© Strojniški vestnik 48(2002)11,601-612
ISSN 0039-2480
UDK 621.983.3:621.979:62-86
Izvirni znanstveni članek (1.01)
© Journal of Mechanical Engineering 48(2002)11,601-612
ISSN 0039-2480
UDC 621.983.3:621.979:62-86
Original scientific paper (1.01)

# Optimiranje pogonskega mehanizma stiskalnice za globoki vlek

## Optimization of Link-Drive Mechanism for Deep Drawing Mechanical Press

**Bojan Vohar - Karl Gotlih - Jože Flašker**

*V prispevku se ukvarjamo z optimiranjem večzgibnega pogona paha stiskalnice za globoki vlek pločevine. Sedanja konstrukcija ne izpolnjuje vseh postavljenih zahtev, zato jo želimo čimbolj prilagoditi idealnim zahtevam tehnološkega postopka. Osnovni namen je prilagoditi sedanjo hitrostno karakteristiko paha zahtevam delovanja v določenem območju. Zato je bilo treba izdelati analizo pogona in njegov matematični model ter izvesti optimizacijo. Uporabljena metoda za nelinearno optimizacijo je sekvenčno kvadratno programiranje. Ker je postopek časovno odvisen, optimizacijskega modela ni moč uporabiti neposredno, ampak je treba primer prevesti v časovno neodvisno obliko, ki je primerna za reševanje s standardnim optimizacijskim postopkom. Cilj optimiranja je določiti takšne izmere pogonskega mehanizma, ki bi čimbolj izpolnile zahteve. V sklepu je prikazana primerjava doseženih rezultatov optimirane konstrukcije večzgibnega pogona z začetnim stanjem pred optimizacijo.*

**(Ključne besede: globoki vlek, stiskalnice za globoki vlek, optimiranje pogoniv, modeliranje)**

*This paper deals with an example of a link-drive for a deep-drawing mechanical press. The existing design has proved unsatisfactory and does not meet all the demands and constraints which are required for this metal-forming process. Optimization of the drive is therefore necessary. The intention of this optimization is to achieve the required velocity characteristics in a defined area of movement. Firstly, the drive is analysed and a mathematical model is made. The whole process is time-dependent, so it cannot be used directly in the optimization algorithm. This mathematical model has first to be transformed into a form suitable for the standard non-linear optimization procedure and then the optimization is carried out. We use the method of sequential quadratic programming. The final objective of the optimization process is to find the dimensions of the link-drive members such that the given requirements are satisfied in the best possible manner. In conclusion, the results are described and compared with the initial design.*

**(Keywords: deep drawing, deep drawing press, link-drive mechanizm, optimization, modelling)**

### 0 UVOD

Globoko vlečenje je zahteven preoblikovalni postopek. Največji vpliv na potek vlečenja ima oblika končnega izdelka in posledično oblika orodja ter vrsta materiala, ki ga obdelujemo. Poleg teh in drugih tehnoloških dejavnikov na kakovost in pravilen potek vlečenja zelo vpliva hitrost vlečenja pločevine. Odvisna je od stiskalnice, na kateri se vlečenje opravlja. Vsak material ima neko optimalno vlečno hitrost. Največja hitrost vlečenja pločevine je tako ena izmed pomembnejših omejitev pri izbiri stiskalnice.

Želje po večji produktivnosti preoblikovalnih strojev narekujejo iskanje novih konstrukcijskih rešitev in izboljšav. Najlažji način povečanja produktivnosti stiskalnic za globoki vlek je povečanje

### 0 INTRODUCTION

Deep drawing is a complex metal-forming process. The quality of the products made with this process is mainly influenced by their required shape and the material used. Many technological parameters influence the quality and the course of the whole process, for example, friction contact and lubrication of the tool. However, the slide velocity is one of the most important factors; this velocity depends only on the press design parameters. Each material has its optimum drawing velocity; therefore, maximum drawing velocity is one of the deciding factors when selecting an appropriate press.

The demands for increased productivity encourage the search for better solutions and improvements in the production of metal-forming machines.

njihove obratovalne hitrosti, torej vrtilne frekvence pogonskega motorja. Vendar hitrosti ni moč poljubno povečevati, ker prevelike vlečne hitrosti povzročajo trganje materiala in druge težave [3], saj material nima na voljo dovolj časa za zadostno tečenje. Hidravlične stiskalnice te probleme zaradi učinkovitega krmiljenja zlahka odpravijo, njihova pomanjkljivost pa je višja cena v primerjavi z mehanskimi stiskalnicami ter drago in zahtevno vzdrževanje. Pri mehanskih stiskalnicah je to težje, ker je pot paha omejena z vnaprej določenim gibom, ki ga določa kinematika in izmere pogonskega mehanizma.

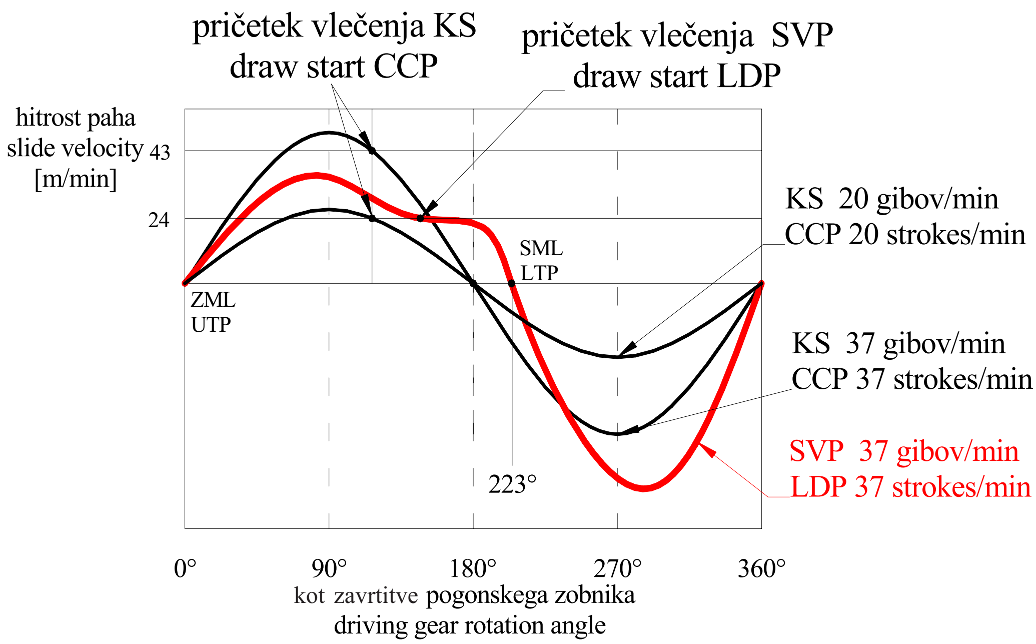
Ena takšnih izboljšav v mehanskih stiskalnicah je večzgibni pogon paha [3]. Takšen pogon je zaradi svojih karakteristik primernejši od običajnega ročičnega pogona, saj je njegovo delovanje mogoče veliko bolje prilagajati zahtevam posameznih preoblikovalnih postopkov. Imenujemo ga tudi mehanizem s pospešenim povratnim gibom. Njegova glavna lastnost je, da je hitrost paha med delovnim gibom tudi za polovico manjša od običajnih ročičnih stiskalnic pri enaki obratovalni hitrosti ter zelo pospešena pri vračanju v izhodno lego. Takšna hitrostna karakteristika je zelo podobna hidravličnim stiskalnicam, le da celoten krog poteka pri mehanskih stiskalnicah občutno hitreje. Ker je največja hitrost vlečenja pločevine hkrati omejitev obratovalne hitrosti stroja, lahko stiskalnice s takšnim pogonom obratujejo veliko hitreje kakor običajne, saj te hitrosti ne bodo prekoračile. Tak primer prikazuje slika 1 [3], kjer je prikazana razlika v obratovalni hitrosti med običajno ročično stiskalnico ter stiskalnico z večzgibnim pogonom. Razvidno je, da lahko v stvarnem primeru na sliki stiskalnica z večzgibnim pogonom obratuje s 37 gibi na minuto, medtem ko običajna ročična stiskalnica obratuje z največ 20 gibi na minuto (da ne pride do prekoračitve vlečne hitrosti); torej dosežemo z večzgibnim pogonom za 85 % večjo produktivnost (na minuto 17 kosov več).

