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Numerični model za določanje dobe trajanja zobnikov

Numerical Model for the Calculation of Service Life of a Gear

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Za analizo zobniških prenosnikov smo izdelali matematični model, da bi lahko čimbolj natančno opisali dejanske razmere. Za analizo smo uporabili verjetnostno mehaniko loma, pri čemer smo upoštevali začetno velikost razpoke, materialne konstante in faktor intenzivnosti napetosti kot naključne funkcije. Na podlagi tega smo razvili algoritem za izračunavanje napetostno deformacijskih porazdelitev in preostale dobe trajanja zobnikov ter računalniški program STAFTAG. Program temelji na metodi končnih elementov ob uporabi verjetnostne mehaniko loma in je namenjen za delo na osebnem računalniku. Rezultati, ki jih daje program STAFTAG, se dobro ujemajo z eksperimentalnimi rezultati.

For the accurate analysis of gear drives, mathematic modeling is used to simulate, as precisely as possible, the actual conditions of operation. Probabilistic fracture mechanics analysis is used whereby the initial crack size, material constants and stress intensity factors are considered as random functions. On the basis of this, the algorithm for calculating the stress/strain distribution and the remaining life of gears is described together with implementation in the computer program STAFTAG. The program is based on the finite element method using probabilistic fracture mechanics, and is intended for work on personal computers. The results of the STAFTAG program are shown to compare well with experimental results.

0. UVOD

Problem utrujanja komponent, obremenjenih z različnimi dinamičnimi obremenitvami, je ena od najzahtevnejših nalog, saj je izredno občutljiv za mehanske lastnosti materialov, geometrijsko obliko komponent, razvoj obremenitev in vpliv okolice. Ker ni mogoče upoštevati vseh teh parametrov z determinističnimi modeli, smo v preračun vpeljali teorijo verjetnostne mehaniko loma [1].

Raziskave smo opravili na zobnikih, ki so najpogosteje uporabljani mehanski deli v strojniški praksi in so vsakemu inženirju dobro znani. Omogočajo velik izkoristek in natančen prenos moči ob razmeroma nizki ceni in nezahtevnem vzdrževanju.

Analizirali smo dva elementa na podlagi elastoplastičnih končnih elementov, in sicer zobnik z razpoko in zobnik brez nje. Praktično se napake — razpoke pojavijo najpogosteje v zobnem korenu. Te napake — razpoke se pojavljajo zaradi utrujanja, termične obdelave ali zaradi začetnih zarez na površini ali pregrobe mehanske obdelave. Rezultati statičnih oziroma dinamičnih analiz so bili uporabljeni kot vstopni podatki pri verjetnostnem izračunu preostale dobe trajanja s programom STAFTAG.

0. INTRODUCTION

The problem of fatigue in components subjected to different dynamic loads is one of the most demanding engineering problems because of its sensitivity to the mechanical properties of materials, geometry of components, load history and the influence of the environment. It is not possible to take into account all these parameters through deterministic approaches, therefore the theory of probabilistic fracture mechanics has been applied to calculations.

The research was performed on gears, which are the most frequently used mechanical parts in automotive engineering. Familiarity with gearing is essential for mechanical engineers. Gears have been and continue to be one of the most important means of mechanical power transmission, and this situation will continue in the future. They offer highly efficient, precise, positive power transmission at relatively low cost and maintenance.

Two dimensional elasto-plastic finite element analyses have been used. Gears with and without defects were taken into account [9]. In practice, defects occur most frequently in the tooth root. These defects are due to the fatigue, heat treatment or initial notching in case of clumsy or too rough machining. The results of the static and/or dynamic analyses were used as input data to the probabilistic calculation of the remain life by the program STAFTAG.

1. MATEMATIČNI MODEL ŠIRJENJA RAZPOKE

Že več ko celo stoletje so raziskave utrujanja kovinskih materialov povezane z uporabo ustreznih krivulj trajanja S-N, vendar je šele v zadnjih časih bila omogočena natančna razlaga teh krivulj, prav zaradi novih spoznanj o lastnostih zelo kratkih razpok, to je razpok z dolžinami od nekaj mikrometrov do nekaj sto mikrometrov.

