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Stabilnost antenskega stebra ob upoštevanju hkratnega delovanja lastne teže in zunanje obremenitve

The Stability of an Antenna Column under the Simultaneous Action of its Own Weight and an Effective Load

Juraj Saucha - Jerko Radoš - Čedomir Ivaković

Prispevek obravnava določitev elastične stabilnosti antenskega stebra ob hkratnem delovanju lastne teže in zunanje vzdolžne tlačne obremenitve. Določitev kritične kombinacije lastne teže stolpa in vzdolžne tlačne sile temelji na rešitvi vodilne diferencialne enačbe stebra v obliki Maclaurinove vrste s hitro konvergenco zaradi uporabljene zamenjave. Dobljeni rezultati s kakovostjo natančne rešitve vodilne diferencialne enačbe so primerni za oceno natančnosti približne rešitve, ki predpostavlja linearno sodelovanje teže stebra in vzdolžne tlačne sile.Ugotovili smo, da so rezultati približne rešitve znotraj dveh odstotkov.

(Ključne besede: stebri antenski, stabilnost, obremenitve kritične, enačbe diferencialne)

This paper deals with the determination of the elastic stability of an antenna column subjected to the simultaneous actions of its own weight and an effective load as an axial compressive force. The determination of the critical combination of the weight of the column and the axial compressive force was based upon the solution to the governing differential equation of the column in the form of a Maclaurin's series with a rapid convergence due to the introduced substitution. The obtained results, with a quality of the exact solution to the governing differential equation, were relevant for the evaluation of the accuracy of the approximate solution which assumed the linear interaction of the weight of the column and the axial compressive force. It has been established that the approximate solution differed from the results obtained in this paper by -2 %.

(Keywords: antenna columns, stability, critical loads, diferential equations)

0 UVOD

Antenski steber smo analizirali in oblikovali kot nosilec, saj je obremenjen upogibno zaradi prečne obremenitve (vihar), in tlak zaradi lastne teže in zunanje obremenitve. V običajni praksi zapleteno analizo stebra drugega reda nadomestimo s približno, ki uporablja povečevalni količnik. Največji moment drugega reda preprosto dobimo z množenjem največjega momenta prvega reda s tem količnikom ([1] in [2]). V naslednjem približku vse druge momente prerezov vzdolž antenskega stebra dobimo z množenjem prvih momentov s povečevalnim količnikom [6]. Da določimo vrednost povečevalnega količnika, moramo najprej preučiti stabilnost stebra, saj je povečevalni količnik odvisen od razmerja dejanske vzdolžne obremenitve in kritične vzdolžne sile stebra.

Določitev kritične vzdolžne sile antenskega stebra izhaja iz preučitve elastične

0 INTRODUCTION

An antenna column has to be analyzed and designed as a beam column because it is subjected to both bending, due to transverse loading, and compression, due to its own weight and an effective load. In practice, the complicated second-order analysis of a beam column is replaced, in an approximate sense, by the use of the magnification or amplification factor. The maximum second-order moment is obtained simply by multiplying the maximum first-order moment by this factor ([1] and [2]). In a further approximation, all the secondary moments along the antenna column can be obtained by increasing the primary moments by the amplification factor [6]. To determine the value of the amplification factor the stability of the column has to be analyzed first, because the amplification factor is a function of the ratio of the actual axial load to the critical axial load of the column.

The determination of the critical axial load of the antenna column results from a study of the elastic stabilnosti nosilnega stebra z zvezno spremenljivim prerezom ob hkratnem delovanju njegove lastne teže (porazdeljena vzdolžna obremenitev) in zunanje obremenitve (vzdolžna tlačna sila, delujoča na prostem koncu). Približno vrednost kritične kombinacije teže stebra in vzdolžne tlačne sile dobimo ob predpostavki linearne povezanosti teže stebra in vzdolžne tlačne sile. To kritično kombinacijo izrazimo s kritično težo stebra in kritično vzdolžno tlačno silo, ki deluje na steber zanemarljive lastne teže [6].

Kritično vzdolžno tlačno silo nosilnega stebra z zvezno spremenljivim prerezom določimo po razcepnem postopku z reševanjem vodilne diferencialne enačbe z uporabo Besselovih funkcij ([4] in [7]). Kritično lastno težo takega stebra približno določimo (vendar z veliko stopnjo natančnosti) z reševanjem vodilne diferencialne enačbe v obliki neskončne potenčne vrste [5], ali pa z uporabo energijskega postopka po Galerkinu [6]. Približne energijske postopke lahko uporabimo za določitev kritične kombinacije lastne teže stebra in vzdolžne tlačne sile, npr. po postopku Rayleigh-Ritz [3].

