UDK 539.374

Vezana termoelastoplastičnost

Coupled Thermo-Elasto-Plasticity

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Neustaljeno temperaturno polje povzroča časovno odvisno deformacijsko in napetostno polje. Prav tako je vsako preoblikovanje telesa vedno povezano s toplotnimi premenami v njem. Zato moramo dinamične probleme vedno obravnavati v medsebojni povezavi pomikov in temperature. Neelastične deformacije pomenijo povezan toplotno-mehanski dogodek, tudi če poteka navidez ustaljeno oziroma izotermno. Prihaja namreč do spremembe mehanskega dela v toploto.

A non-steady temperature field generates time-dependent deformation and stress fields. In turn, any deformation of the body is always associated with a change in temperature of the body. The dynamic problems must consider the evolution of mutually coupled fields of displacements and temperature. Inelastic deformations of solid bodies represent coupled thermo-mechanical processes even if the body deforms quasi-statically and/or isothermally. This is due to dissipation of the mechanical work into heat.

0 UVOD

Navidez ustaljeno poenostavitev smemo uporabiti, če so zunanji vzroki (tj. telesna sila, toplotni vir, površinski vlek, ogrevanje) počasni v primerjavi z značilno frekvenco (c/L), ki je določena z razmerjem napredovanja valov in izmerami obravnavanega telesa. V teh primerih lahko vzamemo, da fizikalni sistem (trdno telo) zvezno prehaja skozi zaporedje ravnovesnih stanj in lahko v gibalnih enačbah opustimo člene, ki podajajo vztrajnost.

0 INTRODUCTION

A quasi-static approximation may be adopted if the external sources (i.e. body force, heat source, surface traction, heating rate) vary slowly with time in comparison with the characteristic frequency (c/L), defined by the propagation velocity of waves and a typical length of the body considered. The physical system (solid body) then passes continually through a sequence of equilibrium states and the inertia terms may be omitted in the equations of motion.

1 TERMOELASTIČNOST

Vodilne diferencialne enačbe Hookove trdnine so podane z Navier-Lamejevimi enačbami:

1 THERMOELASTICITY

Governing differential equations of a Hookean solid body are according to Navier-Lame:

$$\mu \nabla^2 \boldsymbol{u} + (\lambda + \mu)\, \mathrm{grad\,div}\, \boldsymbol{u} + \boldsymbol{X} = \rho \ddot{\boldsymbol{u}} + \gamma\, \mathrm{grad}\, \Theta$$

in s Fourier-Duhamelovo:

and Fourier-Duhamel:

$$\nabla^2 \Theta + Q/\varkappa = \dot{\Theta}/\varkappa + \eta\, \mathrm{div}\, \dot{\mathbf{u}}$$

kjer so: $\boldsymbol{u}$ — pomiki, $\boldsymbol{X}$ — telesne sile, Θ — temperatura, Q — gostota virov toplote, μ — strižni modul $[= E/(2(1+\nu)) = 3K(1-2\nu)/(2(1+\nu))]$, λ — Lamejeva konstanta $[= \nu E/((1+\nu)(1-2\nu)) = 2\mu\nu/(1-2\nu) = 3K\nu/(1+\nu)]$, E — Youngov modul $[= \mu(3\lambda+2\mu)/(\lambda+\mu)]$, ν — Poissonovo število $[=\lambda/(2(\lambda+\mu))]$, K — modul stisljivosti $[=\lambda+2\mu/3]$, ρ — gostota snovi, γ — $[= 3K\alpha_t]$, α_t — toplotna razteznost, $\varkappa$ — prodornost $[= \lambda_0/c_\varepsilon]$, λ_0 — prevodnost, c_ε — specifična toplota, η — $[= \gamma\Theta_0/\lambda_0]$.

where: $\boldsymbol{u}$ — displacements, $\boldsymbol{X}$ — body forces, Θ — temperature, Q — heat source density, μ — shear modulus $[= E/(2(1+\nu)) = 3K(1-2\nu)/(2(1+\nu))]$, λ — Lame constant $[= \nu E/((1+\nu)(1-2\nu)) = 2\mu\nu/(1-2\nu) = 3K\nu/(1+\nu)]$, E — Young's modulus $[= \mu(3\lambda+2\mu)/(\lambda+\mu)]$, ν — Poisson's ratio $[=\lambda/(2(\lambda+\mu))]$, K — bulk modulus $[=\lambda+2\mu/3]$, ρ — mass density, γ — $[= 3K\alpha_t]$, α_t — thermal expansion. $\varkappa$ — diffusivity $[= \lambda_0/c_\varepsilon]$, λ_0 — conductivity, c_ε — specific heat, η — $[= \gamma\Theta_0/\lambda_0]$.

