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Spremembe nihajnih oblik, povzročene z lokalno spremembo konstrukcije
Changes of Modal Shapes Induced by Local Modification of the Structure

MATJAŽ SKRINAR

Proučevanje vpliva spremembe lokalne togosti na lastne frekvence in na nihajne oblike je aktualna inženirska tema. Študije iz literature se omejujejo samo na vpliv poškodovanosti, ki jo v računski model vpeljejo z zmanjšanjem elastičnega modula v delu konstrukcije, kjer se razpoka pojavi. V prispevku je opisan vpliv različnih lokalnih sprememb (razpoke, spremembe mase ali spremembe prereza) na lastne vektorje. Za lažjo primerjavo so lastni vektorji razdeljeni po posameznih komponentah (vzdolžni ter prečni pomiki in zasuki). V študiji so namreč upoštevani tudi vzdolžni pomiki, ki so v primerih iz literature praviloma zanemarjeni, vendar vodijo do pomembnih ugotovitev o spremembi lastnih vektorjev. Primerjava sprememb lastnih vektorjev za različne tipe sprememb namreč pokaže, da je moč postaviti zakonitosti, po katerih je mogoče ugotoviti lokacijo in tip spremembe. Amplitudo spremembe je treba določiti z drugačnimi računskimi postopki. Upoštevanje osnih pomikov pokaže, da je o tipu in lokaciji spremembe konstrukcije najlaže soditi na podlagi sprememb specifičnih deformacij, ki pripadajo posameznim lastnim vektorjem.

Investigation of the influence of local stiffness modification is a currently relevant engineering topic. The literature is limited to studies on the influence of the damage that is introduced into a computational model as a reduction of the elasticity modulus in the part of the structure where the damage occurs. The paper presented considers the influence of various local changes (a crack, a change of mass, a change of cross section) on eigen-vectors. These vectors are separated into three direction components (longitudinal and transverse displacements and rotations). In the presented studies, longitudinal displacements are also considered since these components are neglected in the studies in the literature, although they offer significant conclusions about the change of the eigen-vectors. The comparison of the eigen-vector change for the various types of modifications clearly shows that some principles exist from which it would be possible to determine the location and type of structure change. The magnitude of the change must be further determined by other conventional methods. The introduction of the longitudinal displacements indicates that the type and location of the structure change can be estimated on the basis of the specific deformation changes which belong to the eigen-vectors.

0 UVOD

Vsaka sprememba konstrukcije spremeni njen dinamični (in tudi statični) odziv. Sprememba se kaže v lastnih frekvencah in njim pripadajočim nihajnim oblikam. Iskanje zveze med tipom, lokacijo ter velikostjo spremembe in spremembo odziva konstrukcije (zaradi poškodb) je pomemben inženirski problem. Yuen [11] podaja sistematično študijo, v kateri proučuje spremembe nihajnih oblik zaradi spremembe togosti. Študija, izvedena na primeru preproste konzole, je omejena samo na opazovanje spremembe prve nihajne oblike. V razširjeni študiji Pandey in dr. [6] opazujejo vpliv absolutne spremembe nihajnih oblik (tudi višjih) na primerih konzole in obojestransko vpetega nosilca. Vpeljan je parameter, ki naznačuje lokacijo razpoke. Baruh in Ratan [1] podajata drug kriterij za iskanje nepravilnosti v konstrukciji, saj v analizi kombinirata togostne in masne koeficiente prvotnega sistema z dinamičnimi karakteristikami spremenjenega sistema.

0 INTRODUCTION

Each modification to the structure alters its dynamic (and static) response. The modification is reflected in eigenfrequencies and corresponding eigenvectors. The study of the relation between type, location and magnitude on the one hand, and change of response (as a result of a structure damage) on the other, is a current engineering problem. Yuen [11] offers a systematic study in which he considers the eigenshape changes caused by stiffness variation. The study is performed on a simple cantilever and is limited to the monitoring of the first eigenfrequency change. In the expanded study, Pandey et al. [6] observe the influence of absolute change of the eigen-shapes (even higher) on two examples: a cantilever beam and a clamped-clamped beam. A new parameter that indicates the presence of the crack is introduced. Baruh and Ratan [1] present another criterion for the determination of irregularities in the structure since they combine the coefficients of the intact system with the dynamic characteristics of the modified system.

Proces analize v prispevku zajema sestavo dinamičnega modela fizikalnega sistema s končnimi elementi in numerični algoritem za določitev frekvenc in nihajnih oblik nedušenega linearnega nosilca oz. okvira. Sprememba konstrukcije je sistematično modelirana na vseh elementih in opazovan je vpliv različnih velikosti spremembe na isti lokaciji. V študiji so, v nasprotju s študijami iz literature, zajete vse prostostne stopnje (vzdolžni in prečni pomiki ter rotacijske prostostne stopnje). Sistematičnost študije vodi do ugotovitev, ki sklepe iz literature dopolnjujejo in razširjajo. Poglavitna pozornost je posvečena različnim tipom sprememb, ki se na različne načine kažejo v nihajnih oblikah. O nekaterih spremembah je mogoče soditi že neposredno na podlagi nihajnih oblik, za druge pa je treba poznati odvode komponent nihajnih oblik. Amplituda sprememb nihajnih oblik narašča z velikostjo spremembe in lahko tako rabi kot kazalec velikosti spremembe.

Za ugotavljanje morebitne spremembe konstrukcije med uporabo potrebujemo rezultate meritev, ki jih izvedemo na sami konstrukciji in jih nato primerjamo z rezultati prejšnjih meritev. Če želimo preveriti izvedbo, potem za primerjavo uporabimo podatke, ki jih dobimo z uporabo računskega modela. Pri tem je vedno potrebna previdnost, saj vzroki za razlikovanje izmerjenih in izračunanih vrednosti niso vedno v spremembi konstrukcije, temveč lahko izvirajo bodisi iz neprimerno izbranega računskega modela ali pa iz pomanjkljivo izvedenih meritev. Napakam zaradi neustreznega računskega modela se je običajno mogoče izogniti z zapletenejšimi računskimi modeli, kar omogočajo vedno hitrejši in zmogljivejši računalniki. Drugi vir napak lahko omilimo z nabavo dobre merilne opreme, katere zmogljivost je navadno sorazmerna njeni ceni.

The analysis process in the presented paper consists of determination of an appropriate dynamic model by finite elements of the physical system, determination of the eigenfrequencies and corresponding eigen-vectors (modal shapes) of undamped linear beam or frame-like structure. The modification of the structure is systematically modelled on all finite elements, and the influence of different changes at the same location is supervised. All three degrees of freedom (lateral and transverse displacements and rotational degrees of freedom) are considered in the study. This systematisation is reflected in some conclusions which offer a further advance on results already known from the literature. Main attention is devoted to several types of modifications which differently occur in the eigenvectors. Some changes can be determined directly from eigen-vectors (modal shapes) and the others can be determined on the base of derivatives of the eigenvector components. The amplitude of the modal shape changes increases with the increase of the structure modification, and this can therefore serve as an indicator of the magnitude of the change.

For the identification of a possible change in the structure during its utilisation, the results of the measurements on the structure are needed. These results are compared to the results of previous measurements. If the aim of the measurements is to control the integrity of the structure, then measurements on the structure are compared with the results obtained from the mathematical model. The difference in the results may be caused by a poor choice of the mathematical model selected, or by a measurement error. The errors from a badly conditioned mathematical model can be usually avoided with more complex computational models. The measurements error can be reduced by means of high-quality measurement equipment, which price increases as the quality improves.

1 POSTOPEK ANALIZE

Za konstrukcijo izberemo računski model in z metodo končnih elementov sestavimo togostno matriko $\mathbf{K}$ in masno matriko $\mathbf{M}$, ki sta reda N. Reševanje problema lastnega nihanja za osnovni sistem se lahko prevede v iskanje rešitev problema lastnih vrednosti $(\mathbf{K} - \lambda_r \mathbf{M})\,\mathbf{u}_r = 0$. Rešitve te enačbe so vrednosti λ_r, ki pomenijo lastne vrednosti (povezane z lastnimi frekvencami sistema) in njim pripadajoči lastni vektorji $\mathbf{u}_r$ $(r = 1,2,..N)$. Za spremenjeni sistem lahko podobno zapišemo enačbo: $(\mathbf{K}' - \lambda'_r \mathbf{M}')\,\mathbf{v}_r = 0$, kjer sta $\mathbf{K}'$ in $\mathbf{M}'$ togostna oziroma masna matrika spremenjenega sistema, λ'_r r-ta lastna vrednost in $\mathbf{v}_r$ njej pripadajoči lastni vektor $(r = 1,2,..N)$.

1 THE ANALYSIS PROCESS

For the structure, a mathematical model is chosen and global stiffness matrix $\mathbf{K}$ and global mass matrix $\mathbf{M}$ (both of rank N) are computed. The problem of eigenmotion is transformed into the solution of the eigenvalue problem $(\mathbf{K} - \lambda_r \mathbf{M})\,\mathbf{u}_r = 0$. The solutions of this equation are λ_r, that represent the eigenvalues (directly related to eigenfrequencies of the system) and corresponding eigenvectors $\mathbf{u}_r$ $(r = 1,2,..N)$. For the modified system a similar equation can be written: $(\mathbf{K}' - \lambda'_r \mathbf{M}')\,\mathbf{v} = 0$ where $\mathbf{K}'$ and $\mathbf{M}'$ are stiffness and mass matrices of the modified system, respectively, λ'_r r-th represents the eigenvalues and $\mathbf{v}_r$ is the corresponding eigenvector $(r = 1,2,..N)$.

Opozoriti velja, da sta pri iskanju spremembe sistema matriki $\mathbf{K}'$ in $\mathbf{M}'$ neznani (ni nujno, da hkrati velja $\mathbf{K}' \neq \mathbf{K}$ in $\mathbf{M}' \neq \mathbf{M}$). Z meritvami lahko pridobimo podatke o λ'_r in/oz. $\mathbf{v}_r$ $(r = 1,2,..,Q$, običajno $Q << N)$. Iz znanih podatkov $\mathbf{K}$, $\mathbf{M}$, λ'_r in $\mathbf{v}_r$ (in morda še λ_r in $\mathbf{u}_r$) je treba zaznati lokacijo spremembe in morda še tip spremembe.