Prikazana proučitev elastične stabilnosti antenskega stebra, obravnavanega kot nosilni steber z zvezno spremenljivim prerezom, izpostavljenim hkratnemu delovanju lastne teže in vzdolžne tlačne sile, uporablja razcepni postopek. Določitev kritične kombinacije teže stebra in vzdolžne tlačne sile uporablja rešitev vodilne diferencialne enačbe v obliki Maclaurinove vrste. Določitev kritične obremenitve je izvedena z največjo natančnostjo. Dobljeni rezultati s kakovostjo natančne rešitve vodilne enačbe so primerni za ovrednotenje natančnosti približne rešitve, ki predpostavlja linearno povezanost teže stebra in vzdolžne tlačne sile. To ovrednotenje je namen pričujočega proučevanja.

stability of a cantilevered column with a continuously changing cross-section subjected to the simultaneous action of its own weight (a distributed axial load) and the effective load (an axial compressive force acting at the free end) . The approximate value of the critical combination of the weight of the column and the axial compressive force can be obtained by assuming a linear interaction of the weight of the column and the axial compressive force. This critical combination is expressed by the critical weight of the column and the critical axial compressive force acting on the column with a negligible weight of its own [6].

The critical compressive axial force of the cantilevered column with a continuously changing cross-section can be determined, in a bifurcation approach, by solving the governing differential equation by means of Bessel functions ([4] and [7]). The critical intrinsic weight of such a column can be determined approximately (yet with a high degree of accuracy) by a solution to the governing differential equation in the form of an infinite power series [5] or, using an energy approach, by Galerkin's method [6].The approximate energy methods can be used for determing the critical combination of the intrinsic weight of the column and the axial compressive force, for instance the Rayleigh-Ritz method [3].

The presented study of the elastic stability of an antenna column, treated as a cantilevered column with a continuously changing cross-section subjected to the simultaneous action of its own weight and the axial compressive force, uses the bifurcation approach. The determination of the critical combination of the weight of the column and the axial compressive force is determined from the solution to the governing differential equation in the form of a Maclaurin's series. The determination of the critical load has been worked out with maximum precision. The obtained results, with a quality of the exact solution to the governing equation, are relevant for the evaluation of the accuracy of the approximate solution assuming the linear interaction of the weight of the column and the axial compressive force. This evaluation is the aim of this study.

1 POSTAVITEV VODILNE DIFERENCIALNE ENAČBE

1 FORMULATION OF THE GOVERNING DIFFERENTIAL EQUATION

Slika 1 prikazuje steber z zvezno spremenljivim prerezom. Vztrajnostni moment prereza stebra se vzdolž smeri x spreminja parabolično:

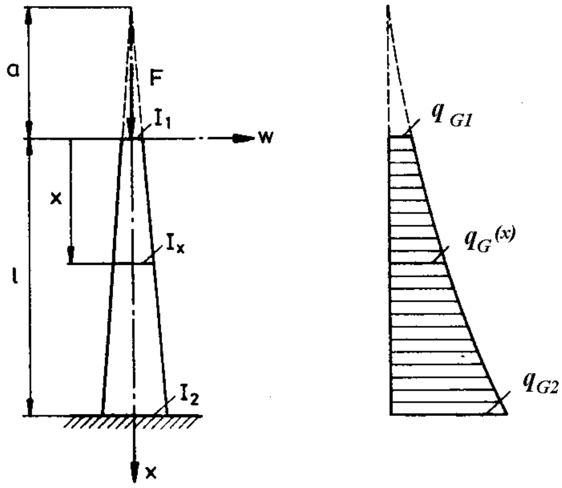
Fig. 1 shows a cantilevered column with a continuously variable cross-section. The moment of inertia of the column cross-section varies along the x axis in accordance with the parabolic law:

$$I(x) = I_2\left(\frac{a+x}{a+l}\right)^n \qquad (1)$$

in porazdeljena vzdolžna sila zaradi lastne teže stebra je prav tako parabolična:

and the distributed axial load due to the intrinsic weight of the column in accordance with the parabolic law:

$$q_G(x) = q_{G2}\left(\frac{a+x}{a+l}\right)^p \qquad (2),$$



Sl. 1. *Steber z zvezno spremenljivim prerezom*
Fig. 1. *Column of variable cross-section*

kjer sta n in p nespremenljiva, odvisno od geometrijske oblike stebra in njegovega prereza.

Ravnovesna enačba stebra ima obliko [6]:

where n and p are constants depending on the geometry of the column and its cross-section.

The equilibrium equation of this column takes the form [6]:

$$\left[EI(x)w''\right]'' + F_a(x)w'' + q_G(x)w' = 0 \quad (3),$$

kjer so $(\ )' = \frac{d}{dx}(\ ), (\ )'' = \frac{d^2}{dx^2}(\ )$ in

where $(\ )' = \frac{d}{dx}(\ ), (\ )'' = \frac{d^2}{dx^2}(\ )$ and

$$F_a(x) = F + \int_0^x q_G(x)dx \quad (4).$$

Z vstavitvijo (4) v (3) dobimo po integraciji (integracija tretjega člena po delih):

Substituting (4) into (3) after integrating (including integrating by parts of the third term) gives:

$$\left[EI(x)w''\right]'' + Fw' + w'\int_0^x q_G(x)dx + C_1 = 0 \quad (5)$$

Ob predpostavitvi majhnih pomikov w, lahko prečno silo F_T izrazimo [1]

Assuming we have small deflections w, the transverse force F_T can be expressed as [1]:

$$F_T = F_L w' + F_Q \quad (6),$$

kjer je vzdolžna sila:

where the longitudinal force is:

$$F_L = -F_a = -F - \int_0^x q_G(x)dx \quad (7)$$

in strižna sila:

and the shearing force is:

$$F_Q = \frac{dM}{dx} = \frac{d}{dx}\left[-EI(x)w''\right] = -\left[EI(x)w''\right]' \quad (8).$$

