design [11] and [12]. A typical firecracker exploding outdoors produces, at distances around 2 m, peak overpressure levels between 140 and 165 dB, with rise time between 1 and 20 μs and duration between 3 and 40 μs. The rarefied period, as described below, is between 4 and 150 ms. Spectral distribution of the energy in the impulse is controlled by both the A -duration and the rise time.

Duration of the A -type impulse (also called A -duration) is determined as the time required for the main wave to reach its unweighted peak sound pressure and return to baseline, (t_2-t_o). This is the time taken for the peak overpressure to fall for the first time to zero value at t_2 in Fig. 1a. In the case of ideal waves it is equal to the positive phase duration

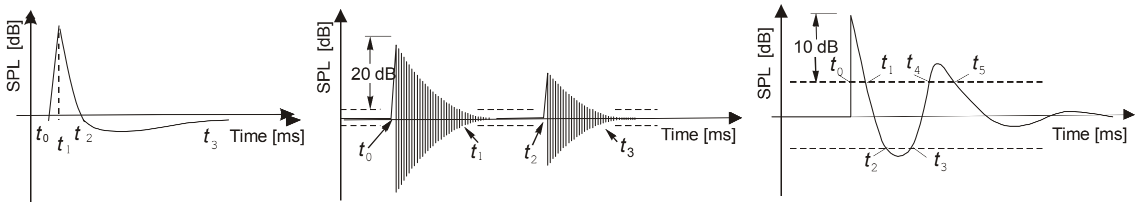


Fig. 1. *Sound pressure histories for different type of impulse noise in time domain; a) A-duration (t_2-t_o), b) B-duration (t_1-t_o)+(t_3-t_2), c) C-duration (t_1-t_o)+(t_3-t_2)+(t_5-t_4)*

used in blast physics. The tail of the impulse extends for approximately six times the A-duration (rarefied period) before the pressure returns again to zero. In practice, this is approx. by 1% of the peak level - at t_3. B-duration is determined as the time required for the main wave to reach -20 dB after oscillation time, (t_1- t_o)+ (t_3- t_2), or, alternatively, when the envelope of the unweighted peak sound pressure decays by 20 dB (Fig. 1*b*). If the path lengths involved are short enough, the reflections may interfere with the original pulse, producing a complex temporal pattern. With C-duration the total time required for the main wave to reach -10 dB, (t_1- t_o)+(t_3-t_2)+(t_5-t_4), or when the peak of pulse level exceeds the criteria -10 dB, as depicted in Fig. 1*c*. The corresponding level is thus about two-thirds of the peak pressure.

The A-type impulse can be described as a combination of a linear function for rising part and exponential function during its decay. The overpressure of such an impulse is thus described as a combination of linear and exponential functions [13]:

$$p(t) = p_r(t) + p_p(t) \tag{1}$$

where

$$p_r(t) = p_{peak}\left(\frac{t-t_o}{t_1-t_o}\right) \quad for \quad t_o \le t \le t_1 \tag{2a}$$

and

$$p_p(t) = p_{peak}\left(1-\frac{t-t_1}{t_2-t_1}\right)e^{-\frac{t-t_1}{t_2-t_1}} \; for \; t \ge t_1. \tag{2b}$$

Here p_{peak} is the overpressure amplitude and t_2 is the time for the overpressure to fall for the first time to its zero value.

A-type impulse caused by a firecracker has the form of the Friedlander pulse occurring as a result of a near instantaneous release of energy from a point source in a free field without reflecting surfaces (see Fig. 1a). Such a pulse is comprised of a rapid rise from ambient levels to a peak pressure followed by a relatively slow decay to ambient levels and, finally, a rarefied period before recovery to ambient pressure again.

In the presence of any reflecting surfaces, the reflections can interfere with the original pulse, producing a more complex pressure time history. The peak and the spectrum of the reflected components are altered by the impedance characteristics of the reflecting surface. On the other hand, within enclosures, there is often a slow, low frequency build-up of pressure upon which reflected wave components are superimposed prior to decay to ambient conditions. In such cases realistic impulses can differ significantly from the ideal Friedlander type.

2 STATISTICAL DISTRIBUTIONS

2.1 General Theory of the Peak Distributions

Firecracker detonations usually take place outdoors, where a free sound field is predominant. The noise of an individual firecracker is highly impulsive in nature. Its peak level and the place of its detonations are almost entirely random. During New Year's Eve hundreds and even thousands of detonations can be registered; by running together, the accompanying noise becomes more continuous and has some statistical properties. It is interesting to point out some observations on Probability Distribution Functions (PDF) of peak sound pressure levels originating from a great number of firecracker explosions, which occur mostly during New Year's time and certain festivals.