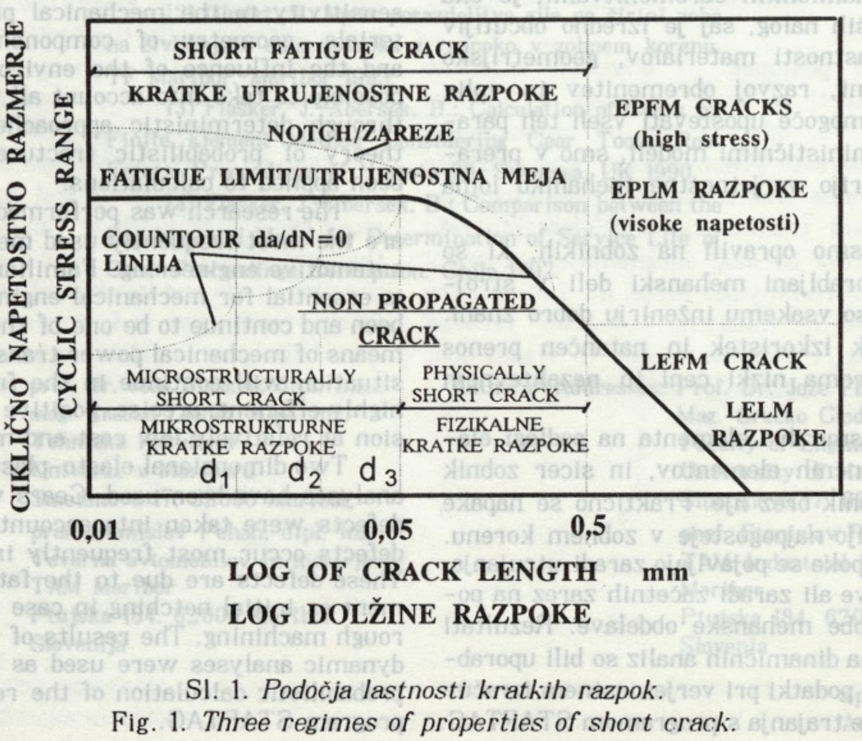
Ta nova spoznanja je omogočilo odkritje elastoplastične mehanike loma (EPML) in novih tehnik zapisovanja rasti majhnih površinskih razpok. Linearno elastična mehanike loma (LEML) je za opisovanje zelo kratkih razpok neuporabna, saj elastično napetostno polje ne more natančno opisati močnih deformacijskih polj ob vrhu majhnih defektov v zelo trdnih materialih.

Če želimo določiti lastnosti kratkih razpok, moramo upoštevati, da je treba v analitičnih modelih, ki opisujejo krivulje S-N, upoštevati posebnosti, ki prevladujejo v polju ob vrhu razpoke, kakor tudi začetno veliko, vendar za tem zmanjšujoče razmerje širjenja razpoke [2]. To lahko razumemo z upoštevanjem posameznih področij v Kitagawa-Takahashijevem diagramu, ki znatno pomaga pri razumevanju lastnosti kratkih razpok. Diagram prikazuje vplivne velikosti napake na mejo utrujanja, kar je prikazano na sliki 1. Za velike napake — razpoke mora biti obratovalna napetost majhna, če želimo imeti dolgo dobo trajanja (izpolnjevati mora veljavnost LELM). Robni pogoj je podan z ravno črto z nagibom — polovico faktorja intenzivnosti napetosti, in sicer:

1. CRACK PROPAGATION MATHEMATICAL MODEL

For more than a century, the research of fatigue of metallic materials has been associated with drafting the appropriate S-N endurance curves, whose precise interpretation has been ensured only recently, due to new findings about the properties of very small cracks, i.e. cracks from some micrometers to some hundreds of micrometers. One important contribution is the emergence of elasto-plastic fracture mechanics (EPFM); others include new techniques to monitor the growth of small surface cracks. Linear elastic fracture mechanics (LEFM) is useless for describing the properties of very small cracks, since the elastic stress field cannot realistically describe high strain fields near the tip of very small defects in high-strength materials.

To be able to determine the properties of very short cracks, it is important in the analytical models describing the S-N curves to consider the dominant properties of the fields of the crack tip as well as the crack growth rate, which is high in the beginning but then decreases [2]. This can be reached by considering the separate regimes in the Kitegawa-Takahashi diagram, which gives a significant advance in the understanding of short crack behavior. This diagram shows the effect of defect size on the fatigue limit stress, for example, figure 1. For large defects, the allowable stress for long life must be low, within linear elastic fracture mechanics regimes and, therefore, the limiting condition is given by straight line of slope minus one half of the threshold stress intensity factor such that:



Sl. 1. *Podočja lastnosti kratkih razpok.*

Fig. 1. *Three regimes of properties of short crack.*

$$\Delta K_{th} = Y\, \Delta\sigma \sqrt{\pi a} \qquad (1),$$

kjer so: Y — oblikovni faktor, $\Delta\sigma$ — napetost, a — kritična dolžina razpoke.

Na drugi strani področja pa so izredno majhne napake, za katere je obratovalna napetost podana kakor za element brez razpoke. Kitagawa-Takahashijev diagram prikazuje vsa področja med tema ekstremoma.

Razpoke so lahko:
— mikrostrukturno kratke,
— fizikalno kratke,
— dolge, pri nizkih napetostih (LEML vrsta razpok).

