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Projektiranje odmikala z uporabo Bézierjeve krivulje
Design of Cam Profile Using a Bézier's Curve

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V prispevku je prikazan postopek računalniško podprtega projektiranja profila odmikala za visokotlačne tlačilke dieselskih vbrizgalnih sistemov. Postopek avtomatično določa optimalni profil glede na predpisan potek hitrosti bata tlačilke ter glede na omejitvene pogoje. Slednji se nanašajo na lokalni polmer profila odmikala in pospešek bata visokotlačne tlačilke. Oblika profila je podana z Bézierjevo krivuljo, pri čemer so nekatere koordinate njenih nadzornih točk privzete kot projektne spremenljivke. Predstavljen postopek je primeren za uporabo pri optimalnem projektiranju celotnega vbrizgalnega sistema. Teorija je ponazorjena s številčnimi zgledi.

Ključne besede: odmikala, dieselski vbrizgalni sistem, krivulje Bézierjeve, projektiranje optimalno

The paper describes a procedure for computer aided design of a cam profile of a high pressure pump in a diesel fuel injection system. The proposed procedure generates automatically an optimum profile with regard to the prescribed history of the pump plunger velocity and to the imposed constraints. The imposed constraints are related to the local radius of the cam profile as well as to the pump plunger acceleration. The shape of the profile is represented by a Bézier's curve, and some of the coordinates of its control points are adopted as design variables. The procedure is suitable for inclusion in the optimal design procedure of the whole fuel injection system. The theory is illustrated by numerical examples.

Keywords: cam profile, diesel fuel injection system, Bézier's curve, optimal design

0 UVOD

Pri konvencionalnem vbrizgalnem sistemu ima potek hitrosti bata tlačilke pomemben vpliv na kakovost vbrizgavanja [1] do [4]. V splošnem ni mogoče določiti idealnega poteka hitrosti in hkrati ohraniti sprejemljivo obliko odmikala. Vendar pa je mogoče določiti obliko odmikala, tako da je potek hitrosti optimalen glede na postavljene omejitve. Primerno orodje za tak postopek ponujajo metode matematičnega programiranja.

Pod ustreznimi predpostavkami (ni izgube stika med odmikalom in valjčkom itn.) je sistem odmikalo - valjček - bat tlačilke dokaj preprost dinamični mehanski sistem. Ker je njegov matematični model podan s sistemom algebrajskih enačb, se problem optimalnega projektiranja profila odmikala lahko precej splošno zapiše kot:

0 INTRODUCTION

In the conventional inline fuel injection equipment (FIE) the shape of the pump plunger velocity history has an important influence on the quality of injection [1] to [4]. In general, it is not possible to obtain an ideal velocity history and to preserve at the same time an acceptable shape of the cam. However, it is possible to determine the shape of the cam so that the shape of the plunger velocity history is optimal with respect to the imposed constraints. An adequate tool for solving this problem is offered by the methods of mathematical programming.

Under some assumptions (no loss of contact between cam and tappet roller, etc.) the system cam - tappet roller - pump plunger represents a rather simple mechanical system with time-dependent response. Since the state equation of this system can be written in algebraic form, the optimum design problem of the cam profile can be generally formulated as:

$$\min \hat{g}_0(\mathbf{b},\mathbf{u})\big|_{s=s_0} \qquad (1),$$

z upoštevanjem pogojev: / subject to constraints:

$$\hat{g}_i(\mathbf{b},\mathbf{u})\big|_{s=s_i} \le 0, i = 1,\ldots, j \qquad (2)$$

in enačbe stanja: / and the state equation:

$$\mathbf{u} = \hat{\mathbf{f}}(\mathbf{b}, s) \qquad (3).$$

Skalarne funkcije $\hat{g}_0$ in $\hat{g}_i$ pomenijo ciljne in omejitvene funkcije, s simbolom $(\hat{\ })$ pa sta v članku ločeni imeni funkcije in ustrezne odvisne spremenljivke. Označba $\mathbf{b} \in R^n$ pomeni vektor projektnih spremenljivk, $\mathbf{u} \in R^m$ je vektor odzivnih spremenljivk, s je neodvisna (času podobna) spremenljivka, $s_0, s_1, \ldots, s_j$ pa so nekatere njene vrednosti, ki so lahko odvisne od $\mathbf{b}$. Simbol j označuje število omejitev, n je število projektnih in m število odzivnih spremenljivk. Enačba stanja (3) določa odvisnost $\mathbf{u}$ od s in $\mathbf{b}$. V primeru, da so funkcije (1) do (3) vsaj enkrat odvedljive po $\mathbf{b}$, je problem (1) do (3) verjetno najučinkoviteje rešljiv z uporabo gradientnih metod matematičnega programiranja. V primeru obravnavanega sistema (odmikalo - valjček - bat tlačilke) je ta pogoj izpolnjen, tako da lahko postopek optimalnega projektiranja temelji na uporabi ustrezne gradientne metode.

V prvih dveh poglavjih članka je predstavljen način opisa profila odmikala in izračun odziva sistema odmikalo-valjček-bat tlačilke. Sledi definicija problema optimalnega projektiranja s potrebnimi preoblikovanji v tretjem in četrtem poglavju. V petem poglavju so prikazani številčni zgledi.

The scalar functions $\hat{g}_0$ and $\hat{g}_i$ are termed the objective and constraint function, respectively, while the symbol $(\hat{\ })$ will be used throughout this paper to distinguish between the name of a function and the name of the corresponding dependent variable. The symbol $\mathbf{b} \in R^n$ is the vector of design variables, $\mathbf{u} \in R^m$ is the vector of the response variables, s is an independent (time-like) variable and $s_0, s_1, \ldots, s_j$ are some given values that may depend on $\mathbf{b}$. The symbol j denotes the number of constraints, n is the number of the design variables and m is the number of the response variables. The state equation (3) establishes the dependence of $\mathbf{u}$ on s and $\mathbf{b}$. If the functions in (1) to (3) are at least once differentiable with respect to $\mathbf{b}$, then the problem (1) to (3) may probably most efficiently be solved by employing gradient based methods of mathematical programming. In the case under consideration (system cam - tappet roller - pump plunger), this requirement is fulfilled so that the optimum design procedure may be based on a suitable gradient method.

In the first two sections of the paper, the shape representation concept of the cam profile and the response analysis of the system cam-tappet roller-pump plunger is considered. Sections 3 and 4 describe the definition of optimum design problems and the necessary transformations. Numerical examples are presented in section 5.

1 PREDSTAVITEV OBLIKE PROFILA ODMIKALA

Naj bo profil odmikala odvisen od vektorja projektnih spremenljivk $\mathbf{b}$. V tem primeru lahko profil v matematični obliki predstavimo s krivuljo:

1 SHAPE REPRESENTATION OF THE CAM PROFILE

Let the cam profile depend on the vector $\mathbf{b}$ of the design variables. In this case, the profile can be mathematically represented by a curve:

$$K = \{\mathbf{r} | \mathbf{r} = \hat{\mathbf{r}}(\mathbf{b}, s), \quad 0 \le s \le 1\} \tag{4}$$

kjer je s neodvisna spremenljivka, ki določa lego točke vzdolž K, simbol $\mathbf{r} \in R^2$ pa označuje krajevni vektor točke na profilu glede na izbran koordinatni sistem (sl. 1).

Pri izbiri definicije funkcije $\hat{\mathbf{r}}$ je dobro upoštevati naslednja dva vidika: a) možnost, da se s krivuljo K opiše poljubna oblika profila in b) oblika krivulje K pri zmerni spremembi vrednosti projektnih spremenljivk ne sme postati nesprejemljiva iz tehničnega vidika. Slednji vidik je pomemben zato, ker lahko v nasprotnem primeru pričakujemo nestabilnosti v postopku optimizacije.

Z upoštevanjem navedenega, lahko K definiramo kot Bézierjevo krivuljo, tako da predpišemo:

where s is an independent variable determining the position along K, and the symbol $\mathbf{r} \in R^2$ denotes the position vector of a point on the profile with respect to a chosen Cartesian coordinate system (Fig. 1).

When deciding how to formulate the function $\hat{\mathbf{r}}$, the following two aspects should be considered: a) it should be possible that K can take virtually any shape and b) the shape of K should not tend to become unacceptable from the engineering point of view if the design variables are varied moderately. The last aspect needs to be taken into account, since otherwise the optimization process tends to become unstable.