S slike 1 [3] je razvidna še ena prednost večzgibnega pogona, namreč skoraj nespremenljiva vlečna hitrost v delovnem področju, kar omogoča občutno boljše razmere za tečenje materiala, izboljša kakovost izdelka ter podaljša dobo trajanja orodja. Takšen večzgibni pogon obravnavamo v prispevku, prikazan je na sliki 2. Pogon je izveden prek pogonskega zobnika z izsrednostjo, na katerega je vezana ojnica, ki je na eni strani prek veznega droga povezana z okrovom stiskalnice, na drugi strani pa se gibanje prenaša na drsnik. Na tega je nato prek drogov pritrjen pah. Mehanizem je 6-zgibni ročični s končnim drsnim členom in je v bistvu sprememba običajnega 4-zgibnega ročičnega mehanizma z drsnikom. Želeno hitrostno karakteristiko dobimo zaradi podaljšane ojnice in vezave le-te na vezni drog in okrov, kar spremeni sinusoidni potek hitrosti v že prej omenjeni in prikazani krog, značilen za večzgibne pogone.

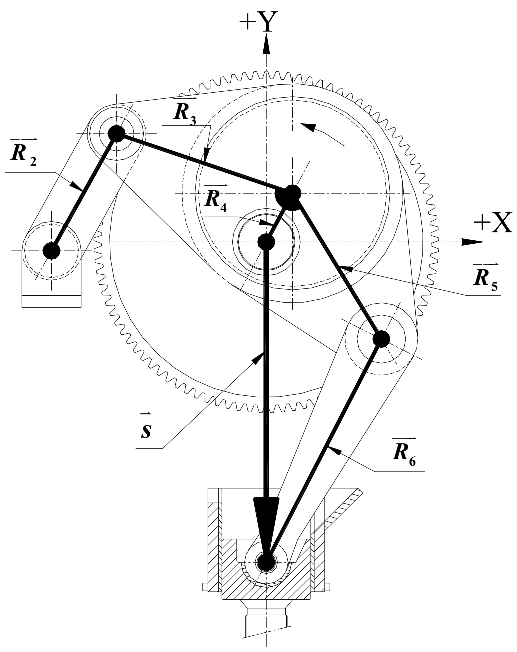
The simplest way to increase the productivity of mechanical presses is to increase their operating speed (the rotational velocity of their driving engine). However, such an increase has its limits, since too high drawing velocities cause tearing of the material and some other problems that occur when there is not enough time available for the material to flow [3]. The issue is not relevant in the case of hydraulic presses due to efficient control and slide control, but their high price and maintenance costs are the main disadvantages. This sort of slide control is hard to implement with mechanical presses, because the motion of the slide is constrained and defined by the dimensions and kinematics of the driving mechanism.

However, during the past few years press manufacturers have begun to incorporate special link-drives into their presses [3]. Because of their characteristics these drives (also called quick-return drives) are much more appropriate than conventional crankshaft and eccentric gear drives, they are more flexible and easier to adapt for each individual metal-forming process. Their main advantage is a much lower slide velocity during the working part of the cycle. This kind of velocity characteristic is very similar to hydraulic presses, except that the whole process runs much faster in mechanical presses. Since the maximum drawing velocity is also the limit of the press’s operational speed, the presses with the link-drive can operate at higher speeds than conventional versions without exceeding the maximum drawing-velocity limit. Figure 1 demonstrates the difference in the velocity characteristics between a classic crankshaft press and one with a link-drive [3]. Whereas a link-drive press operates at 37 strokes/min, the crankshaft version can run at 20 strokes/min – at most – before the maximum drawing-speed limit is exceeded. That means 85% more production with the link-drive press (17 pieces/min more than with the conventional press).

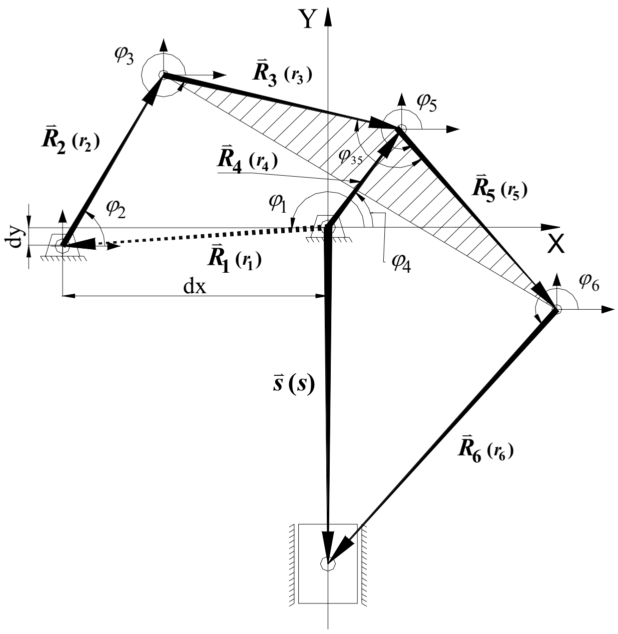
Another important feature of link-drive presses, which is also evident in Figure 1, is the almost constant slide velocity in the working part of the cycle. This considerably improves the conditions for material flow, the quality of the products, and prolongs the lifetime of the tool. Figure 2 presents the link-drive that is considered in this paper. The drive of the press is accomplished through an eccentric driving gear that is linked with a coupler link. On one side the coupler is connected to the press frame via an additional link, and the other end is connected to an output slider-link combination. The mechanism is a 6-bar slider-crank mechanism, which is a modification of a standard 4-bar slider-crank version. A modified velocity characteristic (lower velocity in the working part of the cycle) is achieved through an extended coupler link and its connection to the press frame.




Sl. 1. *Primerjava enega kroga klasične stiskalnice (KS) in stiskalnice z večzgibnim pogonom (SVP)*
Fig. 1. *Comparison of one cycle of the classic crankshaft press (CCP) and the link-drive press (LDP)*




Sl. 2. *Shema večzgibnega pogona*
Fig. 2. *Scheme of deep drawing mechanical press link-drive*




Sl. 3. *Kinematična shema mehanizma*
Fig. 3. *Kinematic scheme of the link-drive mechanism*

Želimo optimirati dani pogon glede na določene kriterije, torej poiskati optimalne izmere ročic in lege vrtišč, da bo ustrezal naslednjim zahtevam:

- Hitrost v delovnem območju naj bo čimbolj nespremenljiva, predvsem v predpisanem območju od okoli 75 do 150 mm pred spodnjo mrtvo lego.
- Mehanizem naj doseže spodnjo mrtvo lego v območju zasuka izsrednika od 190 do 230 stopinj (običajni ročični mehanizem pri 180 stopinjah) – s

The final objective of the optimization is to find the dimensions of the link-drive members such that the following requirements are satisfied in the best possible manner:

- Slide velocity in the working part of the cycle should be as constant as possible, especially in the range from 75 to 150 mm before the lower toggle point (bottom dead centre),
- The mechanism should reach the lower toggle point in the range of the eccentric gear rotation angle from 190° to 230°, so as to achieve a longer

tem je zagotovljen daljši čas dejavnega dela delovnega giba (kakovostno tečenje materiala).