Z uvedbo operatorjev:

Introducing operators

$$C = \nabla^2 - \partial_t^2/c_1^2, \quad A = \nabla^2 - \partial_t^2/c_2^2, \quad D = \nabla^2 - \partial_t/\varkappa, \quad B = CD - m\eta\partial_t\nabla^2,$$

kjer so: ∇^2 – Laplaceov operator [= div grad], ∂_t – časovni odvod = $\partial/\partial t$], c_1 – hitrost vzdolžnih valov [= $\sqrt{(\lambda+2\mu)/\rho}$], c_2 – hitrost prečnih valov [= $\sqrt{\mu/\rho}$], m – [= $\gamma/(\lambda+2\mu) = \alpha_t(3\lambda+2\mu)/(\lambda+2\mu)$], dobimo štiri funkcije φ_i(i = 1, 2, 3) in Ψ, ki izpolnjujejo diferencialne enačbe:

where: ∇^2 – Laplace operator [= div grad], ∂_t – time derivative [= $\partial/\partial t$], c_1 – velocity of dilatational waves [= $\sqrt{(\lambda+2\mu)/\rho}$], c_2 – velocity of distortional waves [= $\sqrt{\mu/\rho}$], m – [= $\gamma/(\lambda+2\mu) = \alpha_t(3\lambda+2\mu)/(\lambda+2\mu)$], four functions φ_i(i = 1, 2, 3) and Ψ, are obtained, obeying differential equations:

$$2\mu AB\varphi_1 + \frac{1-2\nu}{1-\nu}X_1 = 0$$

$$2\varkappa B\Psi + \frac{1-2\nu}{1-\nu}Q = 0.$$

Vsaka od teh parcialnih diferencialnih enačb vsebuje eno samo neznanko. Rešitev dobimo iz partikularnih rešitev nehomogenih enačb ter iz splošnih rešitev homogenih delov:

Each of these partial differential equations contains one unknown function only. The solution consists of particular solutions of the non-homogeneous equations, and of general solutions for homogeneous parts:

$$A\varphi_1 = 0 \quad \text{(prečni val)}$$

$$B\varphi_1 = 0 \quad \text{(vzdolžni val)}$$

$$B\Psi = 0 \quad \text{(toplotni val)}$$

Rešitev mora izpolnjevati predpisane robne pogoje pomikov, vlekov, temperature in toplotnega toka kakor tudi ustrezne začetne pogoje.

Pomike in temperaturo dobimo z:

The solution has to satisfy the prescribed boundary conditions of displacements, tractions, temperature and heat flux, initial conditions.

Displacements and temperature are obtained by:

$$u_1 = (1+a)B\varphi_1 - \text{grad div}(\Gamma\varphi_1) + (1+a)m\ \text{grad}\ \Psi$$

$$\Theta = \eta\partial_t\ \text{div}(A\varphi_1) + (1+a)C\Psi,$$

kjer sta: $\Gamma = aD - (1+a)m\nu\partial_t$ in $a = 1 + \lambda/\mu$.

Z uporabo Laplaceove transformacije pri navidez ustaljenem primeru:

where: $\Gamma = aD - (1+a)m\nu\partial_t$ and $a = 1 + \lambda/\mu$.

Using Laplace transformation for quasi steady case:

$$(\rho \to 0,\ c_1\ \&\ c_2 \to \infty,\ \rho c_1^2 = \lambda + 2\mu,\ \rho c_2^2 = \mu)$$

$$[\nabla^4 - (m\mu - 1/\varkappa)p\nabla^2 + m\eta p^3]\nabla^2\,\overline{\varphi}_1 + \overline{B}_1 = 0,$$

$$(\nabla^2 - p/\varkappa)\nabla^2\overline{\Psi} + \overline{H} = 0$$

ter s Fourierovo transformacijo dobimo:

and by Fourier transformation it follows:

$$[k^4 + (m\eta - 1/\varkappa)pk^2 + m\eta p^3]k^2\widetilde{\varphi}_1 + \widetilde{B}_1 = 0,$$

$$(k^2 + p/\varkappa)k^2\widetilde{\Psi} + \widetilde{H} = 0.$$

2 TERMOPLASTIČNOST

2 THERMOPLASTICITY

Predpostavimo razstavitev tenzorja deformacijske hitrosti:

Assuming the rate of strain tensor to be decomposed:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{T} + \dot{\varepsilon}_{ij}^{P}$$

Z uporabo konstitutivnega zakona: / Making use of constitutive law:

$$\dot{\sigma}_{ij} = C^{e}_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{T}_{kl} - \dot{\varepsilon}^{p}_{kl}\right),$$

kjer je tenzor elastičnih lastnosti v izotropnem primeru: / where the elasticity tensor for isotropic case reads:

$$C^{e}_{ijkl} = \lambda\delta_{ij}\,\delta_{kl} + \mu\left(\delta_{ik}\,\delta_{jl} + \delta_{il}\,\delta_{jk}\right)$$

in v povezavi s funkcijo plastičnega potenciala $F = F(\sigma_{ij}, K, T)$, kjer mora biti utrjevalni količnik K za majhne prirastke plastične deformacije ustaljen: / and combining with the plastic potential function $F = F(\sigma_{ij}, K, T)$, where K is the work hardening parameter, for small increments of plastic deformation required to be stationary:

$$\dot{F} = \frac{\partial F}{\partial \sigma_{ij}}\dot{\sigma}_{ij} + \frac{\partial F}{\partial K}\frac{\partial K}{\partial \varepsilon^{p}_{ij}}\dot{\varepsilon}^{p}_{ij} + \frac{\partial F}{\partial T}\dot{T} = 0,$$

dobimo: / we obtain:

$$\frac{\partial F}{\partial \sigma_{ij}} C^{e}_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{T}_{kl} - \dot{\varepsilon}^{p}_{kl}\right) + \frac{\partial F}{\partial K}\frac{\partial K}{\partial \varepsilon^{p}_{ij}}\frac{\partial F}{\partial \sigma_{ij}}\Lambda + \frac{\partial F}{\partial T}\dot{T} = 0,$$

iz česar izhaja faktor sorazmernosti: / from which the proportionality factor:

$$\Lambda = \frac{1}{S}\left[\frac{\partial F}{\partial \sigma_{ij}} C^{e}_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{T}_{kl}\right) + \frac{\partial F}{\partial T}\dot{T}\right]$$

s / with

$$S = \frac{\partial F}{\partial \sigma_{pq}} C^{e}_{pqrs}\frac{\partial F}{\partial \sigma_{rs}} - \frac{\partial F}{\partial K}\frac{\partial K}{\partial \varepsilon^{p}_{pq}}\frac{\partial F}{\partial \sigma_{pq}}$$

in z / and by

$$\dot{\varepsilon}^{T}_{ij} = \alpha_{ij}\,\dot{T},$$

dobimo: / we get:

$$\Lambda = \frac{1}{S}\left[\frac{\partial F}{\partial \sigma_{ij}} C^{e}_{ijkl}\left(\dot{\varepsilon}_{kl} - \dot{\alpha}_{kl}\,\dot{T}\right) + \frac{\partial F}{\partial T}\dot{T}\right],$$

tako da velja: / so that it follows:

$$\dot{\sigma}_{ij} = C^{e}_{ijkl}\,\dot{\varepsilon}_{kl} - C^{e}_{ijkl}\,\alpha_{kl}\,\dot{T} - \frac{1}{S}C^{e}_{ijvw}\frac{\partial F}{\partial \sigma_{vw}}\left[\frac{\partial F}{\partial \sigma_{tu}} C^{e}_{tukl}\left(\dot{\varepsilon}_{kl} - \alpha_{kl}\,\dot{T}\right) + \frac{\partial F}{\partial T}\dot{T}\right].$$

