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Računalniško simuliranje torzijskih nihanj razvejanih sistemov
Computer Simulation of Torsional Vibrations of Branched Chain Topology Systems

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Prispevek obravnava numerični postopek simuliranja torzijskih nihanj razvejanih linijskih sistemov. Za diskretizacijo smo uporabili metodo prenosnih matrik. Na podlagi te metode razviti programski paket TORZIJA omogoča določanje lastnih frekvenc in pripadajočih lastnih vektorjev linearnega nedušenega sistema ter določitev odziva na harmonsko motnjo dušenih sistemov v ustaljenem stanju. Prikazana sta razvoj in testiranje algoritma ter za uporabnika pripravnega vmesnika. S programom dobljene rezultate smo primerjali z analitičnimi rešitvami in s podatki v virih.

This paper describes the torsional vibrations modelling of branched chain-topology systems using the transfer matrix method. The computer program named TORZIJA, developed for this purpose, allows to predict eigenfrequencies, eigenvectors and the response of a system to harmonic forcing for undamped and damped linear models. The development and testing of the computer program are described. The user-friendly graphic interface for modelling is presented. The results of the simulations have been compared to those obtained analytically as well as those given in references.

0 UVOD

Analitično reševanje dinamike mehanskih sistemov je pogosto nemogoče ali pa časovno zelo potratno. Zato so bile razvite diskretizacijske numerične metode, ki sistem razdelijo na manjše sestavne dele. Te matematično popišejo in zložijo nazaj v celoten sistem kot sistem enačb. Z rešitvijo sistema enačb dobimo numerično rešitev problema.

Ker so torzijski sistemi navadno linijske narave, smo izbrali metodo prenosnih matrik. Metoda je takim sistemom prilagojena in je zato njena uporaba preprosta, računanje pa hitro in natančno.

Za pisanje lastnega programskega paketa — TORZIJA, ki pokriva področje problematike torzijskih nihanj, smo se odločili prav zaradi preprostosti modeliranja sistema in hitrosti ter natančnosti računanja modela. Mnogi komercialni programi, ki slonijo na metodi končnih elementov, omogočajo enake preračune. Vendar je modeliranje težje in bolj zamudno. Pogosto nimajo za torzijska nihanja posebej prilagojenih elementov modela, kakršne nam ponuja TORZIJA.

Za omenjeni programski paket smo posebej razvili algoritem za popis razvejanih torzijskih sistemov, ki omogoča modeliranje tako zobniških in tornih kakor tudi jermenskih in verižnih prenosov moči [3].

0 INTRODUCTION

Analytical evaluation of a mechanical system's dynamics is in many cases impossible, or else a very time consuming task. Hence, a range of numerical methods have been developed for that purpose, based on the idea of breaking up a complicated system into component parts with simple elastic and dynamics properties that can be presented in matrix form. These matrices are considered as building blocks which, when fitted together according to predefined rules, provide the dynamic properties of the entire system.

The transfer matrix method is ideally suited to systems with a predominant chain topology. The torsional systems, continued beams, turbine-generator shafts, crankshafts, etc., always have a chain topology. This was the main reason for using the method for developing an algorithm, upon which the computer program was built.

The program, named TORZIJA, covers the field of linear torsional vibrations of elastic mechanical systems. The ideas of efficient and easy modelling as well as fast and accurate computation were the leading thoughts while developing the program. Many commercial computer applications based on the finite element method provide the same feasibility in calculating, but the modelling of a system is usually far more complicated and time consuming than it is with TORZIJA. The program provides one with building elements designed specially for covering the field of torsional vibrations.

The algorithm of the transfer matrix method was enriched to cover also branched torsional systems, which allows one to model gear, friction, belt and chain drive [3].

1 METODA PRENOSNIH MATRIK (MPM)

Metoda je omejena na linearna nihanja in na elastične dinamske sisteme. Model sistema razdelimo na sestavne dele, imenovane gredi, ki popisujejo eno linijo sistema. Gredi delimo na osnovne sestavne dele modela, elemente, katerih lastnosti lahko popišemo v matrični obliki. Samo pri razvejanih sistemih modeliramo več ko eno gred.

Prenosne matrike definirajo spreminjanje vektorja stanja v elementu. Za popis torzijskih nihanj je vektor stanja, **z**, sestavljen iz amplitude zasuka prereza okoli glavne osi elementa φ in amplitude torzijskega momenta M, ki deluje v težišču prečnega prereza v smeri glavne osi elementa. Obe veličini sta kompleksni. Indeksa r in i označujeta realni in imaginarni del števila ali vektorja:

1 TRANSFER MATRIX METHOD

The method is limited to linear vibrations and elastic dynamic systems. A model of a system is broken up into components named shafts. A shaft represents a one dimensional chain structure in a model that is built of smaller blocks called elements. An element is a basic building block whose dynamic and elastic properties can be expressed in matrix form. Only branched systems have more than one shaft.