Za $x = 0$, $F_L = -F$ in $F_T = 0$, dobimo:

For $x = 0$, $F_L = -F$ and $F_T = 0$:

$$F_Q(0) - Fw'(0) = 0 \quad (9).$$

Prav tako je:

Further:

$$F_Q(0) = \left(\frac{dM}{dx}\right)_{x=0} = \frac{d}{dx}\left[-EI(x)w''\right]_{x=0} = -E\left[\frac{dI(x)}{dx}\right]_{x=0} w''(0) - EI(0)w'''(0) \quad (10).$$

Upogibni moment M je nič na prostem koncu stebra. Tako je:

The bending moment M vanishes at the free end of the column. Hence:

$$M(0) = -EI(0)w''(0) = 0 \tag{11}$$

in zato:

and thus:

$$w''(0) = 0 \tag{12}$$

Z vstavitvijo (12) v (10) dobimo:

Substituting (12) into (10) gives:

$$F_Q(0) = -EI(0)w'''(0) \tag{13}$$

in (16) v (9) daje:

and subsituting (16) into (9) gives:

$$EI(0)w'''(0) + Fw'(0) = 0 \qquad (14).$$

Enačba (5) za $x = 0$ dobi obliko:

Equation (5) for $x = 0$ takes the form:

$$Ew''(0)\left[\frac{dI(x)}{dx}\right]_{x=0} + EI(0)w'''(0) + Fw'(0) + C_1 = 0 \qquad (15).$$

Upoštevajoč (14) in (12) sledi:

Considering (14) and (12) , it follows that:

$$C_1 = 0$$

Integral tretjega člena v enačbi (5), upoštevajoč (2), daje:

The integral in the third term of equation (5), considering (2), yields:

$$\int_0^x q_G(x)dx = \frac{q_{G2}}{(a+l)^p} \cdot \frac{(a+x)^{p+1} - a^{p+1}}{p+1} \qquad (16).$$

Teža stebra je:

The weight of the column is:

$$F_G = \int_0^l q_G(x)dx = \frac{q_{G2}}{(a+l)^p} \cdot \frac{(a+l)^{p+1} - a^{p+1}}{p+1} \tag{17}$$

Vstavitev (1) v (16) daje:

substituting (17) into (16) gives:

$$\int_0^x q_G(x)dx = F_G \frac{(a+x)^{p+1} - a^{p+1}}{(a+l)^{p+1} - a^{p+1}} \qquad (18).$$

Če (1) in (18) vstavimo v (5), dobimo po deljenju z $EI_2/(a+l)^n$

If (1) and (18) are substituited into (5), it follows after dividing by $EI_2/(a+l)^n$ that:

$$\left[(a+x)^n w''\right]' + \frac{F}{EI_2}(a+l)^n w' + \frac{F_G}{EI_2}(a+l)^n \frac{(a+x)^{p+1} - a^{p+1}}{(a+l)^{p+1} - a^{p+1}} = 0 \qquad (19).$$

Če uvedemo konstanti k_1 in k_2, dobimo:

If constants k_1 and k_2 are introduced as follows:

$$k_1 = \frac{F_G}{EI_2}(a+l)^n \frac{1}{(a+l)^{p+1} - a^{p+1}} \qquad (20),$$

$$k_2 = \frac{F}{EI_2}(a+l)^n \qquad (21).$$

Vstavitev k_1 in k_2 v (19) daje:

substitution of k_1 and k_2 into (19) gives:

$$\left[(a+x)^n w''\right]' + \left\{k_1\left[(a+x)^{p+1} - a^{p+1}\right] + k_2\right\} w' = 0 \qquad (22).$$

2 REŠITEV VODILNE DIFERENCIALNE ENAČBE

2 SOLUTION TO THE GOVERNING DIFFERENTIAL EQUATION

Natančna rešitev enačbe (22) ne obstaja. Rešitev v obliki Maclaurinove vrste tudi ni ustrezna.

There is not an exact solution to equation (22). The solution in the form of a Maclaurin's series has not been

Konvergenca vrste je odvisna od zveze $I_2/I_1 < 2^n$, kar omejuje uporabnost rešitve na stebre z majhno koničnostjo. Poleg tega je konvergenca zelo počasna, kar zahteva izračunavanje velikega števila členov vrste.

Zato smo, da bi dobili vrsto s hitro konvergenco, tudi za velika razmerja I_2/I_1, uporabili nadomestitev:

satisfactory. The convergence of the series was conditioned by the relation $I_2/I_1 < 2^n$, which limited the application of the solution to the columns with small conicity. In addition, the convergence was very slow, which meant that a lot of terms in the series had to be calculated.

