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Vpliv Prandtlovega števila na turbulentni prenos toplote ob ravni steni

The Influence of Prandtl Number on Near-Wall Turbulent Heat Transfer

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Za opis turbulentnega prenosa toplote iz stene na tekočino je pri nizkih vrednosti Reynoldsovih in Prandtlovih številih mogoče uporabiti neposredno numerično simulacijo (NNS-DNS), ki opiše vse krajevne in časovne skale pojava. Vpliv Reynoldsovega števila na turbulentni prenos toplote (hitrosti, temperature, fluktuacije itn.) je razmeroma majhen, medtem ko je vpliv Prandtlovega števila veliko večji. Pri naših simulacijah toka v kanalu smo pri Reynoldsovem številu Re ≈ 5000 analizirali tri različna Prandtlova števila, in sicer Pr = 0,025, Pr = 1 in Pr = 5,4.

Ločljivost NNS turbulentnega prenosa gibalne količine je premo sorazmerna z $Re^{3/4}$ v vseh smereh koordinatnega sistema. Pri upoštevanju prenosa toplote, za tekočine s Prandtlovim številom, večjim od ena, velja, da je ločljivost premo sorazmerna z $Re^{3/4}Pr^{1/2}$. Pri Re = 5260 in Pr = 5,4 smo opravili tri numerične simulacije pri različnih ločljivostih. Vse tri simulacije so NNS za hitrostno polje, samo simulacija z največjo ločljivostjo je tudi NNS za temperaturno polje. Rezultati so pokazali, da je mogoče temperaturno polje zelo natančno napovedati tudi z nekoliko slabšo ločljivostjo od teoretično zahtevane.

(Ključne besede: tok ob steni, tok turbulentni, prenos toplote, števila Prandtl, simuliranje numerično)

For describing the heat transfer from a wall to a fluid at low Reynolds and Prandtl numbers we can use a direct mumerical simulation (DNS), which describes all the length and time scales of the phenomenon. The Reynolds number has a weak influence on the turbulent heat transfer (velocities, temperatures, RMS-fluctuations…), whereas the increasing Prandtl number has a stronger influence. In our flow simulations in the channel, three different Prandtl numbers, i.e. Pr = 0.025, Pr = 1 and Pr = 5.4, at a Reynolds number Re ≈ 5000 were analyzed.

The resolution of the DNS for turbulent momentum transfer is proportional to $Re^{3/4}$ in all directions. When considering heat transfer in fluids for a Prandtl number higher than one, the resolution is proportional to $Re^{3/4}Pr^{1/2}$. Three different numerical simulations at different resolutions were performed at Re = 5260 and Pr = 5.4. All three simulations are a DNS for the velocity field, whereas only the simulation at the highest resolution is also a DNS for the thermal field. The results showed that the thermal field could be accurately described with a lower resolution than theoretically required.

(Keywords: near-wall flow, turbulent flow, heat transfer, Prandtl numbers, direct numerical simulation)

0 UVOD

V zadnjih 15 letih se je neposredna numerična simulacija (NNS) uveljavila kot pomembno orodje pri razumevanju mehanizma prenosa toplote v turbulentni mejni plasti. NNS pomeni natančno reševanje Navier-Stokesovih enačb brez dodatnih turbulentnih modelov. Prve NNS hitrostnega polja pri nižjih vrednostih Reynoldsovih številih sta opravila Kim in Moin [5], ki sta raziskovala hitrostno polje ter opazovala turbulentne strukture v kanalu (Re = 5700). Pozneje sta Kim in Moin [6] k enačbam hitrostnega polja dodala še energijsko enačbo za prenos toplote. Tako sta med dvema stenama preučevala tudi

0 INTRODUCTION

Over the past 15 years direct numerical simulation (DNS) has become an important research tool in understanding near-wall turbulent heat transfer. A DNS means a precise solving of the Navier-Stokes equations without any extra turbulent models. The first DNS of a velocity field at a low Reynolds number was performed by Kim and Moin [5], who investigated the velocity field and observed turbulent structures in the channel (Re = 5700). Later, they [6] also added an energy equation to the equations of velocity field for the heat-transfer calculations. When considering heat transfer, the temperaturno polje ter opazovala koherentne strukture, ki so se pojavljale v bližini sten. Uporabila sta nekoliko nenavaden postopek, saj sta predpostavila enakomerno gretje tekočine po celotni prostornini, steni pa sta predstavljali toplotni ponor. Tudi Kasagi [2] je raziskoval temperaturno polje, vendar je za geometrijsko obliko vzel kanal, ki je bil gret z zgornjo in spodnjo steno. Vse te simulacije so potekale pri nizkih vrednostih Reynoldsovih in Prandtlovih števil. Pozneje so raziskovalci (Kawamura [4], Na in Hanraty [7]) mejo Prandtlovega števila dvignili do deset. Kawamura [4] je raziskoval vpliv Reynoldsovega (do Re = 14000) in Prandtlovega števila (do Pr = 5). V območju tik ob steni je ugotovil majhen vpliv Reynoldsovega števila in veliko večji vpliv Prandtlovega števila na statistiko turbulentnega prenosa toplote (hitrostni profili, fluktuacije hitrosti, turbulentni toplotni tokovi).

Prispevek je vsebinsko razdeljen na dva dela. V prvem delu je posebna pozornost namenjena vplivu Prandtlovega števila na prenos toplote. NNS so bile izvedene pri Reynoldsovem številu Re = 4580 in Re = 5260 ter treh različnih Prandtlovih številih: Pr = 0,025, Pr = 1 in Pr = 5,4 (glej preglednico 1).