A transfer matrix describes state vector changes over an element. In the case of torsional vibrations the state vector **z** is a column vector with two components. The first component is the angle of twist φ, and the second is the torque M. Both are complex variables. Here r and i denote real and imaginary parts, respectively.

$$\mathbf{z}=\begin{bmatrix}\varphi\\ M\end{bmatrix}=\begin{bmatrix}\varphi_r\\ M_r\end{bmatrix}+i\begin{bmatrix}\varphi_i\\ M_i\end{bmatrix}=\mathbf{z}^r+i\cdot\mathbf{z}^i \quad (1).$$

Prenosne matrike so določene vedno enosmerno. Model po navadi rešujemo z leve proti desni. Če poznamo vektor stanja na levem robu elementa, lahko izračunamo vektor stanja na desnem robu:

Transfer matrices are always defined according to the solving direction of a system. A model is normally solved from its left towards the right end. If the state vector of the left end of an element is known, one can compute the state vector of the right end of the element:

$$\mathbf{z}_d=\mathbf{PM}\cdot\mathbf{z}_l, \quad (2),$$

kjer je **PM** prenosna matrika elementa, indeksa d in l pa označujeta desno in levo stran. S ponavljanjem enačbe (2) po celotnem modelu izračunamo vse vektorje stanja modela:

where **PM** is the transfer matrix of an element where d and l denote right and left, respectively. All state vectors of a model can be obtained by repeating the procedure of (2) along the model:

$$\mathbf{z}_n=\mathbf{PM}_n\cdot\mathbf{PM}_{n-1}\cdot\mathbf{PM}_{n-2}\cdot\ldots\cdot\mathbf{PM}_2\cdot\mathbf{PM}_1\cdot\mathbf{z}_0 \quad (3).$$

Matrike in vektorje s kompleksnimi elementi predelamo v matrike in vektorje z realnimi komponentami:

Matrices and vectors with complex elements could also be transformed into matrices and vectors with real elements:

$$\begin{bmatrix}\mathbf{z}^r\\ \mathbf{z}^i\end{bmatrix}_d=\begin{bmatrix}\mathbf{PM}^r & -\mathbf{PM}^i\\ \mathbf{PM}^i & \mathbf{PM}^r\end{bmatrix}\cdot\begin{bmatrix}\mathbf{z}^r\\ \mathbf{z}^i\end{bmatrix}_l \quad (4).$$

Pri vsiljenih nihanjih povečamo razsežnost matrike in vektorja za 1:

The order of the matrix and the vector is increased by 1 when dealing with forced vibrations:

$$\begin{bmatrix}\varphi^r\\ M^r\\ \varphi^i\\ M^i\\ 1\end{bmatrix}_d=\begin{bmatrix}a_{11} & a_{12} & a_{13} & a_{14} & a_{15}\\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25}\\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35}\\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45}\\ 0 & 0 & 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}\varphi^r\\ M^r\\ \varphi^i\\ M^i\\ 1\end{bmatrix}_l \quad (5).$$

V primeru lastnih nihanj odvržemo peti stolpec in peto vrstico prenosne matrike ter zadnji element vektorja stanja.

The fifth column and row of the transfer matrix and the fifth row of the state vector in eq. (5) are omitted in the case of free vibrations.

Robna pogoja za rob gredi sta le dva: vpet rob gredi in torzijsko prost rob gredi. Razvejani model vedno rešujemo tako, da začnemo izračun pri gnanih gredeh, na dnu drevesne strukture gredi. Te prispevajo eno prenosno matriko v verigo prenosnih matrik njihovih gonilnih gredi. Postopek ponavljamo, dokler ne pridemo do ravni 1, do gredi na vrhu drevesne strukture.

2 MODELIRANJE RAZVEJIŠČ

Razvejišče povezuje dve ali tudi več gredi. Prenos momenta in zasuka z gonilne na gnano gred modeliramo kot idealen. V tem prispevku se bomo omejili le na obravnavanje zobniških razvejišč [2], [3] pri vsiljenih nihanjih.

