

UDK 62–23

## Alternativni metodi dimenzijske sinteze štiriročičnih mehanizmov

## Alternative Methods of Dimensional Synthesis of Four-Link Mechanisms

VILKO VELIKONJA

V članku opisujem dve preprosti metodi sinteze mehanizmov. S tem mislim na dimenzijsko sintezo štiriročičnega mehanizma, pri katerem predpisujemo gibanje, iščemo pa dolžine ročic, ki bodo sprežno ravnino zapeljale skozi predpisane lege. Metodi krožnice krožnih in središčnih točk lege osnovnega tečaja sta uporabni tam, kjer smo omejeni z lego pritrditve osnovnega tečaja. Uvajati smo ju začeli zaradi neustreznosti sedanjih vratnih mehanizmov (prtlažna in potniška vrata). Zaradi preprostosti metod lahko postopek računanja avtomatiziramo. Tako smo z uvajanjem metod sinteze mehanizmov razvili tudi računalniški program.

The paper deals with two simple methods of mechanism synthesis, i.e. the dimensional synthesis of four-link mechanism with three prescribed positions for motion generation. The Circle-Point and Center-Point Circles, and Ground-Pivot Specification methods are applicable for dimensional synthesis of mechanisms, especially where the location of fixed pivot is limited. We have introduced them mainly because of the unsuitable kinematic performance of the existing door mechanisms (luggage compartment doors, entrance doors). In parallel with the introduction of mechanisms synthesis we have developed a computer programme for analysis and synthesis of mechanisms.

### O UVOD

Inženirske delo temelji na osnovah matematike, fizike in kemije. V veliko primerih se na prej naštetih osnovah analizira prenos gibanja, sile ali energije od vstopa do izstopa. Te naloge v mehaniki togih teles opravlja mehanizmi. Zato lahko rečemo, da so mehanizmi vmesniki, ki so namenjeni za prenos gibanja in/ali sile.

Med najpreprostejše mehanizme spada štiriročični, sestavljen iz treh gibljivih in ene nepremične ročice, ki so med seboj povezane z rotacijskimi členki. Kljub svoji preprostosti pa jih lahko uporabljam za dokaj zahtevne naloge, npr.: nelinljivo gibanje in prenos sile. Zaradi svoje preprostosti in zanesljivosti se ti mehanizmi v strojništvu zelo veliko uporabljajo.

Velikokrat smo postavljeni pred nalogo, da moramo poiskati vmesnik, ki bo zapeljal neko telo skozi želene lege. Pri tem je najtežje delo, ko smo enkrat izbrali tip mehanizma (v tem primeru štiriročični mehanizem), da določimo dolžine ročic, in to tako, da bo izbrani mehanizem zapeljal telo, ki je navadno del ravnine, skozi predpisane lege. S problemi te vrste se ukvarja dimenzijska sinteza mehanizmov.

### 1 MATEMATIČNA SREDSTVA

Matematično sredstvo pri dimenzijski sintezi so navadno kompleksna števila. Ročice mehanizmov ponazarjam z vektorjem:

### O INTRODUCTION

An engineer's work is based on mathematics, physics and chemistry. In numerous cases, motion generation, force or energy transmission from input to output, is analysed on the above mentioned bases. These tasks in mechanics of rigid bodies are carried out by mechanisms. Therefore we can say that mechanisms are mechanical interfaces designed for motion generation and/or for force transmission from input to output.

A four-link mechanism is among the simplest of mechanisms. It consists of three moving links and a fixed one, which are interconnected with rotational joints. Despite its simplicity, a four-link mechanism can be used for doing fairly demanding tasks, e.g. nonlinear motion, force transmission. So such a mechanism (owing to its simplicity and reliability) is widely used in mechanical engineering.

Very often we are given a task to look for an interface that will take a body through the desired positions. Once the mechanism type is chosen (in our case, a four-link mechanism) it is extremely difficult to determine the link lengths in such a way as to enable the chosen mechanism to take a body, usually a part of the coupler plane, through the prescribed positions. Dimensional synthesis of mechanisms deals with problems of this kind.