Po preureditvi členov: / After rearranging the terms:

$$\dot{\sigma}_{ij} = C^{e}_{ijkl}\,\dot{\varepsilon}_{kl} - C^{e}_{ijkl}\,\alpha_{kl}\,\dot{T} - \frac{1}{S}C^{e}_{ijvw}\frac{\partial F}{\partial \sigma_{vw}}\frac{\partial F}{\partial \sigma_{tu}}C^{e}_{tukl}\,\dot{\varepsilon}_{kl} + \frac{1}{S}C^{e}_{ijvw}\frac{\partial F}{\partial \sigma_{vw}}\frac{\partial F}{\partial \sigma_{tu}}C^{e}_{tukl}\,\alpha_{k}\,\dot{T} - \frac{1}{S}C^{e}_{ijkl}\frac{\partial F}{\partial \sigma_{kl}}\frac{\partial F}{\partial T}\dot{T}$$

lahko določimo tenzor plastičnih lastnosti: / we can define the plasticity compliance tensor:

$$C^{p}_{ijkl} = \frac{1}{S} C^{e}_{ijvw} \frac{\partial F}{\partial \sigma_{vw}} \frac{\partial F}{\partial \sigma_{tu}} C^{e}_{tukl} = \frac{1}{S} C^{e}_{ijvw} s_{vw} s_{tu} C^{e}_{tukl}$$

in po vstavitvi dobimo:

and by substituting:

$$\dot{\sigma}_{ij} = C^{e}_{ijkl} \dot{\varepsilon}_{kl} - C^{e}_{ijkl} \alpha_{kl} \dot{T} - C_{ijkl} \dot{\varepsilon}_{kl} + C^{p}_{ijkl} \alpha_{kl} \dot{T} - \frac{1}{S} C^{e}_{ijkl} s_{kl} \frac{\partial F}{\partial T} \dot{T}$$

Z določitvijo tenzorja elasto-plastičnih lastnosti:

Defining the elasto-plasticity compliance tensor:

$$C^{ep}_{ijkl} = C^{e}_{ijkl} - C^{p}_{ijkl},$$

dobimo termo-elasto-plastično konstitutivno enačbo v obliki:

the thermo-elasto-plastic constitutive equation can be expressed as:

$$\dot{\sigma}_{ij} = C^{ep}_{ijkl} \dot{\varepsilon}_{kl} \; C^{ep}_{ijkl} \alpha_{kl} \dot{T} - \frac{1}{S} C^{e}_{ijkl} s_{kl} \frac{\partial F}{\partial T} \dot{T}.$$

Če uvedemo tenzor:

If we introduce the tensor:

$$\zeta_{ij} = C^{ep}_{ijkl} \alpha_{kl} + \frac{1}{S} C^{e}_{ijkl} s_{kl} \frac{\partial F}{\partial T},$$

lahko zapišemo konstitutivno enačbo:

we can write the constitutive equation:

$$\dot{\sigma}_{ij} = C^{ep}_{ijkl} \dot{\varepsilon}_{kl} - \zeta_{ij} \dot{T}.$$

3 SKLEP

Povezava med temperaturo in deformacijami pri plastifikaciji se pokaže v različnih oblikah. Ne le, da toplotno polje vpliva na lastnosti snovi, spreminja obseg plastičnega območja in povzroča zatikanje med ponavljajočim se ogrevanjem itn., temveč deformacija tudi povzroča spremembe temperaturne porazdelitve. V mnogih primerih je povezanost toplotnega in deformacijskega stanja pomembna (npr. preoblikovanje kovin, strig med odrezovanjem, utrujanje itn.).

3 CONCLUSION

The interaction of temperature and deformation during plastic flow appears in various forms. A thermal field influences not only the material properties, modifies the extent of plastic zones and results in ratchetting during cyclic heating etc., but the deformation also induces changes in the temperature distribution. There are many instances when the coupling of thermal and deformation states is of importance (e.g. metal forming, shear during machining, fatigue etc.).

4 LITERATURA
4 REFERENCES

[1] Ziegler, H.: An Introduction to Thermomechanics. North-Holland, Amsterdam, 1983.

[2] Lehman, Th. (ed): The Constitutive Law in the Thermoplasticity. Springer-Verlag, Berlin, 1984.

[3] Nowacki, W.: Thermoelasticity. Pergamon Press, London, 1986.

[4] Lubliner, J.: Plasticity Theory. Macmillan, London, 1990.

[5] Služalec, A.: Introduction to Nonlinear Thermomechanics. Springer-Verlag, Berlin, 1992.

[6] Muschik, W. (ed): Nonequilibrium Thermodynamics with Application to Solids. Springer-Verlag, Berlin, 1993.

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