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Vzporedna analiza toplotnih napetosti v vrtečih se diskih z enakomerno trdnostjo
Simultaneous Analysis of Thermal Stresses in Rotating Disks of Uniform Strength

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V sestavku obravnavamo določitev debeline vrtečega se diska plinske turbine, ki je dodatno izpostavljen temperaturnemu gradientu. Analitična rešitev, ki je v nam znanih virih še nismo zasledili, je dobljena s predpostavljeno temperaturno porazdelitvijo (konstantna, linearna, parabolična ipd.), medtem ko je vezani problem, kjer je temperatura odvisna tudi od debeline, rešljiv le numerično.

The aim of this paper is to determine thickness of a rotating gas turbine disk of uniform strength, if also temperature gradient is encountered. Analytical solution, not known in the searched literature, is obtained by assuming some temperature distribution (being constant, linear, parabolic etc.), while the joint problem, where temperature depends also on the thickness, is left to the numerical procedure.

0. UVOD

Morda je najpomembnejši primer diska v sodobni tehniki pri plinskih turbinah, ki se vrtijo s hitrostmi do $2 \cdot 10^4$ min^{-1}. Takšni diski nosijo lopatje na sorazmerno velikih polmerih. Poleg tega morajo biti tanki, da omogočijo namestitev več enot na isti gredi. Ustrezno se tudi njihova debelina mora spreminjati s polmerom, da se zmanjša količina snovi oziroma njihova teža (centrifugalne sile). Pogosto je uporabljeno gradivo nikljeva zlitina, ki daje veliko trdnost in toplotno obstojnost [1]. Ravnovesna enačba vrtečega se diska spremenljive debeline je znana [2]:

0. INTRODUCTION

Probably the most important example of a disk in modern engineering is that of the gas turbine, running at very high speed up to $2 \cdot 10^4$ *rpm*. Such disks have to carry turbine blades of fairly large radii. They also have to be thin, to enable several units to be carried on a single shaft. Their thickness must therefore be allowed to vary radially to minimise the amount of disk material and its weight (flywheel forces). Materials often used for such disks are nickel based alloys of high strength and heat resistance [1]. The equilibrium equation of a rotating disk with variable thickness is known to be [2]:

$$\frac{d}{dr}(tr\sigma_r) - t\sigma_\varphi + t\rho\omega_o^2 r^2 = 0 \qquad (1)$$

Da čimbolje izkoristimo gradivo, ki naj ima enakomerno porazdeljene napetosti, je pomembno, da je povsod v telesu diska $\sigma_r = \sigma_o$. To pomeni, da je σ_o tudi na notranjem in zunanjem obodu, kar je mogoče, saj mora biti disk znotraj oprt na gred, pa tudi lopatice so pritrjene na vnanjem obroču. Ta predpostavka pomeni, da mora biti izpolnjena naslednja diferencialna enačba za določitev debeline:

To make full use of the material by having the highest possible stresses it seems sensible to put value $\sigma_r = \sigma_o$ everywhere in the disk. That would also necessitate haying σ at the inner and outer radii, which is not unreasonable since the disk has to be secured to the shaft, and the blades are fixed to the outer rim edge of the disk. Such an assumption renders the following differential equation for the required axial thickness:

$$t'/t + \rho\omega_o^2\, r/\sigma_o = 0 \qquad (2)$$

z rešitvijo (če je $a \leqq r \leqq b$), ki je prav tako dobro znana:

with its solution (if $a \leqq r \leqq b$), which is well known to be:

$$t(r) = t(a) \cdot exp\left(-\frac{\rho\omega_o^2}{2\sigma_o}\left(r^2 - a^2\right)\right) \tag{3}$$

1. TOPLOTNA OBREMENITEV

Zaradi visokih temperatur v plinskih turbinah moramo v vrtečem se disku upoštevati nastanek toplotnih napetosti. Zato je ravnovesna enačba (1) posplošena z odvajanjem ter vključitvijo Hookovega zakona in toplotnega raztezanja [3]:

1. THERMAL LOADING

The high temperatures encountered in gas turbines require consideration of the thermal stresses induced in the rotating disk. To this end, the equilibrium equation (1) has to be extended by differentiation, including the Hookean law and thermal expansion [3]:

$$\frac{d^2\sigma_r}{dr^2} + \left(\frac{3}{r} + \frac{t'}{t}\right)\frac{d\sigma_r}{dr} + \left[\frac{t''}{t} + \frac{2+\nu}{r}\cdot\frac{t'}{t} - \left(\frac{t'}{t}\right)^2\right]\sigma_r + (3+\nu)\rho\omega_o^2 + \frac{\alpha E}{r}\cdot\frac{dT}{dr} = 0 \tag{4}$$