Taking these two aspects into account, K may be defined as a Bézier's curve:

$$\mathbf{r} = \sum_{i=1}^{k} B_{ki} \mathbf{q}_i \tag{5}$$

kjer pomenijo: $B_{ki} = \hat{B}_{ki}(s)$ Bézierjev polinom reda $(k-1)$, $\mathbf{q}_i = \hat{\mathbf{q}}_i(\mathbf{b})$ je krajevni vektor nadzorne točke, k pa je število nadzornih točk [5].

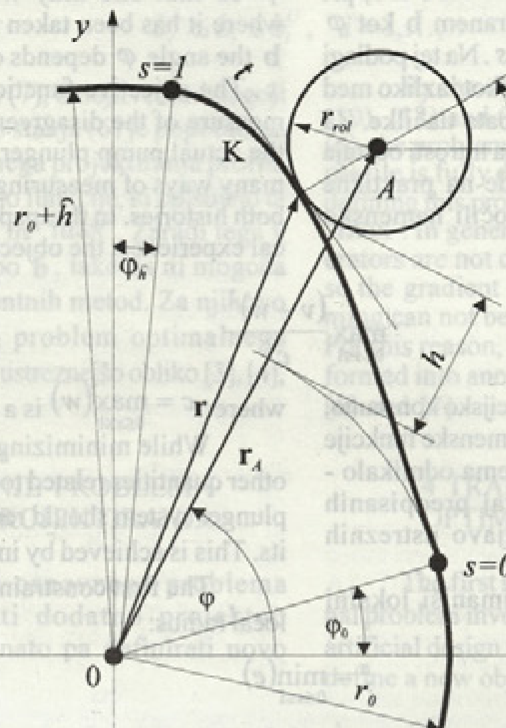
Definicija (5) omogoča zelo veliko prilagodljivost oblike krivulje K. Ta je namreč odvisna od števila nadzornih točk k, ki ga projektant lahko izbere povsem poljubno.

where: $B_{ki} = \hat{B}_{ki}(s)$ is the Bézier's blending polynomial of the order $(k-1)$, $\mathbf{q}_i = \hat{\mathbf{q}}_i(\mathbf{b})$ is the position vector of a control point and k is the number of all control points [5].

The definition (5) offers large flexibility the shape of K. This follows from the fact that the flexibility of a Bézier's curve depends on the number of control points k that may be chosen arbitrarily by the designing engineer.

Poleg velike prilagodljivosti, ima Bézierjeva krivulja tudi druge zelo ugodne lastnosti, na primer, lastnost zmanjševanja variacij in lastnost konveksne lupine [5]. Omenjene lastnosti zadoščajo, da krivulja $\mathcal{K}$ ohrani sprejemljivo obliko pri zmernih spremembah krajevnih vektorjev nadzornih točk.

In addition to large flexibility, the Bézier's curve also offers many other attractive properties, such as, variation diminishing property or the convex hull property [5]. These properties ensures that the curve $\mathcal{K}$ preserves an acceptable shape at moderate variation of the control point position vectors.



Sl. 1. *Parametri na profilu odmikala (t-tangenta, n-normala)*

Fig. 1. *Parameters on the cam profile (t-tangent, n-normal)*

2 ODZIV SISTEMA ODMIKALO - VALJČEK - BAT TLAČILKE

Odziv sistema odmikalo - valjček - bat tlačilke običajno opisujemo z naslednjimi veličinami bata tlačilke: dvig h, relativna hitrost $v = dh/d\varphi$ in relativni pospešek $a = d^2h/d\varphi^2$, kjer pomenijo $\varphi = \arctan(y_A/x_A)$, x_A in y_A pa sta komponenti vektorja $\mathbf{r}_A$ (slika 1). Torej lahko vektor odzivnih spremenljivk zapišemo kot $\mathbf{u} = [h, v, a]^T$.