Drugi del je namenjen preučevanju ločljivosti pri Re = 5260 in Pr = 5,4. Analizirane so bile računske mreže s tremi različnimi ločljivostmi. Teoretično naj bi bila ločljivost NNS pri Prandtlovih številih, večjih od ena, sorazmerna kvadratnemu korenu Prandtlovega števila v vseh smereh koordinatnega sistema v primerjavi z ločljivostjo za Pr = 1 [8]. Zahtevo sta v svojih NNS upoštevala Kawamura [3] in Tiselj [9]. Pri Pr = 5,4 to pomeni, da moramo število točk računske mreže, ki je zadostno za opis hitrostnega polja pri Re= 5260, povečati približno za faktor $\sqrt{5,4}$ v vsaki smeri. Za prvo simulacijo je bila tako vzeta dovolj gosta mreža, ki zmore opisati najmanjše skale hitrostnega in temperaturnega polja (NNS).

Da je zahtevana gostitev mreže pri Pr > 1 nekoliko preostra, sta nakazala že Na in Hanratty [7], ki sta pri Pr = 10 uporabila večjo ločljivost le v smeri pravokotno na steno. Tako je v drugi simulaciji v tem prispevku izboljšana ločljivost v smeri pravokotno na steno, tretja simulacija pa je opravljena z najmanjšo ločljivostjo, ki je zadostna le za NNS hitrostnega polja. Zadnji dve simulaciji tako izpolnjujeta pogoj za NNS hitrostnega polja, na pa tudi za NNS temperaturnega polja.

1 ENAČBE IN NUMERIČNI POSTOPEK

Pri numeričnih simulacijah turbulentnega prenosa toplote sta bili uporabljeni dve različni geometrijski obliki neskončnega kanala. Zaprt kanal (sl. 1-levo), omejen s spodnjo in zgornjo steno, grejemo z nespremenljivim toplotnim tokom, vmes pa zaradi razlike tlakov teče tok tekočine. Pri odprtem kanalu (sl. 1-desno) grejemo spodnjo steno, nad katero teče tok tekočine s prosto površino. Uporabili smo brezrazsežne

temperature field between two walls and the coherent structures near the walls were studied. But this was an unusual approach, because uniform volumetric heat generation was assumed, where the walls represented a heat sink. Kasagi et al [2] also investigated the thermal field, but in this case the channel was heated by the top and bottom walls. All these simulations were performed for low Reynolds and Prandtl numbers. Later, Kawamura et al [3], and Na and Hanratty [7] raised the limit of the Prandtl number to ten, while Kawamura et al [4] studied the influence of Reynolds numbers (up to Re = 14000) and Prandtl numbers (up to Pr = 5). They found a weak influence of the Reynolds number and a stronger influence of the Prandtl number near the wall for turbulent heat transfer (velocity profiles, velocity fluctuations, turbulent heat fluxes).

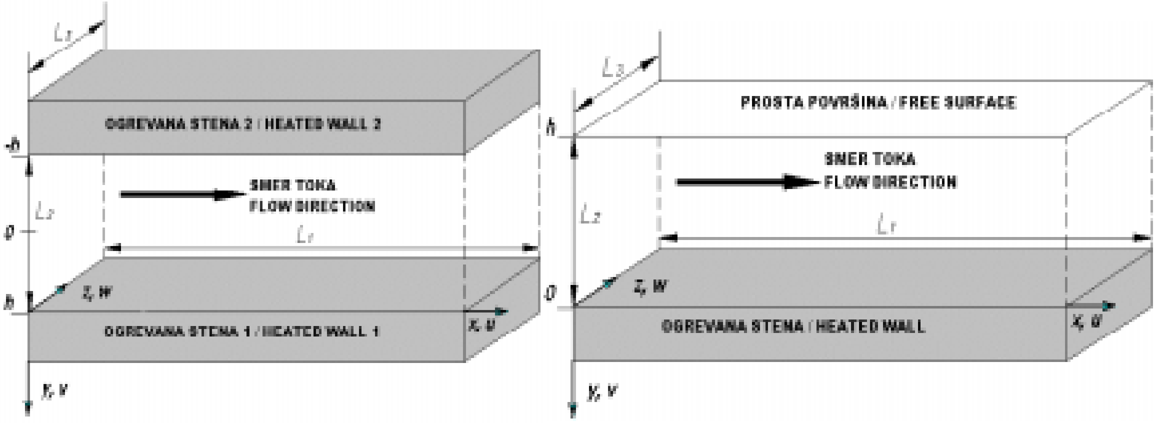
The content of this article is divided into two parts. In the first part we investigate the influence of the Prandtl number on the heat transfer. DNSs were performed at Reynolds numbers Re = 4580 and Re = 5260, and at three different Prandtl numbers: Pr = 0.025, Pr = 1 and Pr = 5.4 (see Table 1).

The resolution requirements at Re = 5260 and Pr = 5.4 are studied in the second part of the paper, where three different resolutions were analyzed. Theoretically, the resolution for Prandtl numbers higher than one should be proportional to the square root of the Prandtl number in all three directions. This requirement was taken into account in the simulations of Kawamura et al [3] and Tiselj et al [9]. Therefore, at Pr = 5.4 the grid should be improved by approximately a factor of $\sqrt{5.4}$ in all directions. In the first simulation sufficient grid was taken so as to be capable of describing the smallest scales of the velocity and thermal fields (DNS).

The grid-refinement study of Na and Hanratty at the Prandtl number Pr = 10 shows that a finer grid is not required in the streamwise and spanwise directions, but it is necessary in the wall-normal direction. This finding was considered in the second simulation, where a finer grid was applied only in the wall-normal direction, whereas the third simulation was performed with the lowest resolution, which was only sufficient for the DNS of the velocity field. Therefore, the last two simulations are sufficient for the DNS of the velocity field and insufficient for the DNS of the thermal field.