ki ob podmeni o enakomerni trdnosti daje enačbo za določitev debline:

which, subject to postulated uniform strength condition, reduces to:

$$\frac{t''}{t} + \frac{2+\nu}{r}\cdot\frac{t'}{t} - \left(\frac{t'}{t}\right)^2 + \frac{3+\nu}{\sigma_o}\rho\omega_o^2 + \frac{\alpha E}{r\sigma_o}\cdot\frac{dT}{dr} = 0 \tag{5}$$

katere rešitev je:

the solution of which is found to be:

$$t(r) = t(a)\cdot exp\left(\frac{\alpha E}{\sigma_o}\left\{\frac{T(a)}{1+\nu}\left[1 - \left(\frac{a}{r}\right)^{1+\nu}\right] - \frac{1}{r^{1+\nu}}\int_a^r T(r)\, r^{\nu}\, dr\right\} - \frac{\rho\omega_o^2}{2\sigma_o}\left(r^2 - a^2\right)\right) \tag{6}$$

ki se s $T(r)$ = konst zbistri v obrazec (3). Seveda pa moramo poznati ustrezno temperaturno porazdelitev, ob upoštevanju hlajenja vzdolž obeh plati diska. Ustrezno diferencialno enačbo [4]:

where for $T(r)$ = const it reduces to (3). However, if the temperature is to be evaluated, heat losses from the sides of the disk must be considered. The corresponding differential equation is [4]:

$$\frac{d^2T}{dr^2} + \left(\frac{1}{r} + \frac{t'}{t}\right)\frac{dT}{dr} - \frac{hT(r)}{kt(r)} = 0 \tag{7}$$

pa v našem primeru lahko razrešimo le numerično z iteracijo zaradi (6), začenši s $T(r) = T(a)$, t.j.:

which can be solved only numerically by an iteration due to eq. (6), initiating by $T(r) = T(a)$:

$$\frac{d^2T}{dr^2} + \left(\frac{1}{r} - \frac{\rho\omega_o^2 r}{2\sigma_o}\right)\frac{dT}{dr} - \frac{hT(r)}{kt(a)}\cdot exp\left(\frac{\rho\omega_o^2}{2\sigma_o}\left(r^2 - a^2\right)\right) = 0 \tag{8}$$

in nadaljujemo z:

and continuing with:

$$\frac{d^2T}{dr^2} + \left\{\frac{1}{r} - \frac{\rho\omega_o^2 r}{2\sigma_o} + \frac{\alpha E}{\sigma_o}\left[\frac{T(a)}{a}\cdot\left(\frac{a}{r}\right)^{2+\nu} + \frac{1+\nu}{r^{2+\nu}}\int_a^r T(r)\, r^{\nu}\, dr - \frac{T(r)}{r}\right]\right\}\frac{dT}{dr} -$$

$$- \frac{hT(r)}{kt(a)}\cdot exp\left(\frac{\rho\omega_o^2}{2\sigma_o}\left(r^2 - a^2\right) - \frac{\alpha E}{\sigma_o}\left\{\frac{T(a)}{1+\nu}\left[1 - \left(\frac{a}{r}\right)^{1+\nu}\right] - \frac{1}{r^{1+\nu}}\int_a^r T(r)\, r^{\nu}\, dr\right\}\right) = 0 \tag{9}$$

2. POENOSTAVLJENI PRIMERI

Da bi ocenili rezultate, predpostavimo temperaturo z linearno in s parabolično porazdelitvijo.

Če je temperaturna porazdelitev linearna $(a \le r \le b)$:

2. CASE STUDIES

In order to assess the results, temperature profiles have been approximated by linear and parabolic distributions.

If the temperature is linear $(a \le r \le b)$:

$$T(r) = T(a) - \frac{T(a) - T(b)}{b - a}\,(r - a) \qquad (10),$$

dobimo ustrezno debelino iz obrazca (6):

the corresponding thickness obtained by eq. (6) is:

$$t(r) = t(a) \cdot exp\left(\frac{\alpha E}{\sigma_o} \cdot \frac{T(a) - T(b)}{1 - b/a}\left\{\frac{1}{1+\nu}\left[1 - \left(\frac{a}{r}\right)^{1+\nu}\right] - \frac{r/a}{2+\nu}\left[1 - \left(\frac{a}{r}\right)^{2+\nu}\right]\right\} - \frac{\rho\omega_o^2}{2\sigma_o}\,(r^2 - a^2)\right) \qquad (11).$$

Ko pa je temperaturna porazdelitev parabolična (upoštevajoč $\frac{dT}{dr}(r = b) = 0$):

If the temperature is parabolic (bearing in mind $\frac{dT}{dr}(r = b) = 0$):

$$T(r) = T(a) - \frac{T(a) - T(b)}{(b - a)^2}\,(a^2 - 2ab + 2br - r^2) \qquad (12),$$