Odzivne spremenljivke so odvisne od projektnih spremenljivk in od parametra s. Torej velja $\mathbf{u} = \hat{\mathbf{u}}(\mathbf{b}, s)$. Pri upoštevanju $\varphi = \hat{\varphi}(\mathbf{b}, s)$ in razmer na sliki 1, je enačba stanja določena z naslednjimi izrazi:

2 RESPONSE OF THE SYSTEM CAM-TAPPET ROLLER-PUMP PLUNGER

The response of the system cam-tappet roller-pump plunger may be described by the following pump plunger quantities: lift h, relative velocity $v = dh/d\varphi$ and relative acceleration $a = d^2h/d\varphi^2$ where $\varphi = \arctan(y_A/x_A)$ and x_A and y_A are the components of the vector $\mathbf{r}_A$ (Fig. 1). This means that the vector of response variables may be written as $\mathbf{u} = [h, v, a]^T$.

The response variables depend on the design variables as well as on the parameter s. One therefore has $\mathbf{u} = \hat{\mathbf{u}}(\mathbf{b}, s)$. Taking into account that $\varphi = \hat{\varphi}(\mathbf{b}, s)$ and the relations on figure 1, the state equation can be defined by as follows:

$$h = \sqrt{x_A^2 + y_A^2} - r_0 - r_{rol}$$

$$v = \frac{\dot{h}}{\dot{\varphi}} \qquad (6),$$

$$a = \frac{\ddot{h}\dot{\varphi} - \dot{h}\ddot{\varphi}}{\dot{\varphi}^3}$$

kjer simbol $(\dot{\ })$ označuje odvod po s, r_0 je polmer osnovnega kroga, r_{rol} pa polmer valjčka (sl. 1).

where the symbol $(\dot{\ })$ denotes differentiation with respect to s, r_0 is the basic radius and r_{rol} denotes the tappet roller radius (Fig. 1).

3 DEFINICIJA PROBLEMA OPTIMALNEGA PROJEKTIRANJA

Označimo z w idealno hitrost bata tlačilke, ki se ji želimo čim bolj približati. Celoten potek idealne hitrosti običajno podajamo v odvisnosti od kota φ, tako da lahko zapišemo $w=\hat{w}(\varphi),\quad 0\leq s\leq 1$, pri čemer smo upoštevali, da je pri fiksiranem $\mathbf{b}$ kot φ odvisen od neodvisne spremenljivke s. Na tej podlagi lahko definiramo namensko funkcijo kot razliko med idealno in dejansko obliko hitrosti bata tlačilke. Za merjenje razlike med obema potekoma hitrosti obstaja kar nekaj različnih možnosti. Glede na praktične izkušnje smo se v tem članku odločili namensko funkcijo definirati kot:

3 DEFINITION OF THE OPTIMUM DESIGN PROBLEM

Let w denote an ideal pump plunger velocity serving as the target velocity. Usually, the ideal velocity history is specified in dependence on the angle φ so that one may write $w=\hat{w}(\varphi),\quad 0\leq s\leq 1$ where it has been taken into account that at a fixed $\mathbf{b}$ the angle φ depends on the independent variable s. The objective function can now be defined as a measure of the disagreement between the ideal and the actual pump plunger velocity history. There are many ways of measuring the disagreement between both histories. In this paper, as a result of the numerical experience, the objective function is defined as:

$$\max_{0\leq s\leq 1}\frac{(v-w)^2}{c^2} \quad (7),$$

kjer pomeni: $c=\max\limits_{0\leq s\leq 1}(w)$ normalizacijsko konstanto.

where: $c=\max\limits_{0\leq s\leq 1}(w)$ is a normalization constant.

Ob minimiziranju vrednosti namenske funkcije morajo nekatere druge veličine sistema odmikalo - valjček - bat tlačilke ostati znotraj predpisanih vrednosti. To je doseženo z vpeljavo ustreznih omejitvenih pogojev.

While minimizing the objective function the other quantities related to the cam-tappet roller-pump plunger system should remain within reasonable limits. This is achieved by imposing constraints.