1 EQUATIONS AND NUMERICAL PROCEDURE

Two different geometries for the numerical simulations of turbulent heat transfer in the infinite channel and the flume were used. Channel (Fig.1 - left) is bounded by top and bottom walls, which are heated with a constant heat source, whereas the pressure gradient drives the fluid flows between them. In the flume (Fig. 1 - right), the fluid flows above the heated bottom wall, whereas the top surface is free.



Sl. 1. *Geometrija toka tekočine: levo) zaprt kanal, desno) odprt kanal*
Fig. 1. *Flow geometry: left) channel, right) flume*

Navier-Stokesove enačbe, normalizirane s polovično višino kanala h v zaprtem kanalu oziroma z višino h v odprtem kanalu, disipativno hitrostjo $u_\tau = \sqrt{\tau_w / \rho}$ in disipacijsko temperaturo $T_\tau = q_w / (u_\tau \rho_f c_{pf})$. Pri tem je τ_w strižna napetost na steni in je enaka $\tau_w = -\mu (du/dy)_w$. Takšen postopek lahko najdemo tudi v objavah Kasagija [2] ali Kawamure [3]:

The dimensionless Navier-Stokes equations, normalized by the flume height h (or the channel half height h), the friction velocity $u_\tau = \sqrt{\tau_w / \rho}$, and the friction temperature $T_\tau = q_w / (u_\tau \rho_f c_{pf})$, were used. τ_w stands for the wall shear stress, defined as $\tau_w = -\mu (du/dy)_w$. Such scaling can be found in the papers of Kasagi et al [2] or Kawamura et al [3]:

$$\nabla \cdot \vec{u}^+ = 0 \tag{1}$$

$$\frac{\partial \vec{u}^+}{\partial t} = -\nabla \cdot \left(\vec{u}^+ \vec{u}^+\right) + \frac{1}{\mathrm{Re}_\tau} \nabla^2 \vec{u}^+ - \nabla p + \vec{l}_x \tag{2}$$

$$\frac{\partial \theta^+}{\partial t} = -\nabla \cdot \left(\vec{u}^+ \theta^+\right) + \frac{1}{\mathrm{Re}_\tau \Pr} \nabla^2 \theta^+ + \frac{u_x^+}{u_B^+} \tag{3}$$

Člena $\vec{l}_x$ (enotski vektor v smeri vzdolž toka) in u_x^+ / u_B^+ se v en. (2) in (3) pojavita zaradi numerične sheme, ki zahteva periodične robne pogoje v vzdolžni (x) in prečni smeri (z). Re_τ je Reynoldsovo število disipacije in je določeno kot:

The terms $\vec{l}_x$ (unit vector in the streamwise direction) and u_x^+ / u_B^+ appear in equations (2) and (3) due to the numerical scheme, which requires periodic boundary conditions in the streamwise and spanwise directions. Re_τ is the friction Reynolds number and is defined as:

$$\mathrm{Re}_\tau = \frac{u_\tau h}{\nu} \tag{4}$$

kjer je h polovica višine kanala. Ne smemo ga mešati z običajnim Reynoldsovim številom, ki je v kanalu definirano kot $\mathrm{Re} = u_B \cdot 2h / \nu$. Iz Reynoldsovega števila disipacije dobimo običajno Reynoldsovo število, če Re_τ pomnožimo z dvakratno vrednostjo povprečne hitrosti u_B v kanalu. Reynoldsovo število $\mathrm{Re} = 4580$ ustreza $\mathrm{Re}_\tau = 150$.

Na osnovi Reynoldsovega števila disipacije priredimo brezrazsežne stenske enote, ki so označene z zgornjim indeksom $^+$. Višina kanala je po definiciji enaka dvakratniku Reynoldsovega števila disipacije. Smisel stenskih enot je v tem, da lahko v območju ob steni med seboj primerjamo turbulentne tokove z različnimi Reynoldsovimi števili. Brezrazsežna temperaturna razlika je določena kot:

where h is the channel half height. It should not be confused with the usual Reynolds number in the channel, which is defined as $\mathrm{Re} = u_B \cdot 2h / \nu$. The usual Reynolds number in the channel can be obtained from the friction Reynolds number multiplied by the double bulk velocity u_B. The Reynolds number $\mathrm{Re} = 4580$ corresponds to $\mathrm{Re}_\tau = 150$.

The dimensionless wall units, denoted by the superscript $^+$, are based on the friction Reynolds number. By definition, the height of the channel is equal to two times the friction Reynolds number. The meaning of the wall units is in the comparison of the turbulent flows near the wall at different Reynolds numbers. The dimensionless wall-temperature difference is defined as:

$$\theta^+(x,y,z,t) = \left(\frac{\langle T_w \rangle - T(x,y,z,t)}{T_\tau} \right) \tag{5}$$

Na prosti površini tekočine sta robna pogoja za komponenti hitrosti, ki sta vzporedni s steno, enaka $du/dz = 0$ in $dw/dz = 0$, hitrost, pravokotno na steno, pa je enaka $v_{prostapovršina}=0$. Takšen robni pogoj ni fizikalen, ker ne dopušča površinskih valov. Vendar so preskusi Hetsronija (1997, 1999) [12] pokazali, da so površinski valovi pri majhnih Reynoldsovih številih skoraj neznatni in ne vplivajo na obnašanje tekočine tik ob steni. Na stiku stene in tekočine so vse tri komponente hitrosti enake nič. Poleg robnega pogoja hitrosti moramo upoštevati še temperaturni robni pogoj. Prosto površino obravnavamo kot adiabatno: $d\theta^+/dy = 0$, na stiku stene in tekočine pa sta mogoča dva različna temperaturna robna pogoja. Prvi, ki ga v tem prispevku ne obravnavamo, je robni pogoj nespremenljive temperature [11], drugi pa robni pogoj nespremenljivega toplotnega toka, za katerega velja:

The boundary conditions for the velocity components on the top, free surface, parallel to the wall, are $du/dz = 0$ and $dw/dz = 0$, whereas the wall-normal velocity is $v_{free\ surface}=0$. The velocity boundary condition at the free surface is not physical since it does not allow surface waves. However, the experiments of Hetsroni et. al. (1997, 1999) [12] show that this is an acceptable approximation at low Reynolds numbers, where the surface waves are negligible and do not affect the near-wall behavior. The velocity components at the interface of the wall and the fluid are set to zero. Besides the velocity boundary conditions, the thermal boundary conditions have to be considered. The free surface is treated as an adiabatic surface, $d\theta^+/dy = 0$, whereas two different thermal boundary conditions can be applied at the wall-fluid interface. The first, which is not presented in this paper, is the isothermal boundary condition [11], and the second isoflux boundary condition is described as:

$$\frac{d\theta^+}{dy^+} = 0 \qquad (6).$$

Pri tem je brezrazsežna temperatura θ na stiku stene in tekočine povprečena po času in koordinatah vzdolž in prečno na tok enaka nič:

The dimensionless temperature θ at the heated wall, averaged by time and the coordinates in the streamwise and spanwise directions is zero:

$$\left\langle \theta^+\left(y=1\right)\right\rangle = 0 \qquad (7).$$

Na vstopu v kanal moramo zagotoviti polno razvit turbulentni tok, ki ga dosežemo s periodičnimi robnimi pogoji v smeri vzdolž (x) in prečno (z) na smer toka. To pomeni, da stanje, ki ga dobimo na izhodu iz kanala, preslikamo na vstop, stanje na levi strani pa na desno stran. Pri tem je treba poskrbeti, da sta dolžina in širina kanala dovolj veliki za mešanje tekočine [11].

Kakor je razvidno iz enačb (1) do (3), je temperatura pasivni skalar, ki ne vpliva na turbulenco. To pomeni, da je vzgon zanemarjen in da lastnosti tekočine (viskoznost, toplotna prevodnost itn.) niso odvisne od temperature. Dobljeni rezultati so točni le za sisteme, pri katerih ni velikih temperaturnih razlik, medtem ko je pri večjih temperaturnih razlikah potrebna večja previdnost. Takšen približek so že uporabili Kasagi [2], Kawamura [3] in Tiselj [11].

Za reševanje enačb je bila uporabljena spektralna shema, ki uporablja Fourierjeve vrste v smereh x in z ter polinome Čebiševa v smeri y. Kontinuitetno, gibalno in energijsko enačbo rešujemo z računalniškim programom, ki temelji na delu Gavrilakisa [1].

Vse numerične simulacije so bile izvedene pri $Re_\tau = 150$ in $Re_\tau = 170{,}8$ ter $Pr = 0{,}025$, 1 in 5,4. Geometrijska oblika toka tekočine je bila v primeru $Re_\tau = 150$, pri katerem je bil uporabljen zaprt kanal, enaka 2356 x 942 x 300 stenskih enot v smereh x, z in y. V primeru $Re_\tau = 170{,}8$ je bil uporabljen odprti kanal, pri katerem je bila geometrijska oblika enaka

Fully-developed turbulent flow must be ensured in the channel entrance. This is achieved with periodic boundary conditions in the streamwise (x) and spanwise (y) directions. It means that the fields at the channel exit are mapped to the channel entrance, and the situation on the left-hand side to the right-hand side, respectively. The length and the width of the channel should ensure sufficient mixing of the fluid [11].

As can be seen from Eqs. (1) to (3) the temperature is assumed to be a passive scalar. This assumption introduces two approximations: neglected buoyancy and neglected temperature dependence of the fluid properties (viscosity, heat conductivity, etc.). The results are thus very accurate, but only for the systems where the temperature differences are not too large; and some caution is required for the systems where the temperature differences are not negligible. Such an approximation was used by Kasagi et al [2], Kawamura et al [3], and Tiselj et al [11].

The equations are solved with a pseudo-spectral scheme using a Fourier series in the x and z directions, and Chebyshev polynomials in the wall-normal y direction. The numerical procedure and the code of Gavrilakis et al[1] are used to solve the continuity, momentum and energy equations.

All the numerical simulations were performed for $Re_\tau = 150$ and $Re_\tau = 170.8$ at $Pr = 0.025$, 1 and 5.4. At $Re_\tau = 150$, where a channel was used, the computational domain was 2356 x 942 x 300 wall units in the x, z and y directions, respectively. At $Re_\tau = 170.8$, where a flume was used, the computational

2146 x 537 x 171 stenskih enot v smereh x, z in y. Rezultate so začeli povprečiti potem, ko je bil dosežen polno razvit turbulentni tok, kar pomeni, da se tok statistično gledano ni več spreminjal.