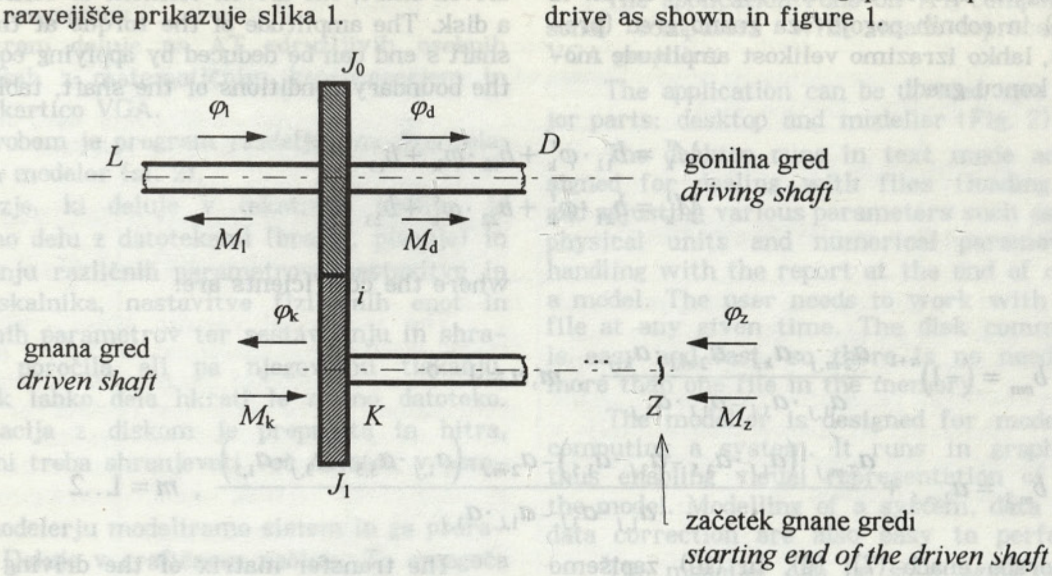
Vsako razvejišče je sestavljeno iz najmanj dveh elementov. Element pogona je vedno na gonilni gredi, elementi odgonov pa so na gnanih gredeh razvejišča. Pogon in odgon sta zobnika. Zobniško razvejišče prikazuje slika 1.

There are only two different boundary conditions of a shaft: fixed or free shaft end. One always starts solving a branched model from the driven shafts on the bottom of the tree structure of shafts. Thus, one transfer matrix is contributed in a chain of transfer matrices of their driving shafts. The procedure is repeated till the shaft on the top of the tree, level 1, is reached.

2 MODELLING OF BRANCHING POINTS

Two or more shafts are joined in a branching point. The transfer of the torque and the angle of twist is modelled ideally. Only a forced vibrations gear drive type [2], [3] of the branching point is considered in this paper.

Each branching point consists of at least two elements. The driving element is always a part of the driving shaft. There may be more than one driven element in the branching point, each belonging to a certain driven shaft. The driving element and the driven element are tooth wheels. In this case the branching point represents a gear drive as shown in figure 1.



Sl. 1. *Zobniško razvejišče*

Fig. 1. *Gear drive*

Prestavno razmerje definiramo kot razmerje med gonilno in gnano krožno frekvenco:

The transmission rate of gear drive is defined as the quotient of driving over driven angular velocity:

$$i = \frac{\omega_{go}}{\omega_{gn}} \tag{6}$$

Postavimo osnovni enačbi razvejišča:

The basic equations of gear drive are:

$$\varphi_l = \varphi_d = -i \cdot \varphi_k$$
$$M_l = M_d + \frac{M_k}{i} \tag{7}$$

Model preračunavamo tako, da vedno najprej preračunamo gnane gredi in šele nato gonilno gred. Tako so prenosna matrika gnane gredi in njeni vektorji stanja znani, ko računamo gonilno gred.

A model is always computed in such a way that the driven shafts are computed before the driving ones. Hence the transfer matrix of the driven shafts and their state vectors are already known when computing the driving shaft.

Gnano gred popišemo z enačbo, ki povezuje konec z začetkom gredi:

The dynamics of the driven shaft is described by:

$$\begin{bmatrix} \varphi^{r} \\ M^{r} \\ \varphi^{i} \\ M^{i} \\ 1 \end{bmatrix}_{k} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \varphi^{r} \\ M^{r} \\ \varphi^{i} \\ M^{i} \\ 1 \end{bmatrix}_{z} \quad (8).$$

Preglednica 1: *Robni pogoji začetka gnane gredi*

Table 1: *Boundary conditions at the starting end (Z) of the driven shaft*

prosto — free	j = 1
vpeto — fixed	j = 2

Pri preračunu prenosne matrike gnane gredi element odgona upoštevamo kot navaden kolut. Iz enačbe (8) in robnih pogojev za gnano gred (preglednica 1), lahko izrazimo velikost amplitude momenta na koncu gredi:

When computing the transfer matrix of the driven shaft, the driven element is considered as a disk. The amplitude of the torque at the driven shaft's end can be deduced by applying eq. (8) and the boundary conditions of the shaft, table 1.

$$M_{k}^{r} = b_{11} \cdot \varphi_{k}^{r} + b_{12} \cdot \varphi_{k}^{i} + b_{13}$$
$$M_{k}^{i} = b_{21} \cdot \varphi_{k}^{r} + b_{22} \cdot \varphi_{k}^{i} + b_{23} \quad (9),$$

kjer so:

where the coefficients are:

$$b_{mn} = (-1)^{n+1} \cdot \frac{a_{2\cdot m,j} \cdot a_{k,l} - a_{2\cdot m,l} \cdot a_{k,j}}{a_{1,j} \cdot a_{3,l} - a_{1,l} \cdot a_{3,j}}, \quad m,n = 1..2$$

$$b_{m,3} = a_{2\cdot m,5} + \frac{a_{2\cdot m,j} \cdot (a_{1,l} \cdot a_{3,5} - a_{3,l} \cdot a_{1,5}) - a_{2\cdot m,l} \cdot (a_{1,j} \cdot a_{3,5} - a_{3,j} \cdot a_{1,5})}{a_{1,j} \cdot a_{3,l} - a_{1,l} \cdot a_{3,j}}, \quad m = 1..2 \quad (10).$$