Therefore, with the aim of obtaining a series with a rapid convergence, which exists even for the big ratia I_2/I_1, a substitution t was introduced:

$$t = -b \cdot \ln\left(1 + \frac{x}{a}\right) \qquad (23),$$

kjer je b ustrezna konstanta. Če zapišemo:

Here b is an arbitrary constant. If it is written:

$$\frac{dw}{dt} = v, \frac{d^2w}{dt^2} = v', \frac{d^3w}{dt^3} = v'' \qquad (24),$$

dobi enačba (22) obliko:

equation (22) takes the form:

$$v'' - \frac{n-3}{b}v' - \frac{n-2}{b^2}v + K_1\left[e^{(n-p-3)\frac{1}{b}} - e^{(n-2)\frac{1}{b}}\right]v + K_2 e^{(n-2)\frac{1}{b}}v = 0 \qquad (25),$$

kjer sta

where:

$$K_1 = k_1 \frac{a^{(p+3-n)}}{b^2} \qquad (26),$$

$$K_2 = k_2 \frac{a^{-(n-2)}}{b^2} \qquad (27).$$

Rešitev enačbe (25) lahko iščemo v obliko Maclaurinove vrste:

The solution to equation (25) can be sought in the form of a Maclaurin's series:

$$v = v_0 + \frac{t}{1!}v_0' + \frac{t^2}{2!}v_0'' + ...+ \frac{t^r}{r!}v_0^{(r)} + ... \qquad (28).$$

Odvode $v_0^{(r)} = \left[v^{(r)}\right]_{t=0}$ lahko določimo po naslednjem postopku. Iz robnih pogojev $w' = C_2$ za $x = 0$ (strmina elastične krivulje na prostem koncu stebra) sledi ob upoštevnaju $t(x = 0) = -b \cdot 1n(1 + 0) = 0$ po (23):

The derivatives $v_0^{(r)} = \left[v^{(r)}\right]_{t=0}$ can be determined as follows. From the boundary condition $w' = C_2$ for $x = 0$ (the slope of the elastic curve at the free end of the column), it follows, considering $t(x = 0) = -b \cdot 1n(1 + 0) = 0$ by (23):

$$\left(\frac{dw}{dx}\right)_{x=0} = -\frac{b}{a}v_0 e^{\frac{0}{b}} = C_2$$

in zato:

and hence:

$$v_0 = -\frac{a}{b}C_2 = C \qquad (29).$$

Iz robnega pogoja $w''(x = 0) = 0$, zaradi $M(x = 0) = 0$ (upogibni moment na prostem koncu stebra), sledi ob upoštevanju $t(x = 0) = 0$:

From the boundary condition $w''(x = 0) = 0$, because $M(x = 0) = 0$ (the bending moment at the free end of the column) , it follows, considering $t(x = 0) = 0$:

$$\left(\frac{d^2w}{dx^2}\right)_{x=0} = \frac{b^2}{a^2}v_0' e^{2\frac{0}{b}} + \frac{b}{a^2}v_0 e^{2\frac{0}{b}} = 0$$

in zato:

and hence:

$$v_0' = -\frac{1}{b}C \qquad (30).$$

Iz (25), upoštevajoč $t(x = 0) = 0$ ter (29) in (30), izhaja:

From (25), considering $t(x = 0) = 0$ and (29) and (30), it follows that:

$$v_0'' = \frac{1}{b^2}C \qquad (31).$$

Zaporedno odvajanje v'', izraženo iz (25) za

Successive derivating of v'' expresed from

$t = 0$, upoštevajoč (29), (30) in (31) daje:

(25) for $t = 0$ considering (29),(30) and (31), produces:

$$v_0''', v_0^{IV}, ..., v_o^{(r)}, ...$$

Nadalje uporabimo robni pogoj $w'(x = l) = 0$. Zaradi (23):

Further, the boundary condition $w'(x = l) = 0$ has to be used. According to (23):

$$t(x = l) = t_1 = -b \cdot \ln\left(\frac{a + l}{a}\right) \quad (32).$$

Iz (1), za $x = 0$ je:

Following (1), for $x = 0$:

$$\frac{a + l}{a} = \left(\frac{I_2}{I_1}\right)^{\frac{1}{n}} \quad (33),$$

kar vstavljeno v (32) daje:

which substituted into (32) gives:

$$t_1 = -b \cdot \ln\frac{I_2}{I_1} \quad (34).$$

Robni pogoj $w'(x = l) = 0$ daje:

The boundary condition $w'(x = l) = 0$ gives:

$$\left(\frac{dw}{dx}\right)_{x=l} = \left(\frac{dw}{dt}\right)_{t=t_1} \cdot \left(\frac{dt}{dx}\right)_{x=l} = 0 \quad (35)$$

ali, upoštevajoč (24) in (23),

or, considering (24) and (23):

$$(v)_{t=t_1} \cdot \left(-\frac{b}{a}\right) \cdot \frac{1}{1 + \frac{l}{a}} = 0 \quad (36)$$

in zato:

and hence:

$$(v)_{t=t_1} = 0 \quad (37).$$

Če (37) izrazimo z (28), dobimo:

If (37) is expressed by (28), it follows that:

$$v_0 + \frac{t_1}{1!} v_0' + \frac{t_1^2}{2!} v_0'' + ... + \frac{t_1^r}{r!} v_0^{(r)} + ... = 0 \quad (38).$$

Izraz (38) vodi k enačbi $P(K_1,K_2) = 0$ (pomeni polinom), če količniki, s katerimi množimo odvode $v_0^{(r)}$ v (38), dajejo konvergentno vrsto.