Prvi pogoj se nanaša na najmanjši lokalni polmer:

The first constraint is related to the minimum local radius:

$$\xi=\min_{0\leq s\leq 1}(e) \quad (8),$$

pri čemer lokalni radij $e=\hat{e}(\mathbf{u})$ izračunamo z izrazom

where the local radius $e=\hat{e}(\mathbf{u})$ is calculated as

$$e=\frac{\left[\left(h+r_0+r_{rol}\right)^2+v^2\right]^{3/2}}{\left(h+r_0+r_{rol}\right)^2+2v^2-\left(h+r_0+r_{rol}\right)a}-r_{rol} \quad (9).$$

Iz tehnoloških razlogov je najmanši lokalni polmer omejen z:

For technological reasons, the minimum local radius is constrained by:

$$\left(\xi^L\right)^2-\xi^2\leq 0 \quad (10),$$

kjer L označuje spodnjo predpisano mejo.

where the superscript L denotes the lower limit.

Naslednja pogoja se nanašata na največjo in najmanšo vrednost pospeška bata visokotlačne tlačilke:

The next two constraints are related to the maximum and minimum pump plunger acceleration

$$\lambda=\max_{0\leq s\leq 1}(a),\qquad \zeta=\min_{0\leq s\leq 1}(a) \quad (11).$$

Za preprečitev prevelikih torzijskih momentov ter izgube stika med odmikalom in valjčkom, sta vpeljana naslednja omejitvena pogoja:

To avoid torsional torque overloading and losing contact between the cam and the follower, the following two constraints are imposed:

$$\lambda-\lambda^U\leq 0,\qquad \zeta^L-\zeta\leq 0 \quad (12),$$

kjer U označuje zgornjo predpisano mejo.

where the superscript U denotes the upper limit.

Poleg zgoraj navedenih omejitvenih pogojev, smo uvedli tudi stranske, ki se nanašajo na spodnje in zgornje meje vrednosti projektnih spremenljivk. Ti pogoji so potrebni zaradi zagotavljanja tehnološke nesprejemljivosti oblike odmikala, zapišemo pa jih lahko kot:

Besides the above mentioned constraints, side constraints limiting the lower and upper bounds of the design variables have been imposed as well. These constraints are necessary to prevent a technologically unacceptable design, and they can be written as:

$$b_i^L \le b_i \le b_i^U, \quad i = 1,\ldots,n \tag{13}$$

Z namensko funkcijo (7), omejitvenimi pogoji (10), (12) in (13) ter enačbo stanja (6) je popolnoma definiran problem optimalnega projektiranja profila odmikala. Na žalost vsebujejo funkcije, ki opisujejo ta problem, operatorja 'min' in 'max'. Zaradi tega v splošnem niso odvedljive po $\mathbf{b}$, tako da ni mogoča neposredna uporaba gradientnih metod. Za njihovo uporabo, je treba opisan problem optimalnega projektiranja preoblikovati v ustreznejšo obliko [3], [4], [6] in [7].

By the objective function (7), the constraints (10), (12) and (13) as well as by the state equation (6), the problem of optimum shape design of the cam profile is fully defined. Unfortunately, the functions defining this problem include the operators 'min' and 'max'. In general, the functions involving these operators are not differentiable with respect to $\mathbf{b}$, and so the gradient methods of mathematical programming can not be used directly to solve this problem. For this reason, the original problem has to be transformed into another, more convenient, form [3], [4], [6] and [7].

4 PREOBLIKOVANJE PROBLEMA OPTIMALNEGA PROJEKTIRANJA

Za preoblikovanje osnovnega problema moramo najprej vpeljati dodatno projektno spremenljivko $b_{n+1} \in \mathrm{R}$, nato pa definirati novo namensko funkcijo:

4 TRANSFORMATION OF THE OPTIMUM DESIGN PROBLEM

The first step in the transformation of the original problem involves the introduction of an additional artificial design variable $b_{n+1} \in \mathrm{R}$. After that one can define a new objective function as:

$$\hat{g}_0(\mathbf{b}) = b_{n+1} \tag{14}$$

in nove dodatne omejitvene pogoje, ki se računajo v stalnih, enakomerno oddaljenih točkah s_i:

and impose the following additional constraints calculated at fixed, equally spaced, points s_i:

$$\left[\frac{(v-w)^2}{c^2}\right]_{s=s_i} - b_{n+1} \le 0, \quad i = 1,\ldots, j_v \tag{15}$$

kjer je $s_i = i/j_v$ in j_v označuje število vseh točk s_i. Poudariti je treba, da je dosežen učinek minimizacije nove namenske funkcije (14) z upoštevanjem dodanih pogojev (15) v bistvu enak kakor pri minimizaciji izvirne namenske funkcije (7).