In the identification process, matrices $\mathbf{K}'$ and $\mathbf{M}'$ are unknown (it is not necessarily valid that $\mathbf{K}' \neq \mathbf{K}$ and $\mathbf{M}' \neq \mathbf{M}$). From the measured data, values for λ'_r and/or $\mathbf{v}_r$ $(r = 1,2,..,Q$, usually $Q < N)$ are obtained. From the known $\mathbf{K}$, $\mathbf{M}$, λ'_r and $\mathbf{v}_r$ (and even λ_r and $\mathbf{u}_r$) the location and type of the modification should be determined.

1.1 Postopek analize s kombiniranjem togostne in masne matrike prvotnega sistema z dinamičnimi karakteristikami spremenjenega sistema

1.1 The process of analysis with the combination of the stiffness and mass matrix of the original system having the dynamic characteristics of the modified system

Zapišimo izraz:

Expression:

$$(\mathbf{K} - \lambda'_r \mathbf{M})\, \mathbf{v}_r = \mathbf{R}_r \neq \mathbf{0} \qquad (1),$$

ki pomeni kombinacijo vrednosti iz osnovnega in spremenjenega sistema. Vpeljimo $\mathbf{M}' = \mathbf{M} - \delta\mathbf{M}_j$ in $\mathbf{K}' = \mathbf{K} - \delta\mathbf{K}_j$, kjer indeks j označuje element, pri katerem se pojavi sprememba. Izraz tako prevedemo v:

represents the combination of values of the original and modified systems. Let us introduce $\mathbf{M}' = \mathbf{M} - \delta\mathbf{M}_j$ and $\mathbf{K}' = \mathbf{K} - \delta\mathbf{K}_j$, where subscript j denotes the element where the deformation occurs. The expression is transferred into:

$$(\delta\mathbf{K}_j - \lambda'_r\, \delta\mathbf{M}_j)\, \mathbf{v}_r = \mathbf{R}_r \qquad (2).$$

Zanima nas, ali (in če, kako) se v vektorju $\mathbf{R}_r$ kažejo posamezne spremembe. Iz enačbe (2) vidimo, da je vektor $\mathbf{R}_r$ odvisen od $\delta\mathbf{K}_j$ in $\delta\mathbf{M}_j$, ki ju ne poznamo, zato v analizi moramo uporabiti enačbo (1).

The question arising is: if (and if, how) different modifications are reflected in vector $\mathbf{R}_r$. Since it is clear from the equation (2) that vector $\mathbf{R}_r$ depends on unknown $\delta\mathbf{K}_j$ and $\delta\mathbf{M}_j$, in the analysis equation (1) must be used.

1.2 Primerjava lastnih vektorjev prvotne in spremenjene konstrukcije

1.2 The comparison of eigenvectors of the original and modified structure

Lastni vektorji so najprej ortonormirani glede na masno matriko, tako da velja $\mathbf{u}_i^T \mathbf{M} \mathbf{u}_i = 1$, nato pa še na pripadajočo krožno frekvenco. Vsak vektor je razdeljen v vektorje po komponentah vzdolžnih pomikov $\mathbf{X}_i$, prečnih pomikov $\mathbf{Y}_i$ in zasukov $\boldsymbol{\varphi}_i$. Takšna ločitev prostostnih stopenj omogoča bolj sistematično opazovanje vpliva spremembe. Nihajne oblike spremenjene konstrukcije so primerjane z nihajnimi oblikami prvotne konstrukcije. Za vsako spremembo konstrukcije se tako izračunajo naslednji vektorji:

Eigenvectors are first normalised over the mass matrix: $\mathbf{u}_i^T \mathbf{M}\, \mathbf{u}_i = 1$, and further to the corresponding eigen circular frequency. Each vector is further divided into three vectors: components of the longitudinal displacements $\mathbf{X}_i$, transverse displacements $\mathbf{Y}_i$ and rotations $\boldsymbol{\varphi}_i$. Such separation allows for a more systematic approach to monitoring of the influence structure modification, the eigenshapes of the modified structure are compared with the eigenshapes of the original structure. For each structural change the following vectors are computed:

$$\mathbf{X}_i^* = \frac{\mathbf{X}_i^u}{\omega_i^u} - \frac{\mathbf{X}_i^v}{\omega_i^v} \qquad \mathbf{Y}_i^* = \frac{\mathbf{Y}_i^u}{\omega_i^u} - \frac{\mathbf{Y}_i^v}{\omega_i^v} \qquad \boldsymbol{\varphi}_i^* = \frac{\boldsymbol{\varphi}_i^u}{\omega_i^u} - \frac{\boldsymbol{\varphi}_i^v}{\omega_i^v} \qquad (3),$$

kjer označujejo: indeks i lastno frekvenco, indeksa u in v pa prvotno oz. spremenjeno konstrukcijo.

where index i stands for the eigenfrequency, while indices u and v indicate the original and modified structure, respectively.

1.3 Kriterija KMU in KMUK

Za primerjavo lastnih vektorjev sta predlagana dva kriterija. Prvi se imenuje kriterij modalne usklajenosti KMU (MAC). Označuje povezavo med dvema vektorjema. Vrednost koeficienta KMU med i-tim vektorjem $\mathbf{v}_i$ in j-tim vektorjem $\mathbf{u}_j$ se izračuna kot:

$$\mathrm{MAC}(\mathbf{u}_j, \mathbf{v}_j) = \frac{\left|\mathbf{u}_j^T \ \mathbf{v}_j\right|^2}{\left(\left(\mathbf{u}_j^T \ \mathbf{u}_j\right)\left(\mathbf{v}_j^T \ \mathbf{v}_j\right)\right)} \quad (4).$$

Vrednost koeficienta blizu 1 označuje, da sta oba vektorja dobro usklajena, vrednost blizu 0 pa označuje dva popolnoma neusklajena vektorja.

Drugi kriterij je kriterij modalne usklajenosti koordinate KMUK (COMAC). Označuje usklajenost posameznih nihajnih oblik za izbrano koordinato. Če z L označimo število lastnih vektorjev, ki jih želimo uskladiti, in z x koordinato, potem se koeficient KMUK (COMAC)izračuna kot:

$$\mathrm{COMAC}(x) = \frac{\left(\sum_{l=1} \mathbf{u}_l(x)\mathbf{v}_l(x)\right)^2}{\sum_{l=1}^{L} \mathbf{u}_l(x)^2 \ \sum_{l=1}^{L} \mathbf{v}_l(x)^2} \quad (5).$$

Vrednost koeficienta KMUK (COMAC) blizu 1 označuje dobro usklajenost na izbranem mestu. Usklajenost se vedno izvede preko več lastnih vektorjev $(L>1)$, saj je pri $L=1$ vrednost KMUK $(\mathrm{COMAC}(x)) \equiv 1$.

2 NUMERIČNE ŠTUDIJE ZA RAZLIČNE SPREMEMBE KONSTRUKCIJE

2.1 Prvi primer

Za demonstracijski primer izberimo preprosto konzolo, vpeto na levem koncu (sl. 1). Dolžina konzole in elastični modul sta privzeta po literaturi [11], izvirno okrogli prerez je nadomeščen s pravokotnim zaradi vpeljave končnega elementa z razpoko. Konzolo diskretiziramo z 21 vozlišči in 20 elementi enakih dolžin (v literaturi s petnajstimi), oštevilčenje vozlišč in elementov poteka od levega roba proti desnemu. Uporabimo standardno formulacijo s končnimi elementi, kjer so kot interpolacijske funkcije uporabljeni hermitovi polinomi, kar omogoča dovolj natančen izračun nižjih frekvenc, ki so zanimive z inženirskega vidika. Za izračun višjih frekvenc in pripadajočih nihajnih oblik je treba uporabiti drugačno formulacijo (p-formulacija ali mešana formulacija [8]). V posameznem vozlišču upoštevamo tri prostostne stopnje. V primerih iz literature so prostostne stopnje, ki pripadajo

1.3 The MAC and COMAC criteria

For the comparison of modal shapes two criteria are suggested. First, the MAC - Modal Assurance Criterion), represents the correlation between the two vectors. The value of the MAC coefficient between vectors $\mathbf{v}_i$ and $\mathbf{u}_j$is computed as:

The value close to 1 indicates good correlation of two vectors, while a value close to 0 indicates completely uncorrelated vectors.

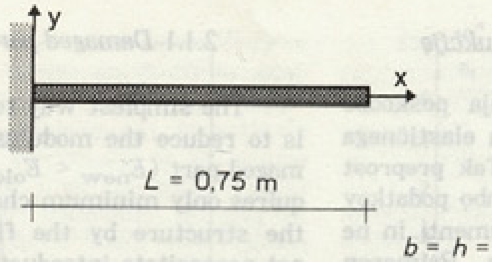
The second criterion is COMAC (Coordinate Modal Assurance Criterion). It indicates the correlation of the different shapes for a chosen coordiante. If L denotes the number of eigenmodal shapes to be correlated, and x denotes the coordinate under observation, COMAC is computed as follows:

A value of the COMAC coefficient close to 1 indicates good correlation at a chosen point. The correlation is always computed over several modal shapes $(L>1)$, since for $L=1$ the $\mathrm{COMAC}(x) \equiv 1$.

2 NUMERICAL STUDIES FOR DIFFERENT STRUCTURE MODIFICATIONS

2.1 First example

As a first demonstration example a simple cantilever fixed at the left end (Fig. 1) is chosen. The length and the modulus of elasticity were taken from the reference [11] and the original circular cross section was replaced by a rectangular one for the implementation of the crack finite element model. The cantilever is discretisised with 21 nodes and 20 elements (in the ref. with only 15), the numeration is performed from the left - the clamped node. For the computation of eigenpairs the standard formulation with finite elements, with Hermitean polynomials as interpolation functions were used. This discretisation allows the computation of lower eigenpairs with satisfactory accuracy. Lower frequencies are more interesting from the engineering point of view. For the computation of higher eigenfrequencies and corresponding eigenvectors a different formulation should be used (p-formulation or mixed formulation [8].) Three degrees of freedom are used at each node. This is the first step forward in comparison to studies from the literature where the degrees of



Sl. 1. *Konzolni nosilec*
Fig. 1. *Cantilever beam*

vzdolžnim pomikom, zanemarjene. Togostna in masna matrika sta sestavljeni z lastnim računalniškim programom, lastne frekvence in pripadajoče lastne vektorje pa izračunamo s programskim paketom MATLAB.