Odvod $v_0^{(r)}$, dobljen kot v'' iz (25), odvajamo $(r - 2)$ – krat zaporedno, dobi obliko:

The expression (38) leads to the equation $P(K_1,K_2) = 0$, (P denotes polynomial) , if the coefficients multiplying the derivatives $v_0^{(r)}$ in (38) form a converging series.

The derivative $v_0^{(r)}$, obtained if v'' expressed from (25) is $(r - 2)$ - times consecutively derivated, takes the form:

$$\begin{aligned} v_0^{(r)} = {} & \frac{n-3}{b} v_0^{(r-1)} + \frac{n-2}{b^2} v_0^{(r-2)} - K_1 \Bigg[v_0^{(r-2)} + \frac{(r-2)}{1!} \cdot \frac{n-p-3}{b} v_0^{(r-3)} + ... \\ & ... + \frac{(r-2)(r-3)...(r-i)}{(i-1)!} \cdot \left(\frac{n-p-3}{b}\right)^{i-1} v_0^{[r-(i+1)]} + ... \\ & ... + \frac{(r-2)(r-3)...[r-(r-1)]}{(r-2)!} \cdot \left(\frac{n-p-3}{b}\right)^{r-2} v_0 \Bigg] + \\ & + (K_1 - K_2) \Bigg[v_0^{(r-2)} + \frac{(r-2)}{1!} \cdot \frac{n-2}{b} \cdot v_0^{(r-3)} + ... \\ & ... + \frac{(r-2)(r-3)...(r-i)}{(i-1)!} \cdot \left(\frac{n-2}{b}\right)^{i-1} v_0^{[r-(i+1)]} + ... \\ & ... + \frac{(r-2)(r-3)...[r-(r-1)]}{(r-2)!} \cdot \left(\frac{n-2}{b}\right)^{r-2} v_0 \Bigg] \end{aligned} \quad (39).$$

Analiza izraza (39) kaže, da lahko (38) zapišemo v obliki:

The analysis of the expression (39) shows that (38) can be written in the form:

$$\sum_{i=1}^{\infty}\left(\sum_{r=0}^{\infty} c_{r,i}\frac{t_1^r}{r!}\right)\cdot\left(K_1^q K_2^s\right)_i = 0 \qquad (40),$$

kjer je $c_{r,i}$ količnik, ki odvod $v_0^{(r)}$ v (38) množi z zmnožkom $\left(K_1^q K_2^s\right)_i$. (Tu je i povezan z določeno dvojico vrednosti eksponentov q in s). Raziskava konvergence vrste:

where $c_{r,i}$ is the coefficient which in the derivative $v_0^{(r)}$, appearing in(38), multiplies the product $\left(K_1^q K_2^s\right)_i$. (Here i is connected with a particular couple of the values of the exponents q and s.) The investigation of the convergence of the series:

$$\sum_{r=0}^{\infty} c_{r,i}\frac{t_1^r}{r!} \qquad (41)$$

je pokazala, da je to hitro konvergentna vrsta, katere konvergentnost je hitrejša, če je I_2/I_1 manjši ter konvergenca obstaja za vsako razmerje I_2/I_1 in zato lahko izberemo b = 1. Upoštevajoč hitro konvergenco (41), lahko vsoto C_i aproksimiramo kot:

showed that this was a rapidly converging series, with a convergence more rapid as I_2/I_1 becomes smaller and that a rapid convergence existed for any ratio I_2/I_1. The convergence does not depend upon the arbitrary constant b introduced in (23) and thus its value can be chosen as b = 1. Considering the rapid convergence of (41) its sum C_i can be approximated as follows:

$$C_i = \sum_{r=0}^{R} c_{r,i}\frac{t_1^r}{r!} \qquad (42),$$

kjer je R dovolj visoka stopnja najvišjega odvoda $v_0^{(R)}$, izračunanega v (39). Prav tako se je izkazalo, če je R dovolj visoka, C_i množi zmnožek $\left(K_1^q K_2^s\right)_i$ z eksponentoma q in s, proti največjim vrednostim v odvodih $v_0^{(r)}$ do $v_0^{(R)}$, praktično izgine. Zaradi tega, ob zadosti visokem R, vrste (40) praktično preide v vrsto:

where R is a sufficiently high order of the highest derivative $v_0^{(R)}$ calculated in (39). Further, it is clear that if R is sufficiently high, C_i multiplying products $\left(K_1^q K_2^s\right)_i$ with exponents q and s tending to the maximum values appearing in derivatives $v_0^{(r)}$ ending by $v_0^{(R)}$, practically vanishes. Considering this, with a sufficiently high R, series (40) is practically equal to the series:

$$\sum_{i=0}^{i_{maks}}\left(\sum_{r=0}^{R} c_{r,i}\frac{t_1^r}{r!}\right)\cdot\left(K_1^q K_2^s\right)_i = 0 \qquad (43),$$

ki ima, zaradi končnega števila členov, obliko polinoma $P(K_1,K_2)$ z vrednostjo nič. Oznaka i_{maks} v enačbi (43) pomeni dvojico eksponentov q in s v kombinaciji z največjimi vrednostmi v odvodih $v_0^{(r)}$ do $v_0^{(R)}$.