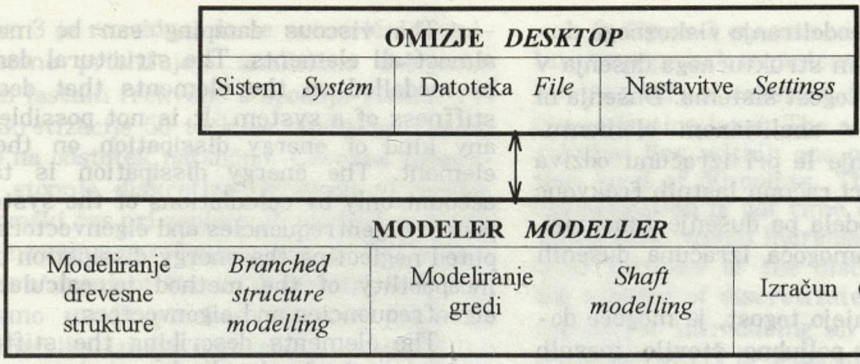
Z uporabo enačb (7), (9), in (10) zapišemo prenosno matriko razvejišča na gonilni gredi z upoštevanjem vztrajnosti in viskoznega dušenja gonilnega zobnika:

The transfer matrix of the driving element can be deduced from equations (7), (9), (10) when taking into account the properties of the element.

$$\begin{bmatrix} \varphi^{r} \\ M^{r} \\ \varphi^{i} \\ M^{i} \\ 1 \end{bmatrix}_{d} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \sum_{j=1}^{n}\left(\frac{b_{11}}{i^{2}}\right)_{j} - \omega^{2} \cdot J_{0} & 1 & \sum_{j=1}^{n}\left(\frac{b_{12}}{i^{2}}\right)_{j} - \omega \cdot c & 0 & \sum_{j=1}^{n}\left(-\frac{b_{13}}{i^{2}}\right)_{j} \\ 0 & 0 & 1 & 0 & 0 \\ \sum_{j=1}^{n}\left(\frac{b_{21}}{i^{2}}\right)_{j} - \omega \cdot c & 0 & \sum_{j=1}^{n}\left(\frac{b_{22}}{i^{2}}\right)_{j} - \omega^{2} \cdot J_{0} & 1 & \sum_{j=1}^{n}\left(\frac{b_{23}}{i^{2}}\right)_{j} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \varphi^{r} \\ M^{r} \\ \varphi^{i} \\ M^{i} \\ 1 \end{bmatrix}_{l} \quad (11).$$

Tu pomenijo: J_0 — masni vztrajnostni moment gonilnega zobnika, i — prestavno razmerje j-te zobniške dvojice, ω — krožno frekvenco gonilne gredi in c — faktor viskoznega dušenja. Vsota po j upošteva prispevke vseh gnanih gredi na danem razvejišču.

Here J_0 is the mass moment of inertia of the driving tooth wheel, i is the transmission ratio of the j tooth wheel couple, ω is the circular frequency of the driving shaft and c is the viscous damping coefficient. Index j denotes summation over all contributions of the driven shafts attached onto a certain branching point.



Sl. 2. *Programska shema*
Fig. 2. *Program chart*

3 OPIS PROGRAMSKEGA PAKETA TORZIJA

Program je napisan v programskem jeziku PASCAL. Izkoristili smo vse prednosti, ki jih ta jezik omogoča. S tem mislimo na objektno in rekurzivno programiranje, ki poenostavi pisanje obsežnih programov.

Program deluje na AT združljivih osebnih računalnikih z matematičnim koprocesorjem in grafično kartico VGA.

V grobem je program razdeljen na dva dela: omizje in modeler (sl. 2).

Omizje, ki deluje v tekstnem načinu, je namenjeno delu z datotekami (branje, pisanje) in nastavljanju različnih parametrov: nastavitve in izbira tiskalnika, nastavitve fizikalnih enot in numeričnih parametrov ter sestavljanju in shranjevanju poročila ali pa njegovemu tiskanju. Uporabnik lahko dela hkrati le z eno datoteko. Komunikacija z diskom je preprosta in hitra, tako da ni treba shranjevati več datotek v pomnilnik.

V modelerju modeliramo sistem in ga preračunamo. Deluje v grafičnem načinu. To omogoča vidni prikaz delov modela ter preprosto modeliranje in vnos podatkov ter njihovo popravljanje.