It is known that sound signals that exhibit Gaussian instantaneous value distribution can be represented by a continuous superposition of sine waves combined in random phase, independent of spectrum shape. However, the peak values will, to a great extent, be influenced by the spectrum shape, especially by its slope. For the peak distribution as a function of sound signal zero crossings, which is further connected with spectrum shape, Rice [14] found a general formula, consisting of two additive terms, one which has a Gaussian character with zero as a mean, and one which has the character of a Rayleigh-like distribution. In a slightly modified form Rice's formula can be written as [15]:

$$f(x)=\frac{\sqrt{1-\beta}}{\sqrt{2\pi}\sigma}e^{-\frac{1}{2\sigma^2}\frac{x^2}{(1-\beta)}}+\frac{\sqrt{\beta}}{2\sigma}\frac{x}{\sigma}\left[1+erf\left(\frac{x}{\sigma}\sqrt{\frac{\beta}{2(1-\beta)}}\right)\right]e^{-\frac{x^2}{2\sigma^2}}, \tag{3}$$

where: $f(x)$ is probability density of x, x is peak values of the sound observation, σ is standard deviation (rms value), erf is error function defined as

$$erf\ x=\frac{2}{\sqrt{\pi}}\int_0^x e^{-w^2}dw, \tag{4}$$

and β is a factor that depends on the spectrum shape and has the following form

$$\beta=\left(\frac{z}{2m}\right)^2, \tag{5}$$

where z is the total number of sound signal zero crossings per second (Fig. 2) and m is the total number of noise maxima per second.

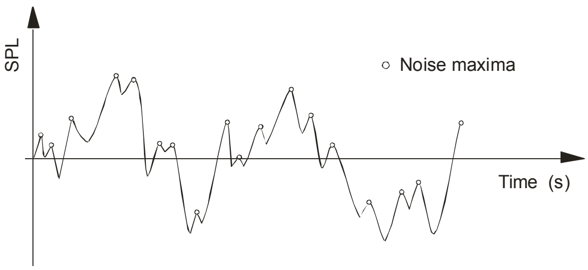


Fig. 2. *The number of peaks between two sound pressure zero crossings, which influence the peak distribution*

The Rayleigh probability density function is defined by

$$f(x,\sigma)=\frac{x}{\sigma^2}e^{-\frac{x^2}{2\sigma^2}} \quad for\ x\geq 0, \sigma>0, \tag{6a}$$

$$f(x,\sigma)=0, \quad \text{elsewhere.} \tag{6b}$$

There is a low probability density for very small and very large values of x, signifying that these values are uncommon and that most of the values lie in the region close to $x = \sigma$.

The peaks originating from small background level fluctuations must be considered separately or excluded in contrast to high peak values belonging to firecracker explosions. Here, we are interested in high values of peak levels due to firecracker explosions only. It can be seen that the final result of these two terms in Eq. (3) depends upon the value of parameter β. When there are infinitely many peaks per sound signal zero crossing we have a wide band spectrum, $\beta = 0$, and the peak distribution is Gaussian while in the extreme case, i.e., when there are exactly two sound signal zero crossings per peak, which is, for instance, the characteristic of a modulated sine wave, $\beta = 1$, and the Rayleigh distribution is obtained.

It must be pointed out that to obtain a true Rayleigh distribution only one noise maximum (or minimum) can occur between two succeeding zero crossings of the noise signal, while in the case of a Gaussian type peak distribution a large number of smaller noise maxima (and minima) occur between the zeros. The deviation from the Rayleigh distribution is due to the peaks and notches contained in each main half cycle of the noise signal (Fig. 3).

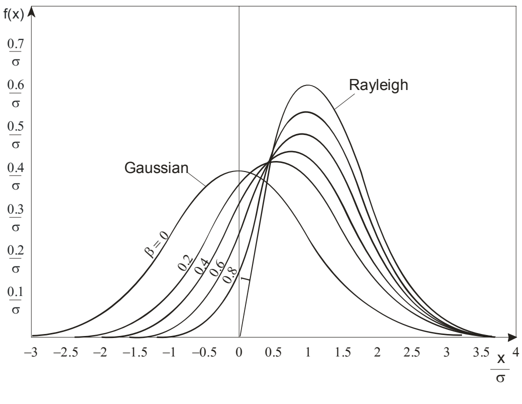


Fig. 3. *Transition from Rayleigh to a Gaussian distribution*

This shows that the spectrum defines the peak probability density curve, while the converse is not true, i.e. the peak probability density curve does not define the spectrum [15]. So, different frequency spectra may produce the same peak probability density curve. Transition to a Gaussian distribution is accelerated when reflections are present, as explained below.

In the limiting case, there is one peak for each sound signal zero crossing, $\beta = 1$, and Eq. (3) is reduced to the Rayleigh distribution, whereas a broadband random process can have many peaks for each sound signal zero crossing, $\beta \rightarrow 0$, and Eq. (3) is reduced to a normal distribution.