Kje so meje med njimi, ni mogoče natančno določiti ali izmeriti. V splošnem obstajata dve metodi za opisovanje tega matematičnega pojava. Prva metoda (teoretična matematika) sloni na teoriji, druga (eksperimentalna matematika) pa na preizkusu. Glavna lastnost teoretične matematike je njena abstraktnost in splošnost, kar pa je takoj povezano s pomanjkljivostmi, bolj ko je teorija splošna in formalna, manj je primerna za reševanje posameznih numeričnih problemov. Prav zaradi tega so kot alternativo začeli uporabljati eksperimentalno matematiko. Z njo lahko dobimo rezultate na več načinov, in to z:
— napetostno-deformacijsko analizo,
— varianco — kovarianco,
— neposredno sintezo,
— metodo Monte Carlo (MC).

V naših raziskavah smo uporabljali slednjo, to je metodo MC, saj se je pokazala kot zelo primerna. Uporabljamo jo lahko tako za deterministične kakor tudi za verjetnostne probleme, kar je odvisno od izhodnih podatkov naključnih procesov.

1.1 Metoda Monte Carlo

Simuliranje z metodo MC se pogosto uporablja za nastajanje numeričnih rezultatov tudi v verjetnostnih modelih lomne mehanike. Vsestranost in preprosta uporaba sta glavna razloga, da je ta metoda tako primerna.

Z uporabo metode MC [3] je mogoče aproksimirati deterministični problem tako, da uporabimo za osnovo teoretično matematiko, pomanjkljivost te teorije pa nadomestimo s preizkusi povsod tam, kjer je to potrebno in mogoče. Bistvena lastnost vseh preračunov z metodo MC je, da zamenjamo verjetnostno porazdelitev eksperimentalnih vzorcev z naključnimi števili, ki imajo zahtevane statistične lastnosti.

1.1.1 Osnove metode Monte Carlo

Katerikoli izračun, narejen po metodi MC, katerega rezultati so kvantitativne vrednosti, imamo

where Y is shape factor, $\Delta\sigma$ stress ratio and a treshold crack length.

At the other end of the spectrum, for vanishingly small defects, the allowable stress level must relate to the uncracked specimen fatigue limit. The Kitagawa — Takahashi diagram shows the observed behavior between these extremes.

The cracks can be divided into:
— microstructurally short cracks,
— physically small cracks,
— long cracks at low stress (LEFM type crack).

The individual regimes and the boundaries between them cannot be accurately measured and determined. Therefore, in general, two methods are available for describing this mathematical phenomenon. The first method is based on the theory, i.e., theoretical mathematics and the other on the tests, i.e., experimental mathematics. One of the main bases of theoretical mathematics is its relation to abstractness and generality. That basis is inseparably accompanied by disadvantages: the more the theory is general and formal, the less it is suitable for solving particular numerical problems. For this reason experimental mathematics was introduced as an alternative to theoretical mathematics. In case of the experimental mathematics, the numerical results can be obtained in several ways. The methods of calculation include:
— stress-strain analysis,
— variance/covariance,
— direct synthesis and
— Monte Carlo method (MC).

As a tool for describing the phenomena in experimental mathematics, the MC method will be used, particularly because in our previous research it has proved to be very convenient.

It can be used for two kinds of problems, for both probabilistic and deterministic problems, depending on the properties of output data of a random process.

1.1 Monte Carlo Method

The MC simulation is also often used in the generation of the numerical results for probabilistic fracture mechanics (PFM) models. The versatility and simple applicability are the reasons why the MC simulation is particulary suitable.

By means of the MC method [3], it is possible to approximate the deterministic problems, so that the bases of theoretical mathematics are applied, but any disadvantages are eliminated by replacing the theory by experiments wherever possible and feasible. The essential feature common to all MC computations is that at some point we have to substitute for a probability distribution a sample of random numbers having the required statistical properties.

1.1.1 Basic principle of Monte Carlo method

Any calculation made by the MC method, whose results are the quantitative data, can be za rešitev večkratnega integrala. Predpostavimo, da noben izračun ne zahteva več ko N naključnih števil (npr. 10^{10}). Rezultat zapišemo v vektorski obliki kot funkcijo:

considered to be the solution of a multiple integral. It is assumed that no calculations require more than N random numbers (e.g. 10^{10}). The results are expressed in vector form as a function:

$$R(\xi_1, \xi_2, \ldots \xi_N) \quad (2).$$

Rešitev večkratnega integrala potem simuliramo kot:

The solution of the multiple integral, can be simulated as:

$$\int_0^1 \ldots \int_0^1 R(x_1, x_2, \ldots, x_N)\,dx_1 \ldots dx_N \quad (3).$$

Metoda reševanja teh integralov ni vedno najpreprostejša, vendar pa jo lahko uporabimo kot podlago nadaljnjim rešitvam tudi za uporabo različnih tehnik MC ali drugih numeričnih metod, da bi dosegli splošno uporabnost.

V realnem verjetnostnem problemu lomne mehanike je poglavitni numerični problem določitev integrala:

The method of solving such an integral is not always the simplest, but it can be used as a basis for further solutions and for introducing various MC techniques or other numerical methods to achieve general applicability.