– Hitrost v delovnem krogu naj bo čim manjša (manjša stična hitrost in manjša preoblikovalna hitrost).
– Če je le mogoče, naj bodo spremembe izmer čim manjše; izmere ročic naj variirajo v območju okoli ±20%, prav tako naj se čim manj spreminja gib paha.

Za sam postopek optimiranja je treba izdelati matematični model mehanizma ter izbrati namensko funkcijo, ki jo bomo optimirali. Zato moramo najprej opraviti kinematično analizo mehanizma, kjer bodo prikazane funkcijske odvisnosti med vsemi spremenljivkami, ki bodo udeležene v optimizacijskem postopku.

duration of the working part of the cycle and therefore higher quality of deformed material (classic crankshaft mechanism at 180°),

– The slide velocity in the working part of the cycle should be as low as possible, within the given constraints,
– The changes of the member lengths should not exceed ±20% of the original values, and the change of the stroke length should be as small as possible.

For the optimization procedure a mathematical model of the link-drive has to be made, and an appropriate cost function has to be chosen. To do this the kinematic analysis of the drive has to be carried out first in order derive the functional relations between all the variables included in the optimization process.

## 1 KINEMATIČNA ANALIZA POGONA

Na podlagi kinematične sheme (sl. 3) izpeljemo kinematično analizo mehanizma. Z $\vec{R}_2$ do $\vec{R}_6$ smo označili vse ročice ($\vec{R}_3$ in $\vec{R}_5$ sta skupaj ena ročica – ojnica $\vec{R}_{3-5}$), $\vec{R}_1$ pomeni podlago, $\vec{s}$ pa vektor lege paha (dolžina $\vec{s}$ se spreminja s časom). V oklepajih so označene dolžine posameznih vektorjev. Mehanizem ima samo eno prostostno stopnjo – torej je njegovo gibanje moč opisati s funkcijo ene same spremenljivke; v našem primeru bo to kot pogonske ročice $\vec{R}_4$-$\varphi_4$. Zanima nas potek gibanja paha ter njegove hitrosti in pospeškov, zato moramo poiskati zvezo med vhodno in izhodno veličino, torej funkcijsko odvisnost $\vec{s} = \vec{s}(\varphi_4)$. Analizo gibanja mehanizma smo izvedli s kompleksnimi števili. Mehanizem obravnavamo v dveh stopnjah: prva stopnja je 4-zgibni ročični mehanizem ($\vec{R}_1 - \vec{R}_2 - \vec{R}_3 - \vec{R}_4$), druga pa 4-zgibni ročični mehanizem z drsnikom ($\vec{R}_4 - \vec{R}_5 - \vec{R}_6 - \vec{s}$), pri čemer je kot ročice $\vec{R}_5$ odvisen od kota ročice $\vec{R}_3$. Najprej analiziramo prvo stopnjo, ker je rešitev tega mehanizma vhodni podatek za drugo stopnjo.

Za sklenjeno zanko prve stopnje lahko zapišemo (sl. 3):

## 1 KINEMATIC ANALYSIS OF THE DRIVE

Figure 3 shows the kinematic scheme of the drive. Vectors $\vec{R}_2$ to $\vec{R}_6$ represent the drive members ($\vec{R}_3$ and $\vec{R}_5$ together represent one member – the coupler link $\vec{R}_{3-5}$), $\vec{R}_1$ is the foundation (press frame) and $\vec{s}$ is the vector of the slide position (variable length – time dependent). In the brackets are the appropriate member lengths. The mechanism has one degree of freedom and its motion can therefore be described with a function of a single variable. In our case this variable is the rotational angle of the driving eccentric gear $\vec{R}_4$-$\varphi_4$. The slide motion is of concern here, so the desired relation we are looking for is the dependency of the output variable $\varphi_4 : \vec{s} = \vec{s}(\varphi_4)$. The kinematic analysis is made with complex-numbers notation. The mechanism is analyzed in two steps: the first step is the analysis of the 4-bar mechanism ($\vec{R}_1 - \vec{R}_2 - \vec{R}_3 - \vec{R}_4$); and the second is the analysis of the 4-bar slider-crank mechanism ($\vec{R}_4 - \vec{R}_5 - \vec{R}_6 - \vec{s}$), where the output results of the first analysis are the input data for the second analysis.

For a closed loop in the first step we can write the following relation (figure 3):

$$\vec{R}_1 + \vec{R}_2 + \vec{R}_3 - \vec{R}_4 = 0 \quad (1)$$

oz. v kompleksnem zapisu:

or in complex-numbers notation:

$$r_1 e^{i\varphi_1} + r_2 e^{i\varphi_2} + r_3 e^{i\varphi_3} - r_4 e^{i\varphi_4} = 0 \quad (2).$$

Po ločitvi na realni in imaginarni del dobimo sistem dveh enačb z dvema neznankama (kota $\varphi_2$ in $\varphi_3$), znane so vse dolžine ročic $r_2$ do $r_4$, $\varphi_4$ je kot pogonske ročice $r_4$ in je odvisen od časa $t$ in vrtilne frekvence $\omega$ ($\varphi_4 = \omega t$). Rešitev tega sistema je:

After separation of the real and imaginary parts we get a system of two equations with two unknowns (rot. angles $\varphi_2$ and $\varphi_3$). The member lengths $r_2$, $r_3$, $r_4$ are known, the rotational angle of the driving eccentric gear $\varphi_4$ is the input variable, which is dependent on the time t and the rotational velocity $\omega$ ($\varphi_4 = \omega t$). The solution of this system is:

$$\varphi_{2_{1,2}} = 2\arctan\left(\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right), \varphi_{3_{1,2}} = 2\arctan\left(\frac{-E \pm \sqrt{E^2 - 4DF}}{2D}\right) \quad (3),$$

kjer so uporabljene oznake: / where the following notations are used:

$$A = K_1 - K_2\cos(\varphi_1 - \varphi_4) - K_4\cos\varphi_1 + \cos\varphi_4$$
$$B = 2(K_4\sin\varphi_1 - \sin\varphi_4)$$
$$C = K_1 - K_2\cos(\varphi_1 - \varphi_4) + K_4\cos\varphi_1 - \cos\varphi_4$$
$$D = K_5 + K_4\cos\varphi_1 + K_3\cos(\varphi_1 - \varphi_4) - \cos\varphi_4$$
$$E = 2(K_4\sin\varphi_1 + \sin\varphi_4)$$
$$C = K_5 - K_4\cos\varphi_1 + K_3\cos(\varphi_1 - \varphi_4) + \cos\varphi_4$$

$$K_1 = \frac{r_1^2 + r_2^2 - r_3^2 + r_4^2}{2r_2r_4}$$
$$K_2 = \frac{r_1}{r_2}$$
$$K_3 = \frac{r_1}{r_3}$$
$$K_4 = \frac{r_1}{r_4}$$
$$K_5 = \frac{-r_1^2 + r_2^2 - r_3^2 - r_4^2}{2r_3r_4} \quad (4).$$

Za vsak kot ($\varphi_2$ in $\varphi_3$) sta mogoči dve rešitvi (obe realni in enaki, kompleksno konjugirani ali realni in različni), ustrezno določimo s kinematično shemo. Prva stopnja je v celoti določena, za poljubni čas $t$ lahko izračunamo vse potrebne parametre in sledi analiza druge stopnje.