Uspešnost izbranega modela in računskega postopka preverimo s primerjavo z znanimi rešitvami za konzolo [13].

freedom belonging to the longitudinal motion were neglected. The stiffness and mass matrices are assembled with own software, eigenfrequencies and eigenvectors are further computed with the program package MATLAB. The efficiency of both chosen model and computational procedures was verified by comparing obtained eigenfrequencies with known theoretical solutions for cantilever [13].

Preglednica 1: *Primerjava lastnih frekvenc osnovnega sistema*
Table 1: *Comparison of eigenfrequencies of the original system*

n	ω analitično analytical s^{-1}	ω MKE FEM s^{-1}
1	10096.83948	10097.78
2	63275.81291	63276.37
3	177174.1084	177183.25
4	347190.5396	347258.275
5	573930.8394	574232.58
6 osna longitudinal	609832.693	610111.379
7	857353.294	858342.81

Preglednica 1 podaja najnižjih sedem lastnih frekvenc za prečno in vzdolžno nihanje. Šest (prvih pet in sedma) jih pripada prečnemu nihanju, šesta pa vzdolžnemu nihanju. Vidno je, da je natančnost uporabljenega modela dovolj velika, da omogoči uporabo (vsaj) prvih sedem lastnih frekvenc in nihajnih oblik za analizo. Primerjava rezultatov potrjuje pravilnost izbrane diskretizacije in uspešnost numeričnega postopka.

Obravnavajmo naslednje primere sprememb:
— poškodovanost oziroma razpokanost dela konstrukcije,
— dodatno maso na konstrukciji — povečanje mase konstrukcije brez vpliva na njeno togost,
— povečanje dela prereza — hkratno povečanje mase in togosti konstrukcije.

Obravnavamo torej variacije togostne matrike, masne matrike in njuno kombinacijo. Vsako izmed naštetih sprememb simuliramo s spremembo lastnosti enega elementa in izračunamo pripadajoče vektorje $\mathbf{R}_i$, $\mathbf{X}_i^*$, $\mathbf{Y}_i^*$ in $\boldsymbol{\varphi}_i^*$ $(i=1,2,..,7)$.

Table 1 compares the first seven eigenfrequencies for longitudinal and transverse motions. Six of them (first five and seventh) belong to transverse motions, and the sixth belongs to longitudinal motion. The accuracy of the chosen model is evidently high enough to allow (at least) for the first seven eigenpairs in the analysis. The comparison of the results confirms both the choice of the mathematical model and the numerical procedures.

Let us consider the following modifications:
— damaged or cracked part of the structure,
— added mass on the structure — the increase of the mass on the structure without affects on the stiffness of the structure,
— enlargement of the cross-section of the structure — with simultaneous increase of the mass and stiffness of the structure.

The variations of stiffness matrix, mass matrix and simultaneous variation of both matrices are considered. Each modification is simulated with a change of properties of a single element, and the corresponding vectors $\mathbf{R}_i$, $\mathbf{X}_i^*$, $\mathbf{Y}_i^*$, $\boldsymbol{\varphi}_i^*$ $(i=1,2, ..., 7)$ are computed.

2.1.1 *Poškodovanost dela konstrukcije*

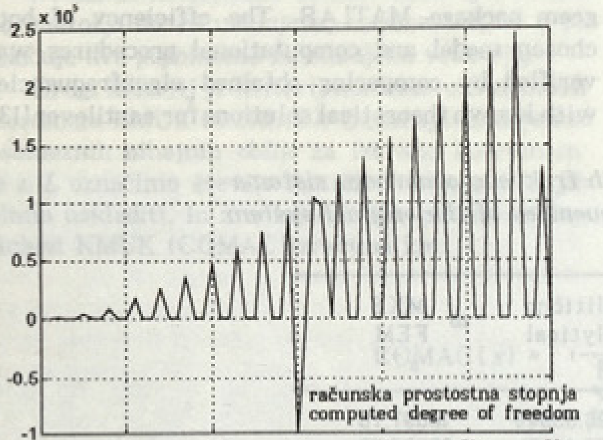
Najpreprostejši način modeliranja poškodbe konstrukcije izvedemo z zmanjšanjem elastičnega modula elementa ($E_{novi} < E_{stari}$). Tak preprost način terja samo minimalno spremembo podatkov pri opisu konstrukcije s končnimi elementi in ne zahteva vpeljave posebnega elementa. Primeren je predvsem za opis plastifikacije dela konstrukcije. Takšno modeliranje poškodbe konstrukcije je razširjeno v literaturi.

Slika 2 prikazuje vektor $\mathbf{R}_1$, ki pripada prvi nihajni obliki. Poškodba je modelirana na enajstem elementu.

2.1.1 *Damaged part of the structure*

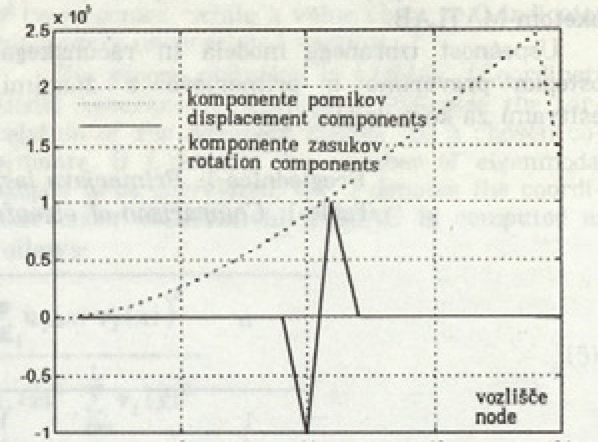
The simplest way to model the damaged part is to reduce the modulus of elasticity of the damaged part ($E_{new} < E_{old}$). Such an approach requires only minimum change in the description of the structure by the finite elements, and does not necessitate introduction of a special element. It is efficient especially for the description of the plastification of the part of the structure. This approach is widely used in the literature.

Figure 2 shows vector $\mathbf{R}_1$, which belongs to the first modal shape (transverse displacements). The damage is modeled at the eleventh element.



Sl. 2. *Vektor* $\mathbf{R}_1$

Fig. 2. *Vector* $\mathbf{R}_1$



Sl. 3. *Vektor* $\mathbf{R}_1$*, razdeljen po komponentah - motnja je očitna*

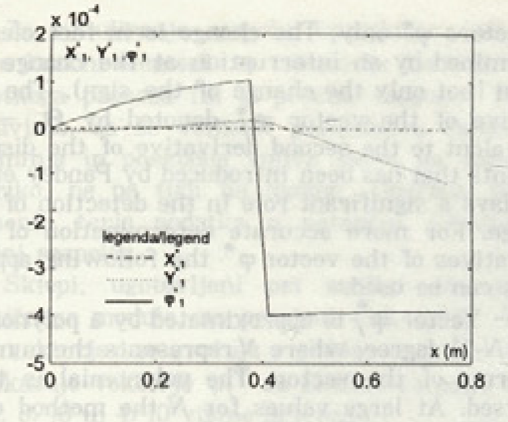
Fig. 3. *Vector* $\mathbf{R}_1$*, divided into components - the presence of the damage is obvious*

S slike 2 je razvidno, da se motnja pojavi v 30. (zasuk vozlišča 11) in 33. (zasuk vozlišča 12) računski prostostni stopnji. Še jasnejša je slika 3, kjer je vektor $\mathbf{R}_1$ razdeljen po posameznih komponentah. Ker prva nihajna oblika pripada prečnemu nihanju, so komponente vzdolžnih pomikov enake nič. S slike 3 je dalje razvidno, da komponente pomikov monotono naraščajo (izjema je zadnji element). Komponente zasukov dosežejo ekstremni vrednosti v vozliščih 11 in 12, torej v vozliščih, ki omejujeta element 11, kjer se sprememba togosti dejansko pojavi. Vektorji, ki pripadajo višjim nihajnim oblikam, izkazujejo podobno obnašanje. Vrednosti pomikov so numerično večje, vendar ne omogočajo identifikacije motnje.

Slika 4 prikazuje vektorje $\mathbf{X}_1^*$ $\mathbf{Y}_1^*$ in $\boldsymbol{\varphi}_1^*$. Vektor $\boldsymbol{\varphi}_1^*$ jasno kaže vpliv spremembe konstrukcije: na plastificiranem elementu naredi vektor $\boldsymbol{\varphi}_1^*$ preskok. Yuen [11] ne analizira višjih nihajnih oblik, vendar trdi, da poškodbe dela konstrukcije v višjih nihajnih oblikah ni moč zaznati. Vendar je sprememba konstrukcije opazna tudi v višjih

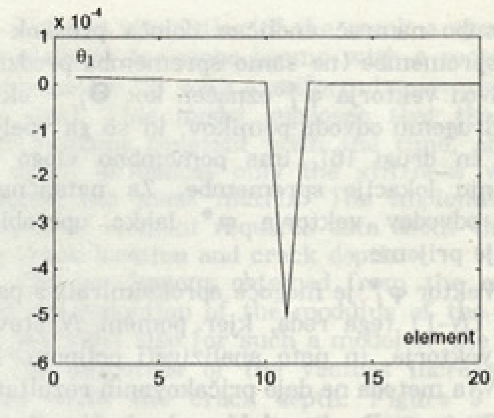
From figure 2 it is evident that the damage is introduced in the 30th (rotation of node 11) and 33rd (rotation of node 12) computational degree of freedom. Figure 3 confirms this fact most clearly - vector $\mathbf{R}_1$ is divided into components. Since the first modal shape belongs to the transverse motion, all components of longitudinal motions are equal to zero. From figure 3 it is further evident that the components of the displacements are monotonously increasing (the exception is the last element). The components of the rotations achieve their maximums at nodes 11 and 12. These are the nodes that bound the element 11, where the change of stiffness is actually modelled. The vectors that belong to higher modal shapes exhibit similar behaviour. The values of the displacements are numerically higher but they do not allow for identification of the change.