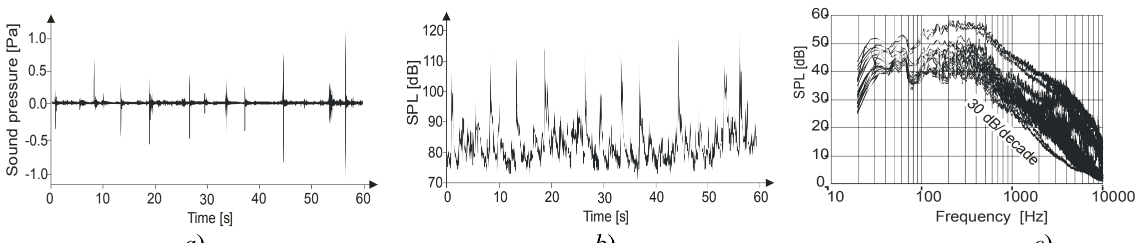
2.2 Gaussian Distribution as a Result of Multiple Reflections

Measurements, made during New Year's Eve (between 23:10 h, on 31. December, and 01:02 h, on 1. January) in a suburb of the city Ljubljana (only peaks exceeding the surroundings levels for more than 10 dB), have shown over 6,000 detonations as a result of firecracker explosions.

Their intensity was different, depending on the type of a firecracker and environmental conditions. In Fig. 4 the time dependence of sound pressure for the explosion of 12 firecrackers is presented. Their sound pressure level can be seen in Fig. 4b. It has full characteristics of the impulse noise, which tends to a background level at the end. Apart from the main peaks, the reverberant pulses appear as well as a result of the reflections from nearby walls and other obstacles.

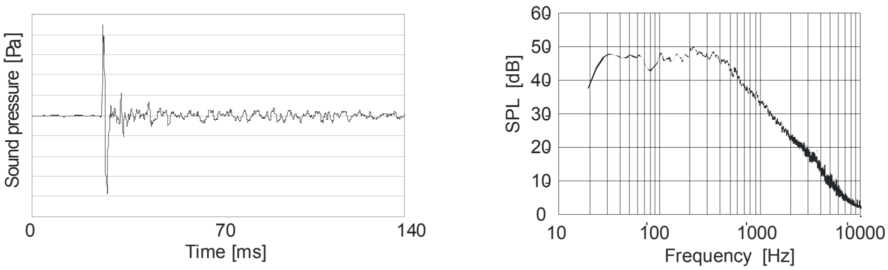
Due to very low time interval between these reflections, such peaks cannot usually be heard separately. The human ear is not able to discriminate two sound events in a time shorter than 30 ms [16]. For this reason, the main peaks and its reflections are usually perceived as only one peak, except when the explosion, receiver and obstacle are at very distant positions. Alternatively, such reverberant peaks can be recorded with sophisticated measuring equipment. The simple impulse can be compared with the A-type and the multiple (double and triple) with the B- or C-type. Sound pressure of a single firecracker impulse noise versus time is presented in Fig. 5a and the corresponding spectrum in Fig. 5b. For double impulses these characteristics are shown in Figs. 6a and 6b. In Figs. 5b and 6b the sampling rate is 44,100 Hz and FFT size is 2,048 with 90% overlap. The impulses, due to a still higher order of reflection, are hard to detect, since they interfere with small fluctuations of background noise. In general, the shape of the spectrum of an impulse is determined by the time history of the impulse, i.e., on the rise time, peak, and duration [17]. According to this, the spectrum of firecracker explosions differs from type to type too, type to type too, but the main spectral content is within the frequency range between 200 and 1,000 Hz (Figs. 5 and 6), and by making A-weighting it would be between 800 and 1,000 Hz. It is likely that such spectral content is chosen intentionally by producers using different type of additives in order to make the firecracker explosion as loud as possible.This could be inferred from the high pressure gradient (30 dB/decade) in the most audible part of the spectra (between 800 and 4,000 Hz).

Although sound pressure level amplitudes of multiple reflections decrease quickly, their tendency to a Gaussian distribution can also be proven.



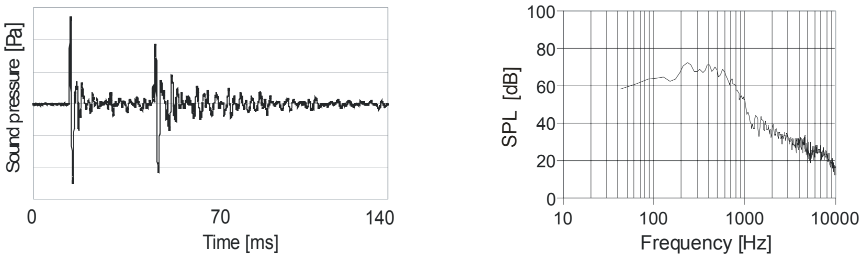
a) b) c)

Fig. 4. *Explosions of 12 firecrackers in time domain: a) sound pressure in Pa and b) SPL in dB, c) corresponding noise spectra*



a) *b)*

Fig. 5. *Single detonation of a firecracker: a) sound pressure, b) corresponding sound spectrum*



a) *b)*

Fig. 6. *Double detonation of a firecracker: a) sound pressure, and b) corresponding sound spectrum in dB*

A part of sound wave energy striking the obstacle is reflected and the rest is absorbed. The quantity of energy absorbed depends on its surface and is proportional to incident intensity. The intensity of the reflected sound is smaller than that of the incident sound by the quantity absorbed. Firecracker explosions are generally accompanied by secondary and higher order reflections (Figs. 4 to 6).