Iz kinematične sheme 2. stopnje (sl. 4) je razvidno, da je kot $\varphi_5$ vezan na kot $\varphi_3$, saj ročici $\vec{R}_3$ in $\vec{R}_5$ sestavljata eno ročico – ojnico $\vec{R}_{3-5}$. Zato ga zapišemo kot:

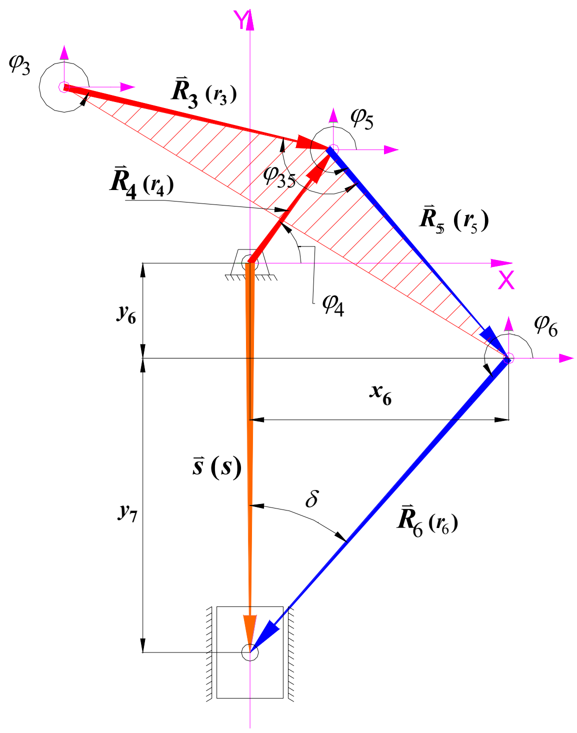
For every angle $\varphi_2$ and $\varphi_3$ two solutions are possible (both real and equal, complex conjugated or real and different). The appropriate solution is chosen according to the kinematic scheme. The first stage of the mechanism is completed, for an arbitrary time $t$ all the parameters can be computed.

The second step of the kinematic analysis follows according to the kinematic scheme of the second stage of the mechanism (Fig. 4). The members $\vec{R}_3$ and $\vec{R}_5$ together form one member – the coupler link $\vec{R}_{3-5}$, therefore the angle $\varphi_5$ is dependent on the angle $\varphi_3$, and can be written as:

$$\varphi_5 = \varphi_3 + \varphi_{35} - \pi \quad (5),$$

kjer je $\varphi_{35}$ kot trikotne ojnice, torej kot med ročicama $\vec{R}_3$ in $\vec{R}_5$. Izhodna veličina, katere funkcijsko

where $\varphi_{35}$ is the angle of the triangular coupler (the angle between members $\vec{R}_3$ and $\vec{R}_5$). The output quan-




Sl. 4. *Kinematična shema 2. stopnje mehanizma*
Fig. 4. *Kinematic scheme of the second stage*

odvisnost potrebujemo, je gib oz. pozicija paha (drsnika) – dolžina vektorja $\vec{s}$ .

Zapišemo lahko naslednje zveze:

tity, whose functional relation we are looking for, is the slide position – the length of the vector $\vec{s}$ .

The following relations can be written:

$$s = y_6 + y_7 \,, \qquad y_6 = r_4 \sin\varphi_4 + r_5 \sin\varphi_5 \,, \qquad y_7 = \frac{x_6}{\tan\ \delta} \,, \qquad x_6 = r_4 \cos\varphi_4 + r_5 \cos\varphi_5 \,,$$

$$\delta = \arcsin\left(\frac{x_6}{r_6}\right), \qquad y_7 = \frac{x_6}{\tan\ \delta} = \frac{r_4 \cos\varphi_4 + r_5 \cos\varphi_5}{\tan\left(\arcsin\left(\frac{x_6}{r_6}\right)\right)} \tag{6}$$

oziroma:

hence:

$$s = r_4 \sin\varphi_4 + r_5 \sin\varphi_5 + \frac{r_4 \cos\varphi_4 + r_5 \cos\varphi_5}{\tan\left(\arcsin\left(\frac{x_6}{r_6}\right)\right)} \tag{7}$$

Vse odvisne veličine na desni strani enačbe (7) so funkcije kota $\varphi_4$ in smo tako dobili želeno zvezo med vhodno ter izhodno veličino: $\vec{s} = f(\varphi_4)$ .

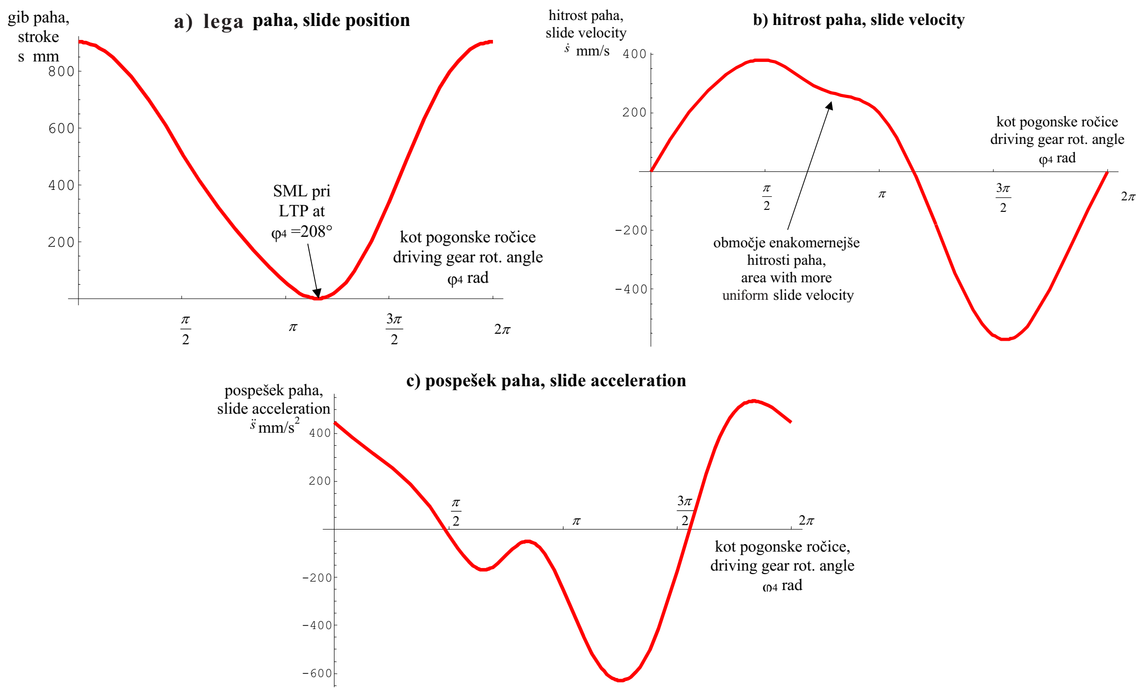
Hitrosti in pospeške paha je sedaj moč preprosto dobiti z odvajanjem (7). Iz rezultatov analize je razvidno, da je zveza (7) nelinearna, kar pogojuje način optimiranja. Rezultate kinematične analize prikazujejo krivulje: gib, hitrost in pospešek pehala v odvisnosti od vhodnega kota $\varphi_4$ (in s tem posredno časa *t*) na sliki 5.