Figure 4 shows vectors $\mathbf{X}_1^*$ $\mathbf{Y}_1^*$ and $\boldsymbol{\varphi}_1^*$. Vector $\boldsymbol{\varphi}_1^*$ clearly indicates the influence of the modification on the structure: on the plastificated element there is a jump of vector $\boldsymbol{\varphi}_1^*$. Yuen [11] does not analyse higher flexural modes since he states that the damage cannot be determined in higher modes. However, the modification of the



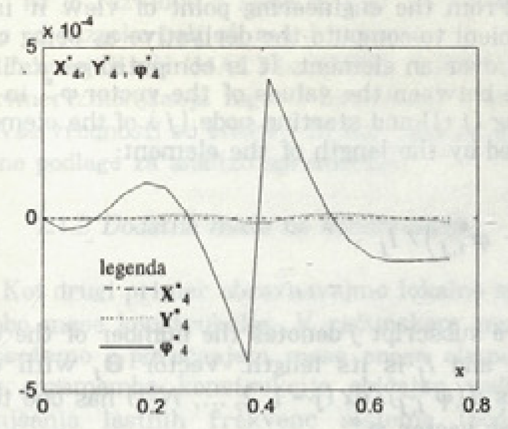
Sl. 4. *Vektorji* $\mathbf{X}^*_1$, $\mathbf{Y}^*_1$ *in* φ^*_1 *za plastifikacijo elementa 11*

Fig. 4. *Vectors* $\mathbf{X}^*_{11}$, $\mathbf{Y}^*_1$ *and* φ^*_1 *for the plastification of the 11th element*



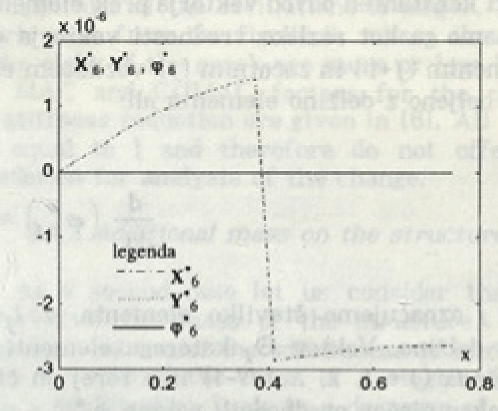
Sl. 5. *Odvodi vektorja* φ^*_1 *po elementih*

Fig. 5. *The derivatives of vector* φ^*_1 *over the elements*



Sl. 6. *Vektorji* $\mathbf{X}^*_4$, $\mathbf{Y}^*_4$ *in* φ^*_4

Fig. 6. *Vektors* $\mathbf{X}^*_4$, $\mathbf{Y}^*_4$ *and* φ^*_4



Sl. 7. *Vektorji* $\mathbf{X}^*_6$, $\mathbf{Y}^*_6$ *in* φ^*_6

Fig.7. *Vectors* $\mathbf{X}^*_6$, $\mathbf{Y}^*_6$ *and* φ^*_6

nihajnih oblikah (sl. 6, 7). Slika 7, ki prikazuje razliko nihajnih oblik za vzdolžno nihanje, potrjuje, da se sprememba konstrukcije kaže tudi v vzdolžnem nihanju (vektor $\mathbf{X}^*_6$).

Jasno je vidno, da tudi višje modalne oblike kažejo lokacijo poškodovanega dela. V analizi, ki jo podajajo Pandey in drugi [6], so v računskem modelu upoštevani samo prečni pomiki brez zasukov in vzdolžnih pomikov, lastni vektorji pred primerjavo nihajnih oblik niso normalizirani na lastno frekvenco, opazovana pa je absolutna vrednost vektorjev $\mathbf{Y}^*$. Za lociranje poškodb je uporabljena aproksimacija drugih odvodov prečnih pomikov. Največja absolutna vrednost spremembe nihajne oblike naznačuje lokacijo spremembe na konstrukciji. Absolutne vrednosti razlik nihajnih oblik in aproksimacij odvodov zasukov sicer poudarijo lokacijo spremembe, vendar za račun z absolutnimi vrednostmi ni fizikalne osnove.

S slik 4, 6 in 7 je razvidno, da je spremembo togosti mogoče locirati že s slike vektorjev φ^*.

structure is evident even in higher modal shapes (Figs. 6, 7). Figure 7, showing the difference of the modal shapes for the longitudinal motions, confirms that the modification on the structure is evident even in longitudinal motion (vector $\mathbf{X}^*_6$).

It is clearly evident that even higher modal shapes expose the location of the damaged part. In the analysis given by Pandey et al. [6] only transverse displacements (neglecting longitudinal displacements and rotations) are considered. Eigenvectors are not normalised by the eigenfrequency prior to the comparison of modal shapes, and only absolute values of vectors $\mathbf{Y}^*$ are considered. To locate the damage the approximation of second derivatives of transverse displacements is used. The maximum absolute difference of the modal shapes' change indicates the location of the damage on the structure. The absolute values of the modal shapes' differences emphasise the location of the change, but there is no physical interpretation for absolute values.

From Figures 4, 6 and 7 it is evident that the stiffness change can be determined from the figure

Spremembo namreč enolično določa preskok na mestu spremembe (ne samo sprememba predznaka). Odvod vektorja φ_i^* označen kot Θ_i – ekvivalent drugemu odvodu pomikov, ki so ga vpeljali Pandey in drugi [6], ima pomembno vlogo pri odkrivanju lokacije spremembe. Za natančnejšo analizo odvodov vektorja φ^* lahko uporabimo naslednje prijeme.

– Vektor φ_i^* je mogoče aproksimirati s polinomom (N-1) tega reda, kjer pomeni N število členov vektorja, in nato analizirati polinom. Pri večjih N ta metoda ne daje pričakovanih rezultatov.

– Po vzoru Pandeya lahko odvod vektorja φ^* izračunamo iz vektorja $\mathbf{Y}_j^*$, vendar se je treba pri tem zavedati, da ne moremo izračunati vrednosti odvoda v najmanj štirih točkah (zaradi postopka numeričnega odvajanja).

Izkaže se, da je za analizo dovolj približno izračunati konstanten odvod vektorja prek elementa. Izračunamo ga kot razliko vrednosti vektorja φ_i^* med končnim (j+1) in začetnim (j) vozliščem elementa, deljeno z dolžino elementa ali:

of vectors φ^* only. The change is in fact clearly determined by an interruption at the change location (not only the change of the sign). The derivative of the vector φ_i^* denoted by Θ_i – an equivalent to the second derivative of the displacements that has been introduced by Pandey et al. [6] plays a significant role in the detection of the change. For more accurate determination of the derivatives of the vector φ^* the following approaches can be used.

– Vector φ_i^* is approximated by a polynomial of a (N-1) degree, where N represents the number of terms of the vector. The polynomial is then analysed. At large values for N the method does not exhibit the expected results.

– Following the idea by Pandey the derivative of the vector φ^* can be computed from the vector $\mathbf{Y}_j^*$ (bearing in mind that the value of the derivative cannot be computed in four points due to the process of the numerical derivation).

From the engineering point of view it is sufficient to compute the derivative as being constant over an element. It is computed as a difference between the values of the vector φ_i^* in the ending (j+1) and starting node (j) of the element, divided by the length of the element:

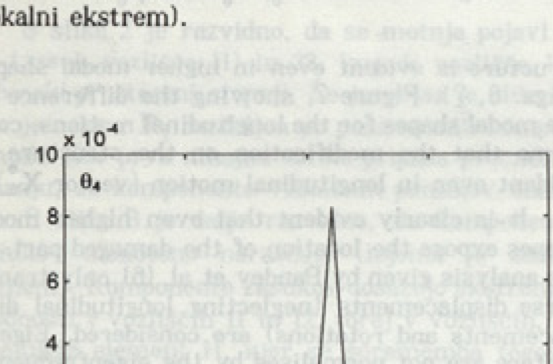
$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\varphi_{i,j}^*\right) \approx \left(\varphi_{i,j+1}^* - \varphi_{i,j}^*\right)/\,l_j \tag{6}$$

kjer z j označujemo številko elementa, z l_j pa njegovo dolžino. Vektor Θ_i katerega elementi so $d(\varphi_{i,j}^*)/dx$ (j = 1, 2, ..., N-1) ima torej en člen manj kakor njegov predhodnik vektor φ_i^*.

Sliki 5 in 8 jasno izkazujeta element 11 pri katerem se vrednost odvoda sunkovito spremeni (lokalni ekstrem).

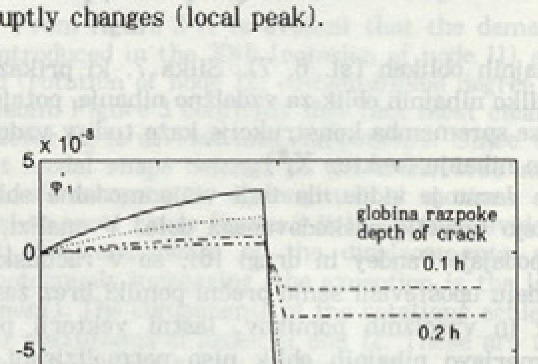
where subscript j denotes the number of the element and l_j is its length. Vector Θ_i with elements $d(\varphi_{i,j}^*)/dx$ (j = 1, 2, ..., N-1) has one term less as vector φ_i^*.

Figures 5 and 8 clearly indicate element 11 as the element where the value of the derivative abruptly changes (local peak).



Sl. 8. *Odvod vektorja* φ_4^*

Fig. 8. *Derivative of the vector* φ_4^*



Sl. 9. *Primerjava vektorjev* φ_1^* *za različne globine razpok*

Fig. 9. *Comparison of vectors* φ_1^* *for several crack depths*

Za simuliranje razpoke uporabimo tudi poseben končni element za ravninske konzole pravokotnega prereza [9] s prečno razpoko. Predpostavimo, da se velikost poškodbe s časom ne spreminja in poškodba vpliva samo na togostno matriko, ne pa tudi na masno. Uporaba takega elementa terja podatka o natančni lokaciji in globini razpoke.

Sklepi, ugotovljeni pri analizi zmanjšanja elastičnega modula, so potrjeni tudi tukaj. Velikost vektorjev se zvečuje z globino razpoke. Slika 9 prikazuje vektorje φ_1^* za globine razpoke 1/10, 2/10, 3/10 in 4/10 višine prereza.

Globoke razpoke so zelo prilagodljive in se obnašajo bolj ali manj kot členki. V primeru obravnavane konzole se to kaže tako, da del desno od razpoke oscilira v približno enaki obliki kakor pri nepoškodovani konstrukciji, le z večjo amplitudo. To je razvidno s slike 9, kjer so vrednosti desno od razpoke bolj ali manj linearne.

Faktorji KMU (MAC) in KMUK (COMAC) so za primer zmanjšanja togosti izračunani v študiji [6]. Vse vrednosti so enake 1 in kot take ne dajejo nobene podlage za analizo spremembe.