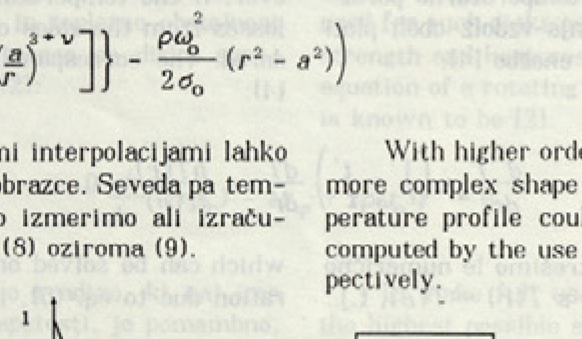
dobimo ustrezno debelino iz obrazca (6):

the corresponding thickness obtained by eq. (6) is:

$$t(r) = t(a) \cdot exp\left(\frac{\alpha E}{\sigma_o} \cdot \frac{T(a) - T(b)}{(b - a)^2}\left\{\frac{a^2 - 2ab}{1+\nu}\left[1 - \left(\frac{a}{r}\right)^{1+\nu}\right] + \frac{2br}{2+\nu}\left[1 - \left(\frac{a}{r}\right)^{2+\nu}\right] - \frac{r^2}{3+\nu}\left[1 - \left(\frac{a}{r}\right)^{3+\nu}\right]\right\} - \frac{\rho\omega_o^2}{2\sigma_o}(r^2 - a^2)\right) \qquad (13)$$

Z višjimi in drugačnimi interpolacijami lahko dobimo tudi bolj zapletene obrazce. Seveda pa temperaturni profil tudi lahko izmerimo ali izračunamo z uporabo enačb (7), (8) oziroma (9).

With higher order interpolations, formulae of more complex shape may be obtained. The temperature profile could also be measured and/or computed by the use of eqs. (7), (8) and (9) respectively.



Sl. 1. *Oblika diska (vrtilna frekvenca 19100 min^{-1}).*
Fig. 1. *Disk Profile (19100 rpm).*

Slika 1 prikazuje izračunane oblike diska pri podmeni o konstanti in parabolični porazdelitvi temperature, ob uporabi podatkov: $\omega_o = 2\cdot10^3\,s^{-1}$, $E = 200\ GPa$, $\sigma_o = 300\ MPa$, $\rho = 7850\ kg/m^3$, $\nu = 0{,}3$, $\alpha = 1\cdot10^{-5}\,K^{-1}$, $\Delta T = \pm 200\ K$, $a = 0{,}05$ m, $b = 0{,}30$ m.

Figure 1 shows the disk profiles due to constant and parabolic temperature distributions, using the following sets of data: $\omega_o = 2\cdot10^3\,s^{-1}$, $E = 200\ GPa$, $\sigma_o = 300\ MPa$, $\rho = 7850\ kg/m^3$, $\nu = 0.3$, $\alpha = 1\cdot10^{-5}\,K^{-1}$, $\Delta T = \pm 200\ K$, $a = 0.05$ m, $b = 0.30$ m.

3. PREOBREMENITEV

3. OVERLOADING

Kadar disk izbrane oblike zavrtimo hitreje ($\omega > \omega_o$), moramo namesto enačbe (4) razrešiti:

If the disk of selected profile is run faster ($\omega > \omega_o$), the equation (4) must be replaced by:

$$\frac{d^2\sigma_r}{dr^2} + \left\{\frac{3}{r} - \frac{\rho\omega_o^2 r}{2\sigma_o} + \frac{\alpha E}{\sigma_o}\left[\frac{T(a)}{a}\cdot\left(\frac{a}{r}\right)^{2+\nu} + \frac{1+\nu}{r^{2+\nu}}\int_a^r T(r)\,r^{\nu}\,dr - \frac{T(r)}{r}\right]\right\}\frac{d\sigma_r}{dr} -$$

$$-\left(\frac{3+\nu}{\sigma_o}\,\rho\omega_o^2 + \frac{\alpha E}{r\sigma_o}\cdot\frac{dT}{dr}\right)\sigma_r + (3+\nu)\,\rho\omega^2 + \frac{\alpha E}{r}\cdot\frac{dT}{dr} = 0 \qquad (14),$$

ki se s $T(r)$ = konst zbistri v obrazec:

which for $T(r)$ = const reduces to:

$$\frac{d^2\sigma_r}{dr^2} + \left(\frac{3}{r} - \frac{\rho\omega_o^2 r}{2\sigma_o}\right)\frac{d\sigma_r}{dr} - \left(\frac{3+\nu}{\sigma_o}\,\rho\omega_o^2\right)\sigma_r + (3+\nu)\,\rho\omega^2 = 0 \qquad (15).$$