Enačba $P(K_1,K_2) = 0$ omogoča določitev kritične kombinacije podkritične teže F_G stebra in tlačne sile F'_{cr} ki povzroči uklon sttebra določene geometrijske oblike. Zaradi (26), (27), (20), (21) in (1), upoštevajoč b = 1, dobimo:

which, due to a finite number of terms, has the form of a polynomial $P(K_1,K_2)$ equal to zero. The mark i_{max} appearing in (43) denotes a couple of exponents q and s in a combination of their maximum values appearing in derivatives $v_0^{(r)}$ ending by $v_0^{(R)}$.

The equation $P(K_1,K_2) = 0$ makes it possible to determine the critical combination of the subcritical weight F_G of the column and the compressive force F'_{cr} which will cause buckling of a column with a particular geometry. According to (26),(27),(20),(21) and (1), considering b = 1 it follows that:

$$K_1 = \frac{F_G}{EI_2}\cdot\frac{I_2}{I_1}\cdot\frac{a^2}{\left(\frac{I_2}{I_1}\right)^{\frac{p+1}{n}}-1} \qquad (44),$$

$$K_2 = \frac{F}{EI_2}\cdot\frac{I_2}{I_1}\cdot a^2 \qquad (45).$$

Veličino a^2 lahko spremenimo, upoštevaje (1) v:

The quantity a^2 can be transformed, considering (1):

$$a^2 = \frac{a^2}{l^2}\cdot l^2 = \frac{l^2}{\left(\frac{a+l}{a}-1\right)^2} = \frac{l^2}{\left[\left(\frac{I_2}{I_1}\right)^{\frac{1}{n}}-1\right]^2} \qquad (46).$$

Vstavitev (46) v (44) in (45) daje:

Substituting (46) into (44) and (45) gives:

$$K_1 = \frac{F_G l^2}{EI_2}\cdot\frac{\frac{I_2}{I_1}}{\left[\left(\frac{I_2}{I_1}\right)^{\frac{1}{n}}-1\right]^2}\cdot\frac{1}{\left(\frac{I_2}{I_1}\right)^{\frac{p+1}{n}}-1} \qquad (47)$$

$$K_2 = \frac{Fl^2}{EI_2} \cdot \frac{\frac{I_2}{I_1}}{\left[\left(\frac{I_2}{I_1}\right)^{\frac{1}{n}} - 1\right]^2} \quad (48).$$

Videli smo, da K_1 in K_2 vsebujeta F_G, F, I_2/I_1 in eksponenta n in p, ter zato enačba $P(K_1,K_2) = 0$ kakor je bilo povedano, omogoča določitev kombinacije F_G in F'_{cr}, ki povzroča uklon stebra znane geometrijske oblike.

Določitev kritične kombinacije $(F_G + F)_{cr}$ je bila izvedena kakor sledi. Najprej za $F = 0$ je enačba $P(K_1,K_2) = 0$ spremenjena v $P(K_1) = 0$. Iz najmanjšega korena K_{10} te enačbe, je določena kritična teža F_{Gcr} stebra, brez dodatne vzdolžne tlačne sile F, z uporabo enačbe (47).

Nato je za različna razmerja F_G/F_{Gcr}, ki določajo vrednost F_G in tako tudi vrednost K_1, enačba $P(K_1, K_2) = 0$ spremenjena v posebno enačbo $P(K_2) = 0$.

Iz najmanjšega korena K_{20} teh enačb, so bile kritične sile F'_{cr}, ki skupaj z dano F_G povzročajo uklon, določene z enačbo (48). Na ta način, za $F_G = 0$, je bila določena F_{cr0} kot kritična tlačna sila, ki deluje v stebru zanemarljive teže.

Vrednosti F_{Gcr}, F_{cr0} in F'_{cr} so bile ovrednotene za naslednje stebre z geometrijskimi oblikami, ki jih določata n in p:

1. $n = 4$, $p = 2$ (steber v obliki prisekanega stožca ali piramide oz. votlega prisekanega stožca ali piramide);
2. $n = 3$, $p = 1$ (to se nanaša na steber nespremenljive debeline in linearno odvisne širine prereza oziroma približno na steber v obliki stožčaste cevi nespremenljive debeline stene);
3. $n = 2$, $p = 0$ (to se približno nanaša na steber, zgrajen iz štirih palic nespremenljivega prereza, postavljenih vzdolž robov navidezne prisekane piramide, združenih z mrežno polnitvijo zanemarljive teže).

Veličina d_G je narisana v odvisnosti od razmerja I_2/I_1 (sl.1) ter za omenjene vrednosti n in p v diagramu 1. Kritična lastna teža stebra F_{Gcr} je povezana z d_G kot:

We can see that K_1 and K_2 contain F_G, F, I_2/I_1 and exponents n and p , and thus the equation $P(K_1,K_2) = 0$, as was mentioned above, makes possible the determination of the combination of F_G and F'_{cr} causing the buckling of a column of a particular geometry.

The determination of the critical combination $(F_G + F)_{cr}$ was carried out as follows. First, for $F = 0$ the equation $P(K_1,K_2) = 0$ transformed to $P(K_1) = 0$. From the smallest root K_{10} of this equation the critical weight F_{Gcr} of the column, which had not been loaded by the axial compressive force F, was determined with (47).

Then, for several ratia F_G/F_{Gcr}, which determined the value of F_G and thus the value of K_1, the equation $P(K_1, K_2) = 0$ transformed to particular equations $P(K_2) = 0$.