2.1.2 *Dodatna masa na konstrukciji*

Kot drugi primer obravnavajmo lokalno spremembo mase konstrukcije. V računskem modelu jo dosežemo s povečanjem mase enega elementa. Taka sprememba konstrukcije običajno vodi do zmanjšanja lastnih frekvenc sistema (podobno kakor vpeljava poškodbe), zato lastne frekvence same zase ne morejo biti enolični pokazatelj spremembe konstrukcije. V računskem zgledu je masa šestega elementa povečana skoraj za 100 odstotkov.

For the simulation of the crack a special finite element for plane beams with a rectangular cross-section [9] with a uniform transverse crack was used. This model supposes that the crack depth remains constant with the time, and that the crack influences only the stiffness without affecting the mass matrix. The implementation of such an element requires data about the precise crack location and crack depth.

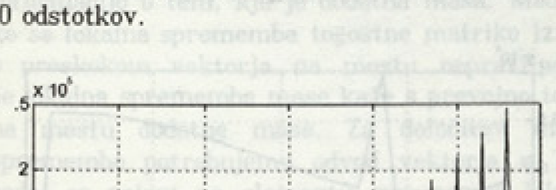
The conclusions obtained from the analysis with the reduction of the modulus of the elasticity are valid also for such a model of the damage. The amplitude of the vectors increases together with the crack depth. Figure 9 shows vectors φ_1^* for crack depths 1/10, 2/10, 3/10 and 4/10 of the beam height.

Deep cracks are very flexible and they behave more or less as pins. In the cantilever under consideration this is reflected in the fact that the part to the right of the crack oscillates in approximately equal shape as in the undamaged case. Only the amplitude of the oscillations is larger. This is clearly seen from the figure 9, where the values to the right of the crack are more or less linear.

MAC and COMAC factors for the case of the stiffness reduction are given in [6]. All values are equal to 1 and therefore do not offer any foundation for analysis of the change.

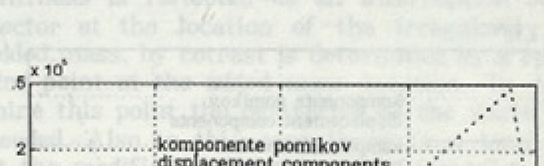
2.1.2 *Additional mass on the structure*

As a second case let us consider the local variation of the mass of the structure. In the computational model the added mass is introduced as an increase of the mass of a single element. Such a modification leads to a decrease of eigenfrequencies (similar as damage). Therefore eigenfrequencies alone cannot be an unique indicator of the structure change. In the numerical example the mass of element 6 is enlarged by almost 100 %.



Sl. 10. *Vektor* $\mathbf{R}_1$ *za primer dodatne mase na elementu 6*

Fig. 10. *Vector* $\mathbf{R}_1$ *for the case of the added mass on the element 6*

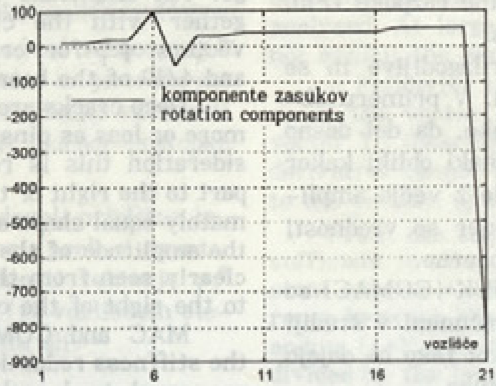


Sl. 11. *Vektor* $\mathbf{R}_1$ *za primer dodatne mase, razdeljen po komponetah*

Fig. 11. *Vector* $\mathbf{R}_1$ *divided into components for the case of the added mass*

S slike 10 je ponovno praktično nemogoče ugotoviti motnjo, njeno lokacijo ali vzrok. Slika 11 daje nekaj več informacij: lokacija motnje je zelo grobo nakazana. Opozoriti je treba, da so komponente prečnih zasukov tako majhne, da so v primerjavi s komponentami pomikov predstavljene kot premica z vrednostjo 0 (vse komponente so prikazane v istem merilu). Šele ločena slika zasukov (sl. 12) daje pravo sliko o lokaciji spremembe.

From figure 10 it is practically impossible to identify the change, neither the location nor its cause. Figure 11 offers more information: the location of the change is very roughly indicated. Note that the components of the transverse displacements are so small that the vector of transverse displacements appears as a straight line with a value 0 (all components are plotted on the same scale). Only a separated picture of the displacements (Fig. 12) exhibits the real situation about the change locations.

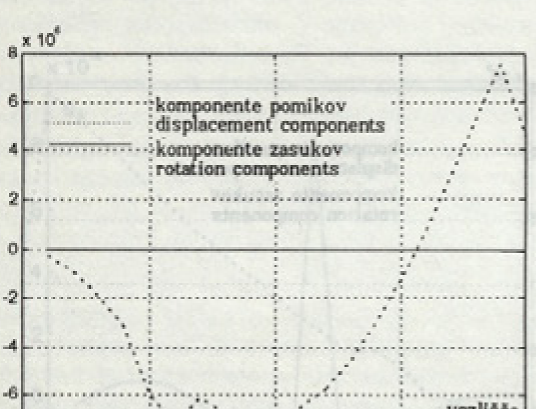


Sl. 12. *Komponente zasukov vektorja $\mathbf{R}_1$ za primer dodatne mase*

Fig. 12. *The components of the rotations of the vector $\mathbf{R}_1$ for an added mass*

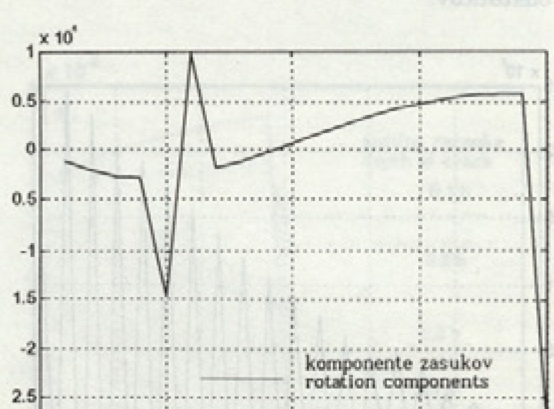
Lokacija spremembe je sedaj očitna. Če zanemarimo skok na prostem koncu, pozornost takoj vzbudi obnašanje funkcije med vozliščema 6 in 7, torej na elementu 6, ki ima tudi povečano maso. Podobno obnašanje kažeta tudi sliki 13 in 14, ki prikazujeta vektor $\mathbf{R}_2$. Ker na sliki 13 komponente pomikov popolnoma prekrijejo sliko komponent zasukov, so ti ločeno prikazani na sliki 14.

The change location is now obvious. Neglecting the jump at the free end, full attention is immediately directed towards the behaviour of the function between the nodes 6 and 7 that bound element 6, where the mass was actually increased. Similar behaviour is also exhibited in figures 13 and 14, which present vector $\mathbf{R}_2$. Since in figure 13 the components of the displacements totally overlay the components of rotations, they are presented separately in figure 14.



Sl. 13. *Vektor $\mathbf{R}_2$ za primer dodatne mase, razdeljen po komponetah; pomiki spet prevladujejo*

Fig. 13. *Vector $\mathbf{R}_2$ divided into components for the case of an added mass; the displacements components are favorised*

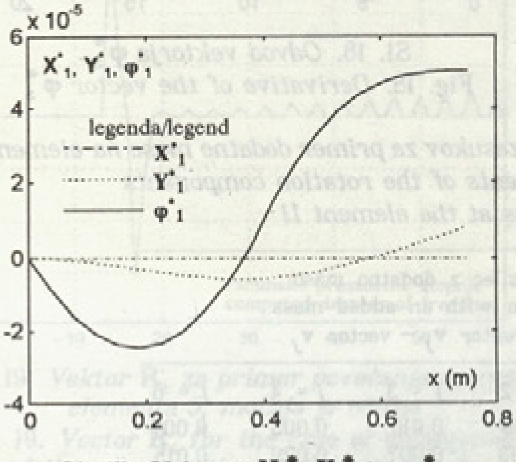


Sl. 14. *Komponente zasukov vektorja $\mathbf{R}_2$ za primer dodatne mase*

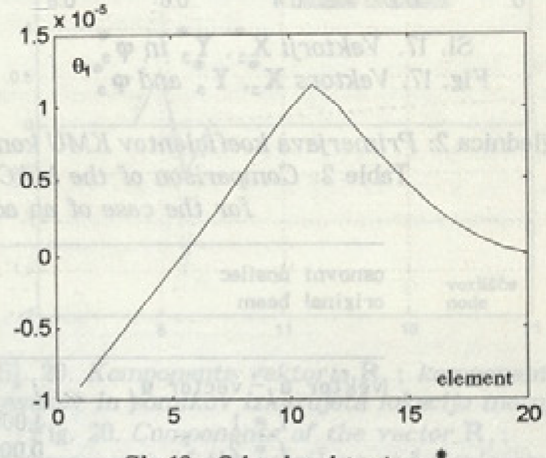
Fig. 14. *Rotational components of the vector $\mathbf{R}_2$ for the case of an added mass*

Tudi v primeru dodatne mase je sprememba konstrukcije opazna v višjih nihajnih oblikah v komponentah zasukov. Motnja se delno kaže tudi v sliki komponent pomikov, vendar ni tako očitna, da bi se lokacija motnje dala natančno določiti – pojavi se namreč v sosednjem elementu. Dokaj grobo lahko ocenimo lokacijo, ki je lahko namenjena za nadaljnje iskanje. Komponente pomikov, risane v istem merilu kakor komponente zasukov, popolnoma zakrijejo komponente zasukov, podobno kakor se to zgodi pri poškodbi konstrukcije.

Even in the case of an added mass the change on the structure is demonstrated in higher flexural modes in the rotational components. The change is slightly indicted also in the picture of the displacement components. However, it is not so evident as to allow for identification of the change; it appears namely in the neighbouring element. The location can therefore be very roughly indicated. The displacement components presented in the same scale as the rotation components completely overlie the components of the rotations, similarly as in the case of a damaged structure.