Simulacije lahko glede na vrednosti Prandtlovih števil razdelimo na tri glavne skupine (preglednica 1). V prvem delu so prikazane NNS za Pr = 0,025, kjer je bila uporabljena ločljivost, zadostna za hitrostno in temperaturno polje. Ko je Prandtlovo število manjše od 1, so najmanjše krajevne skale temperaturnega polja večje od najmanjših krajevnih skal temperaturnega polja. Drugi del vsebuje NNS za Pr = 1, kjer je bila izbrana ločljivost zadostna za hitrostno in temperaturno polje. Tretji del vsebuje tri različne NNS za Pr = 5,4. Ločljivost prve študije NNS je bila zadostna za hitrostno in temperaturno polje, druga študija je imela približno 2-krat manjšo ločljivost v vzdolžni (x) in prečni smeri (z), tretja študija pa je imela v primerjavi s prvo približno 2-krat manjšo ločljivost v vseh treh smereh. Zadnjih dveh numeričnih simulacij pravzaprav ne smemo več imenovati NNS, ker ne popišeta najmanjših skal temperaturnega polja, ki se pojavijo pri visokih valovnih številih. Takšne simulacije sva poimenovala ″navidez″ NNS.

domain was 2146 x 537 x 171 wall units in the x, z and y directions, respectively. The results were averaged after the fully-developed turbulent flow was achieved, which means that the flow did not change from the statistical point of view.

The simulations can be divided into three main parts, according to the Prandtl numbers (see table 1). In first part the DNS at Pr = 0.025 is shown, where the applied resolution was sufficient for the velocity and thermal fields. If the Prandtl number is less than 1, the smallest length scales of the temperature field are larger than the smallest length scales of the velocity field. The second part involves a DNS at Pr = 1, where the chosen resolution was sufficient for the velocity and thermal fields. The third part involves three different DNSs at Pr = 5.4. The first DNS resolution was sufficient for the velocity and thermal fields, the second resolution had an approximately two-times smaller resolution in the streamwise (x) and spanwise (z) directions, and the third resolution had a two-times smaller resolution in all three directions. Strictly speaking, the last two numerical simulations cannot be called DNS due to the smallest scales of the thermal field at high wave-number modes. Such simulations were named "quasi" DNSs.

Preglednica 1. *Izračuni pri različnih Pr in mrežah*
Table 1. *Computational conditions at different Pr and grids*

geometrija geometry	Re_τ	Pr	mreža grid	Δt^+	Δx^+	Δz^+	Δy^+	čas povprečenja averaging time
zaprt kanal channel	150	0,025	128x128x97	0,09	18,41	7,36	0,08-4,9	4500
odprt kanal flume	170,8	1	128x72x65	0,0512	16,77	7,46	0,10-4,19	3074
	170,8	5,4	256x128x129	0,0256	8,38	4,19	0,05-2,10	2560
	170,8	5,4	128x72x129	0,0427	16,77	7,46	0,05-2,10	2562
	170,8	5,4	128x72x65	0,0683	16,77	7.46	0,10-4,19	4099

2 REZULTATI

2.1 Vpliv Prandtlovega števila

Prvi sklop rezultatov so NNS, kjer smo analizirali vpliv naraščanja Prandtlovega števila na temperaturne profile, fluktuacije temperatur in turbulentne toplotne tokove pri treh različnih Prandtlovih številih: Pr = 0,025, Pr = 1 in Pr = 5,4.

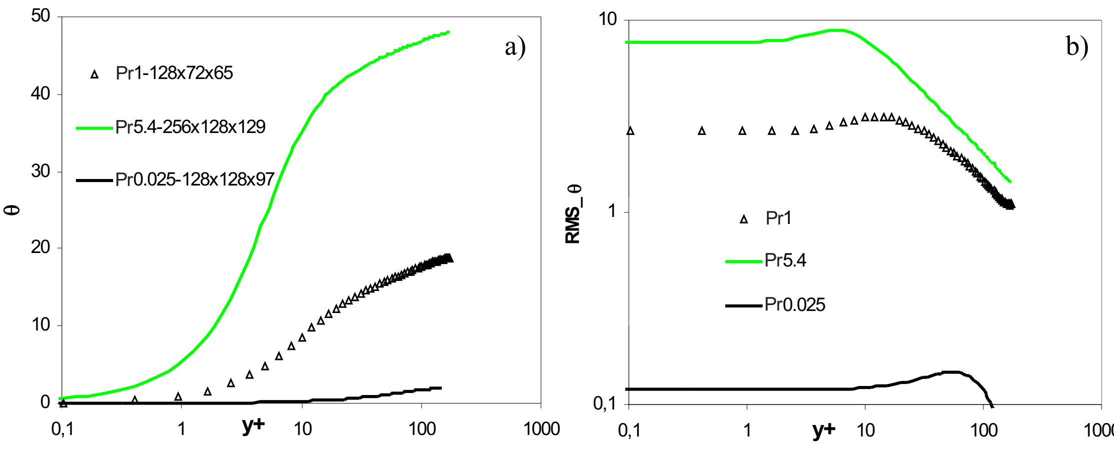
Na sliki 2a so prikazani povprečni brezrazsežni temperaturni profili za tri različne vrednosti Prandtlovih števil. Brezdimenzijsko temperaturo θ dobimo s povprečenjem po ravninah, vzporednih z greto steno in po času (~ 10000 časovnih korakov). Poudariti je treba, da je θ negativna brezrazsežna temperaturna razlika, kar pomeni, da se največje brezrazsežne temperaturne razlike med tekočino in greto steno pojavijo v sredini zaprtega kanala oziroma na prosti površini odprtega kanala. Večje ko je Prandtlovo število, večja je brezrazsežna temperaturna razlika med steno in površino tekočine. Slika 2b

2 RESULTS

2.1 Prandtl number influence

The first part of the results presents the influence of the Prandtl number on the temperature profiles, temperature fluctuations and turbulent heat fluxes at Pr = 0.025, Pr =1 and Pr = 5.4.