It can be proven in this case, that multiple mechanisms exist and that peaks follow a normal distribution. Denoting I_o as the sound intensity of firecracker explosion just before striking the first reflecting surface with an average sound absorption of α_1 and I_1 the reflected sound intensity, one can write

$$I_1 = I_o(1-\alpha_1)\ . \tag{7}$$

By taking into account that, at relatively short distances involved, the effect of air absorption is negligible in comparison with divergence and reflections [18], therefore only geometric divergence must be considered after the second reflection. The corresponding attenuation can be approximated by a potential function (see Eq. (13)):

$$I_2 = I_1\left(\frac{s_2}{s_1}\right)^{-2\gamma}(1-\alpha_2) = I_o\left(\frac{s_2}{s_1}\right)^{-2\gamma}(1-\alpha_1)(1-\alpha_2), \tag{8}$$

where s_1 and s_2 are total paths travelled by sound wave when striking the first and second surfaces respectively. γ is an attenuating coefficient, taking geometric divergence into consideration and also the effect of air and ground absorption to some extent. Sound intensity after the n-th reflection I_n can be written as

$$I_n = I_o(1-\alpha_1)(1-\alpha_2).....(1-\alpha_n)\left(\frac{s_n}{s_1}\right)^{-2\gamma} = I_0\left(\frac{s_n}{s_1}\right)^{-2\gamma}\prod_{i=1}^{n}(1-\alpha_i). \tag{9}$$

Taking logarithms of Eq. (9) the product can be transformed into a sum

$$\log(I_n) = \log(I_o) - 2\gamma\ \log\left(\frac{s_n}{s_1}\right) + \sum_{i=1}^{n}\log(1-\alpha_i). \tag{10}$$

Sound absorption coefficients of reflective surfaces and corresponding distances to them are random variables, depending on incident angle, frequency and surface characteristics. Therefore, the logarithms of their complements are random variables as well. Their exact values are hard to predict and they are not of interest at this stage. Of much greater importance is the distribution of corresponding intensities. According to the central limit theorem, the sum on right side of Eq. (10) is approximately normally distributed, [12] and [19]. Alternatively, sound pressure levels can be expressed by logarithms of sound intensity, yielding

$$L_n = L_o - 20\gamma \ \log\left(\frac{s_n}{s_1}\right) + 10\sum_{i=1}^{n}\log\left(1-\alpha_i\right) + C, \quad (11)$$

where C is a constant quantity, expressing dominant meteorological conditions during observation. In this way, it can be shown that peak sound pressure levels of multiple reflected impulses follow a Gaussian distribution.

2.3 Statistical Description of Noise from Firecrackers

The immission of a peak sound pressure level at a given location due to an individual firecracker depends mainly on its sound power and the distance from the place of its explosion. Instead of sound power levels, it is easier to deal with emitted sound pressure levels at a reference distance from the explosion. As shown above, peak sound pressure levels of randomly chosen firecrackers at a standard reference distance can usually be approximated by the Rayleigh distribution. When the number of explosions in the time unit is high, this distribution tends to a normal distribution pattern [8]. Besides, the most appropriate spatial probability density function appears to be a triangular distribution. Using a combination of these two distributions, a Joint Probability Density Function (JPDF) of immission peak sound pressure levels at a given location can be created. Here, a divergent part of sound attenuation during outdoor propagation can be employed as the most important component. Sound absorption is considered as well. Some practical examples are provided based on measured results, carried out over a New Year's Eve.

Usually, we only investigate high impulses with levels l_p, exceeding the background level l_b by at least 10 dB [9], that means

$$l_p \geq l_b + 10\,dB, \quad (12)$$

where l_p is the peak sound pressure level, defined by $l_p = 10 \log (p^2/p_o^2)$, with p as the peak sound pressure in Pa and p_o as the reference sound pressure.

In addition to spherical divergence, an excess attenuation due to air and ground absorption should be considered. The peak pressure decreases with distance r generally in an exponential manner

$$p(r) = p(r_{ref})\,(r/r_{ref})^{-\gamma}, \quad (13)$$

where $p(r_{ref})$ is the peak pressure at reference distance and γ is the attenuation coefficient. According to the ANSI model [20], the peak pressure for small explosive charges attenuates as $r^{-1.1}$ in the far field, which corresponds to a level decay of 6.6 dB per doubling of distance.

2.3.1 Statistical Methods

One of the simplest problems, considering only geometrical divergence and absorption, can be described in statistical terms and depends on two quantities: the sound power of the firecrackers used and the distance between the explosion and location of interest (usually a noise-sensitive location). These are two random variables, which must be considered simultaneously. For this reason, we will deal with some statistical quantities, where the Joint Cumulative Distribution Function (JCDF) and Joint Probability Density Function (JPDF) are most important. Of interest here is the influence of the distribution of sound emission peak levels radiated from the firecrackers and the distribution of their distances on the distribution of immission peak sound pressure levels at one location inside the region of interest.