Modeler se nadalje deli na tri večje dele. Prvi del je namenjen modeliranju drevesne strukture modela. Tu model razdelimo na manjše sestavne dele, imenovane gredi. Modeliranje je shematično, drevesna struktura modela pa je enaka kakor DOS-ova drevesna struktura imenikov. V drugem delu modelerja modeliramo gredi. Gredi so sestavljene iz elementov, ki so osnovni gradniki modela. Program pozna devet različnih elementov, s katerimi popisujemo vztrajnosti in togosti ter topologijo modela. Med njimi je tudi element, ki popisuje harmonsko motnjo v sistemu. Zadnji del modelerja omogoča preračun lastnih frekvenc in lastnih vektorjev ter odziv modela na harmonsko motnjo v ustaljenem stanju. Program je narejen tako, da omogoča preračun celotnega modela, posamezne gredi v modelu ali pa določene veje modela.

3 DESCRIPTION OF THE COMPUTER PROGRAM TORZIJA

The code is written in PASCAL computer language, using the object oriented and recursive programming technique.

The application runs on AT compatible personal computers with math-coprocessor and VGA graphics.

The application can be divided into two major parts: desktop and modeller (Fig. 2).

The desktop runs in text mode and is designed for dealing with files (loading, saving) and adjusting various parameters such as: printer, physical units and numerical parameters, and handling with the report at the end of computing a model. The user needs to work with only one file at any given time. The disk communication is easy and fast, so there is no need to have more than one file in the memory.

The modeller is designed for modelling and computing a system. It runs in graphic mode, thus enabling visual representation of parts of the model. Modelling of a system, data input and data correction are also easy to perform.

The modeller can be split again into three main parts. The first is designed to support user friendly modelling of the tree structure of shafts of the model. The model is divided into smaller components called shafts. Modelling is schematic, so the tree structure of shafts of the model is presented in the same manner as the DOS tree structure of directories. In the second part of the modeller the shaft is modelled. Shafts are built of elements that form the basic building blocks of a model. There are nine different elements available. One can specify the properties of a system - such as inertia, stiffness, topology and torques and kinematic forcing - by using the elements available in the application. The last part of the modeller is the modulus, designed to compute eigenfrequencies and eigenvectors as well as the response of the model in the steady state at harmonic excitation. The application has been written in such a way that it is possible to compute a model as a whole, a branch of the model, or a single shaft.

Program omogoča modeliranje viskoznega dušenja v vseh elementih in strukturnega dušenja v elementih, ki popisujejo togost sistema. Dušenja ni mogoče modelirati le v analitičnem elementu. Program upošteva dušenje le pri izračunu odziva na harmonsko motnjo, pri računu lastnih frekvenc in lastnih vektorjev modela pa dušenje zanemari, ker sama metoda ne omogoča izračuna dušenih lastnih frekvenc.

Elemente, ki popisujejo togost, je mogoče dodatno diskretizirati na poljubno število masnih točk (kolutov). Tako poleg togosti popišemo še maso elementa.

Programski paket TORZIJA

- je uporabniku lahko razumljiv,
- omogoča modeliranje razvejanih sistemov,
- omogoča modeliranje strukturnega in viskoznega dušenja,
- omogoča modeliranje elastičnega podprtja,
- vsebuje devet različnih elementov,
- dovoljuje največ 255 elementov na gredi,
- dovoljuje največ 255 gredi modela,
- zaseda 400 kB pomnilnika na trdem disku,
- potrebuje grafično kartico VGA.

4 TESTIRANJE PROGRAMA

4.1 **Primerjava z analitično rešitvijo**

Natančnost rezultatov smo najprej preverjali z analitično rešitvijo parcialne diferencialne enačbe torzijskih nihanj. Na sliki 3 je prikazana konvergenca numeričnih rezultatov za prve tri lastne frekvence v odvisnosti od stopnje diskretizacije v primeru obojestransko togo vpete gredi nespremenljivega okroglega prereza.

The viscous damping can be modelled on almost all elements. The structural damping can be modelled on the elements that describe the stiffness of a system. It is not possible to model any kind of energy dissipation on the analytic element. The energy dissipation is taken into account only by calculations of the system's response. Eigenfrequencies and eigenvectors are computed neglecting the energy dissipation due to the incapability of the method to calculate damped eigenfrequencies and eigenvectors.

The elements describing the stiffness of a system can be further broken up into a number of disks joined by elastic fields. So the mass of the elastic field should be taken into account. This feature is automatic.