Poleg v namenski funkciji, vsebujejo operatorja 'min' in 'max' tudi omejitveni pogoji, ki se nanašajo na krivinski polmer in pospešek bata. Na srečo lahko pri teh pogojih ugotovimo, da preoblikovanje ni potrebno zaradi naslednjih dejstev: funkciji $\hat{e}$ in $\hat{a}$ imata lahko glede na s sicer več ekstremov, katerih lega in velikost se spreminjata pri spremembi vrednosti $\mathbf{b}$. Vendar so praktične izkušnje pokazale, da isti ekstremi ostanejo globalni pri zmernem spreminjanju vrednosti projektnih spremenljivk. Z drugimi besedami, operatorja 'min' in 'max' v (8) in (11) vrneta vedno ista ekstrema. Pod to predpostavko so veličine ξ, λ in ζ odvedljive po $\mathbf{b}$, tako da pogojev (10) in (12) ni treba preoblikovati.

where $s_i = i/j_v$ and j_v is the number of all points s_i. It should be noted that the effect of minimizing the new objective function (14) and taking into account the constraints (15) is essentially the same as the effect of minimizing the old objective function (7).

The operators 'min' and 'max' are also present in the constraints related to the local radius and the plunger acceleration. Fortunately these constraints do not require any transformation because of the following facts: the functions $\hat{e}$ and $\hat{a}$ may have with respect to s several minima and maxima whose position and level is changes as $\mathbf{b}$ is changes. However, practical experience has shown that usually the same minima and maxima remain global if the design variables are varied in a reasonable range. In other words, the operators 'min' and 'max' in (8) and (11) always return the same extremes. Under this assumption, the quantities ξ, λ and ζ are differentiable with respect to $\mathbf{b}$ and no transformation of the constraints (10) and (12) is necessary.

Glede na omenjeno je končna oblika problema optimalnega projektiranja, ki se dejansko rešuje, podana z namensko funkcijo (14), omejitvenimi pogoji (10), (12), (13) in (15) ter z enačbo stanja (6). Za reševanje tega problema se načelno lahko uporabi katerakoli gradientna metoda matematičnega programiranja. V tem članku je v ta namen uporabljena učinkovita aproksimacijska metoda [8] in [9], ki je v splošnem zelo preprosta za uporabo.

Based on the transformation and assumptions stated above, the final form of the optimal design problem that is actually solved, is given by the objective function (14), the constraints (10), (12), (13) and (15) and by the state equation (6). This problem may be solved by virtually any gradient based method of mathematical programming. In this paper, an effective and simple-to-use approximation method [8] and [9] was adopted for this purpose.

5 ŠTEVILČNI ZGLEDI

Za potrditev teorije smo naredili veliko številčnih zgledov. Štirje, morda najbolj predstavni, so prikazani v nadaljevanju. Zgledi so označeni s simboli A1, A2, B1 in B2.

V primerih A1 in A2 je krivulja $\mathcal{K}$ definirana s 4 nadzornimi točkami, ki vključujejo 2 projektni spremenljivki. Koordinate nadzornih točk so:

5 NUMERICAL EXAMPLES

To illustrate the theory, many numerical examples have been performed. In this paper, four of perhaps most representative examples are presented. This examples are denoted by the symbols A1, A2, B1 and B2.

In examples A1 and A2, the curve $\mathcal{K}$ is defined by 4 control points involving 2 design variables. The coordinates of these control points are:

$$
\begin{aligned}
q_{1x} &= r_0\cos(\varphi_0), & q_{1y} &= r_0\sin(\varphi_0)\\
q_{2x} &= q_{1x} - b_1, & q_{2y} &= \frac{r_0^2 - q_{1x}q_{2x}}{q_{1y}}\\
q_{3x} &= q_{4x} + b_2, & q_{3y} &= \frac{\left(r_0+\hat{h}\right)^2 - q_{4x}q_{3x}}{q_{4y}} \qquad (16),\\
q_{4x} &= \left(r_0+\hat{h}\right)\sin\left(\varphi_{\hat{h}}\right), & q_{4y} &= \left(r_0+\hat{h}\right)\cos\left(\varphi_{\hat{h}}\right),
\end{aligned}
$$

kjer je $\hat{h}$ maksimalni dvig bata tlačilke, φ_0 pomeni kot odmične gredi pri ničelnem dvigu bata, $\varphi_{\hat{h}}$ je kot odmične gredi pri največjem dvigu bata tlačilke, q_{ix} in q_{iy} sta koordinati x in y i-te nadzorne točke. Začetni vrednosti projektnih spremenljivk sta v obeh primerih enaki: ${}^0b_1 = {}^0b_2 = 4$.