The starting point is a spatial cumulative distribution function $F_R(r)$, which is the ratio between the area inside of which an individual explosion occurs and the whole area of investigation inside of which all these explosions occur. The latter is described by the radial distance $r_o(\varphi)$, which is generally directionally dependent on the most distant explosion of interest. It is supposed that the place of an explosion occurs with equal probability anywhere inside the investigated area. In other words, equal probabilities are assigned to equal areas in this region, which presents a constant probability density function.

Within this distance, peak levels can be investigated, which exceed certain values. The expression for $F_R(r)$ depends on boundary conditions and can be fairly complicated, especially in the presence of sound barriers and other obstacles on propagation paths. Since such explosions take place mainly outdoors, this kind of problem is less obvious, especially for our purposes where only peak levels are observed. However, it was shown that time intervals between reflections of one explosion outdoors are very short and higher order peaks cannot be usually perceived by ears. Therefore, it can be expressed as the ratio of two annular areas:

$$F_R(r)=\frac{\int_0^{2\pi}(r^2(\varphi)-r_{\min}^2(\varphi))d\varphi}{\int_0^{2\pi}(r_o^2(\varphi)-r_{\min}^2(\varphi))d\varphi}. \quad (14)$$

This ratio can be simplified when the curve of investigation $r(\varphi)$ and boundary curve $r_o(\varphi)$ are similar; in an open area without obstacles these are concentric circles. This is a reasonable assumption for outdoor sound propagation. Due to isotropy in outdoor sound propagation, the problem can be dealt with as a one-dimensional phenomenon and to be estimated as the ratio of two circular annuluses:

$$F_R(r)=\frac{\pi\,(r^2-r_{\min}^2)}{\pi\,(r_0^2-r_{\min}^2)}=\frac{r^2-r_{\min}^2}{(r_o^2-r_{\min}^2)}, \quad (15)$$

where r is the distance between the exposed position of investigation and the explosion of the firecracker, $r_{\min}$ is the shortest distance from an explosion at which an investigation makes sense and r_o is the maximal distance. The probability density function can be obtained by derivation [12]:

$$f_R(r)=\frac{d\,F_R(r)}{dr}, \quad (16)$$

and

$$f_R(r)=\frac{2\,r}{(r_0^2-r_{\min}^2)} \quad \text{for } r_{\min}\le r\le r_0, \quad (17a)$$

$$f_R(r)=0 \quad \text{elsewhere.} \quad (17b)$$

This is a triangular distribution. The value $r_{\min} = 0$ does not make sense, because of a singularity caused by the model of the point source. On the other hand, the place of an explosion is usually at least some decimetres away from the nearest ear or microphone, which are the most likely receivers of the noise.

In addition to applying triangular distribution for spatial coordinates, it is reasonable to assume that the probability density function of the emitted impulsive impulse noise generally has a Rayleigh-like form that tends to a normal one for a large number of explosions. Normal (Gauss) distribution of peak overpressure sound levels (power or pressure at a reference distance) is thus:

$$f_{L_{p,ref}}(l_{p,ref})\approx\frac{1}{\sqrt{2\pi}\,\sigma_{L_{p,ref}}}\,e^{-\frac{1}{2}\left(\frac{l_{p,ref}-\mu_{L_{p,ref}}}{\sigma_{L_{p,ref}}}\right)^2}. \quad (18)$$

The above function refers to the peak noise levels $l_{p,ref}$ at reference distance r_{ref} from the explosion (for instance 1 m); $\mu_{Lp,ref}$ presents its mean value and $\sigma_{Lp,ref}$ is a standard deviation of emission peak levels at the reference distance. Peak levels above 175 dB (at distances above $r_{\min}$) almost never occur. At this distance explosions whose peak levels do not exceed 125 dB are rare. Furthermore, low immission peak levels not exceeding the background level are of no interest; this has already been partially considered in Eq. (12).

The logical choice of minimum actual peak noise level $l_{p,ref\ \min}$ depends on the maximum distance r_o and immission background noise level l_b at the location of interest. Providing that the most distant explosion (in the order of 1 km) is clearly audible, the following relationship must hold true: $l_{p,ref\ \min} = l_b + 10 + 22 \log (r_o/r_{ref}) = l_b + 76$ dB. At usual background levels of around 50 dB, the minimum emission reference level must be 50 dB + 76 dB = 126 dB. In the following calculation we used 125 dB for simplification. Although strong detonations can often be heard clearly for many kilometres, additional effects must be taken into consideration and this complicates the use of a point source model. For the large majority of detonations, the values $l_{p,ref\ \min}$ = 125 dB and $l_{p,ref\ \max}$ = 175 dB can be taken. Similar limits were confirmed by some other investigations, [1] and [21]. This provides an approximate upper and lower limit for variables used.