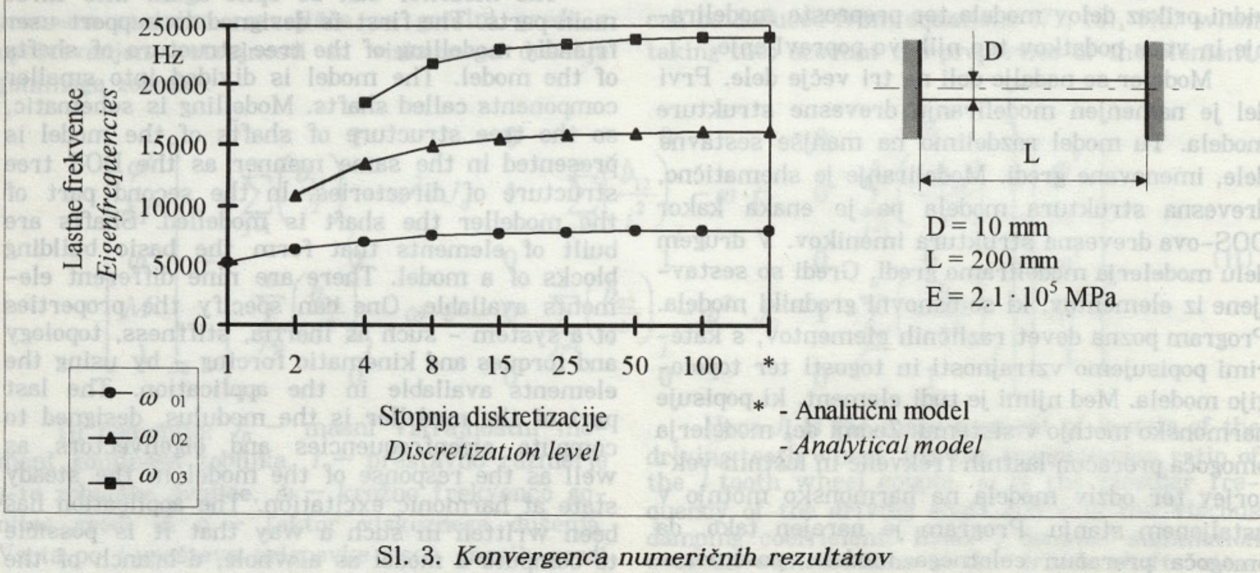
The main characteristics of the computer program could be presented as follows:

- user-friendly,
- modelling of branched systems,
- modelling of viscous and structural damping,
- modelling of elastic supports,
- nine different elements,
- max. 255 elements on a shaft,
- max. 255 shafts on a model,
- application takes 400 kB of hard disk drive,
- application needs VGA graphics.

4 THE APPLICATION TESTING

4.1 **Comparison with the analytical solution**

The numerical results were compared to those obtained by analytical solutions of partial differential equation of the torsion vibrations. The convergence of the numerical results for the first three eigenfrequencies of the system, consisting of a shaft with both ends fixed and with constant circular cross-section, is shown in figure 3.



Sl. 3. *Konvergenca numeričnih rezultatov*
Fig. 3. *Convergence of numerical results*

S slike 3 je razvidno, da se numerični rezultati monotono približujejo analitično dobljenim vrednostim lastnih frekvenc s spodnje strani. Pri stopnji diskretizacije 50 točk se analitični rešitvi približamo na odstotek natančno. Časovna zahtevnost take stopnje diskretizacije sicer ni velika. Ker bi računski čas pri zapletenih modelih naraščal približno linearno s povečanjem stopnje diskretizacije in s povečevanjem števila diskretiziranih elementov, smo uvedli poseben element, katerega prenosna matrika izhaja neposredno iz analitične rešitve problema [1], [3].

V preglednici 2 je prikazana primerjava rezultatov za prvo lastno frekvenco med analitičnimi rešitvami gredi nespremenljivega premera in rezultati, dobljenimi z ustreznim diskretnim modelom. Uporabili smo 50 točk diskretizacije pri diskretnem modelu in poseben element pri analitičnem modelu.

In figure 3 one can see that the numerical solutions approach monotonously to the analytic ones for the shaft of constant diameter with increasing discretization level. The accuracy of the numerical solution lies within one percent at a discretization level of 50 points. The computation of such discretization is not time consuming. The computation time would increase approximately linearly with increase of the discretization level and of the number of discretizated elements. This is the reason for introducing an analytical element. Its transfer matrix is deduced directly from the analytical solution of the problem [1], [3].

Table 2 shows results of the first eigenfrequency obtained by analytical approach of the shaft with constant diameter. The results are compared to those of the discrete model of the same shaft. The discretization level of 50 points was used for the discrete model, and the analytical element was used for the analytical model.

Preglednica 2: *Primerjava rezultatov med analitičnim in diskretnim modelom*
Table 2: *Comparison between the analytical and discrete model*

Model *Model*	**Analitični model** *Analytical model*	**Diskretni model** *Discrete model*	
	rezultat *results*	rezultat *results*	napaka *error*
prosto - prosto *free - free*	8019,1 Hz	8097,6 Hz	0,98 %
vpeto - prosto *fixed - free*	4009,6 Hz	4009,2 Hz	≈ 0 %
vpeto - vpeto *fixed - fixed*	8019,1 Hz	7938,9 Hz	-1 %

Rezultati v preglednici so med seboj primerljivi. Odstopanja so v redu velikosti enega odstotka. Upoštevati je treba, da diskretni model sestavlja 101 prenosna matrika, analitičnega pa le ena sama. Računski čas je pri diskretnem modelu približno 10-krat daljši od časa, ki ga za izračun porabi analitični model. Prednosti analitičnega elementa so očitne. Pomanjkljivost je le, da ne popisuje dušenja, tako da elementa ni mogoče uporabiti tam, kjer dušenje v sistemu ni zanemarljivo.