### 1 MATHEMATICAL MEANS

Mathematical means of dimensional synthesis are usually complex numbers. Mechanism links are expressible as a complex vector:

$$\vec{Z}_k = | \vec{Z}_k | e^{i\varphi_k} \quad \text{H-Gnd T: Local} \quad \text{KGS-23} \quad (1).$$

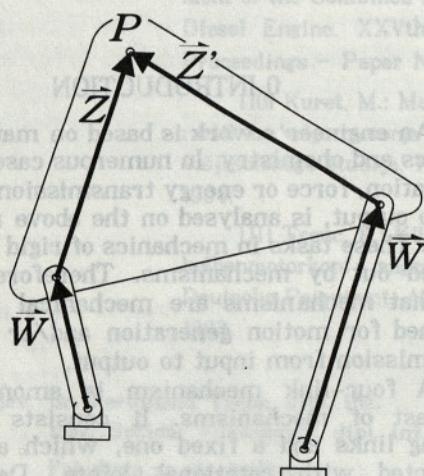
Če ta vektor v ravnini zavrtimo za kot  $\Phi$  in se pri tem dolžina ročice ne spremeni (kar velja za štiriročične mehanizme), dobljeni nov vektor zapišemo kot:

$$\vec{Z}'_k = |\vec{Z}_k| e^{i(\varphi_k + \Phi)} = \vec{Z}_k e^{i\Phi} \quad (2).$$

Faktor  $e^{i\Phi}$  v zgornji enačbi imenujemo rotacijski operator, ker zavrti vektor  $\vec{Z}_k$  za kot  $\Phi$ . Če položimo kompleksne vektorje na štiriročični mehanizem (sl. 1), dobimo dve dvojici vektorjev  $\vec{Z}$ ,  $\vec{W}$  in  $\vec{Z}'$ ,  $\vec{W}'$ . Za določitev neznane dvojice moramo poznati  $\vec{R}_i$ ,  $\vec{R}_j$ ,  $\alpha_j$  in  $\beta_j$  (sl. 2). Vrednosti  $\vec{R}_i$ ,  $\vec{R}_j$  in  $\alpha_j$  pomenijo predpisane lege, skozi katere želimo, da se telo na ravnini zapelje.

If this vector is rotated by the angle, without changing the length of the link (that applies to four-link mechanisms), the new vector is expressible as:

Factor  $e^{i\Phi}$  in the above equation is termed the rotational operator, because it rotates the vector  $\vec{Z}_k$  by the angle  $\Phi$ . If we place complex vectors onto the four-link mechanism (Fig. 1), we obtain two dyads of vectors  $\vec{Z}$ ,  $\vec{W}$  and  $\vec{Z}'$ ,  $\vec{W}'$ . For determining unknown dyads, it is necessary to know  $\vec{R}_1$ ,  $\vec{R}_j$ ,  $\alpha_j$  and  $\beta_j$  (Fig. 2). The values  $\vec{R}_1$ ,  $\vec{R}_j$  and  $\alpha_j$  represent prescribed positions, through which we want the body to move on the coupler plane.



Slika 1  
Figure 1

Slika 2 prikazuje dvojico vektorjev v dveh predpisanih legah. Zapišimo vektorsko enačbo po zaprti zanki:

$$\vec{W} e^{i\beta_j} + \vec{Z} e^{i\alpha_j} - \vec{W} - \vec{Z} - \vec{R}_+ + \vec{R}_- = 0 \quad (3)$$

Figure 2 represents dyads in two prescribed positions. Our vector equation can be put down as a loop closure equation:

$$\vec{W}(e^{i\beta_j} - 1) + \vec{Z}(e^{i\alpha_j} - 1) = \vec{\delta}. \quad (4)$$

where:

$$\vec{\delta}_j = \vec{R}_j - \vec{R}_1 \quad \text{! MATHEMATICA SHREDS THIS!} \quad (5).$$

Enačba (4) pomeni dvojico vektorjev v dveh želenih legah. Za vsako nadaljnjo lego dobimo dodatno enačbo, ki je podobna enačbi (4).

Equation (4) represents dyads in two desired positions. For each further position, we obtain an additional equation that is similar to equation (4).

Preglednica 1

Table 1

Število predpisanih leg (n)	število skalarnih enačb	število skalarnih neznank	število neznank, ki jih lahko poljubno izberemo
Number of prescribed positions (n) $j = 1 \dots n$	number of scalar equations	number of scalar unknowns	number of unknowns of free choice
2	2	5 ( $\bar{Z}, \bar{W}, \beta_2$ )	3
3	4	6 (prejšnje + $\beta_3$ ) – 6 (the upper + $\beta_3$ )	2
4	6	7 (prejšnje + $\beta_4$ ) – 7 (the upper + $\beta_4$ )	1
5	8	8 (prejšnje + $\beta_5$ ) – 8 (the upper + $\beta_5$ )	0

Preglednica 1 prikazuje število enačb, neznank in veličin, ki jih lahko poljubno izbiramo v odvisnosti od števila predpisanih leg. Največje število predpisanih leg pri nastajanju gibanja je pet. V tem sistemu štirih kompleksnih oziroma osmih skalarnih enačb so  $R_{xj}$ ,  $R_{yj}$ ,  $\alpha_j$  predpisane vrednosti (želene lege),  $Z_x$ ,  $Z_y$ ,  $W_x$ ,  $W_y$ ,  $\beta_j$  ( $j = 2 \dots 5$ ) pa so neznane vrednosti.