The relationship between immission peak levels L_p and emission peak level at a reference distance $L_{p,ref}$, and the distance R to the firecracker exploded, is based on a point source propagation model:

$L_p = L_{p.ref} - 22 \log(R/r_{ref})$. (19)

In this way the expression for JCDF with L_p as a variable can be derived, which presents the probability P that peak sound pressure level L_p at a given location does not exceed a definite value l_p. This can be written in the following formula, [12] and [19]:

$$F_{Lp}(l_p) = P[L_p \le l_p] = \int_{R_{l_p}} fL_{p,ref,R}\,(l_{p,ref}, r) dl_{p,ref} dr. \quad (20)$$

Here, R_{lp} is a region in the plane $[l_{p,ref}, r]$, where the Eq. (19) holds true and its value is less than l_p. For a given l_p the region in the plane $[l_{p,ref}, r]$ is qualitatively pictured in Fig. 7.

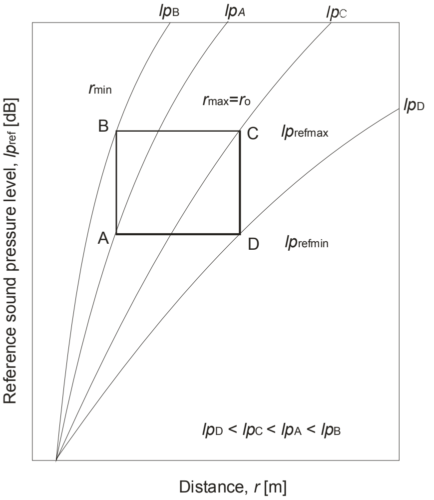


Fig. 7. *The line bounding region where $l_{p,ref} < l_p + 22 \log(r/rref)$*

Here, the lower boundaries of this region are considered as well. The expression under the integral sign $f_{Lpref,R}$ $(l_{p,ref}, r) dl_{p,ref}\, dr$ presents the probability of the peak emission level of some detonation to be in the interval $[l_{p,ref}, l_{p,ref}+dl_{p,ref}]$ and that this explosion occurs at a distance $[r, r+dr]$ from the point of investigation. In general, the capital letter $L_{p,ref}$ will be used for peak pressure level at the reference location as a random variable, and the same letter in lowercase $l_{p,ref}$ will represent the continuum of its possible values, which it may assume. For a fixed values of l_p, it is possible to solve the Eq. (20) of the boundary curve as the function of $l_{p,ref}$ and l_p by using equation (19):

$l_{p,ref} - 22 \log(r/r_{ref}) < l_{p,}$ (21)

stating that along this curve immission peak sound pressure level of detonations has a constant value. In Fig. 7 the region where this value is lower than a limit value, is shown as a hatched area. It is bounded by the curve $r_b\,(l_{p,ref})$. r_b is plotted as a function of $l_{p,ref}$ with l_p as a parameter. The cumulative distribution function can be generally obtained for nonnegative values of R and $L_{p,ref}$, by integration in the plane $[r\ l_{p,ref}]$. Both variables are independent since the sound power level of firecrackers used does not generally depend on the location of their explosion. In this way $f_{Lp,ref,R}(l_{p,ref},r)$ can be written as a product of two functions $f_{Lp,ref}(l_{p,ref})$ and $f_R(r)$. By considering the boundary conditions already mentioned, the limits of the integral become finite, which leads to

$$F_{Lp}(l_p) = \int_{l_{p,ref}\min}^{l_{p,ref}\max} f_{Lp,ref}(l_{p,ref}) \left(\int_{h(l_{p,ref},lp)}^{r_o} f_R(r) dr \right) dl_{p,ref}. \quad (22)$$

Finally, JPDF can be found by derivation of this expression with variable of l_p. After some calculus the distribution of immission peak pressure levels is obtained being very close to the Gaussian form. Solutions of Eq. (22) are presented in reference [3]. There exist three different solutions for JPDF depending on how the particular value of l_p creates a bounding curve in Fig. 7.

Depending on the magnitude it can pass through the top of the rectangle, through its ides or not at all.

2.3.2 Practical Results

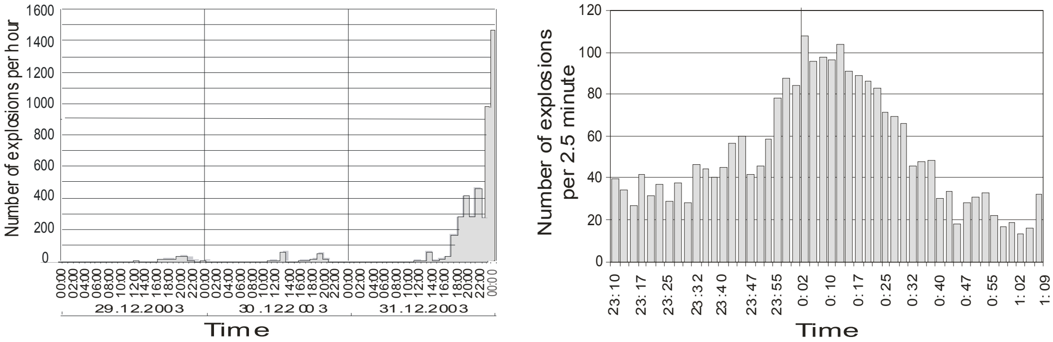
During New Year's Eve a high density of peak pressures was recorded as the result of firecracker explosions. Fig. 8a shows the number of peaks corresponding to the number of firecracker explosions (and their reflections) per hour for three successive days before New Year Eve and Fig. 8b the number of peaks per 2.5 minute for an hour before and after New Year's. We can see how the number of peaks during New Year's Eve rapidly increases. Between 23:10 and 23:30 h there were approximately 12 detonations per minute, whereas between 00:00 and 00:15 h this density is increased to approximately 40 per minute, i.e., more than tripled.