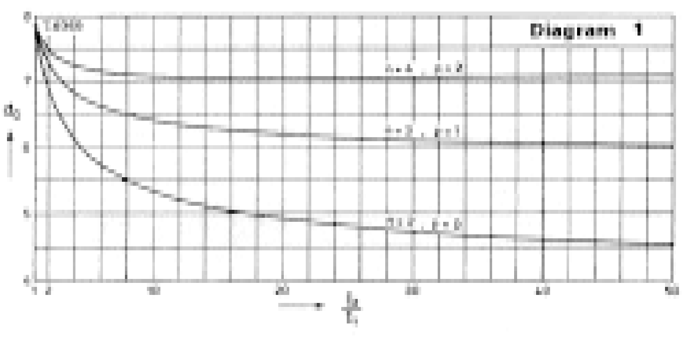
From the smallest root K_{20} of these equations the critical forces F'_{cr} which, in combination with the given F_G would cause buckling, were determined through (48). In this way, for $F_G = 0$,F_{cr0} was determined as the critical compressive force acting on the column of negligible weight.

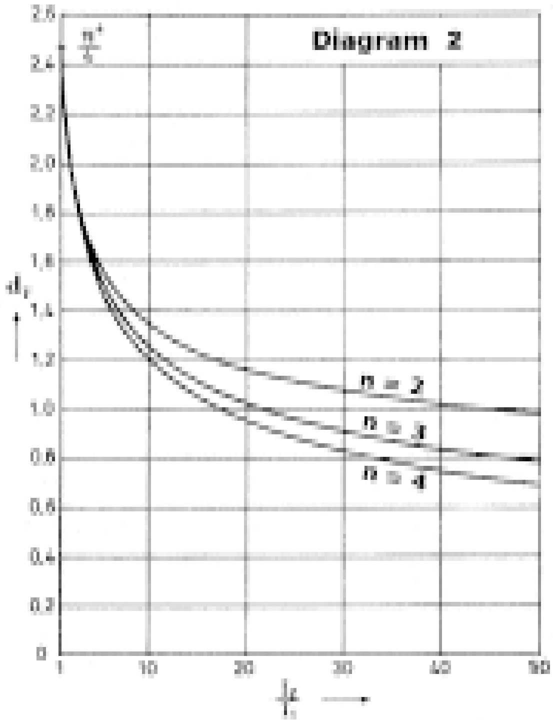
The values of F_{Gcr}, F_{cr0} and F'_{cr} were determined for the following columns with geometries defined by n and p :

1. $n = 4$, $p = 2$ (the column in the form of a truncated cone or pyramid resp. hollow truncated cone or pyramid);
2. $n = 3$, $p = 1$ (this is referring to the column with a constant width and linearly varying height of the cross-section or approximately to the column in the form of a conical tube of constant wall thickness);
3. $n = 2$, $p = 0$ (this is approximately referring to the column constructed from four rods of constant cross-section laid along the edges of a virtual truncated pyramid connected by the lattice filling of negligible weight).

The quantity d_G plotted as a function of the ratio I_2/I_1 (Fig.1) and for the values of n and p as mentioned above is shown in Diagram 1. The critical intrinsic weight of the column F_{Gcr} is expressed by d_G as follows:

$$F_{Gcr} = d_G \frac{EI_2}{l^2} \quad (49).$$





Veličina d_F je narisana v odvisnosti od razmerja I_2/I_1 ter za omenjene vrednosti n v diagramu 2. Kritična sila F_{cr0}, ki deluje na steber zanemarljive teže, je povezana z d_F kot:

The quantity d_F plotted as a function of the ratio I_2/I_1 and for the values of n mentioned above is shown in Diagram 2. The critical force F_{cr0} acting on the column of negligible weight is expressed by d_F as follows:

$$F_{cr0} = d_F \frac{EI_2}{l^2} \qquad (50).$$

3 PRIMERJAVA S PRIBLIŽNO REŠITVIJO

3 COMPARISON WITH THE APPROXIMATE SOLUTION

Približna rešitev problema s predpostavljeno linearno povezanostjo teže stebra in vzdolžne tlačne sile F je prikazana na sliki 2 z Dunkelreyevo premico (črtkana črta)

The approximate solution to the problem assuming a linear interaction of the weight of the column and the axial compressive force F, is shown in Fig.2 by Dunkelrey's straight line (dashed line)

$$\frac{F'_{crD}}{F_{cr0}} + \frac{F_G}{F_{Gcr}} = 1 \qquad (51),$$

kjer je F'_{crD} vzdolžna tlačna sila, ki bi skupaj s podkritično težo stebra F_G, povzročila uklon. Po postopku, ki je opisan v tem prispevku, so bile za ista razmerja F_G/F_{Gcr} dobljene večje vrednosti F'_{cr}. Te vrednosti so podane kot krivulja na sliki 2, ki se razlikuje od Dunkelreyeve premice za razliko Δ. Zato lahko zapišemo