Če si še ogledamo sisteme enačb, vidimo, da dobimo za štiri in pet predpisanih leg sistem ne-linearnih enačb za neznanke  $\beta_j$ ,  $\bar{Z}$  in  $\bar{W}$ , medtem ko je sistem za tri predpisane lege linearen za neznanke  $\bar{Z}$  in  $\bar{W}$ . Zaradi tega se pri nezahtevnih nalogah gibanja odločamo oz. predpisujemo tri lege. Sistem dveh enačb za tri predpisane lege je naslednji:

$$(II) \quad \begin{aligned} \bar{W}(e^{i\beta_2} - 1) + \bar{Z}(e^{i\alpha_2} - 1) &= \delta_2 \\ \bar{W}(e^{i\beta_3} - 1) + \bar{Z}(e^{i\alpha_3} - 1) &= \delta_3 \end{aligned} \quad (6).$$

Če želimo izračunati dolžini vektorjev  $\bar{Z}$  in  $\bar{W}$ , moramo zraven predpisanih leg izbrati še vrednosti  $\beta_2$  in  $\beta_3$  (zasuki vektorja  $\bar{W}$ ).

Izbira kotov  $\beta_2$  in  $\beta_3$  je na splošno poljubna, vendar je iskanje ustreznih vrednosti lahko tudi zamudno in nas včasih tudi ne pripelje do želenega rezultata. Zaradi tega so se razvile metode, ki nas zelo hitro pripeljejo do rešitve. Omejil bi se na metodi:

- krožnice krožnih in središčnih točk,
- lego osnovnega tečaja.

## 2 KROŽNICE KROŽNIH IN SREDIŠČNIH TOČK

Krožnice krožnih in središčnih točk so alternativna metoda za izbiro dveh kotnih neznank, od katerih sta odvisni legi osnovnega in gibljivega tečaja, v primeru treh predpisanih leg.

Če poljubno izberemo vrednost enega nepredpisanega kota in pri tem drugi kot zavzame vse možne vrednosti, sestavlajo lege osnovnega tečaja  $m$  in gibljivega tečaja  $k$  krožnice. Vektorske enačbe zaprete zanke (sl. 3) za dvojice v treh predpisanih legah so:

Table 1 shows the number of equations, unknowns and free choices with regard to the number of prescribed positions. The maximum number of prescribed positions at motion generation is 5. In a system of four complex or eight scalar equations,  $R_{xj}$ ,  $R_{yj}$ ,  $\alpha_j$  are prescribed values (desired positions), whereas  $Z_x$ ,  $Z_y$ ,  $W_x$ ,  $W_y$ ,  $\beta_j$  ( $j = 2 \dots 5$ ) are unknown values.

If we examine the systems of equations, we can see that for four and five prescribed positions, there is a system of nonlinear equations for unknowns  $\beta_j$ ,  $\bar{Z}$  and  $\bar{W}$ , whereas the system of three prescribed positions is linear for the unknowns  $\bar{Z}$  and  $\bar{W}$ . So we prescribe three positions for undemanding tasks of motion. The system of two equations for three prescribed positions is as follows:

If we want to compute the lengths of vectors  $\bar{Z}$  and  $\bar{W}$ , we have to choose values for  $\beta_2$  and  $\beta_3$  (turns of the vector  $\bar{W}$ ) in addition to the prescribed positions.

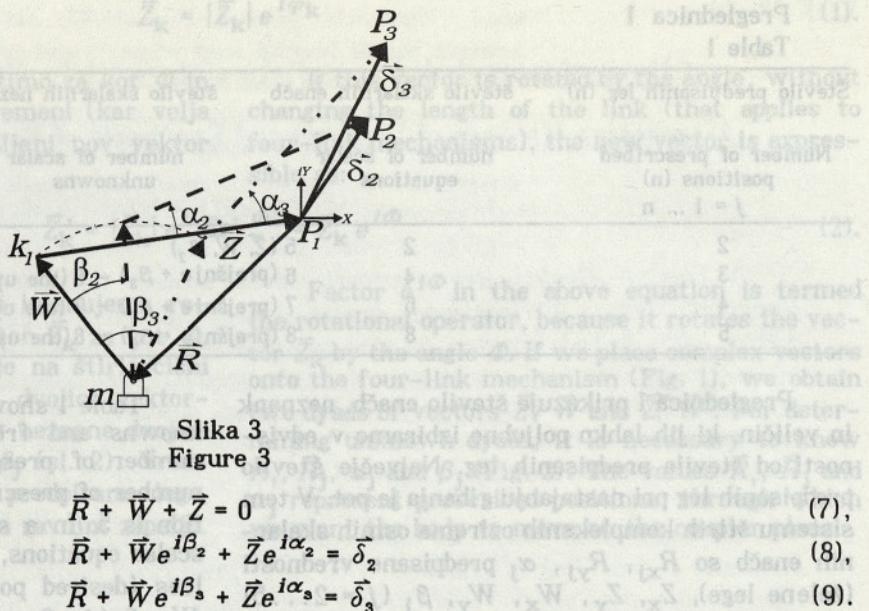
The choice of angles  $\beta_2$  and  $\beta_3$  is generally arbitrary, but looking for suitable values can also be time-consuming and sometimes it does not lead to the desired result. Some methods have been developed for that very reason. They lead to a solution very quickly. The paper deals with two methods only:

- circle-point and centre-point circles,
- ground-pivot specification.

## 2 CIRCLE-POINT AND CENTRE-POINT CIRCLES

Circle-point and centre-point circles is an alternative method for choosing the two angular unknowns on which the locations of the ground and moving pivots will depend if there are three prescribed positions.

If an arbitrary value is chosen for one unpreserved angular parameter, while the other angular parameter is allowed to assume all possible values, the resulting positions of the ground pivot  $m$  and the moving pivot  $k$  are found to be circles. The loop-closure equations (Fig. 3) for the dyad in three prescribed positions are:



$$\vec{R} + \vec{W} + \vec{Z} = 0 \quad (7),$$

$$\vec{R} + \vec{W} e^{i\beta_2} + \vec{Z} e^{i\alpha_2} = \vec{\delta}_2 \quad (8),$$

$$\vec{R} + \vec{W} e^{i\beta_3} + \vec{Z} e^{i\alpha_3} = \vec{\delta}_3 \quad (9).$$

Neznana lega gibljivega tečaja  $k_1$  je definirana z vektorjem  $-\vec{Z}$  glede na točko  $P_1$ , na katero je tudi pripelj koordinatni sistem. Neznana lega nepremičnega tečaja  $m$  pa je določena z vektorjem  $\vec{R}$ . Vektorja  $-\vec{Z}$  in  $\vec{R}$  izračunamo iz enačb (7) do (9) z uporabo Kramerjevega pravila:

$$\vec{R} = \frac{\begin{vmatrix} 0 & 1 & 1 \\ \vec{\delta}_2 e^{i\beta_2} & e^{i\alpha_2} \\ \vec{\delta}_3 e^{i\beta_3} & e^{i\alpha_3} \end{vmatrix}}{\begin{vmatrix} 1 & 1 & 1 \\ 1 & e^{i\beta_2} & e^{i\alpha_2} \\ 1 & e^{i\beta_3} & e^{i\alpha_3} \end{vmatrix}} = \frac{\vec{\delta}_2 (e^{i\beta_3} - e^{i\alpha_3}) - \vec{\delta}_3 (e^{i\beta_2} - e^{i\alpha_2})}{e^{i\beta_2} e^{i\alpha_3} + e^{i\alpha_2} + e^{i\beta_3} - e^{i\beta_2} - e^{i\beta_3} e^{i\alpha_2} - e^{i\alpha_3}} \quad (10)$$

in

$$-\vec{Z} = \frac{\begin{vmatrix} 1 & 1 & 0 \\ 1 & e^{i\beta_2} & \vec{\delta}_2 \\ 1 & e^{i\beta_3} & \vec{\delta}_3 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 1 & e^{i\beta_2} & -e^{i\alpha_2} \\ 1 & e^{i\beta_3} & -e^{i\alpha_3} \end{vmatrix}} = \frac{-\vec{\delta}_2 (e^{i\beta_3} - 1) + \vec{\delta}_3 (e^{i\beta_2} - 1)}{-e^{i\beta_2} e^{i\alpha_3} - e^{i\alpha_2} - e^{i\beta_3} + e^{i\beta_2} + e^{i\beta_3} e^{i\alpha_2} + e^{i\alpha_3}} \quad (11).$$

Če so vsi parametri na desni strani enačb konstantni, razen kotnega parametra  $\Theta$ , ki zavzame vse možne vrednosti, sta vektorja  $-\vec{Z}$  in  $\vec{R}$  izražena v odvisnosti od  $\Theta$  v naslednji obliki:

$$\vec{R}(\Theta) = \frac{a\Theta + b}{c\Theta + d} \quad (12),$$

$$-\vec{Z}(\Theta) = \frac{e\Theta + f}{g\Theta + h} \quad (13).$$

The unknown location of the moving pivot  $k_1$  is defined by the vector  $-\vec{Z}$  with respect to  $P_1$ , to which the coordinate system is fixed. The unknown location of fixed pivot  $m$  is defined by vector  $\vec{R}$ . Vectors  $-\vec{Z}$  and  $\vec{R}$  may be obtained from equations (7) to (9) using Cramer's rule:

and

If all the parameters on the right-hand side of equations are fixed except for an angular parameter  $\Theta$ , which ranges over all possible values, the equations for  $-\vec{Z}$  and  $\vec{R}$  can be expressed with respect to  $\Theta$  in the following way:

V enačbah je  $\Theta = e^{i\vartheta}$  ( $\vartheta$  pomeni kot, ki ga spremenjamo ( $\beta_2, \beta_3$ )). Vrednosti za  $a$  do  $h$  so znane, ker pomenijo predpisane lege. Če se  $\vartheta$  spreminja med 0 in  $2\pi$  opisuje pri tem enoto krožnice. Enačbi (12) in (13) pa pomenita enačbo krožnice v kompleksni obliki. Konstante od  $a$  do  $h$  dobimo tako, da enačbi (10) in (11) preuredimo v obliko enačb (12) in (13).

Za primer vzemimo, da spremenjamo kot  $\beta_2$  med 0 in  $2\pi$ . Pri tem se izražata vektorja  $-\vec{Z}$  in  $\vec{R}$  v odvisnosti od kota  $\beta_3$  kot:

$$\vec{R}(e^{i\beta_3}) = \frac{(\vec{\delta}_2)e^{i\beta_3} + (-\vec{\delta}_2 e^{i\alpha_3} + \vec{\delta}_3(e^{i\beta_2} - e^{i\alpha_2}))}{(1 - e^{i\alpha_2})e^{i\beta_3} + (e^{i\beta_2}e^{i\alpha_3} + e^{i\alpha_2} - e^{i\beta_2} - e^{i\alpha_3})} = \frac{ae^{i\beta_3} + b}{ce^{i\beta_3} + d} \quad (14)$$

in

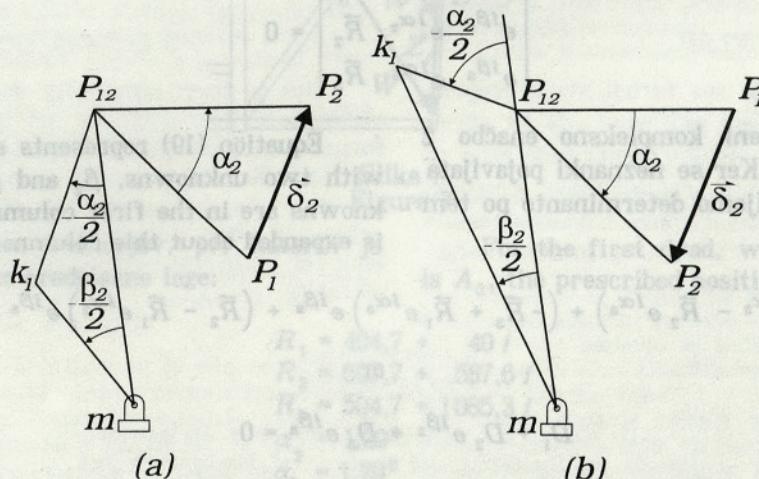
$$-\vec{Z}(e^{i\beta_3}) = \frac{(-\vec{\delta}_2)e^{i\beta_3} + (\vec{\delta}_2 + \vec{\delta}_3(e^{i\beta_2} - 1))}{(e^{i\alpha_2} - 1)e^{i\beta_3} + (-e^{i\beta_2}e^{i\alpha_3} - e^{i\alpha_2} + e^{i\beta_2} + e^{i\alpha_3})} = \frac{ee^{i\beta_3} + f}{ge^{i\beta_3} + h} \quad (15)$$

Središča krožnic  $C_M$  in  $C_K$  lahko izračunamo na več načinov, najlaže je, če izračunamo vrednosti za  $-\vec{Z}$  in  $\vec{R}$  pri treh različnih kotih  $\vartheta$ , s tem je določena krožnica. Ko smo narisali krožnice, je treba samo še izbrati lege nepremičnega in gibljivega tečaja. To naredimo tako, da izberemo lego enega tečaja na ustrezni krožnici, lego drugega tečaja pa dobimo s povezavo s polom (sl. 4).

Here  $\Theta = e^{i\vartheta}$  ( $\vartheta$  stands for the angle to be varied ( $\beta_2, \beta_3$ )), and  $a$  through  $h$  are known, because they represent the prescribed positions. When  $\vartheta$  varies from 0 to  $2\pi$ , describes the unit circle. Equations (12) and (13) represent a circle equation in its complex form. The complex constants  $a$  through  $h$  can be obtained by rearranging the equations (10) and (11) in the form of equations (12) and (13).