Iz grafov je vidna tipična karakteristika večzgibnega pogona: SML se pojavi kasneje kakor pri običajnem ročičnem pogonu, v območju delovnega giba je hitrost manjša in ima enakomernejši potek, čemur sledi skokovito povečanje hitrosti pri vračanju paha navzgor. Za obravnavani pogon smo izdelali računalniški program v programskem jeziku

The desired relation $\vec{s} = f(\varphi_4)$ is derived, and all the quantities on the right-hand side of the equation are dependent only on the angle $\varphi_4$.

The slide velocity and the acceleration can now be easily obtained with a derivation of (7). It is clear that the relation (7) is highly nonlinear and therefore a nonlinear optimization procedure has to be chosen. The results of the kinematic analysis are shown in Figure 5.

The typical characteristics of the link-drive are clear from the graphs: the lower toggle point occurs much later than with the classic crankshaft drive; the slide velocity in the working part of the cycle is lower and more constant; there is a rapid velocity increase in the returning part of the cycle. A FORTRAN algorithm was made which, for given input




Sl. 5. *Rezultati kinematične analize večzgibnega pogona (a - lega, b – hitrost, c -pospešek)*
Fig. 5. *Results of the kinematic analysis (a - position, b - velocity, c - acceleration)*

FORTRAN, ki za dane vhodne podatke izračunava kinematične veličine (lego, hitrost, pospešek in gib paha, lego SML, največji absolutni pospešek in njegovo lego). Izračunani podatki za obravnavani mehanizem so naslednji:

- SML pri $\varphi_4 = 208°$,
- gib mehanizma = 903,48 mm,
- predpisano območje (okoli 75 do 150 mm pred SML): $\varphi_4 = 155° \div 176°$,
- največji absolutni pospešek na predpisanem območju = 198,8 mm/s$^2$ ($\varphi_4 = 176°$).

Kinematična analiza pogona je končana, znani so vsi potrebni parametri, ki jih potrebujemo za optimizacijo.

data (the drive geometry), calculates the required kinematic quantities (slide position, velocity and acceleration, stroke, position of the lower toggle point, maximum absolute acceleration and its position) in the required resolution step. The calculated values are:

- lower toggle point at $\varphi_4 = 208°$
- stroke = 903.48 mm
- desired working part of the cycle approximately 75 to 150 mm before the lower toggle point): $\varphi_4 = 155°$ to $176°$
- maximum absolute slide acceleration in the desired range = 198.8 mm/s$^2$ ($\varphi_4 = 176°$)

The kinematic analysis of the drive is finished, and all the parameters required for the optimization procedure are known.

## 2 OPTIMIRANJE POGONA

## 2 OPTIMIZATION OF THE LINK-DRIVE

### A) Splošni optimizacijski model

Osnovni cilj vsakega optimizacijskega postopka, ki temelji na metodah matematičnega programiranja, je poiskati odgovor na vprašanje "kaj je najboljše?" pri problemih, pri katerih lahko kakovost odgovora izrazimo kot numerično vrednost. Ali drugače: poiskati takšno kombinacijo parametrov (projektnih spremenljivk), ki bodo minimizirali izbrano veličino (namensko funkcijo). Optimizacijski problem je najprej treba spremeniti v matematično obliko.

Splošni model optimizacijskega problema zapišemo v obliki:
poišči tak vektor projektnih spremenljivk $\mathbf{x} \in R^n$,
ki bo minimiziral namensko funkcijo $f(\mathbf{x},\mathbf{z}) \in R^n$,
ob upoštevanju pogojev:

### A) General optimization model

The basic objective of every optimization procedure based on methods of mathematic programming is to find an answer to the question "What is the best?" concerning problems where the quality of the solution can be expressed and evaluated as a numerical value. In other words: to find such a combination of parameters (design variables) that will minimize the chosen quantity (cost function). But first, the optimization problem has to be transformed into a mathematical formulation.

The general optimization model can be expressed in the following form:
find such a vector of design variables $\mathbf{x} \in R^n$
which minimizes the cost function $f(\mathbf{x},\mathbf{z}) \in R^n$,
subjected to following constraints:

$$x_i^{sp} \le x_i \le x_i^{zg} \quad i = 1,2,...,k$$
(omejitve projektnih spremenljivk/constraints of design variables, lower/upper bounds)
$$g_j(\mathbf{x},\mathbf{z}) \le 0 \qquad j = 1,2,...m$$
(omejitvene funkcije/inequality constraints) (8),
$$h_l(\mathbf{x},\mathbf{z}) = 0 \qquad l = 1,2,...m'$$
(odzivne enačbe sistema/equality constraints, system-response equations)

kjer $\mathbf{z}$ pomeni vektor odzivnih oz. sistemskih spremenljivk (hitrosti, pospeški, reakcije itn.), $\mathbf{x}$ pa vektor projektnih spremenljivk (parametri, katerih optimalne vrednosti iščemo, da zadostimo zahtevanim kriterijem). Namenska funkcija je tista, ki jo izberemo kot merilo za uspešnost optimiranja in katere minimum iščemo. Če je katerakoli od definiranih funkcij (namenska, omejitvene itn.) nelinearna, imamo opravka z nelinearnim optimiranjem. Optimizacijski problem rešujemo iterativno, za njegov zagon pa potrebujemo začetno točko $\mathbf{x}^0$ (ocena ali sedanja varianta projekta, ki ga želimo izboljšati).

Optimizacijski problem v obliki (8) ni neposredno uporaben, kadar imamo opravka z dinamičnimi sistemi, kakor je npr. večzgibni pogon, ki ga obravnavamo. Pri takšnih sistemih se pojavlja nova neodvisna spremenljivka – čas, zaradi česar postane

where $\mathbf{z}$ is the state variable vector of the system-response variables (velocity, acceleration, reaction forces, etc.) and $\mathbf{x}$ is the vector of the design variables (parameters for which optimum values are to be found to satisfy given criteria and constraints). The cost function is the one which is chosen accordingly as a measure of the efficiency of the optimization procedure. If any of the defined functions (cost, constraint, etc.) are nonlinear, the nonlinear optimization algorithm has to be used. The optimization problem is solved iteratively, for its start the starting point $\mathbf{x}^0$ is required (current project status or estimation).