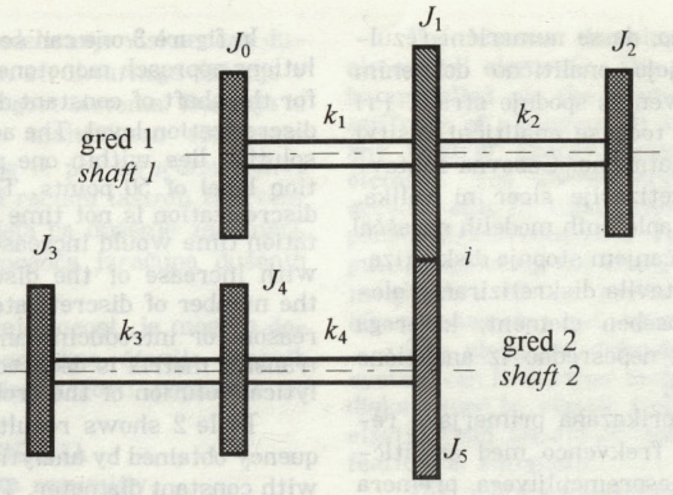
4.2 **Preračun razvejanega sistema**

Model na sliki 4 je diskreten sistem brez analitičnega elementa. Sestavljata ga dve gredi.

There is a pleasant agreement between the results in the table 2. The errors do not exceed 1%. One should bear in mind that the discrete model consists of 101 transfer matrices, and that the analytical model is represented only by one matrix. Therefore the computing time of the discrete model is 10 times longer than that of the analytical model. The advantages of the analytical element are obvious. But one can not model any kind of energy dissipation on the analytical element, so it can be used only in those places in a system where energy dissipation could be neglected.

4.2 **The computation of a branched system**

A model without the analytical element is shown in fig. 4. The model consists of two shafts.



Sl. 4. *Razvejani model*

Fig. 4. *Branched model*

Razvrstitev drevesne strukture sistema lahko modeliramo tako, kakor poteka tok moči. Gred 1 je v tem primeru na ravni 1 (gonilna gred), gred 2 pa na ravni 2 (gnana gred). Lahko pa sistem modeliramo prav nasprotno. Če se lastna frekvenca gnane gredi ali veje ujame z lastno frekvenco modela, program take frekvence ne zazna, saj pomeni singularnost modela. Zato je včasih model treba preračunati na oba načina.

V preglednici 3 so prikazane lastne frekvence modela, dobljene z analitičnim preračunom diskretnega sistema in s programom dobljene numerične vrednosti. Razlik v rezultatih ni, če modeliramo gred 1 kot gonilno gred. Kadar modeliramo gred 2 kot gonilno gred, program ne izračuna druge lastne frekvence modela, ker se ta ujame s prvo lastno frekvenco gredi 1.

The system' s shaft tree structure can be modelled in the same direction as the power flow runs. In such a case shaft 1 is at level 1 (driving shaft), and shaft 2 is at level 2 (driven shaft). It is also possible to model the system in reverse order. The application can not compute the eigenfrequency of a system if a shaft or a branch of the system have the same eigenfrequency. This singularity can be avoided by reversing the power flow of the model.

Table 3 shows eigenfrequencies obtained analytically and numerically. The results show no differences if the shaft 1 is modelled as a driving shaft. If shaft 2 is modelled as the driving shaft, the application can not calculate the second eigen frequency of the model, because the second eigenfrequency of the model is the same as the first eigenfrequency of shaft 1.

Preglednica 3: *Lastne frekvence modela na sliki 4*

[1]*gonilna gred je gred 1 in gnana gred je gred 2.* [2]*gonilna gred je gred 2 in gnana gred je gred 1*

Table 3. *Eigenfrequencies of model from figure 4*

[1]*driving shaft is shaft 1 and driven shaft is shaft 2.* [2]*driving shaft is shaft 2 and driven shaft is shaft 1*