In a realistic probabilistic fracture mechanics problem, the fundamental numerical problem is to compute the integral:

$$\int_0^\infty R(\sigma_f) \int_0^\infty R(\sigma) \int_0^1 R(a/c) \int_0^\infty R(K_{Ic}) \int_{a_c}^t R(a)\,da\,dK_{Ic}\,d(a/c)\,d\sigma\,d\sigma_f \quad (4),$$

kjer so: a — dolžina razpoke, K_{Ic} — lomna žilavost, σ_f — napetost tečenja, a_c — kritična dolžina razpoke, σ — imenska napetost in $R(a)$ — ustrezna normalizirana verjetnostna funkcija. Ta integral lahko rešimo s simuliranjem MC. Neposredna uporaba metode MC daje enake rezultate kakor numerični preizkus, npr. iz vzorcev razpok izberemo za določeno dolžino razpoke neko razpoko v odvisnosti od verjetnostne porazdelitve.

where a is the crack depth, K_{Ic} the fracture toughness, σ_f yield stress, a_c critical crack depth, σ nominal stress, and $R(a)$ the respective normalized probability density. This can be performed by MC simulation. Direct MC usage is equivalent to the performance of numerical experiments. For example, a crack of fixed depth is selected from a sample of cracks generated according to the probability distribution.

1.1.2 Monte Carlo in doba trajanja zobnika

Eksperimentalne podatke dobimo najpogosteje z majhnim številom preizkušancev. Na podlagi tega ni mogoče zanesljivo predpostavljati, kako bi se obnašalo veliko število podobnih preizkušancev. Iz teh podatkov lahko izračunamo približno povprečje in standardni odmik. Da pa bi povečali zanesljivost izračuna, uporabimo generator naključnih števil, ki ga najdemo v vsaki računalniški knjižnici. Z njim lahko ob uporabi metode MC navidezno povečamo število preizkušancev v okviru predpisanega raztrosa na podlagi standardnega odmika. Iz velikega števila simuliranj (od 10^3 do 10^7) lahko dobimo novo porazdelitev vrednosti ter novo povprečje z ustreznim razponom raztrosa.

Kot primer imamo lahko imensko napetost za naključno normalno porazdeljeno spremenljivko. Potem lahko zapišemo:

1.1.2 Monte Carlo and service life of gears

The data obtained by experiments are most frequently acquired from a small population of specimens on the basis of which it is not possible to conclude reliably how the entire population behaves. However, from these data it is possible to calculate the approximate mean value and the standard deviations. In order to increase the reliability of our calculations, the generator of random numbers, which can be found in any computer library, is used. By means of it, and by using the MC method, it is possible to increase the number of specimens in the frame of the prescribed scatter depending on the standard deviation. On the basis of a large number of simulations (from 10^3 to 10^7), it is posible to obtain the new distribution of the desired value and/or the new mean value with the respective scatter.

For example, the stress can be considered to be a random variable which is normally distributed. Therefore, it is possible to write:

$$\log \sigma = \overline{N}(\mu_\sigma, S_\sigma) \quad (5),$$

kjer je μ_σ povprečje, ki ga dobimo eksperimentalno in ga izračunamo:

In this equation, μ_σ is the mean value obtained by experiment and amounts to:

$$\mu_\sigma = \sum_{i=1}^{n} \frac{\sigma_i}{n} \quad (6),$$

kjer je n število vseh preizkusov.

Eksperimentalni standardni odmik je potem:

where n is the number of experiments.

The standard deviation can be calculated:

$$S_\sigma = \sqrt{\frac{\sum_{i=1}^{n} \left(\sigma_i - \mu_\sigma\right)^2}{n-1}} \quad (7).$$

V splošnem je znano, da so rezultati tem bolj zanesljivi, čim večje je število vstopnih podatkov. Z generatorjem naključnih števil lahko numerično povečamo število podatkov. Na podlagi tega novega, večjega števila podatkov, izračunamo novo povprečje μ_σ, ki se lahko razlikuje od stare največ za vrednost predpisanega standardnega odmika.

Vse druge naključne spremenljivke v preračunu lahko izračunavamo enako, kakor imensko napetost.

It is known that the results are more accurate, if the number of data is large. By means of the generator of random numbers, the number of input data can be numerically increased and on the basis of that new population it is possible to calculate the new mean value μ_σ, which can deviate from the original value for not more than the specified standard deviation.

All other random variables in the calculation can be treated in the same way as the stress.

1.2 Kratke razpoke

Za kratke razpoke se integralska enačba (3) za posamezni primer poenostavi, in sicer:

1.2 Short cracks

For a short crack, the integral equation (3) is simplified in particular cases into the form:

$$\int_D \frac{f(x)}{x - x_0} \, \mathrm{d}x = \frac{P(x_0)}{A} \quad (8).$$

V tej enačbi je $A = (G b)/(2\pi(1 - \nu))$, kjer je G strižni modul in b Burgersov vektor, $P(x_0)$ pa je rezultirajoča strižna napetost.