Fig. 2a shows the average dimensionless temperature profiles at three different Prandtl numbers. The dimensionless temperature θ is averaged in the planes parallel to the heated wall and in time (~ 10000 time steps). It should be emphasized that the temperature θ is a negative dimensionless difference. This means that the maximum temperature differences appear in the middle of the channel or on the top free surface in the flume. A higher Prandtl number means a higher dimensionless temperature difference between the wall and the middle of the fluid in the channel or on the top surface in the flume, respectively. Fig. 2b shows the temperature



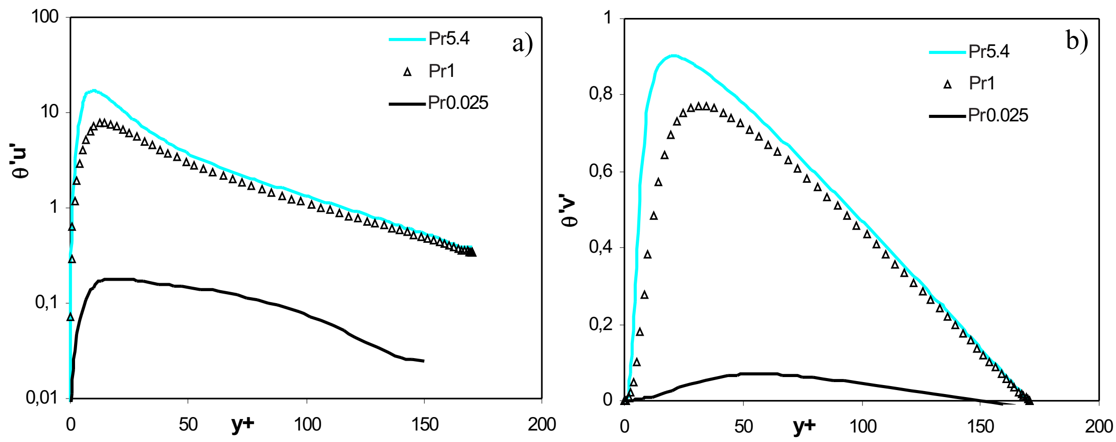
Sl. 2. *a) Povprečni brezrazsežni temperaturni profili, b) fluktuacije temperatur*
Fig. 2. *a) Dimensionless profiles of mean temperature, b) temperature fluctuations*

prikazuje fluktuacije temperatur. Fluktuacije temperatur *RMS_θ* dobimo tako, da trenutne temperature odštejemo od povprečnih, nato pa kvadrate razlik povprečimo po ravninah, vzporednih s steno, in času. Večje ko je Prandtlovo število, večje so fluktuacije. Pri preučevanem robnem pogoju temperature (robni pogoj nespremenljivega toplotnega toka) so fluktuacije opazne že ob steni in ostanejo konstantne v območju laminarne podplasti. Največjo vrednost dosežejo v vmesni plasti, med laminarno in turbulentno podplastjo, nato pa se zmanjšujejo do sredine zaprtega kanala oziroma do proste površine odprtega kanala.

Slika 3a prikazuje turbulentni vzdolžni toplotni tok $\theta'u'$ v odvisnosti od razdalje od stene za različna Prandtlova števila: Pr = 0,025, Pr = 1 in Pr = 5,4. Izračunamo ga tako, da zmnožek fluktuacij temperature θ in fluktuacij komponente hitrosti u v smeri vzdolž kanala (x) povprečimo po ravninah, vzporednih s steno, in po času. Vidimo, da večje Prandtlovo število pomeni večjo največjo vrednost toplotnega toka in manjšo razdaljo do stene. Zelo podobne ugotovitve sledijo za turbulentni toplotni tok v smeri pravokotno na steno $\theta'v'$ (sl. 3b).

fluctuations. The temperature fluctuations *RMS_θ* are obtained as the root mean square difference of the instantaneous and averaged temperatures averaged in planes parallel with the heated wall and in time. The temperature fluctuations already appear near the wall and remain constant through the viscous sublayer for the applied thermal boundary condition (isoflux boundary condition). In the buffer sublayer, between the turbulent and viscous sublayers, the maximum is reached; afterwards the *RMS*_θ is decreasing towards the middle of the channel or the top free surface of the flume.

Fig. 3a shows the profiles of the turbulent axial heat fluxes $\theta'u'$ versus the dimensionless distance from the wall at different Prandtl numbers: Pr = 0.025, Pr = 1 and Pr = 5.4. It is calculated as a product of the temperature fluctuations and the streamwise (x) velocity fluctuations averaged by planes parallel with the wall and by time. It is seen that the higher Prandtl number means a higher maximum value of heat flux and a smaller distance to the wall. Very similar conclusions were found for the turbulent wall-normal heat flux $\theta'v'$ (Fig. 3b).



Sl. 3. *Turbulentni toplotni tok: a) vzdolžno, b) v smeri pravokotno na steno*
Fig. 3. *Profiles of turbulent heat flux: a) axial, b) wall normal*

2.2 Vpliv ločljivosti pri Pr = 5,4

Drugi sklop rezultatov predstavljajo numerične simulacije pri Prandtlovem številu Pr = 5,4 in treh različnih računskih mrežah: 256x128x129, 128x72x129 in 128x72x65. Zaradi Prandtlovega števila, večjega od ena, samo ločljivost prve mreže ustreza teoretičnim zahtevam NNS za hitrostno in temperaturno polje, preostali dve mreži zadostujeta le za hitrostni polji.

Rezultati na sliki 4a kažejo, da različno število mrežnih točk ne vpliva na povprečne temperaturne profile. Razlike, ki se pojavijo, so kvečjemu istega reda velikosti kot negotovosti zaradi statistične obravnave rezultatov (~0,5%). Podobno lahko povzamemo za temperaturne fluktuacije na sliki 4b. V tem primeru je statistična napaka manjša od 2%.