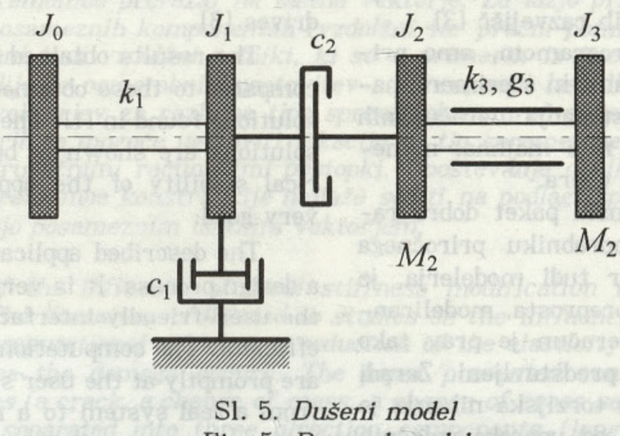
Lastne frekvence *Eigen frequencies* ω	s torzijo [1] *by torzija [1]* rad/s	s torzijo [2] *by torzija [2]* rad/s	analitično *analytically* rad/s
1.	266,44	266,44	266,44
2.	387,30	/	387,30
3	482,12	482,12	482,12
4.	639,59	639,59	639,59

4.3 Preračun dušenega sistema

Model na sliki 5 vsebuje vse vrste dušenj, ki jih program omogoča. Kolut 1 lahko ponazarja ladijski vijak z viskoznim dušenjem. Elastično polje 2 nima modelirane torzijske togosti, ampak le viskozno dušenje in je preprost model elektromagnetne sklopke. V elastičnem elementu 3 je modelirano strukturno dušenje materiala. Koluta 2 in 3 sta vzbujana s harmonskim torzijskim momentom.

4.3 The computation of a damped system

The model shown in figure 5 contains practically all the features that a simple torsion system may possibly possess, and all damping models that the application enables. Disk 1 may represent a propeller. The elastic field 2 has no torsional stiffness, and may represent an electromagnetic coupling. The elastic field 3 has rather strong structural damping. Disks 2 and 3 are exited by torques.



Sl. 5. *Dušeni model*
Fig. 5. *Damped model*

V preglednici 4 so prikazane amplitudne vrednosti realnega in imaginarnega dela zasuka odziva modela na harmonsko vzbujanje v ustaljenem stanju. Rezultati, dobljeni s programom, se ujemajo z rezultati po viru [1].

Table 4 shows the amplitudes of real and imaginary parts of the angle of twist of stationary response to the harmonic excitation. The results show an excellent agreement with those in [1].

Preglednica 4: *Odgovor modela na sliki 5 na harmonsko motnjo*
Table 4: *Response of the model from figure 5 on a harmonic external excitation*

	zasuk *angle of twist* φ	po viru [1] *by [1]* φ rad	s torzijo *by torzija* φ rad
Realni del *Real part*	0	0,2271	0,2271
	1	0,0908	0,0908
	2	-0,1034	-0,1034
	3	-0,0801	-0,0801
Imaginarni del *Imaginary part*	0	-0,3826	-0,3826
	1	-0,1530	-0,1530
	2	-0,4106	-0,4106
	3	-0,3191	-0,3191

5 SKLEP

V prispevku podajamo algoritem za preračun razvejanih linijskih torzijskih sistemov z metodo prenosnih matrik [1], [3]. Na algoritmu te metode smo zgradili programski paket za obravnavo linearnih torzijskih nihanj razvejanih mehanskih sistemov [3]. Programski paket TORZIJA odlikuje predvsem preprosto modeliranje samega sistema, še posebej pa preprosto modeliranje razvejišč in dušenja v sistemu. Poleg že omenjenega zobniškega razvejišča omogoča program tudi modeliranje jermenskih, verižnih in tornih razvejišč [3].

Rezultate, dobljene s programom, smo primerjali z analitičnimi rešitvami in s primeri, navedenimi v virih [1]. Odstopanja izračunanih vrednosti od referenčnih so zelo majhna, numerična stabilnost metode pa je dobra.

Sklepamo, da je programski paket dobro računsko pomagalo. Zaradi uporabniku priročnega vmesnika, tako omizja kakor tudi modelerja, je sama uporaba programa zelo preprosta, modeliranje je hitro in pregledno. Preračun je prav tako hiter in rezultati pregledno predstavljeni. Zaradi usmerjenosti programa le na torzijska nihanja je njegova uporaba preprostejša od uporabe splošnih komercialnih programov za preračun dinamike sistemov. Program ne rešuje problema prehoda od dejanskega sistema na matematični model. To je še vedno prepuščeno izkušnjam in občutku konstrukterja.

5 CONCLUSION

In this paper, the algorithm for branched chain topology torsional systems based on the transfer matrix method is given [1], [3]. It was on this basis that the computer program named TORZIJA was developed [3]. The advantages of the application are reflected in user-friendly interface and easy modelling of a system, particularly of the branching points and of damping. The application enables one to use not only the gear drive but also the friction, belt and chain drives [3].

The results obtained by the application were compared to those obtained analytically and to the solutions found in [1]. The errors of the numerical solutions are shown to be small, and the numerical stability of the application is found to be very good.

The described application is a helpful tool in a design process. It is very easy to be used due to the user-friendly interface. Modelling is fast and efficient. The computation is rapid, and the results are promptly at the user's disposal. The transition from a real system to a model is left to be done by the user depending on her or his experiences and feelings.

6 LITERATURA
6 REFERENCES

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