Obstajata dve različni rešitvi enačbe (8), odvisno od tega, ali uporabimo za rešitev omejeno ali neomejeno funkcijo na koncih plastičnih con [1], [2]. Ta integralska enačba določa ravnotežje vsake dislokacije [4], ki se kaže v zvezi med dolžino razpoke a in velikostjo plastične cone c, to je legi dejanskega vrha razpoke, kar lahko v poenostavljeni obliki zapišemo kot:

Above $A = (G b)/(2\pi(1 - \nu))$, where G is the shear modulus, b is the Burger's vector and $P(x_0)$ is the resulting shear stress.

There are two different solutions of this equation, depending on whether a bounded or an unbouned solution function at the ends of the plastic zones is considered [1], [2]. The integral equation defines the equilibrium of every dislocation [4], which results in the relation between the crack length a and the plastic zone size c, i.e., the location of the original crack tip, which can be written in a simplified form as follows:

$$\frac{a}{c} = d = \cos\left(\frac{\pi}{2} \frac{\tau}{\sigma_f}\right) \quad (9).$$

Rezultat je plastični pomik na vrhu razpoke [10]:

The result is the plastic displacement at the tip of the crack [10]:

$$\Phi = \frac{2\,b}{\pi^2 A} \, \sigma_f \, a \ln(1/d) \quad \Phi = \frac{2\,b}{\pi^2 A} \, \sigma_f \, a \ln\left(\frac{1}{d}\right) \quad (10).$$

Če vpeljemo v enačbo (10) faktor intenzivnosti napetosti K, lahko zapišemo plastični pomik v obliki:

If the stress intensity factor is taken into account, equation (10) can be written as follows:

$$\Phi = \frac{b}{\pi^2 A}\frac{(1-d^2)^{1/2}}{\tau}K^2 \qquad (11),$$

kjer so $d = a/c$ — brezdimenzijska dolžina, σ_f — imenska napetost in τ strižna napetost.

Ob predpostavki, da je razmerje širjenja razpoke sorazmerno plastičnemu pomiku Φ, lahko v posplošeni obliki zapišemo model za računanje tega razmerja:

where $d = a/c$ gives the dimensionless size of the crack tip, σ_f is the friction stress and τ is the shear stress.

By assuming that the crack growth rate is proportional to plastic displacement Φ, the present theory describes the model for calculating this rate:

$$\frac{da}{dN} = M(\Delta\Phi)^m \qquad (12),$$

kjer je $\Delta\Phi = \Phi_{maks} - \Phi_{min}$, pri čemer sta Φ_{maks} in Φ_{min} plastična pomika vrha razpoke, ki ustrezata največjim in najmanjšim napetostim. Faktor M in eksponent m sta funkciji materiala. Nazadnje lahko izračunamo dobo trajanja za področje kratkih razpok z integralsko enačbo:

where $\Delta\Phi = \Phi_{max} - \Phi_{min}$ and Φ_{max} and Φ_{min} are the crack tip displacement corresponding to maximum and minimum respectively. The factor M and exponent m are the function of material shear stresses. Finally, lifetime calculations of short cracks are carried out with the following integral equation:

$$\Delta N_1 = \int \frac{1}{M(\Delta\Phi)^m}\,da \qquad (13).$$

1.3 Zvečanje dolgih razpok

Deterministične metode za dolge razpoke smo razširili s statistično spremenljivostjo razmerja zvečanja razpoke $X(t)$. Če v to razmerje zvečanja razpoke vpeljemo naključne spremenljivke, dobimo:

1.3 Long crack propagation

Deterministic models were extended for long cracks with the statistical variability of the ratio of the crack growth $X(t)$. If it is applied as a random variable, it is possible to write as follows:

$$\frac{da(t)}{dt} = X(t)\,L(\Delta K,\ K_{maks},\ R,\ S,\ a,\ldots) \qquad (14),$$

kjer je $X(t)$ pozitiven stacionaren stohastični lognormalni naključni proces. Z integracijo dolžine razpoke po enačbi (14) od 0 do t dobimo porazdelitev $a(t)$. Za rešitev tega problema s programom STAFTAG uporabimo simulacijsko metodo MC [3].

Here, $X(t)$ is the positive stationary stochastic log-normal random process. By integrating the equation (14) from 0 to t, the crack size distribution $a(t)$ is obtained. For solving this problem in the program package STAFTAG, we also used the MC simulation method [3].