Sl. 15. *Vektorji* X_1^*, Y_1^* *in* φ_1^*
Fig.15. *Vektors* X_1^*, Y_1^* *and* φ_1^*



Sl. 16. *Odvod vektorja* φ_1^*
Fig. 16. *Derivative of the vector* φ_1^*

Slika 15 prikazuje vektorje X_1^*, Y_1^* in φ_1^* za primer, ko se na elementu 11 pojavi dodatna masa. Slika ne kaže lokacije spremembe tako očitno, kakor je to v primeru poškodbe konstrukcije. Vendar tudi tu vektor φ_1^* ponuja zadostno informacijo o tem, kje je dodatna masa. Medtem ko se lokalna sprememba togostne matrike izraža s preskokom vektorja na mestu nepravilnosti, se lokalna sprememba mase kaže s prevojno točko na mestu dodatne mase. Za določitev mesta spremembe potrebujemo odvod vektorja φ. Tudi sedaj se pojavi na elementu spremembe lokalni ekstrem. Razlika je samo v načinu: pri spremembi togosti se največja vrednost pojavi skokovito na elementu, ki je spremenjen, pri spremembi mase pa funkcija enakomerno narašča (oz. pojema) do ekstremne vrednosti. Podobno obnašanje se opazi tudi v višjih nihajnih oblikah. Slika 17 prikazuje vektorje X_3^*, Y_3^* in φ_3^*, slika 18 pa odvod vektorja φ_3^*.

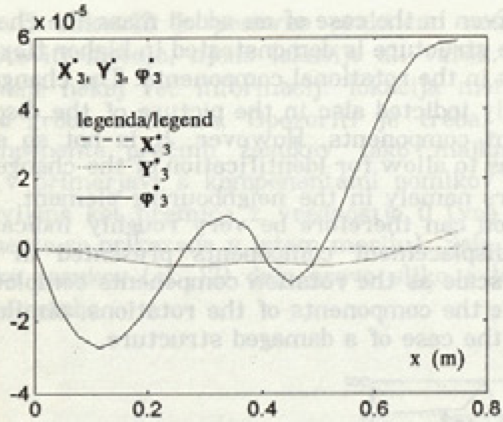
Ker se razpoka v koeficentih KMU (MAC) in KMUK (COMAC) ne kaže, je bila izvedena študija koeficientov za primer dodatne mase.

Izračunana je tudi vrednost koeficienta KMU (MAC) za vektorja vzdolžnih pomikov; vrednost znaša 1.

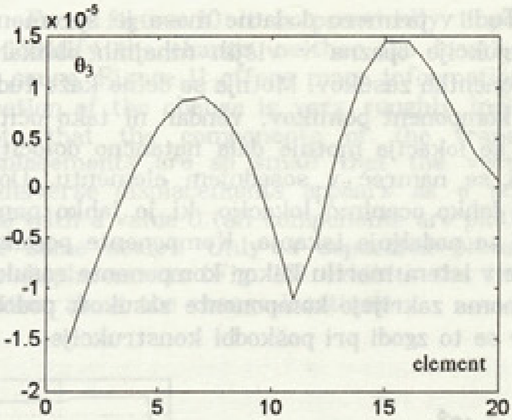
Figure 15 shows vectors X_1^* Y_1^* and φ_1^* for the case of an added mass on the element 11. The figure does not exhibit the location of the change so evidently as in the case of the damaged structure. However, also in this case the vector φ_1^* offers enough information about the location of the added mass. While the local reduction of the stiffness is reflected as an interruption of the vector at the location of the irregularity, the added mass, by cotrast is determined by a reflection point at the added mass location. To determine this point the derivative of the vector φ is needed. Also in this case there is a local peak at the modification location. The essential difference lies in the way in which this extreme is reached. While in the case of the stiffness reduction the peak is reached only at the location of the modification, in the case of an added mass the values are smoothly increasing (or decreasing) to finally reach the peak value at the element where the added mass actually appears. Similar behaviour is detected also in higher modal shapes. Fig. 17 presents vectors X_3^*, Y_3^* and φ_3^* and fig. 18 displays the derivative of the vector φ_3^*.

Since the damage is not reflected in the MAC and COMAC coefficients, a study of the coefficients for the added mass was performed.

The MAC coefficients for the case of longitudinal displacements were also calculated; the value is equal to 1.



Sl. 17. *Vektorji* $\mathbf{X}^*_3$, $\mathbf{Y}^*_3$ *in* φ^*_3
Fig. 17. *Vektors* $\mathbf{X}^*_3$, $\mathbf{Y}^*_3$ *and* φ^*_3



Sl. 18. *Odvod vektorja* φ^*_3
Fig. 18. *Derivative of the vector* φ^*_3

Preglednica 2: *Primerjava koeficientov KMU komponent zasukov za primer dodatne mase na elementu 11*
Table 2: *Comparison of the MAC coefficients of the rotation components for the case of an added mass at the element 11*

osnovni nosilec original beam	nosilec z dodatno maso beam with an added mass vektor $\mathbf{v}_j$ – vector $\mathbf{v}_j$				
vektor $\mathbf{u}_i$ – vector $\mathbf{u}_i$	j = 1	j = 2	j = 3	j = 4	j = 6
i = 1	1.000	0.019	0.010	0.005	0.007
i = 2	0.009	0.998	0.007	0.029	0.015
i = 3	0.009	0.011	1	0.015	0.012
i = 4	0.010	0.011	0.010	0.993	0.005
i = 5	0.010	0.011	0.011	0.011	0.998

Preglednica 3: *Primerjava koeficientov KMU komponent pomikov za primer dodatne mase na elementu 11*
Table 3: *Comparison of the MAC coefficients of the transverse displacement components for the case of an added mass at the element 11*

osnovni nosilec original beam	nosilec z dodatno maso beam with an added mass vektor $\mathbf{v}_j$ – vector $\mathbf{v}_j$				
vektor $\mathbf{u}_i$ – vector $\mathbf{u}_i$	j = 1	j = 2	j = 3	j = 4	j = 6
i = 1	1.000	0.396	0.060	0.095	0.033
i = 2	0.383	1.000	0.229	0.085	0.103
i = 3	0.062	0.235	1	0.167	0.044
i = 4	0.088	0.065	0.148	0.998	0.091
i = 5	0.035	0.101	0.045	0.120	0.999

Opazimo lahko, da so vrednosti zunaj diagonalnih členov pri zasuku mnogo manjše v primerjavi s komponentami pomikov, diagonalne vrednosti pa so praktično enake 1, kar ni primerno za ugotavljanje spremembe na konstrukciji. Vrednosti koeficientov KMUK so bile izračunane v 20 vozliščih (razen v vpetem). Upoštevanih je šest nihajnih oblik (L=6), ki pripadajo prečnemu nihanju. Izračunane vrednosti so bile praktično enake 1 (vrednost, ki je najbolj odstopala je bila 0,99965104440336).

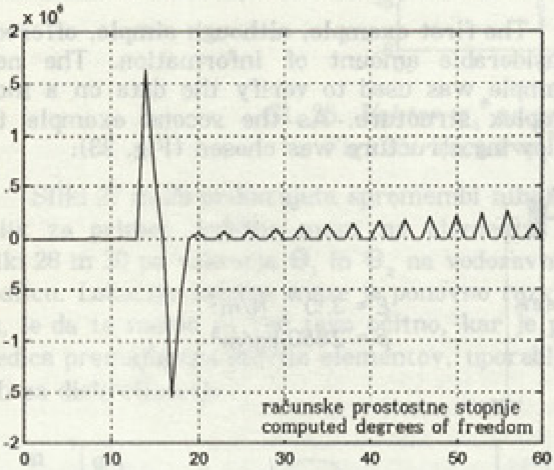
It is noted that the off-diagonal terms for the displacements are essentially smaller than in the comparison of rotations. The diagonal values are more or less practically equal to 1, and that is a very poor basis for the determination of an eventual modification of the structure. The COMAC coefficients were computed in 20 nodes (except in the clamped one) and the first six (L=6) transverse eigen modes were considered. The computed values were more or less equal to 1 (the best distinguished value was 0.99965104440336).

2.1.3 *Oslabitve/okrepitve prereza*

Obravnavane so bile tudi oslabitve in okrepitve prereza, kjer se pojavi hkrati sprememba togostne in masne matrike. Pojavijo se lahko kot rezultat proizvodnega procesa zaradi nenatančne izdelave konstrukcije, npr. kot povečanje oz. zmanjšanje prereza. Taka študija je torej zanimiva predvsem z vidika potrditve računskega modela.

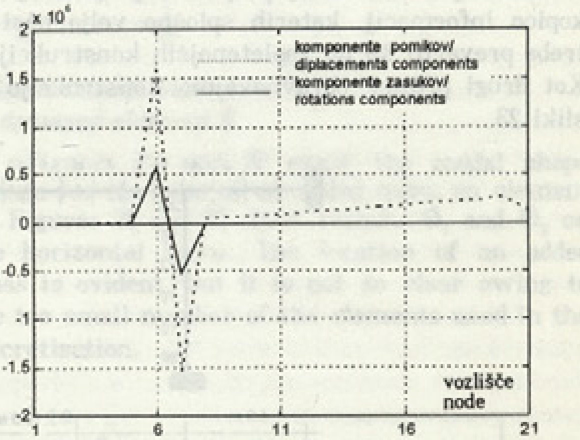
2.1.3 *The weakening/hardening of the cross-section*

The weakening and hardening of the cross-sections were also considered. In such cases the change of stiffness occurs simultaneously with the variation of the mass matrix. They may be the result of a manufacturing process due to an unprecise production as a reduction or enlargement of the cross-section. Such a study is therefore interesting mostly from the point of supervision and validation of the computational model chosen.