The cumulative PDF and JPDF of peak pressures was analysed for two observation intervals: between 23:10 and 23:15h and between 0:00 and 0:05 h. Results are presented in Figs. 9a and b). From these figures the background noise can be estimated as well. According to Slovenian environmental noise regulations, L_{99} level is considered to be background noise and L_1 to be long-term peak levels (absolute peak values described in this article are not considered in environmental legislation). The background level L_{99} between 23:10 and 0:00 h increased from 52 to 63 dB and L_1 from 72 to 96 dB.

In Figs. 9a and b the cumulative density distribution of peak levels for two different time periods are presented, at Figs. 10a and 10b JPDF, which were derived from the first two with respect to variable l_p. In the first case probability density distribution resembles a Rayleigh distribution, and a Gaussian in the second. Generally, the peak distribution is closer to a Rayleigh form, but when the number of explosions approaches 40 per minute, it tends to a Gaussian form, which can also be proved by inserting Eqs. (17a) and (18) into Eq. (24) and derivation of this expression by l_p. Thus, when the number of explosions is small, their peak levels follow a Rayleigh distribution. This can be seen from experimental data as shown in Fig. 8. Between 23:10 and 23:15 there were approximately 12 detonations per minute.

Between 00:00 and 00:05 this density more than tripled. For this reason many explosions took place, before the influence of the earlier had disappeared. In such cases, the probability distribution approached a Gaussian form.

Due to firecracker explosions, three days before and during New Year's Eve the typical daily exposure to noise changed dramatically (Fig. 11a). The maximum exposure usually occurs in the late morning and afternoon period, while the minimum is typically present in the early morning hours (around 4 am).



a) b)

Fig. 8. *Firecracker explosions density versus time: a) for 3 days before, b) for an hour before and after New Year's Eve*

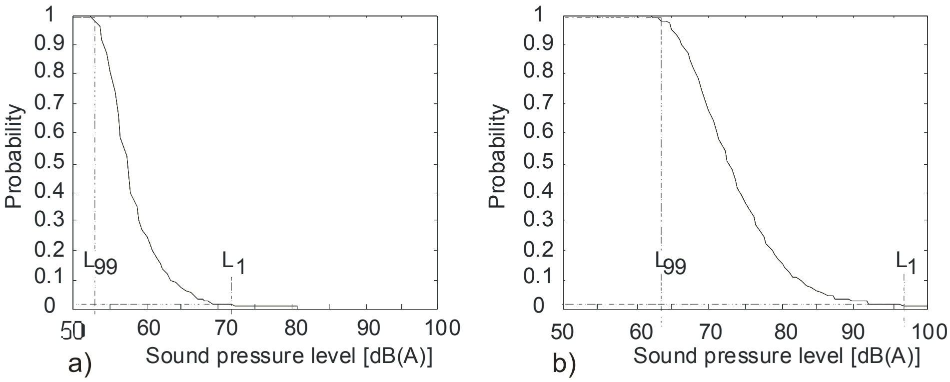


Fig. 9. *Cumulative density distribution of peak levels: a) during 23:10 to 23:15, b) during 0:00 to 0:05*

Fig. 11b shows results measured in weekdays without firecracker explosions (Tuesday, Wednesday and Thursday) in February for comparison with the days before and during New Year's Eve. This noise is mainly due to traffic. Such noise is usually considered as random and stationary, with differences in amplitudes during day and night time. The distribution is approximately normal. However, the whole day samples do not essentially change from day to day. Figs. 11 a and b show equivalent and impulse noise for comparison in order to see how impulse noise, which is measured with a dynamic impulse (35 ms), differs from the equivalent noise, which is measured with a dynamic fast (125 ms), and accounts for the estimated value of environmental noise. In earlier Slovene environmental noise regulation [22] a penalty was given according to the factor, which was determined by the difference between simultaneously measured A-weighted impulse noise level and A-weighted equivalent level (rating level). When greater than 2 dB(A) it was added to the equivalent level of noise for comparison with the limit value. Since the beginning of 2009 requirements of the new Slovenian regulation must be considered, namely a penalty of 12 dB(A) for such types of explosions must be added to measured values [23] and [24]. Such estimations have shown that, due to firecracker explosions, noise levels rise by more than 20 dB during the day and by more than 50 dB during the night in comparison with other days.