1.4 Model računanja dobe trajanja

Splošno enačbo verjetnostne mehanike loma za računanje dobe trajanja mehanskih delov, to je računanje časa od nastanka do kritične dolžine razpoke, zapišemo:

1.4 Model for calculation of service life

The equation of probabilistic fracture mechanics for the entire service life of a mechanical part, i.e., for the time of the initiation and subsequent crack propagation, can be written:

$$N = N_1(\mathrm{A},\ M,\ \Delta\Phi) + N_p(\mathrm{B},\ \Delta\sigma,\ \mathrm{A},\ \mathrm{C}) \qquad (15).$$

Za točno simuliranje uporabimo metodo MC, pri čemer za lažjo uporabo razbijemo osnovno enačbo (15) na dva dela in jo potem zapišemo za:

— nastanek po enačbah (12) in (13):

For accurate simulation, the MC method will be used. For easier application the basic equation (15) should be written in the form for:

— for initiation according to eq. (12) and (13):

$$\log N_1 = \log A + p \log M + m \log(\Delta\Phi) \qquad (16),$$

— in zvečanje razpoke po enačbi (14):

$$\log N_p = \log B + p \log C + q \log(\Delta\sigma) + r \log a \qquad (17),$$

kjer sta A in B konstanti osnovne enčbe, p, q in r so konstantni koeficienti, a, C, $\Delta\varphi$ in $\Delta\sigma$ pa naključne spremenljivke, za katere smo predpostavili normalno porazdelitev. Faktor intenzivnosti napetosti K je bil simuliran z naključnim spreminjanjem vrednosti $\Delta\sigma$ in Y, plastični pomik pa s spreminjanjem d in τ.

2. PROGRAM STAFTAG

Po zgoraj opisani teoriji je bil razvit program STAFTAG, ki je namenjen za preračunavanje dobe trajanja evolventnih zobnikov za dva osnovna primera:

— izračun dobe trajanja zobnikov, na katerih nismo odkrili nobene poškodbe;

— izračun preostale dobe trajanja zobnikov, na katerih smo med periodičnim pregledom odkrili razpoke.

V prvem primeru moramo upoštevati tako teorijo nastanka in širjenja kratkih (mikro) razpok po enačbi (16) in za tem širjenje te razpoke do kritične dolžine po enačbi (17). Celotna doba trajanja je vsota obeh. Ta izračun rabi predvsem za optimiranje zobnikov oziroma zobniških prenosnikov.

V drugem primeru, to je v primeru zvečanja razpoke, iz znanega defekta uporabljamo le teorijo dolgih razpok po enačbi (17), saj kratkih razpok z običajnimi defektoskopskimi metodami sploh ne moremo odkriti. Tako izračunavamo časovno obratovanje oziroma, kdaj moramo poškodovani del zamenjati.

V programu STAFTAG [5] upoštevamo celoten obremenitveni kolektiv, to je primarne (zunanje) obremenitve in sekundarne obremenitve, ki se pojavijo zaradi zaostalih napetosti. Primarne obremenitve dobimo z matematičnimi modeli zobniških prenosnikov [5]. Za računanje napetosti v kritičnem prerezu korena zoba pa uporabljamo dvodimenzionalno elasto-plastično analizo končnih elementov [8].

3. DEFINIRANJE PROBLEMA

Uporabnost modela oziroma programa bomo prikazali na pastorku za pogon žerjava, to je valjasti zobnik z ravnimi zobmi, številom zob $z = 18$, modulom $m = 10$ mm in vpadnim kotom $\alpha = 20^0$.

— and propagation according to eq. (14):

where A and B are constant parts of the basic equation; p, q and r are constant coefficients, a, C, $\Delta\varphi$ and $\Delta\sigma$ are random variables for which the normal distribution was assumed. The stress intensity factor was simulated by randomly varying the variables $\Delta\sigma$ and Y and plastic displacement by varying d and τ.

2. PROGRAM STAFTAG

On the basis of the theory described above, the STAFTAG program has been developed, with special emphasis on application to evolvent gear for two basic problems:

—calculation of service life of gears without defects found;

— calculation of service life of gears on which cracks were found with non-destructive testing during periodic inspections.

In the first case, it is necessary to take into account the theory of occurence, i.e., initiation of the crack and its propagation in the micro conditions according to equation (16) or in the macro conditions according to equation (17). The entire service life is obtained by a combination of both equations. This calculation serves particularly for optimization of gearings or gear drives.

In the second case, i.e., in case of propagation of cracks from the existing defects — crack only the equation (17) is used. That calculation ensures timely manufacture and/or replacement of damaged components.