V primerih B1 in B2 je krivulja $\mathcal{K}$ definirana s 6 nadzornimi točkami, ki vključujejo 6 projektnih spremenljivk. Koordinate teh nadzornih točk so naslednje:

where $\hat{h}$ is the maximum pump plunger lift, φ_0 denotes the angle of zero pump plunger lift, $\varphi_{\hat{h}}$ denotes the angle of maximum pump plunger lift, q_{ix} and q_{iy} are x and y-coordinates of the i-th control point. The initial values of the design variables in both examples have been chosen as ${}^0b_1 = {}^0b_2 = 4$.

In examples B1 and B2 the curve $\mathcal{K}$ is defined by 6 control points involving 6 design variables. The coordinates of these control points are:

$$
\begin{aligned}
q_{1x} &= r_0\cos(\varphi_0), & q_{1y} &= r_0\sin(\varphi_0)\\
q_{2x} &= q_{1x} - b_1, & q_{2y} &= \frac{r_0^2 - q_{1x}q_{2x}}{q_{1y}}\\
q_{3x} &= q_{2x} - b_2, & q_{3y} &= q_{2y} + b_3 \qquad (17).\\
q_{4x} &= q_{3x} - b_4, & q_{4y} &= q_{3y} + b_5\\
q_{5x} &= q_{6x} + b_6, & q_{5y} &= \frac{\left(r_0+\hat{h}\right)^2 - q_{6x}q_{5x}}{q_{6y}}\\
q_{6x} &= \left(r_0+\hat{h}\right)\sin\left(\varphi_{\hat{h}}\right), & q_{6y} &= \left(r_0+\hat{h}\right)\cos\left(\varphi_{\hat{h}}\right)
\end{aligned}
$$

V zgledih B1 in B2 so izbrane naslednje začetne vrednosti projektnih spremenljivk:

In both examples the initial values of the design variables have been chosen as:

$$ {}^0b_1 = {}^0b_6 = 3\,,\ {}^0b_2 = {}^0b_3 = {}^0b_4 = {}^0b_5 = 1\,. $$

Zgleda A1 in A2 kakor tudi B1 in B2 se razlikujeta po različno izbranem poteku idealne hitrosti bata tlačilke, preostali vhodni podatki pa so enaki:

The examples A1 and A2, as well as the examples B1 and B2, differ by the shape of the ideal pump plunger velocity history, while all other input data remain the same:

$$r_0 = 17\text{mm},\ r_{rol} = 11.35\text{mm},\ \varphi_0 = 35^\circ\text{OG},\ \varphi_{\hat{h}} = 7^\circ\text{OG},\ \hat{h} = 11\text{mm},\ j_v = 100,$$
$$\xi^L = 3\text{mm},\ \lambda^U = 80\text{mm},\ \zeta^L = -80\text{mm},\ b_l = 0.001\text{mm},\ b_u = 10\text{mm},\ i = 1,\dots,n.$$

Vsi zgledi so bili uspešno rešeni z uporabljenim optimizacijskim podprogramom. Učinek optimizacije vrednosti projektnih spremenljivk, podanih v preglednici 1, je najbolje ponazorjen s primerjavo začetne, želene in optimalne relativne hitrosti bata tlačilke na slikah 2 do 5.

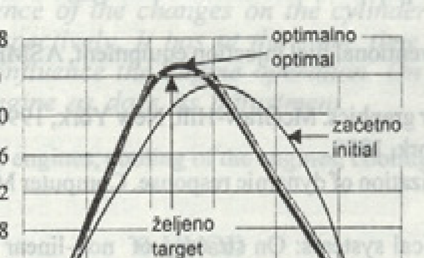
All examples have been solved nicely with the employed optimizer. The effect of optimization of the design variable values, given in Table 1, is best illustrated by comparing the initial, target and optimum relative pump plunger velocity histories given in figures from 2 to 5.