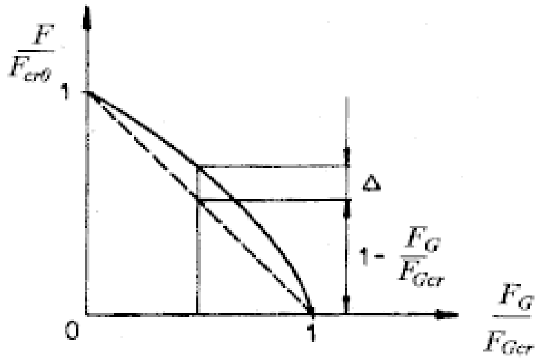
where F'_{crD} is the axial compressive force that would, in a linear interaction with a subcritical weight F_G of the column, cause buckling. With the procedure described in this paper for the same ratia F_G/F_{Gcr} larger values of F'_{cr} have been determined. These values are shown by the curve in Fig.2. This curve differs from Dunkelrey's line by a difference Δ. Thus it can be written that:

$$\frac{F'_{cr}}{F_{cr0}} + \frac{F_G}{G_{Gcr}} = 1 + \Delta \qquad (52).$$

Razmerje:

The ratio:

$$f = \frac{\Delta}{F'_{crD} / F_{cr0}} = \frac{F'_{cr} / F_{cr0}}{1 - F_G / F_{Gcr}} - 1 \qquad (53),$$



Sl.2. *Povezanost teže stebra in vzdolžne tlačne sile*
Fig. 2 *Interaction of the weight of the column and the axial compressive force*

ki ga izberemo kot merilo razlike med F'_{cr} in F'_{crD}, narašča za večja razmerja I_2/I_1 in F_G/F_{Gcr} ter večje vrednosti eksponentov n in p. Največje vrednosti f presegajo 1,3 (tj. F'_{cr} prek 30 odstotkov nad F'_{crD}).

Toda, če upoštevamo razmerje kritične obremenitve $(F+F_G)_{cr}$, kakor ga daje rešitev ravnovesne enačbe, opisane v tem prispevku, ter celotna kritična obremenitev $(F+F_G)_{crD}$ približne rešitve, je največja vrednost razmerja pod 1,02, tj. celotna dejanska kritična obremenitev se od približne rešitve razlikuje le za +2 odstotka. Vzrok za tako majhno razliko je v dejstvu, da je F_{Gcr} 3 do 10-krat večja od F_{cr0}, tako da je vpliv večje dejanske kritične sile F'_{cr} zanemarljiv v primerjavi z veliko večjo vrednostjo F_G, tembolj ker največja različnost med F'_{crD} in F'_{cr} nastaja v območju velikih razmerij F_G/F_{Gcr}, ko je vrednost F_G veliko večja od vrednosti F'_{cr}.

which has been taken as a measure of differing F'_{cr} from F'_{crD} was growing as the ratia I_2/I_1 and F_G/F_{Gcr} were larger and the exponents n and p were larger. The highest values of f are over 1.3 (i.e., F'_{cr} over 30 % more than F'_{crD}).

But if the ratio of the total critical load $(F+F_G)_{cr}$ determined by the solution to the equilibrium equation described in this paper to the total critical load $(F+F_G)_{crD}$ using the approximate solution is considered, a maximum obtained value of this ratio extends under 1.02, i.e., the total real critical load differs from the one obtained by the approximate solution by less than +2 %. The reason for such a small difference is the fact that F_{Gcr} is 3 to 10 times bigger than F_{cr0}, and so the influence of the larger real critical force F'_{cr} vanishes in the addition to the much larger value of F_G, the more so as the largest differing of F'_{crD} from F'_{cr} takes place in the area of the big ratia F_G/F_{Gcr} when the value of F_G is much larger than the value of F'_{cr}.

4 SKLEP

To poročilo ugotavlja, da približna rešitev, ki predpostavlja linearno povezanost teže stebra in vzdolžne tlačne sile, daje celotno kritično obremenitev $(F_G+F)_{crD}$, ki se razlikuje od dejanske celotne kritične sile $(F_G+F)_{cr}$ znotraj -2 odstotka in zato približna rešitev ponuja rezultate velike natančnosti in na varni strani. Zato je povečevalni količnik:

4 CONCLUSION

This paper establishes that the approximate solution, assuming a linear interaction of the weight of the column and the axial compressive force, gives the total critical load $(F_G+F)_{crD}$ which differs from the real total critical load $(F_G+F)_{cr}$ within -2 % and therefore the approximate solution offers great accuracy and is on the side of safety. Thus the amplification factor:

$$\alpha = \frac{1}{1 - \dfrac{(F_G + F)}{(F_G + F)_{cr}}} \tag{54}$$

zelo dobro približno podan z [6]:

can be very well approximated by [6]:

$$\alpha = \frac{1}{1 - \dfrac{(F_G + F)}{(F_G + F)_{crD}}} \tag{55}$$

kjer je:

where:

$$(F_G + F)_{crD} = \left(1 + \frac{F}{F_G}\right) \frac{F_{Gcr} + F_{cr0}}{\dfrac{F}{F_G} F_{Gcr} + F_{cr0}} \tag{56}$$

Tu je F/F_G dejansko razmerje delujoče obremenitve v razmerju s težo antenskega stebra. Vrednosti F_{Gcr} in F_{cr0} lahko določimmo po (49) in (50).

Here F/F_G is the real ratio of the effective load to the weight of the antenna column. The values of F_{Gcr} and F_{cr0} can be determined by (49) and (50).

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