Let us take an example of changing the angle  $\beta_2$  between 0 and  $2\pi$ . Vectors  $-\vec{Z}$  and  $\vec{R}$  are expressed with respect to the angle  $\beta_3$  as follows:

The centres of the circular loci  $C_M$  and  $C_K$  can be computed in several ways. The easiest way is to compute the values for  $-\vec{Z}$  and  $\vec{R}$  at three different values  $\vartheta$ . This defines the circles. When the circles have been drawn, the associated fixed and moving pivots on a pair of circles remain to be coordinated. This is done by choosing a position of one pivot on the appropriate circle. The position of the other pivot is obtained by using the pole relationship (Fig. 4).



Slika 4  
a) pomik v nasprotni smeri, b) pomik v isti smeri

Figure 4

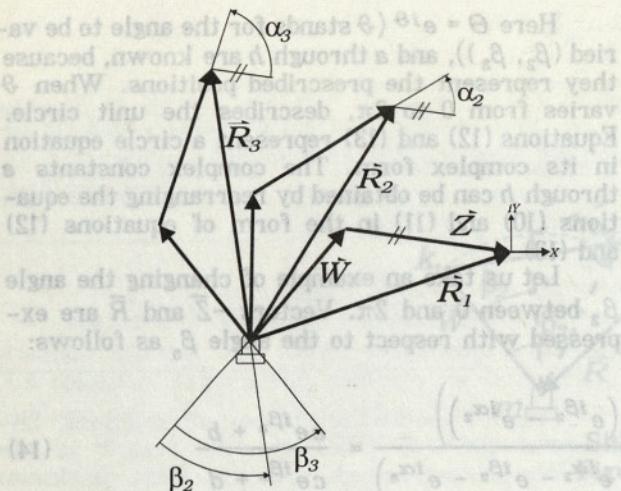
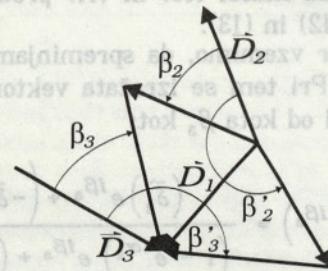
a) movement in the opposite direction, b) movement in the same direction

### 3 LEGA OSNOVNEGA TEČAJA

Lega osnovnega tečaja je še ena izmed zelo uporabnih metod za določitev kotov  $\beta_2$  in  $\beta_3$ . S to metodo vnaprej določimo lego osnovnega tečaja. Slika 5 prikazuje dvojice vektorjev v treh legah.

### 3 GROUND-PIVOT SPECIFICATION

Ground-pivot specification is yet another useful method for determining the angles  $\beta_2$  and  $\beta_3$ . By using this method, a ground pivot location may be specified directly. Figure 5 shows dyads in three positions.

Slika 5  
Figure 5Slika 6  
Figure 6

Enačbe sinteze so:

The synthesis equations can be written as:

$$\vec{W} + \vec{Z} = \vec{R}_1 \quad (16),$$

$$\vec{W} e^{i\beta_2} + \vec{Z} e^{i\alpha_2} = \vec{R}_2 \quad (17),$$

$$\vec{W} e^{i\beta_3} + \vec{Z} e^{i\alpha_3} = \vec{R}_3 \quad (18).$$

Enačbe (16) do (18) so linearne in nehomogene za kompleksni neznanki  $-\vec{Z}$  in  $\vec{R}$ . Sistem teh enačb ima rešitev le v primeru, če je matrika koeficientov enaka nič:

Equations (16) to (18) are linear and nonhomogeneous in the two complex unknowns  $-\vec{Z}$  and  $\vec{R}$ . This set has a solution for  $-\vec{Z}$  and  $\vec{R}$  only if the determinant of the augmented matrix of the coefficients vanishes:

$$\begin{vmatrix} 1 & 1 \\ e^{i\beta_2} & e^{i\alpha_2} \\ e^{i\beta_3} & e^{i\alpha_3} \end{vmatrix} = 0 \quad (19).$$

Enačba (19) pomeni kompleksno enačbo z neznankama  $\beta_2$  in  $\beta_3$ . Ker se neznanki pojavljata v prvem stolpcu, razvijemo determinanto po tem stolpcu:

Equation (19) represents a complex equation with two unknowns,  $\beta_2$  and  $\beta_3$ . Since the unknowns are in the first column, the determinant is expanded about this column:

$$(\vec{R}_3 e^{i\alpha_2} - \vec{R}_2 e^{i\alpha_3}) + (-\vec{R}_3 + \vec{R}_1 e^{i\alpha_3}) e^{i\beta_2} + (\vec{R}_2 - \vec{R}_1 e^{i\alpha_2}) e^{i\beta_3} = 0 \quad (20)$$

ali

or

$$D_1 + D_2 e^{i\beta_2} + D_3 e^{i\beta_3} = 0 \quad (21).$$

Koeficienti  $D_1$ ,  $D_2$  in  $D_3$  pomenijo predpisane legi. Enačbo (21) lahko rešujemo grafično ali analitično. Slika 6 prikazuje grafično reševanje, kjer so  $D_1$ ,  $D_2$  in  $D_3$  predstavljeni kot vektorji. Vektor  $D_1$  je stalen in na njega sta pripeta  $D_2$  in  $D_3$ . Vektorja  $D_2$  in  $D_3$  sta pomnožena z operatorjem rotacije  $e^{i\beta_2}$  in  $e^{i\beta_3}$ . Rešitev dobimo, ko vektorji  $D_1$ ,  $D_2 e^{i\beta_2}$  in  $D_3 e^{i\beta_3}$  sestavljajo zaprto zanko, pri tem dobimo dve dvojici rešitev  $\beta_2$ ,  $\beta_3$  in  $\beta'_2$ ,  $\beta'_3$ . Ena rešitev je trivialna:  $\beta_2 = \alpha_2$  in  $\beta_3 = \alpha_3$ ; to preverimo, če te vrednosti vstavimo v enačbo (19).

Coefficients  $D_1$ ,  $D_2$  and  $D_3$  represent prescribed positions. Equation (21) yields either a graphical or analytical solution. Figure 6 shows a graphical solution, where  $D_1$ ,  $D_2$ , and  $D_3$  are represented as vectors. Notice that  $D_2$  and  $D_3$  are pinned to  $D_1$ , but vector  $D_1$  is fixed. Vectors  $D_2$  and  $D_3$  are multiplied by rotation operators  $e^{i\beta_2}$  and  $e^{i\beta_3}$ . When the vectors  $D_1$ ,  $D_2 e^{i\beta_2}$  and  $D_3 e^{i\beta_3}$  form a closed loop, the equation will be satisfied. There are two sets of solutions for the triangle:  $\beta_2$ ,  $\beta_3$ ,  $\beta'_2$ , and  $\beta'_3$ . One set of solutions will be trivial, however. This solution is  $\beta_2 = \alpha_2$  and  $\beta_3 = \alpha_3$ . This can be

Ko najdemo netrivialno rešitev kotov  $\beta_2$  in  $\beta_3$ , vstavimo njuni vrednosti v dve poljubno izbrani enačbi iz sistema enačb (16) do (18) in s tem izračunamo vektorja  $\vec{Z}$  in  $\vec{W}$ .

#### 4 SINTEZA VRATNEGA MEHANIZMA

Kot primer uporabe metode *lega osnovnega tečaja* poglejmo sintezo mehanizma za odpiranje oz. zapiranje prtljažnih vrat avtobusa B3 090 TL. Vrata lahko, brez dodatnih elementov, pritrdimo le na okvir avtobusa. Ko smo izbrali legi osnovnih tečajev (točki  $A_0$  in  $B_0$ ) še izberemo lege vrat. Te so (sl. 7):

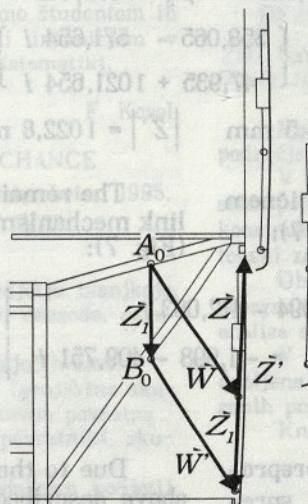
- zaprta,
- ko so vrata najbolj oddaljena od avtobusa,
- odprta.

verified by plugging the trivial roots back into equation (19). When a non-trivial solution for the angles  $\beta_2$  and  $\beta_3$  is found, their values may be plugged into two arbitrarily chosen equations from the system of equations (16) to (18). The vectors  $\vec{Z}$  and  $\vec{W}$  are thus calculated.

#### 4 DOOR MECHANISM SYNTHESIS

In relation to an example of applying the ground-pivot specification method, we may examine the synthesis of mechanisms for opening and closing the luggage compartment doors of bus B3 090 TL. Without any additional elements, the doors may be fixed to the bus frame only. When the ground pivot locations have been chosen (points  $A_0$  and  $B_0$ ), the door positions have to be chosen. These are (Fig. 7):

- closed,
- doors are at the most distant point from the bus,
- opened.