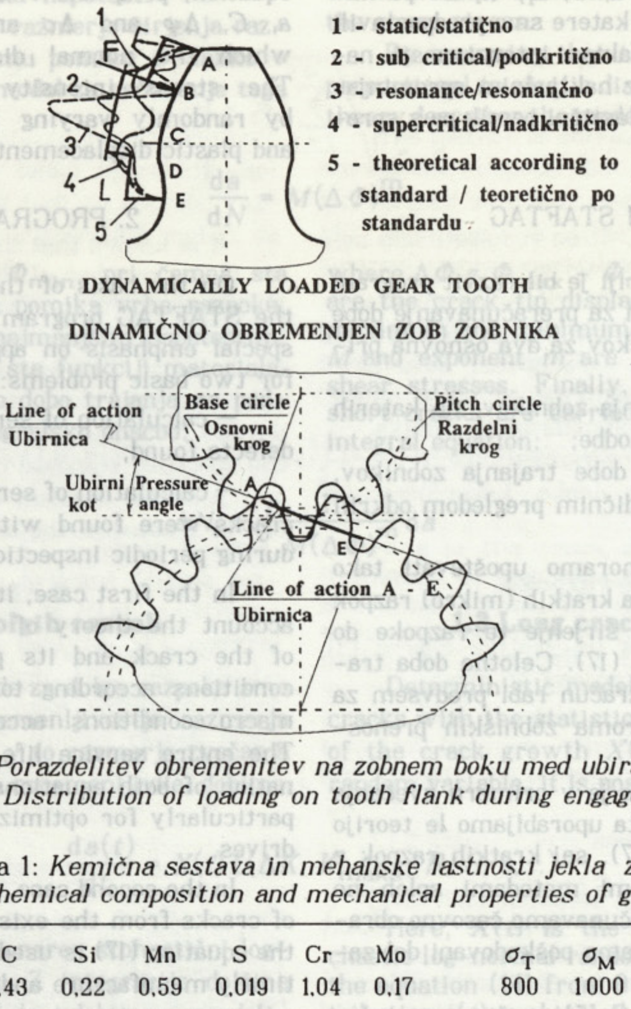
Loadings on gears are most frequently of variable amplitude. Therefore we incorporated in our program STAFTAG [5] the possibility of generating the loading by means of a mathematical model of a random gearing. For calculation of stresses, plane elasto-plastic finite element analysis [8] is used.

3. DEFINING THE PROBLEM

The subject of the present research is the pinion of the gear drive of cranes, namely involute cylindrical gear with straight toothing, teeth number $z = 18$, module $m = 10$ mm, inclination angle $\alpha = 20^0$.

Slika 2 prikazuje detajl tega zobnika med ubiranjem. Obremenitev smo simulirali v točki enojnega ubiranja in znaša F = 1000 N/mm. Zobnik je izdelan iz materiala AISI 4130 (4140), kar je Č.4732. Lomna žilavost za ta material znaša K_{Ic} = = 2620 N/(mm)$^{3/2}$ [11]. Kemična sestava in mehanske lastnosti materiala zobnika so podane v preglednici 1.

Figure 2 shows a detail of this gear during engagement. The loading acted on the internal point of enagagement and was equal to F = 1000 N/mm. The gear was made of steel AISI 4130 (4140) corresponding to DIN 4732. The fracture toughness, i.e. the critical value of stress intensity factor was equal to K_{Ic} = 2620 N/(mm)$^{3/2}$. The chemical composition and mechanical properties of the material are shown in table 1.



Sl. 2. *Porazdelitev obremenitev na zobnem boku med ubiranjem.*
Fig. 2. *Distribution of loading on tooth flank during engagement.*

Preglednica 1: *Kemična sestava in mehanske lastnosti jekla zobnika.*
Table 1: *Chemical composition and mechanical properties of gear steel.*

	C	Si	Mn	S	Cr	Mo	σ_T	σ_M	
%	0.43	0.22	0.59	0.019	1.04	0.17	800	1000	MPa

Izbrani material je bil toplotno obdelan po naslednjem postopku:

— plamensko segrevanje pri 810 °C, 2 minuti;
— kaljenje v olju, 3 minute;
— popuščanje na temperaturi 180 °C, 2 uri.

Po teh podatkih smo izračunali tako faktorje intenzivnosti napetosti in plastične pomike kakor tudi dobo trajanja zobnika pri različnih dolžinah razpoke, pri čemer smo upoštevali 95-odstotno zanesljivost.

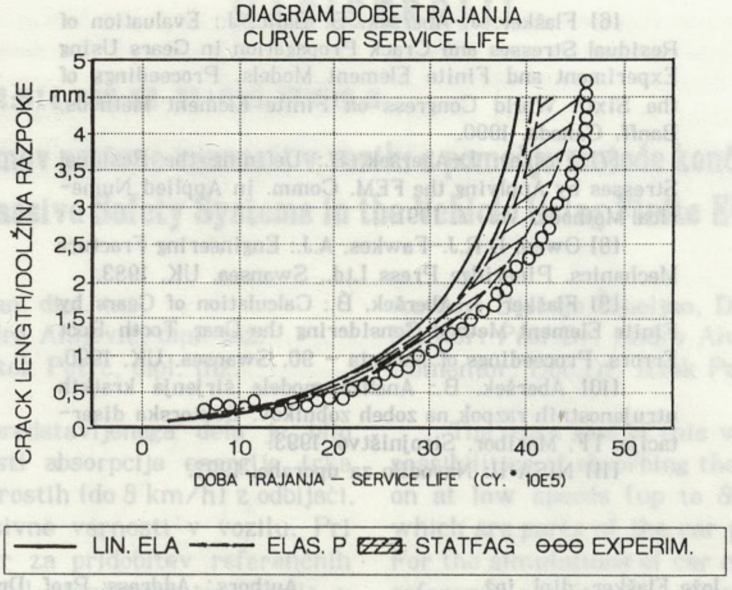
Naš algoritem in program STAFTAG smo potrdili tako z elastično kakor tudi z elastoplastično analizo s programom NISA II [12], prav tako pa tudi z eksperimentalnimi rezultati [11]. Primerjavo rezultatov prikazuje slika 3.