Če je temperaturna porazdelitev linearna (10), velja enačba:

If the temperature distribution is linear (10), the above equation transforms to:

$$\frac{d^2\sigma_r}{dr^2} + \left\{\frac{3}{r} - \frac{\rho\omega_o^2 r}{2\sigma_o} + \frac{\alpha E}{\sigma_o}\cdot\frac{T(a)-T(b)}{1-b/a}\left\{\frac{1}{1+\nu}\left[1-\left(\frac{a}{r}\right)^{1+\nu}\right] - \frac{r/a}{2+\nu}\left[1-\left(\frac{a}{r}\right)^{2+\nu}\right]\right\}\right\}\frac{d\sigma_r}{dr} -$$

$$-\left(\frac{3+\nu}{\sigma_o}\,\rho\omega_o^2 + \frac{\alpha E}{r\sigma_o}\cdot\frac{T(a)-T(b)}{1-b/a}\right)\sigma_r + (3+\nu)\,\rho\omega^2 + \frac{\alpha E}{r}\cdot\frac{T(a)-T(b)}{a-b} = 0 \qquad (16).$$

Ko pa je temperaturna porazdelitev parabolična (12), velja enačba:

When the temperature distribution is parabolic (12), it alternatively follows:

$$\frac{d^2\sigma_r}{dr^2} + \left\{\frac{3}{r} - \frac{\rho\omega_o^2 r}{2\sigma_o} + \frac{\alpha E}{\sigma_o}\cdot\frac{T(a)-T(b)}{(b-a)^2}\left\{\frac{a^2-2ab}{1+\nu}\left[1-\left(\frac{a}{r}\right)^{1+\nu}\right] + \frac{2br}{2+\nu}\left[1-\left(\frac{a}{r}\right)^{2+\nu}\right] -\right.\right.$$

$$\left.\left.-\frac{r^2}{3+\nu}\left[1-\left(\frac{a}{r}\right)^{3+\nu}\right]\right\}\right\}\frac{d\sigma_r}{dr} - \left(\frac{3+\nu}{\sigma_o}\,\rho\omega_o^2 + \frac{\alpha E}{r\sigma_o}\cdot\frac{T(a)-T(b)}{(b-a)^2}\cdot 2\,(r-b)\right)\sigma_r +$$

$$+ (3+\nu)\,\rho\omega^2 + \frac{\alpha E}{r}\cdot\frac{T(a)-T(b)}{(a-b)^2}\cdot 2\,(r-b) = 0 \qquad (17).$$

Reševanje enačb (14) do (17) je mogoče npr. numerično, ob ustreznih robnih pogojih pri notranjem in zunanjem polmeru.

Solution of eqs. (14) to (17) is possible by numerical means, subject to boundary conditions at the inner and outer radii.

4. SKLEP

Posebno pozornost moramo posvetiti izbiri delovne napetosti, da zagotovimo ustrezno dobo trajanja turbinskega diska ob zvišani temperaturi, upoštevajoč pojavljanje termičnega lezenja pri veliki toplotni obremenitvi. Podrobnosti gradnje turbin so odvisne od mnogih vplivov, ki si utegnejo tudi nasprotovati, toda oblikovanje rotorjev je prav gotovo med najpomembnejšimi dejavniki z vidika tveganj in varnosti.

Določeno obliko diska po navadi skušamo poenostaviti, npr. z ravnimi boki. Takšna trapezna oblika je preprostejša za izdelavo in nadzor kakovosti, kakor pa natančen eksponentni potek, toda reševanje enačb (4) in (7) trapeznega profila v analitični obliki [3] sploh ni mogoče.

Obročna napetost eksponencialnega profila:

4. CONCLUSION

Care must be taken to select a working stress which will give an adequate length of life for the turbine disk at its maximum operating temperature, bearing in mind that some creep of the material will inevitably occur at the high temperatures involved. The design of turbines depends on many factors, often conflicting, but the rotor design is certainly one of the most important from the point of view of hazards and safety.

Disk profile determined may actually be approximaed by straight lines. Such a trapezoidal shape is easier to manufacture and to check by quality control that the precise exponential form, but the solution of eqs. (4) and (7) for trapezoidal profile cannot be gained be analytically [3].

Circumferential stress of exponential profile:

$$\sigma_\varphi = \sigma_0 + \alpha E\left\{\left[T(a)a^{1+\nu} + (1+\nu)\int_a^r T(r)r^\nu dr\right]/r^{1+\nu} - T(r)\right\} \neq f(\omega) \qquad (18).$$

je v soglasju s [5] in [6].

is in agreement to [5] and [6].

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6. REFERENCES

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