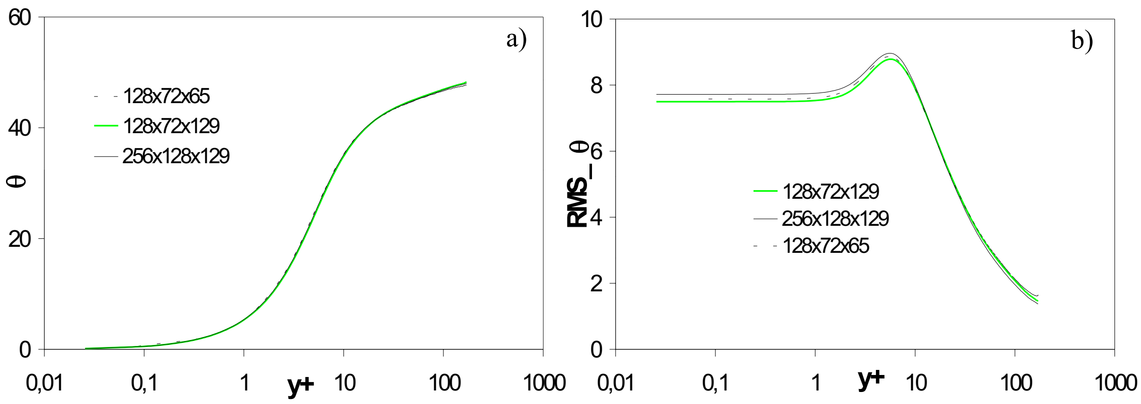
Pri turbulentnih toplotnih tokovih (sl. 5) so razlike nekoliko večje kakor v prejšnjih primerih, vendar so ocenjene napake še vedno znotraj statistične negotovosti, razen odstopanja vzdolžnega turbulentnega toplotnega toka pri $y^+ > 100$ na najbolj grobi mreži (sl. 5a).

2.2 Resolution at Pr = 5.4

The second part of the results presents numerical simulations at the Prandtl number Pr = 5.4, and three different computational grids: 256x128x129, 128x72x129 in 128x72x65. Because the Prandtl number is bigger than one, only the resolution of the first grid corresponds to the theoretical requirements of the DNS for the velocity and thermal field, while the other grids are sufficient only for the DNS of the velocity fields.

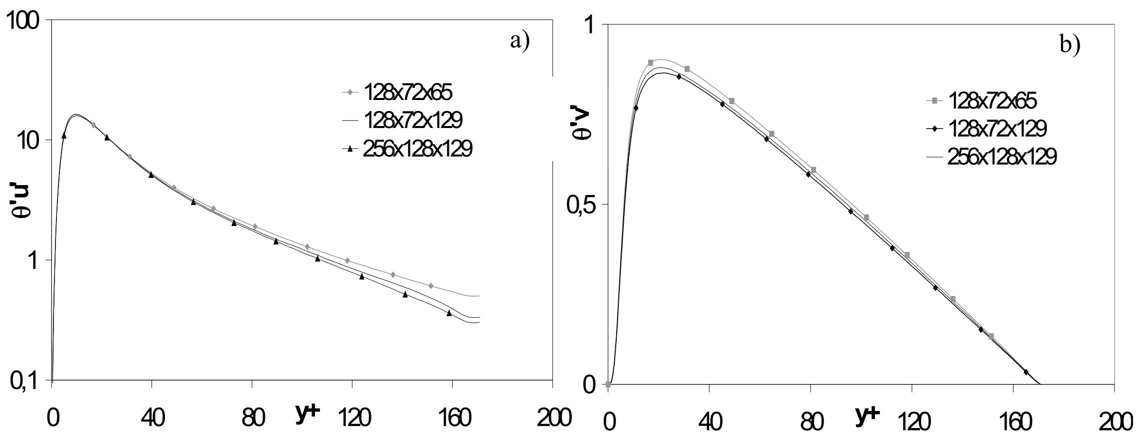
The results in Fig. 4 show that the number of grid points does not influence the mean temperature profiles. All the differences that appear are of the same order of magnitude as the uncertainties. A similar conclusion can be made for the temperature fluctuations (Fig. 4b), where the statistical uncertainties are less than 2%.

The differences are slightly larger in the turbulent heat fluxes (Fig. 5), but the estimated errors are still within the statistical uncertainties, except the turbulent axial heat flux deviations of the smaller grid at $y^+ > 100$ (Fig. 5a).



Sl. 4. *a) Povprečni brezrazsežni temperaturni profili, b) fluktuacije temperatur pri Pr = 5,4 in treh različnih ločljivostih*

Fig. 4. *a) Dimensionless profiles of mean temperatures, b) temperature fluctuations at Pr = 5.4 and three different resolutions*



Sl. 5. *Turbulentni toplotni tok: a) vzdolžno, b) v smeri pravokotno na steno*

Fig. 5. *Turbulent heat fluxes: a) axial, b) wall normal*

Pri preučevanju ločljivosti računske mreže se uporabljajo spektri, ki prikazujejo, kako so v toku zastopane različne krajevne skale. Spekter fluktuacij temperature (komponente hitrosti) izračunamo z avtokorelacijsko funkcijo temperature (komponente hitrosti). Tako npr. avtokorelacijsko funkcijo temperature vzdolž kanala (x) pri izbrani razdalji od stene y_0 dobimo kot:

In a study of the resolution requirements the spectra are usually used to show how different length scales are represented. The spectrum of the temperature (velocity component) fluctuations is derived from an auto-correlation function of the temperature (velocity component). For example, the auto-correlation function of the temperature in the streamwise direction at a given distance from the wall y_0 is:

$$R(x_1) = \sum_{j=-\frac{N_2}{2}}^{\frac{N_2}{2}} \sum_{i=-\frac{N_1}{2}}^{\frac{N_1}{2}} \theta(x_i, y_0, z_j)\cdot\theta(x_i + x_1, y_0, z_j) \qquad (8).$$

Če avtokorelacijske funkcije preslikamo po Fourier-ju in rezultat ustrezno normiramo [6], dobimo spekter temperature pri izbrani oddaljenosti od stene y_0 v smeri vzdolž toka x. Na podoben način pridemo do spektrov v prečni smeri z.