The selected material was thermally treated:

— flame heated at 810 °C for 2 minutes,
— hardened in oil for 3 minutes and
— tempered at 180 °C for 2 hours.

On the basis of this data by means of the program STAFTAG, the stress intensity factors and the direction of the crack propagation [8] were obtained and service life of gears by taking into account 95% confidence band was calculated accordingly.

Our algorithm was verified by using the NISY II [12] program with finite linear elastic and elastoplastic element and the experimental results [11]. Comparison of results is indicated in figure 3.



Sl. 3. *Krivulja širjenja razpoke.*

Fig. 3. *Crack growth curve.*

4. SKLEP

Iz primerjave rezultatov s slike 3 je razvidno, da je linearno elastična analiza primerna le za začetne osnovne analize, saj se rezultati razlikujejo od realnih, to je eksperimentalnih rezultatov, pri kritičnih dolžinah razpoke tudi več ko 25 odstotkov.

Z uporabo programa STAFTAG pa smo dosegli odstopke 5 do 12 odstotkov, kar je dober približek za dobo trajanja. Iz tega je razvidno, da smo razvili dober model za preračun dobe trajanja ozkih zobnikov, to je zobnikov, pri katerih je razmerje med širino in zunanjim premerom (b/d) < 0,5, saj za takšne zobnike lahko predpostavimo, da se bo razpoka pomikala po vsej širini zobnega boka enakomerno. Zato lahko tak problem poenostavimo in uporabimo dvodimenzionalno analizo.

4. CONCLUSION

The comparison of results of all mentioned methods according to figure 3 shows that the linear elastic analysis can be used only for rough preliminary analysis, since the deviation of results in comparison with real condition, i.e., experiment in case of critical crack length is up to 25% and more.

By means of the programme STAFTAG we reached the deviation within 5 to 12% which is a good result for estimation of the service life. The comparison of results shows that we have developed a good model for calculating the service life of narrow gears, i.e., gears on which the width to gear diameter ratio is b/d < 0.5, since on such gears the assumption applies that the crack propagates uniformly on the entire width of the tooth flank. Therefore, the problem can be treated two-dimensionally.

5. LITERATURA
5. REFERENCES

[1] Prowan, J.W.: Probabilistic Fracture Mechanics and Reliability, Martinus Nijhoff Publ. Dordrecht, Boston, Lancaster, 1987.

[2] Miller, K.J.: The Behaviour of Short Fatique Crack and their Initiation. Part I – FFEMS, Vol. 10, No. 1, 1987, Part II – FFEMS, Vol 10, No. 2, 1987.

[3] Hammersley, J.M.–Handschomb, D.C.: Monte Carlo Methods. Chapman and Hall, London, New York, 1983.

[4] Bilby, B.A.–Cottrel, A.H.–Swinden, K.H.: The Spread of Plastic Yield from a Notch.Proc.Royal Soc.272/63.

[5] Flašker, J.–Aberšek, B.–Jezernik, A.: Numerical Model for the Determination of Service Life of Gear. Proceedings of the Symposium on Finite Element Methods FEMSA 92, South Africa.

[6] Flašker, J.–Aberšek, B.–Balič, J.: Evaluation of Residual Stresses and Crack Propagation in Gears Using Experiment and Finite Element Models. Proceedings of the Sixth World Congress on Finite Element Methods. Banff, Canada, 1990.

[7] Flašker, J.–Aberšek, B.: Defining the Residual Stresses by Applying the FEM. Comm. in Applied Numerical Methods, Vol. 7, 1991.

[8] Owen, D.R.J.–Fawkes, A.J.: Engineering Fracture Mechanics. Pineridge Press Ltd., Swansea, UK, 1983.

[9] Flašker, J.–Aberšek, B.: Calculation of Gears by Finite Element Method Considering the Gear Tooth Root Errors. Proceedings of Numeta – 90, Swansea, UK, 1990.

[10] Aberšek, B.: Analiza modela širjenja kratkih utrujenostnih razpok na zobeh zobnikov. Doktorska disertacija, TF, Maribor, Strojništvo, 1993.

[11] NISA II: Navodilo za uporabo, 1992.

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