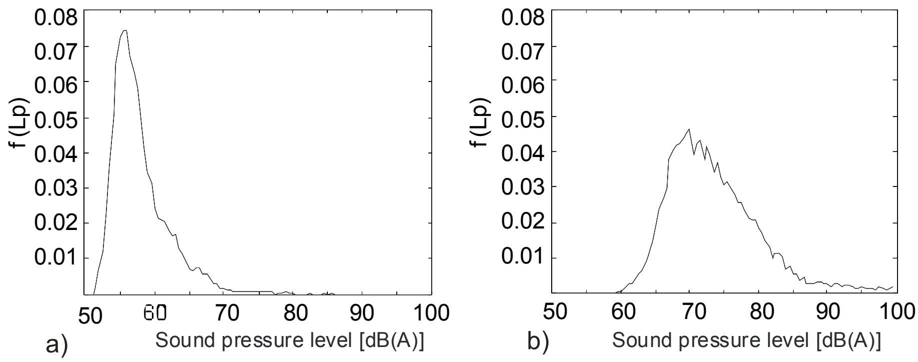


Fig. 10. *Peak level density distribution: a) between 23:10 and 23:15, b) between 0:00 and 0:05*

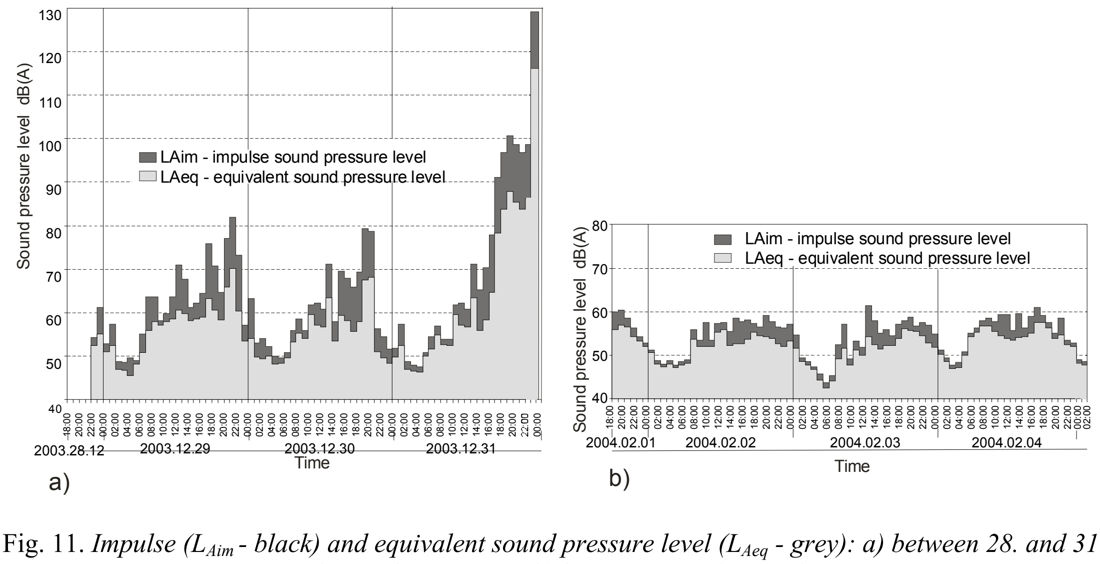


Fig. 11. *Impulse (L_{Aim} - black) and equivalent sound pressure level (L_{Aeq} - grey): a) between 28. and 31. December and 1. January, b) between 8. and 12. February*

According to Slovenian legislation [22], the estimated values, with 79 dB(A) during the day and 105 dB(A) during the night, exceed the limit values for a mixed (residential and business) area by 19 dB(A) during the day and by 55 dB(A) during the night. According to the new legislation this penalty value is even higher, achieving 12 dB(A) for high impulsive noise, typical for firecracker explosions. Aside from legislation, this also presents a very serious threat to many people and animals. Fig. 11 also shows that an increase in the number of firecracker explosions, which are purely impulse in character, does not adequately increase the contribution to the impulse noise. The main reason is that by increasing the number of firecracker explosions, the sound becomes more and more continuous, decreasing impulse penalty.

3 CONCLUSION

During New Year's Eve the density of firecracker explosions is enormous, both in time and space. Therefore, some statistical properties of firecracker explosions were investigated. Among them the basic one, the probability distribution function of peak overpressure levels, was developed. According to the results obtained, the authors revealed to which probability distributions the series of firecrackers explosions are best fitted. The results have shown that, when firecracker explosions are rare, their distribution showed a Rayleigh form. As New Year approached, the number of explosions increased almost exponentially, and reached up to 40 detonations per minute just some minutes after midnight as measured in a suburb of the city Ljubljana. In this way, noise maxima moved towards the late night hours and the Rayleigh distribution tended to a Gaussian one. Taking into consideration sound reflections in the regions where reflective obstacles are present, this transition is accelerated. As shown by measurements, firecracker explosions increase rating noise levels by more than 20 dB during the day and by more than 50 dB during the night in comparison with other days. This can have serious consequences for people and animals. The statistical method described in this study can be used for predicting noise due to different type of impulse noise sources, such as during New Year's Eve and other festivals, where a series of impulsive noise events occur. The principles of these investigations can be extended to factories and metalworking shops, to battlefield etc., with the aim to prevent hazardous effects of impulse noise.

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