Preglednica 1 *Optimalne vrednosti projektnih spremenljivk*
Table 1 *Optimum values of design variables*

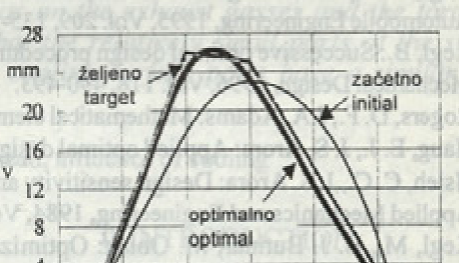
Zgled Case	b_1	b_2	b_3	b_4	b_5	b_6	b_{n+1}
A1	6,218	7,136					0,00210
A2	6,805	7,482					0,00529
B1	3,549	0,395	3,538	1,411	2,225	6,108	0,00146
B2	4,239	0,228	4,500	1,014	3,046	5,753	0,00414

V zadnjem stolpcu preglednice 1 so podane vrednosti namenske funkcije v optimalni točki. Po pričakovanjih so rezultati optimizacije v zgledih B1 in B2 boljši kakor v zgledih A1 in A2. To je posledica večje prilagodljivosti krivulje K pri uporabi večjega števila nadzornih točk. Seveda pa je treba poudariti, da nadaljnje povečevanje števila nadzornih točk prek 6 kakovosti končnega projekta ne bi več bistveno izboljšalo.

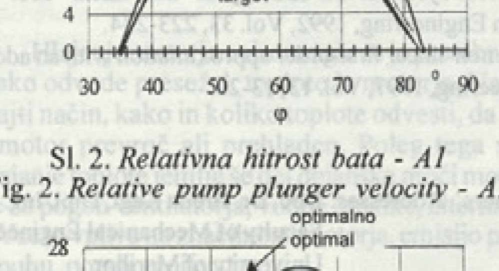
In the last column of Table 1, the values of the objective function at optimum points are given. As expected, examples B1 and B2 always yield a slightly better result than the corresponding examples A1 and A2. This is a consequence of the larger flexibility of K, facilitated by using more control points. It should be noticed, however, that increasing the number of control points beyond 6 would bring no significant further improvement to the final design.



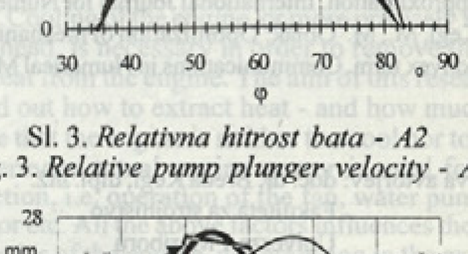
Sl. 2. *Relativna hitrost bata - A1*
Fig. 2. *Relative pump plunger velocity - A1*



Sl. 3. *Relativna hitrost bata - A2*
Fig. 3. *Relative pump plunger velocity - A2*



Sl. 4. *Relativna hitrost bata - B1*
Fig. 4. *Relative pump plunger velocity - B1*



Sl. 5. *Relativna hitrost bata - B2*
Fig. 5. *Relative pump plunger velocity - B2*

6 SKLEP

Glede na prikazane rezultate lahko sklenemo, da daje uporaba predlaganega postopka dobre rezultate z relativno malo računskega časa. Predstavljen problem optimalnega projektiranja odmikala je mogoče zelo preprosto prilagoditi usmeritvam razvoja tlačilk za dieselske vbrizgalne sisteme. Tako lahko dopolnjujemo ali spreminjamo namensko funkcijo, omejitvene pogoje ter izbiro projektnih spremenljivk glede na trenutne zahteve.

6 CONCLUSION

In accordance with the results presented, one may conclude that the proposed procedure yields good results with a relative small computational effort. The proposed problem of cam optimum design can be quite straightforwardly adjusted or modified in order to follow the trend of development of diesel fuel injection systems. In other words, it is easily possible to supplement or to modify the objective function, the constraints and the choice of the design variables according to the immediate requirements.

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