Slika 7  
Figure 7

Za prvo dvojico vektorjev, pri katerih je osnovni tečaj  $A_0$ , so predpisane lege:

For the first dyad, where the ground pivot is  $A_0$ , the prescribed positions are as follows:

$$\begin{aligned} R_1 &= 404,7 + 40i \\ R_2 &= 697,7 + 587,6i \\ R_3 &= 504,7 + 1085,3i \\ \alpha_1 &= 1,29^\circ \\ \alpha_2 &= 1,29^\circ \\ \alpha_3 &= 1,29^\circ \end{aligned}$$

Vektorji  $D_1$ ,  $D_2$  in  $D_3$  so:

$$\begin{aligned} D_1 &= -204,15 + 493,23i \\ D_2 &= -101,01 + 1036,19i \\ D_3 &= 294,01 + 538,49i \end{aligned}$$

Iz enačbe (21) izračunamo zasuka  $\beta_2$  in  $\beta_3$ :

$$\beta_2 = 54,816^\circ$$

Nato iz (17) in (18) izračunamo še dolžine ročic:

Vectors  $D_1$ ,  $D_2$  and  $D_3$  are:

The turns  $\beta_2$  and  $\beta_3$  may be calculated from the equation (21):

$$\beta_3 = 100,947^\circ$$

The link lengths are then calculated from (17), (18):

$$\begin{Bmatrix} \vec{W} \\ \vec{Z} \end{Bmatrix} = \begin{bmatrix} e^{i\beta_2} & e^{i\alpha_2} \\ e^{i\beta_3} & e^{i\alpha_3} \end{bmatrix}^{-1} \begin{Bmatrix} \vec{R}_2 \\ \vec{R}_3 \end{Bmatrix} = \begin{Bmatrix} 374,162 - 569,3 i \\ 30,538 + 609,3 i \end{Bmatrix}$$

$$|\vec{W}| = 681,2 \text{ mm} \quad |\vec{Z}| = 610,1 \text{ mm}$$

Za drugo dvojico so predpisane lege naslednje:

The other dyad has prescribed positions:

$$R'_1 = 406 + 450 i$$

$$R'_2 = 699 + 997,6 i$$

$$R'_3 = 566 + 1495,3 i$$

$$\alpha'_2 = 1,29^\circ$$

$$\alpha'_3 = 1,29^\circ$$

Izračunane vrednosti so:

The calculated values are:

$$D'_1 = -204,16 + 493,23 i$$

$$D'_2 = -110,23 - 1036,27 i$$

$$D'_3 = 33,23 + 538,57 i$$

Enačbe sinteze so:

$$\beta'_2 = 55,825^\circ \quad \beta'_3 = 102,443^\circ$$

$$\begin{Bmatrix} \vec{W}' \\ \vec{Z}' \end{Bmatrix} = \begin{Bmatrix} 358,065 - 571,654 i \\ 47,935 + 1021,654 i \end{Bmatrix}$$

$$|\vec{W}'| = 674,5 \text{ mm} \quad |\vec{Z}'| = 1022,8 \text{ mm}$$

Drugi dve ročici  $\vec{Z}_3$  in  $\vec{Z}_1$  v štiriročičnem mehanizmu lahko izračunamo iz enačb (sl. 7):

$$\vec{Z}_3 = \vec{Z} - \vec{Z}' = -18,094 - 412,093 i$$

$$\vec{Z}_1 = \vec{W} + \vec{Z}_3 - \vec{W}' = -1,998 - 409,751 i$$

The remaining two links  $\vec{Z}_3$  and  $\vec{Z}_1$  in a four-link mechanism may be calculated from equations (Fig. 7):

$$|\vec{Z}_3| = 412,5 \text{ mm}$$

$$|\vec{Z}_1| = 409,7 \text{ mm}$$

## 5 SKLEP

Prej opisani metodi sta zaradi svoje preprostosti in uporabnosti zelo primerni, da ju sprememimo v računalniški jezik. Zaradi tega smo se tudi odločili, da napišemo računalniški program za analizo in sintezo mehanizmov. Z dosego takšne avtomatizacije lahko nato iščemo optimalne mehanizme za določen tip naloge.

## 5 CONCLUSION

Due to their simplicity and applicability, the above described methods are suitable for transforming into computer language. For that reason, we have decided to write a computer programme for analysis and synthesis of mechanisms. By achieving such automatization, it is possible to look for optimum mechanisms for a definite type of task.

## 6 LITERATURA 6 REFERENCES

[1] Norton, R.L.: Design of Machinery. An Introduction to the Synthesis and Analysis of Mechanisms and Machines, McGraw-Hill, 1992.

[2] Grosjean, J.: Kinematics and Dynamics of Mechanisms, McGraw-Hill, 1992.

[3] Erdman, A.G.-Sandor, G.N.: Mechanism Design, Vol. 1, Analysis and Synthesis, Prentice-Hall, 1991.

Naslov avtorja: Vilko Velikonja, dipl. inž.  
TAM Maribor d.d. Razvoj  
Ptujska 184  
62000 Maribor

Author's Address: Vilko Velikonja, Dipl. Ing.  
TAM Maribor Ltd. Development  
Ptujska 184  
62000 Maribor