However, the optimization model in the form of (8) cannot be used directly in the cases of dynamical systems, for example, in a link-drive. With these systems a new independent variable arises, time $t$, which makes the vector of the system variables, $\mathbf{z}$, vektor odzivnih spremenljivk **z** odvisen od časa $t$. Obstaja več metod, kako časovno odvisen problem predelati v splošno obliko, ki bo primerna za reševanje. Ena od možnosti ([1] in [2]) je npr., da tak problem prevedemo v zaporedje časovno neodvisnih problemov. V našem primeru smo izbrali metodo z uvedbo nove spremenljivke [1]. Ker nas pri analizi dinamičnih sistemov v nekem časovnem razponu običajno zanima največja vrednost odziva sistema, ki jo želimo zmanjšati oz. optimirati (največji pospeški, hitrosti itn.), lahko namensko funkcijo za tak problem zapišemo kot:

time dependent. There are various methods for transforming time-dependent problems into a suitable form for the general optimization model. Some methods ([1] and [2]) translate the time-dependent problem into a sequence of time-independent problems. In our case the method of introducing a new artificial variable [1] was chosen. In the analyses of dynamical systems the maximum values of a measure of the system response in a certain time interval are usually the required quantities (maximum velocity, accelerations, etc.) and these also tend to be the quantities to be optimized. Therefore, the cost function for such a dynamic system can be expressed as:

$$\psi_0 = \max_{0\le t\le\tau} \; f_0\left(\mathbf{x},\mathbf{z}(t),t\right) \qquad (9).$$

Da bi takšno namensko funkcijo lahko uporabili v splošnem optimizacijskem modelu (8), se moramo znebiti funkcije max iz namenske funkcije ter časovne odvisnosti iz omejitev. To dosežemo z uvedbo nove umetne spremenljivke $x_{k+1}$, ki pomeni zgornjo mejo $f_0$, za katero velja:

To use the above form of the cost function in the general optimization model (8), the maximum value function from the cost function and the time dependency of the constraint functions have to be removed. This can be achieved with the introduction of a new, artificial variable $x_{k+1}$, which represents the upper limit $f_0$, subjected to:

$$f_0\left(\mathbf{x},\mathbf{z}(t),t\right) - x_{k+1} \le 0 \quad 0\le t\le\tau \qquad (10).$$

S tem postane vektor konstrukcijskih spremenljivk oblike:

Hence the new form of the vector of project variables is now:

$$\mathbf{x} = \left[x_1, x_2, x_3, \ldots x_k, x_{k+1}\right]^T \qquad (11),$$

ponovno pa tudi določimo namensko funkcijo:

and the new definition of the cost function is now:

$$\overline{\psi_0} = x_{k+1} \qquad (12).$$

Omejitvene funkcije, odzivne enačbe sistema ter omejitev namenske funkcije (10) zamenjamo z ustreznimi integralskimi omejitvami. Za poljubno zvezno fukcijo $f$(t) lahko neenakost v obliki $f\left(t\right)\le 0$ za $0\le t\le\tau$ zamenjamo z ustrezno integralsko omejitvijo [1]:

The constraint functions, the system-response equations and the constraint of the cost function (10) are replaced with equivalent integral constraints, as follows. For an arbitrary continuous function *f(t)* the inequality in the form of $f\left(t\right)\le 0, 0\le t\le\tau$ can be replaced with the equivalent integral constraint [1]:

$$\int_0^\tau \left\langle f(t)\right\rangle dt = 0, \quad \left\langle f(t)\right\rangle = \begin{cases} f(t) & ;\ f(t)\ge 0 \\ 0 & ;\ f(t)<0 \end{cases} \qquad (13).$$

Spremenjeni problem dobi s tem naslednjo obliko: poišči tak vektor $\mathbf{x}\in \mathrm{R}^n$, ki bo minimiziral $x_{k+1}\in \mathrm{R}$, ob upoštevanju pogojev:

The transformed problem is now stated in the following form: find such a vector $\mathbf{x}\in \mathrm{R}^n$, which minimizes $x_{k+1}\in \mathrm{R}$, subjected to following constraints:

$$\begin{aligned} & x_i^{sp} \le x_i \le x_i^{zg} && i=1,2,\ldots,k,k+1 \\ & \psi = \int_0^\tau \left\langle f_0\left(\mathbf{x},\mathbf{z},t\right) - x_{k+1}\right\rangle dt = 0 && \\ & \psi_j = \int_0^\tau \left\langle g_j(\mathbf{x},\mathbf{z},t)\right\rangle dt = 0 && j=1,2,\ldots m \\ & h_l(\mathbf{x},\mathbf{z}(t),t) = 0 && l=1,2,\ldots m' \end{aligned} \qquad (14).$$

Tako smo dobili obliko, ki jo predpisuje splošni optimizacijski model (8) in jo lahko uporabimo v postopku optimiranja.

The optimization problem is now formulated according to the general optimization model (8), and can be used with the standard optimization procedure.

**B) Optimizacijski model večzgibnega pogona**

Najprej je treba izbrati namensko funkcijo. Osnovni cilj našega optimiranja je določiti parametre mehanizma tako, da bo hitrost paha v predpisanem območju čim bolj nespremenljiva. Zato bi morali biti pospeški na tem območju enaki nič ali pa se temu čim bolj približati. Kot namensko funkcijo zato določimo največji absolutni pospešek na predpisanem območju, naš cilj pa je ta pospešek zmanjšati v največji mogoči meri. Namenska funkcija za obravnavani problem se torej glasi:

**B) Link-drive optimization model**

First, the cost function has to be chosen. The primary objective of this link-drive optimization is to determine the link-drive parameters in such a way that the slide velocity in the defined range of movement is as constant as possible. This means that the slide acceleration in this range has to be zero or close to zero. Therefore, a suitable cost function for this problem is defined as the maximum absolute slide acceleration, and the purpose is to lower this acceleration as much as possible. The cost function is:

$$f = \max \left|\ddot{s}\left(\mathbf{x},\varphi_4\right)\right| \quad \varphi_{4\min} \le \varphi_4 \le \varphi_{4\max} \quad \varphi_4 = \varphi_4(t) \quad t_{\min} \le t \le t_{\max} \tag{15}$$

Kot omejitve smo postavili omejitve dolžin ročic $r_2$ do $r_6$, lego vrtišča ročice $\overline{R}_2$ (izmere $dx$, $dy$) ter omejitev giba pehala. Pri tem je pomembno poudariti, da je zadnja omejitev giba nelinearna, saj izvira iz nelinearne zveze $\vec{s} = \vec{s}(\varphi_4)$.

Vektor projektnih spremenljivk dobi s tem obliko:

The constraints are as follows: the bounds of the link-drive members' lengths ($r_2$ to $r_6$), the position of the member $\overline{R}_2$ cylindrical joint (dimensions $dx$, $dy$), and the nonlinear constraint for the stroke, which is derived from the nonlinear relation $\vec{s} = \vec{s}(\varphi_4)$.

The vector of the project variables takes the following form:

$$\mathbf{x} = \left[r_2, r_3, r_4, r_5, r_6, dx, dy\right]^T \tag{16}$$

Ker pa problem v takšni obliki ni neposredno uporaben za optimizacijski postopek (namenska funkcija posredno odvisna od časa), uvedemo novo dejansko spremenljivko $x_8$ in spremenimo namensko funkcijo v:

Because of the dynamic nature of the problem this model cannot be used directly in the optimization procedure (time-dependent cost function), and has to be transformed. Therefore, a new artificial, real variable $x_8$ is introduced, which changes the cost function into:

$$\overline{\psi} = x_8 \tag{17}$$

Spremenjeni problem se tako glasi: poišči tak vektor $\mathbf{x} = \left[r_2, r_3, r_4, r_5, r_6, dx, dy, x_8\right]^T$, ki minimizira $x_8 \in \mathrm{R}$, ob upoštevanju pogojev:

The transformed problem is then: find such a vector $\mathbf{x} = \left[r_2, r_3, r_4, r_5, r_6, dx, dy, x_8\right]^T$, which minimizes $x_8 \in \mathrm{R}$, subjected to:

$$\begin{array}{lll} r_{i,\min} \le r_i \le r_{i,\max} & i = 2,3,4,5,6 & \\ dx_{\min} \le dx \le dx_{\max} & dy_{\min} \le dy \le dy_{\max} & gib_{\min} \le gib \le gib_{\max} \\ \int\limits_{t,\min}^{t,\max} \left\langle \left|\ddot{s}\left(\varphi_4(t)\right)\right| - x_8 \right\rangle dt = 0 & & \end{array} \tag{18}$$

Zaradi oblike namenske funkcije ter obeh nelinearnih omejitev (omejitev giba ter namenske funkcije), smo se odločili za uporabo optimizacijskega postopka, ki temelji na metodi sekvenčnega kvadratnega programiranja (SQP). Ta metoda spada med prilagojene Newton-ove metode, je torej gradientna. Uporabili smo algoritem iz knjižnice numeričnih rutin in programov NAG. Velika prednost uporabljenega algoritma je, da uporabniku ni treba podati vseh odvodov funkcije, torej Jacobijevega gradienta in Hessejeve matrike, če so ti preveč zapleteni. Program si jih sam aproksimira z metodo končnih razlik. Uporabnik mora zagotoviti vse potrebne podprograme, v katerih so določene namenska in omejitvene funkcije, druge omejitve ter kolikor je mogoče prvih odvodov funkcije. Vse potrebne podprograme smo napisali v programskem

Because of the form of the cost function and both nonlinear constraints (constraints of stroke and cost function) we decided to use an optimization procedure based on the method of sequential quadratic programming (SQP). This method is a modified Newton method and belongs to the group of gradient methods. The algorithm was chosen from the NAG library of numerical routines and programs NAG (The Numerical Algorithms Group). A major advantage of this algorithm is that the user does not need to supply all the derivatives of the cost function (the Jacobian gradient vector and the Hessian matrix). Instead, the program approximates them with numerical differentiation. The only thing the user needs to supply is the subroutines that define and calculate the cost and constraint functions (equality and inequality), the variable constraints, and as many dejeziku FORTRAN, v enakem jeziku kakor je napisana tudi knjižnica NAG.

Kot začetno točko optimiranja smo vzeli sedanje stanje pogona. Za konstrukcijske spremenljivke smo določili njihove začetne omejitve z mejami ±20% glede na njihovo trenutno vrednost; gib optimiranega pogona naj bo čim bližje sedanjemu, zato smo ga omejili z mejama ±3% začetne vrednosti. Najpomembnejša je zagotovo prva nelinearna omejitev, ki v praksi predstavlja razliko med izbrano namensko funkcijo (največjim absolutnim pospeškom) ter uvedeno umetno spremenljivko. Da bi funkcijo minimirali, mora biti ta razlika čim manjša, v idealnem primeru enaka nič. Zato smo jo omejili na vrednost nič, kateri se bo algoritem poskušal čimbolj približati. S tem so bili določeni vsi začetni pogoji in izvedeno je bilo optimiranje.

rivative functions as possible. All the required subroutines were written in FORTRAN, the same as the NAG library.

The current design of the drive was the starting point for the optimization. The design parameters can vary within ±20% of the original values and for the stroke a ±3% limit was chosen. The most important constraint is the nonlinear constraint of the cost function (the difference between the maximum absolute slide acceleration and the artificial variable). To minimize the cost function in the best possible manner, this difference has to be as close to zero as possible (zero in the ideal case); therefore, the constraint value was set to zero.

## 3 REZULTATI IN RAZPRAVA

Začetni vektor projektnih spremenljivk za sedanji pogon ima naslednje vrednosti:

## 3 RESULTS AND REMARKS

The initial vector of the design variables had the following numerical values:

$$\mathbf{x} = \left[774.6466,\ 1067,\ 320,\ 987,\ 1453,\ 1233,\ 51,\ 198.8\right]^T$$

Prvih 7 vrednosti pomeni dolžine ročic v mm, zadnja pa vrednost umetne spremenljivke $x_8$ v mm/s$^2$. Torej je $x_{8,zač} = 198,8$ mm/s$^2$ naša začetna vrednost, ki jo moramo čimbolj zmanjšati in katere končna vrednost bo merilo za uspešnost optimiranja. Optimiranje smo izvedli v več poskusih, z različnimi nastavitvami algoritma (toleranca konvergence) in uporabo prejšnjih rešitev v novem poskusu. Potek optimiranja prikazuje slika 6.

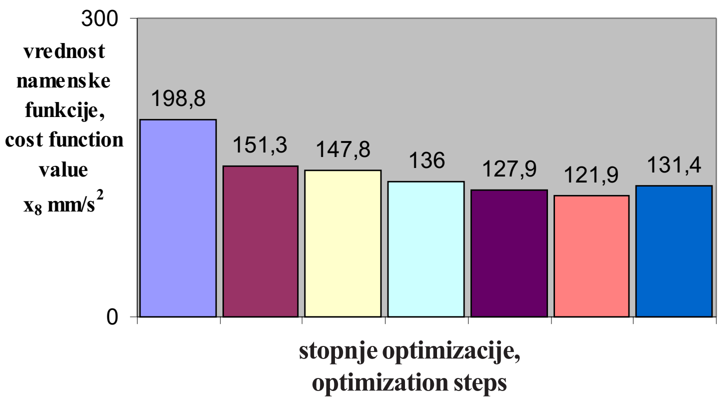
Optimiranje v okviru zastavljenih ciljev je bilo uspešno, saj nam je v 5. poskusu uspelo znižati vrednost namenske funkcije na približno 61% prvotne vrednosti. Vsaka od teh 6 rešitev da eno kombinacijo parametrov, ki določajo obravnavani večzgibni pogon.

Optimalno rešitev (s stališča kinematike) pomeni rešitev 5:

The first seven components are the link-drive member lengths in mm, the last one is the value of the artificial variable $x_8$ in mm/s$^2$. Hence, $x_{8,zač} = 198.8$ was the initial value that the optimization algorithm was trying to minimize. The optimization process was run in several attempts, with different options (convergence tolerance, the use of previous solutions in the subsequent steps). The optimization history is shown in Figure 6.

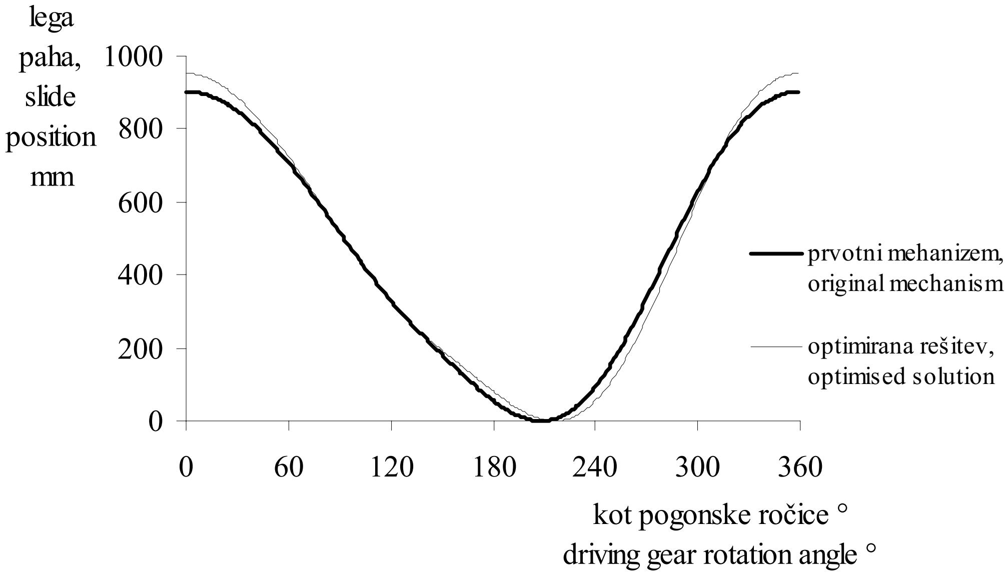
It is clear that the cost function was minimized to be approximately 61% of its original value at the fifth attempt. Thus the optimization was quite successful. Each of these six solutions represents one set of design variables that define the geometry of the link-drive.