Sl. 19. *Vektor* $\mathbf{R}_1$ *za primer povečanja prereza elementa 5; motnja je očitna*

Fig. 19. *Vector* $\mathbf{R}_1$ *for the case of enlargement of the cross section at the element 5; the disturbation is obvious*

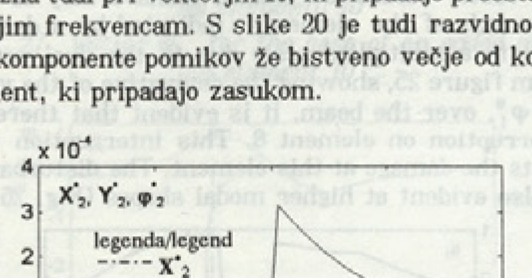


Sl. 20. *Komponente vektorja* $\mathbf{R}_1$; *komponenti zasukov in pomikov izkazujeta lokacijo motnje*

Fig. 20. *Components of the vector* $\mathbf{R}_1$; *the components of the rotations and displacement exhibit the location of the disturbance*

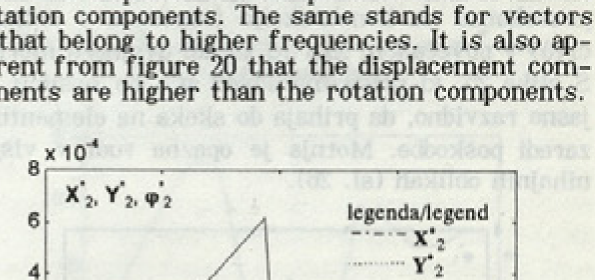
Na sliki 20 je motnja zelo očitna. Bistvena razlika med sliko 20 in drugimi slikami, ki prikazujejo vektorje $\mathbf{R}$ po komponentah, je, da tudi komponente pomikov na sliki 20 jasno (če ne celo jasneje) izkazujejo lokacijo motnje. Enaka težnja je opazna tudi pri vektorjih $\mathbf{R}$, ki pripadajo preostalim višjim frekvencam. S slike 20 je tudi razvidno, da so komponente pomikov že bistveno večje od komponent, ki pripadajo zasukom.

The location of the disturbance from fig. 20 is evident. The essential difference between fig. 20 and other figures, presenting vectors $\mathbf{R}$ by their components, is that also the displacement components clearly exhibit the location of the disturbance. The location is somehow even more evident from the displacement components than from the rotation components. The same stands for vectors $\mathbf{R}$ that belong to higher frequencies. It is also apparent from figure 20 that the displacement components are higher than the rotation components.



Sl. 21. *Vektorji* $\mathbf{X}_2^*$, $\mathbf{Y}_2^*$ *in* φ_2^* *za primer oslabitve prereza*

Fig. 21. *Vectors* $\mathbf{X}_2^*$, $\mathbf{Y}_2^*$ *and* φ_3^* *for reduction of the cross-section*



Sl. 22. *Vektorji* $\mathbf{X}_2^*$, $\mathbf{Y}_2^*$ in φ_2^* *primer okrepitve prereza*

Fig. 22. *Vectors* $\mathbf{X}_2^*$, $\mathbf{Y}_2^*$ *and* φ_2^* *for enlargement of the cross-section*

Slika 21 prikazuje drugi vektor za primer oslabitve prereza, slika 22 pa prikazuje prvi vektor za primer okrepitve prereza. Razvidno je, da je sprememba togostne matrike prevladujoča v primerjavi s spremembo masne matrike, saj so rezultati podobni kakor pri spremembi togosti.

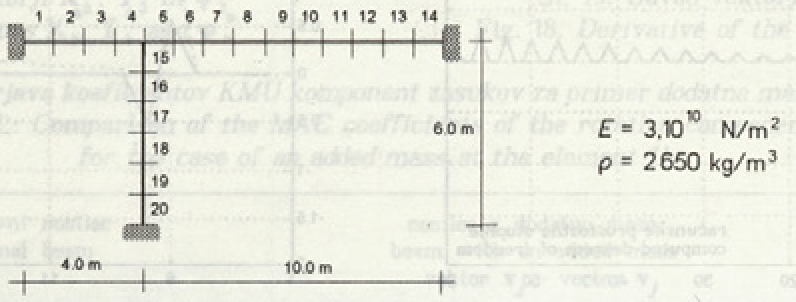
2.2 Drugi primer

Prvi primer, dokaj preprost, je postregel s kopico informacij, katerih splošno veljavnost je treba preveriti še na zapletenejših konstrukcijah. Kot drugi primer obravnavajmo konstrukcijo na sliki 23.

Figure 21 shows the second vector for the reduction of the cross section, and figure 22 displays the first vector for the case of enlargement of the cross section. It is evident from both figures that the change of the stiffness is dominant over the change of the mass. The results obtained are in fact similar to those obtained for the modification of the stiffness only.

2.2 Second example

The first example, although simple, offered a considerable amount of information. The next example was used to verify the data on a more complex structure. As the second example the following structure was chosen (Fig. 23):



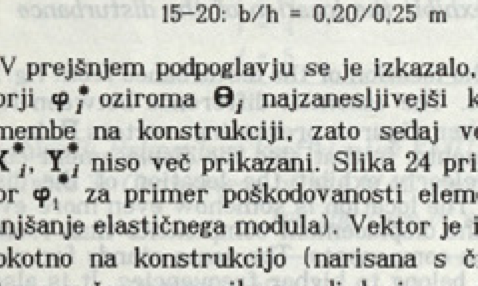
Sl. 23. *Drugi primer*
Fig. 23. *Second structure*

izmere: elementi 1–4: b/h = 0.20/0.20 m nosilec
5–14: b/h = 0.20/0.30 m
15–20: b/h = 0.20/0.25 m steber

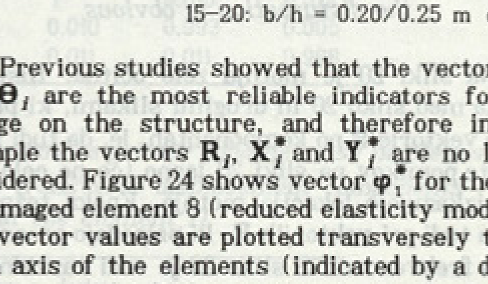
dimensions: elements 1–4: b/h = 0.20/0.20 m beam
5–14: b/h = 0.20/0.30 m
15–20: b/h = 0.20/0.25 m column

V prejšnjem podpoglavju se je izkazalo, da so vektorji φ_i^* oziroma Θ_i najzanesljivejši kazalec spremembe na konstrukciji, zato sedaj vektorji R_i, X_i^*, Y_i^* niso več prikazani. Slika 24 prikazuje vektor φ_1 za primer poškodovanosti elementa 8 (zmanjšanje elastičnega modula). Vektor je izrisan pravokotno na konstrukcijo (narisana s črtkano črto) v povečanem merilu zaradi dvojnega merila. S slike 25, ki prikazuje odvod φ_1^*, po nosilcu, je jasno razvidno, da prihaja do skoka na elementu 8 zaradi poškodbe. Motnja je opazna tudi v višjih nihajnih oblikah (sl. 26).

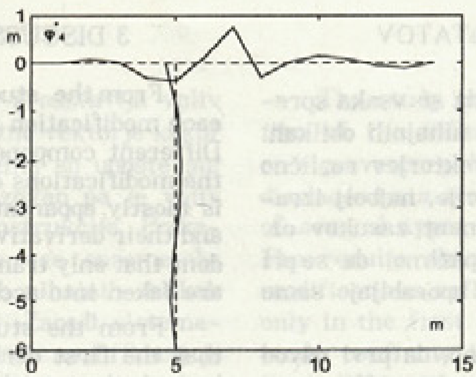
Previous studies showed that the vectors φ_i^* and Θ_i are the most reliable indicators for the change on the structure, and therefore in this example the vectors R_i, X_i^* and Y_i^* are no longer considered. Figure 24 shows vector φ_1^* for the case of damaged element 8 (reduced elasticity modulus). The vector values are plotted transversely to the main axis of the elements (indicated by a dashed line) in a multiplied scale due to the dual scale. From figure 25, showing the derivative of the vector φ_1^*, over the beam, it is evident that there is interruption on element 8. This interruption reflects the damage at this element. The disturbance is also evident at higher modal shapes (Fig. 26).



Sl. 24. *Vektor* φ_1^*, *za primer poškodovanosti elementa 8*
Fig. 24. *Vector* φ_1^*, *for the case of damaged element 8*



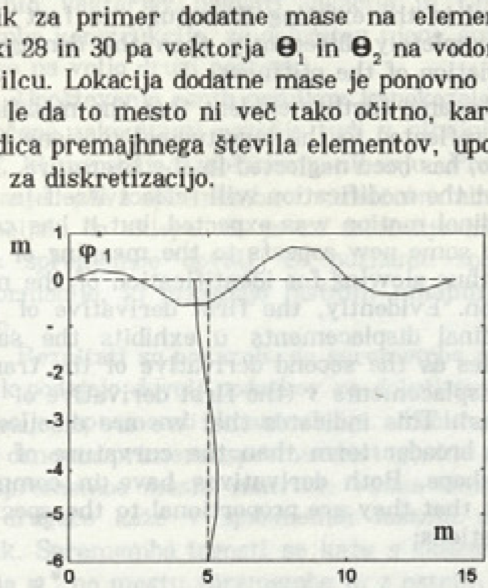
Sl. 25. *Odvod vektorja zasukov po prečki*
Fig. 25. *The derivative of the vector of rotation components over the element 8*



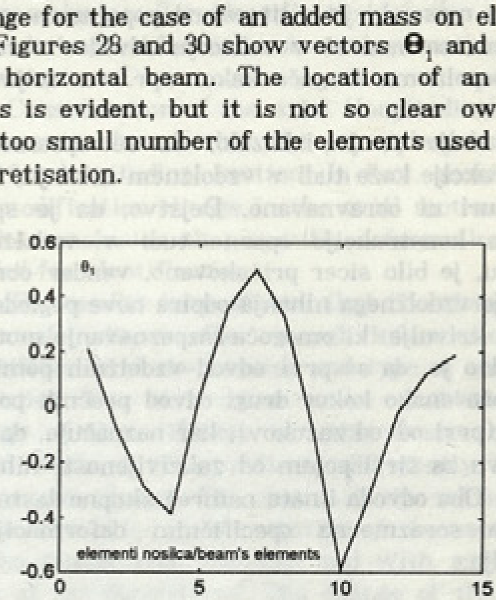
Sl. 26. *Vektor φ_4^* za primer poškodovanosti elementa 8*
Fig. 26. *Vector φ_4^* for the damaged element 8*

Sliki 27 in 29 prikazujeta spremembi nihajnih oblik za primer dodatne mase na elementu 10, sliki 28 in 30 pa vektorja Θ_1 in Θ_2 na vodoravnem nosilcu. Lokacija dodatne mase je ponovno razvidna, le da to mesto ni več tako očitno, kar je posledica premajhnega števila elementov, uporabljenih za diskretizacijo.