The fifth solution is the optimum one (from the kinematic point of view):

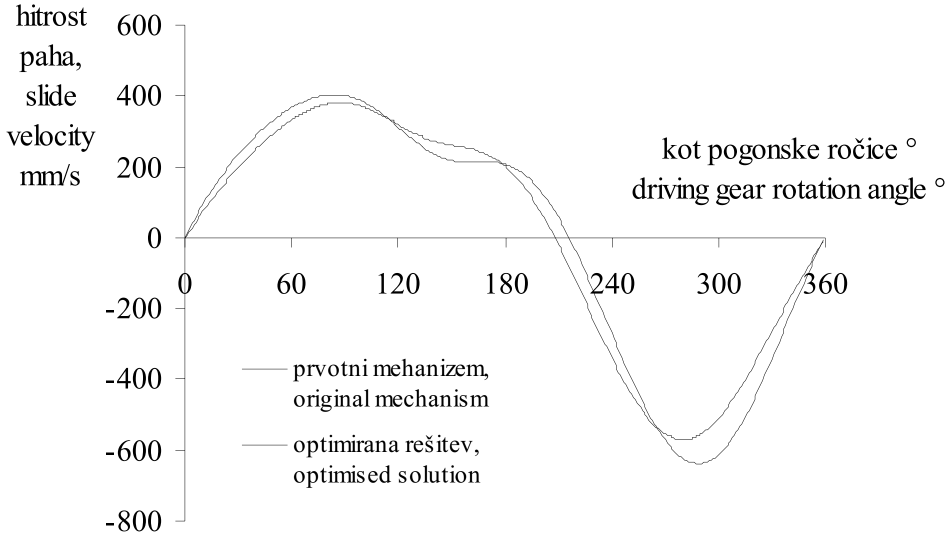



Sl. 6. *Potek optimizacije - optimalna rešitev je v 5. poskusu*
Fig. 6. *Optimization process – optimal solution obtained in 5th step*

| | | |
|---|---|---|
| $r_2$ = 730 mm (-44,6466 mm) | $r_3$ = 985 mm (-82 mm) | $r_4$ = 280 mm (- 40 mm) |
| $r_5$ = 1250 mm (+ 263 mm) | $r_6$ = 1198 mm (+ 255 mm) | |
| $dx$ = 1150 mm (- 83 mm) | $dy$ = 43,5 mm (- 7,5 mm) | |




Sl. 7. *Gib paha v odvisnosti od kota pogonske ročice (primerjava rešitev)*
Fig. 7. *Slide position curve (comparison between original and optimised solution)*




Sl. 8. *Hitrost paha v odvisnosti od kota pogonske ročice (primerjava rešitev)*
Fig. 8. *Slide velocity curve (comparison between original and optimised solution)*

V oklepajih je navedena sprememba proti začetnim vrednostim.

Dejanski pomen dobljene rešitve se pokaže, če jo prikažemo v obliki grafov poti in hitrosti pehala za en sklenjen krog ter primerjamo z začetnim stanjem pogona (slika 7).

In the brackets the difference from the original values is shown.

The true meaning of this minimization appears if we look at the slide position and the slide-velocity curves (Figure 7) and compare them with the original drive.

Iz grafov vidimo, da je optimirana rešitev zelo blizu začetnemu stanju, iz česar lahko sklepamo, da je sedanji pogon že dokaj blizu lokalnemu optimumu. SML je pri vseh rešitvah pomaknjena proti večjim zasukom pogonske gredi 215 do 220°. Vseh 6 rešitev ima v dejavnem delu kroga veliko bolj nespremenljivo in nižjo hitrost – v povprečju za okoli 15%. Torej je bila izbira namenske funkcije in načina optimiranja ustrezna. Ne smemo pa pozabiti, da smo se v delu omejili samo na kinematični vidik večzgibnega pogona, nič pa ni bilo govora o preoblikovalnih silah, ki se prenašajo čezenj, o njihovih karakteristikah, kje morajo doseči svoj vrh, na kakšnem območju morajo delovati in podobno. Zato ni nujno, da bo optimalna rešitev s kinematičnega vidika hkrati tudi optimalna v končni fazi, kjer bo treba upoštevati tudi dinamiko postopka. Dobljene optimirane rešitve so le podlaga za nadaljnje analize, pri katerih bo upoštevana celotna dinamika postopka.

It is clear that all the obtained solutions are very close to the original state. Thus we can conclude that the original geometry of the drive was already very well chosen. The slide velocity in the working part of the cycle is, in all six solutions, approximately 15% lower and more constant than with the original drive. The lower toggle point has shifted towards the larger rotation angles of the driving gear (from 215° to 220°; original drive 208°), which causes the extended range of the constant slide velocity and subsequently more time is available for the deformation and flow of the material. Hence, the choice of the cost function and the optimization algorithm was appropriate. However, this work considered only the kinematic aspects of the link-drive, no dynamic characteristics were considered (the operational forces in the process of deep drawing, which the link-drive has to transmit, the maximum forces required and their dependence on the slide position, the joint forces, etc.). It is possible that the kinematic optimum solution may not satisfy the dynamic constraints. Hence, the obtained solutions are a good basis for further analyses, optimization and development of the link-drive, where all the above-mentioned features are to be explored.

## 4 LITERATURA
## 4 REFERENCES


[1] Haug, E.J., J.S.Aurora (1979) Applied optimal design: Mechanical and structural systems, 1.izd., *John Wiley & Sons*, New York.

[2] Kegl, M. (1990) Optimiranje mehanskih sistemov z metodo kriterija optimalnosti, magistrsko delo, *Tehniška fakulteta*, Maribor.

[3] Vohar, B. (2001) Optimiranje in sinteza pogona stiskalnice za globoki vlek, diplomsko delo, *Fakulteta za strojništvo*, Maribor.

[4] Baze podatkov na internetu: www.aida-america.com, www.metalforming-online.com



Naslov avtorjev: Bojan Vohar
dr. Karl Gotlih
dr. Jože Flašker
Univerza v Mariboru
Fakulteta za strojništvo
Smetanova 17
2000 Maribor
bojan.vohar@uni-mb.si
gotlih@uni-mb.si
joze.flasker@uni-mb.si

Author's Address: Bojan Vohar
Dr. Karl Gotlih
Dr. Jože Flašker
University of Maribor
Faculty of Mechanical Eng.
Smetanova 17
2000 Maribor, Slovenia
bojan.vohar@uni-mb.si
gotlih@uni-mb.si
joze.flasker@uni-mb.si



Prejeto:
Received: 27.12.2002

Sprejeto:
Accepted: 31.1.2003