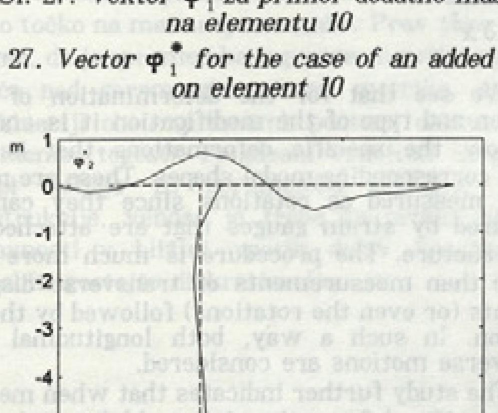
Figures 27 and 29 show the modal shape change for the case of an added mass on element 10. Figures 28 and 30 show vectors Θ_1 and Θ_2 on the horizontal beam. The location of an added mass is evident, but it is not so clear owing to the too small number of the elements used in the discretisation.



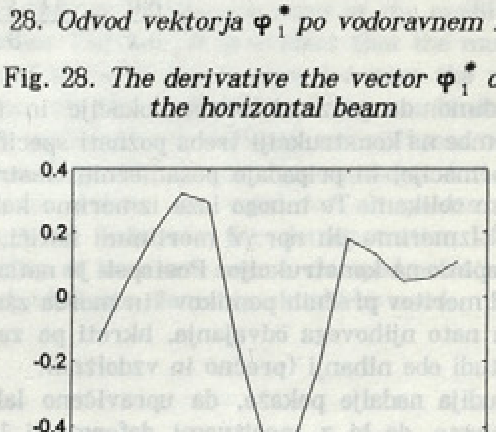
Sl. 27. *Vektor φ_1^* za primer dodatne mase na elementu 10*
Fig. 27. *Vector φ_1^* for the case of an added mass on element 10*



Sl. 28. *Odvod vektorja φ_1^* po vodoravnem nosilcu*
Fig. 28. *The derivative the vector φ_1^* over the horizontal beam*



Sl. 29. *Vektor φ_2^* za primer dodatne mase na elementu 10*
Fig. 29. *Vector φ_2^* for the case of an added mass on element 10*



Sl. 30. *Ovod vektorja φ_2^* po vodoravnem nosilcu*
Fig. 30. *The derivative the vector φ_2^* over the horizontal beam*

Podobne rezultate dobimo, če modeliramo spremembo na stebru.

Similar results are obtained if the modification is modelled on the column.

3 OBRAVNAVA REZULTATOV

Iz študije je jasno razvidno, da se vsaka sprememba konstrukcije kaže tudi v nihajnih oblikah. Posamezne komponente lastnih vektorjev različno prikazujejo spremembo konstrukcije, najbolj izrazito pa je ta razvidna iz komponent zasukov oz. njihovih odvodov. V literaturi opazimo, da se pri numeričnih simuliranjih pogosto uporabljajo samo komponente prečnih pomikov.

V pričujoči študiji je pokazano, da prvi odvod zasukov (ki je ekvivalent drugemu odvodu komponent pomikov) daje zadostne informacije ne samo o lokaciji spremembe konstrukcije, temveč je iz oblike mogoče sklepati o tipu spremembe. Sprememba mase, ki je v literaturi popolnoma zanemarjena, se namreč v vektorju odvodov zasukov kaže popolnoma drugače kakor npr. variacija togosti.

Nadalje se je izkazalo, da se sprememba konstrukcije kaže tudi v vzdolžnem nihanju, ki v literaturi ni obravnavano. Dejstvo, da je sprememba konstrukcije opazna tudi v vzdolžnem nihanju, je bilo sicer pričakovano, vendar obravnavanje vzdolžnega nihanja odpira nove poglede na pomen krivulje, ki omogoča razpoznavanje motnje. Razvidno je, da se prvi odvod vzdolžnih pomikov u obnaša enako kakor drugi odvod prečnih pomikov v (prvi odvod zasukov), kar naznačuje, da gre v bistvu za širši pojem od zakrivljenosti nihajne oblike. Oba odvoda imata namreč skupno lastnost, da sta sorazmerna specifičnim deformacijam, saj velja:

3 DISCUSSION OF THE RESULTS

From the study presented it is evident that each modification is reflected in the modal shapes. Different components of the eigenvectors exhibit the modifications differently, but the modification is mostly apparent from the rotation components and their derivatives. From the literature it is evident that only transverse displacement components are taken into account in numerical simulations.

From the study presented it is also evident that the first derivative of the rotations (which is equivalent to the second derivative of the displacement components) gives enough information not only about the location of the structure modification, but also about the type of modification. The change of the mass, which has been totally neglected in the existing literature, in fact reflects completely differently than the damage — the variation of the stiffness.

It became further evident that the modification is reflected in the longitudinal motions, and this, too, has been neglected in the literature. The fact that the modification will reflect itself in the longitudinal motion was expected, but it has contributed some new aspects to the meaning of the curve, thus alowing for identification of the modification. Evidently, the first derivative of the longitudinal displacements u exhibits the same properties as the second derivative of the transverse displacements v (the first derivative of the rotations). This indicates that we are discussing a much broader term than the curvature of the modal shape. Both derivatives have in common the fact that they are proportional to the specific deformations:

$$\varepsilon = \frac{\partial u}{\partial x} = \frac{\partial(-y\,\varphi)}{\partial x} = -y\,\frac{\partial^2 v}{\partial x^2} \qquad (7).$$

Vidimo, da je za določitev lokacije in tipa spremembe na konstrukciji treba poznati specifične deformacije, ki pripadajo posameznim lastnim nihajnim oblikam. Te mnogo laže izmerimo kakor zasuke. Izmerimo jih npr. z merilnimi lističi, ki jih prilepimo na konstrukcijo. Postopek je natančnejši od meritev prečnih pomikov (in morda zasukov) in nato njihovega odvajanja, hkrati pa zajamemo tudi obe nihanji (prečno in vzdolžno).

Študija nadalje pokaže, da upravičeno lahko pričakujemo, da bi z meritvami deformacij laže razpoznali dodatno maso kakor spremembo togosti, saj bi v tem primeru morali imeti merilna mesta skoraj v neposredni bližini poškodbe, kar bi zahtevalo veliko število merilnih mest. Ta problem pri razpoznanju dodatne mase ni tako izrazit. Ko sta lokacija in tip spremembe na konstrukciji znana, je treba poiskati še podatke o velikosti spremembe. V ta namen uporabimo prilagojene postopke iz literature, saj je lokacija spremembe že znana.

We see that for the determination of the location and type of the modification it is enough to know the specific deformations that belong to the corresponding modal shapes. These are more easily measured as rotations since they can be measured by strain gauges that are attached to the structure. The procedure is much more accurate than measurements of transverse displacements (or even the rotations) followed by the derivation. In such a way, both longitudinal and transverse motions are considered.

The study further indicates that when measuring specific deformation it would be easier to identify an added mass as a local change of stiffness. The identification of local stiffness change would actually require the transducers to be located very near the damaged part, thus requiring a large number of transducers. This problem is not present in the identification of an added mass. Once the location and the type of the modification are known, the magnitude of the modification can be identified by adapted methods known from the references.

4 SKLEP

Predstavljena študija širše predstavlja vpliv spremenjene konstrukcije na lastne vektorje kakor je to znano iz literature. V analizi so upoštevani tudi vzdolžni (osni) pomiki, opazovan pa je vpliv različnih lokalnih sprememb konstrukcije. Prikazani rezultati potrjujejo, da se vse spremembe konstrukcije kažejo tudi v višjih nihajnih oblikah in ne samo v prvi nihajni obliki. Zaradi sistematičnosti študije z uporabo različnih modifikacij je razvidno, da se različne spremembe različno kažejo v posameznih nihajnih oblikah oziroma v posameznih komponentah prostostnih stopenj. V idealnem primeru je moč iz oblike spremembe lastnih vektorjev določiti lokacijo in tip spremembe konstrukcije, za določitev njene amplitude pa so na voljo drugi postopki.

Upoštevanje osnih pomikov je pokazalo, da se določene zakonitosti poznajo tudi v nihajnih oblikah, ki pripadajo vzdolžnemu nihanju. Prav opazovanje vpliva spremembe v vzdolžnem nihanju je pripeljalo do sklepa, da je za določanje mesta in tipa spremembe dovolj, če poznamo specifične deformacije, ki pripadajo lastnim nihajnim oblikam.

Rezultati so pokazali, da spremembe nihajnih oblik podajajo dovolj podatkov za določitev tipa in lokacije sprememb konstrukcije. Ločimo lahko dva osnovna primera: spremembo togostne matrike in spremembo masne matrike. Vsaka izmed njiju se drugače kaže v spremembi lastnih nihajnih oblik. Sprememba togosti se kaže s skokom vektorja φ_j^* na mestu spremembe in z ostrim vrhom v sliki odvodov. Sprememba mase se izraža s prevojno točko na mestu spremembe. Prav tako lahko trdimo, da je sprememba togostne matrike prevladujoča nad spremembo masne matrike, saj pri kombinaciji obeh sprememb postane očitna samo sprememba togosti. Prikazani rezultati in podani sklepi veljajo za spremembo poljubne točke konstrukcije, vendar je treba za prikaz njihove veljavnosti v bližini vpetih delov konstrukcije uporabiti gostejšo diskretizacijo.

4 CONCLUSION

The study presented offers a wide illustration of the influence of structure modification on eigenvectors. In the analysis longitudinal (axial) displacements are also included, and the influence of several types of modification is investigated. The results obtained confirm the fact that all modifications reflect in all modal shapes, and not only in the first one. The systematisation of the study confirms that different modifications reflect differently in modal shapes or components of displacements (degrees of freedom). In the ideal case it is possible to determine the location and the type of modification from the eigenvectors change. For identification of the magnitude of the modification some other methods are known.

Consideration of the axial (longitudinal) displacements showed that the principles are valid also for longitudinal motion. In fact, monitoring the modification change in the axial motion leads to the conclusion that specific deformations are needed for identification.

The results have shown that the changes of the modal shapes offer enough information for identification of the location and type of the modification. Two basic cases can be considered: the change of the stiffness and the change of the mass. Each of them reflects differently in the eigenvector change. The change of the stiffness is characterised by an interruption of the vector φ_j^* at the modification location and with a narrow peak at the derivatives. The change of the mass is marked by a reflection point at the modification location. Further, it is evident that the modification of the stiffness is superior over the modification of the mass since the combination of both modifications reflects as stiffness modification only. The results presented and the conclusions drawn are valid for all points of the structure; however for demonstration of their validity at locations near a clamped point, a finer mesh of finite elements should be used.

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5 REFERENCES

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