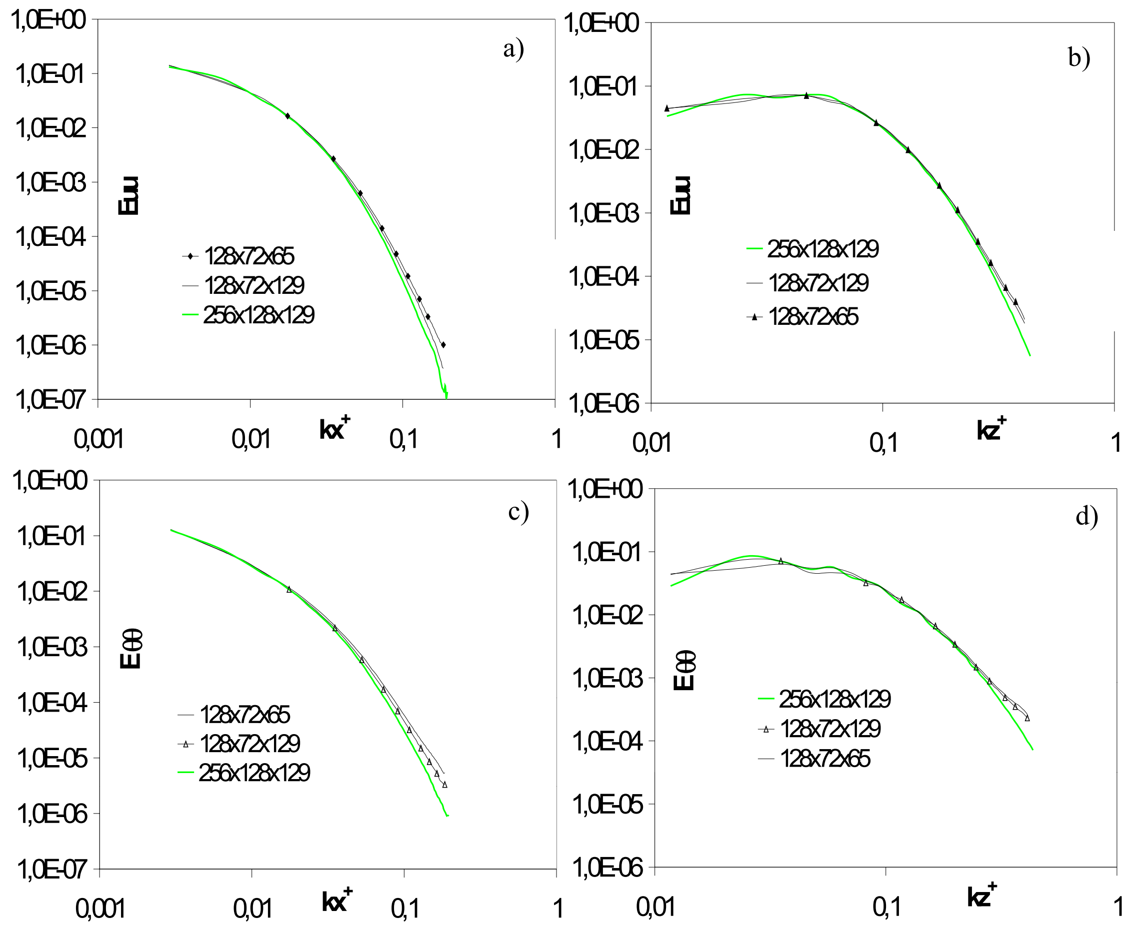
Turbulentno gibanje sestavljajo vrtinci različnih izmer. Geometrijska oblika sistema povzroča največje vrtince, najmanjši pa so določeni z viskoznimi silami. Za najmanjše vrtince je značilna velik raztros kinetične energije turbulentnega gibanja v toploto. Viskozna strižna napetost opravi deformacijsko delo, ki poveča notranjo energijo tekočine na račun kinetične energije turbulence. Povedano drugače, večji vrtinci, ki jih v diagramih spektrov opisujejo nižje vrednosti valovnih števil, difundirajo v manjše vrtince, ki jih opisujejo višje vrednosti valovnih števil. Energija manjših vrtincev je manjša od energije večjih vrtincev, zato se spektri z večanjem valovnih števil zmanjšujejo. Primerjave spektrov fluktuacij hitrosti (na slikah so spektri za najbolj pomembno komponento hitrosti u_x) in temperatur kažejo pričakovane rezultate: hitrostni spektri se hitreje zmanjšujejo proti nič kakor temperaturni spektri pri Pr > 1. Približno enako stopnjo zmanjševanja dobimo za Pr = 1. Večja vrednost Prandtlovega števila pomeni počasnejši razpad nižjih valovnih števil v višja, kar zahteva večjo ločljivost za opis vseh najmanjših temperaturnih skal.

To potrjuje slika 6, ki prikazuje spektre fluktuacij temperatur in hitrosti vzdolžno in prečno na smer toka pri razdalji y^+=3,7 od stene. Manjše razlike so vidne v spektrih fluktuacij hitrosti (6a, 6b), medtem ko so te razlike večje v spektrih fluktuacij temperatur (6c, 6d). Največje razlike med numeričnimi simulacijami različnih ločljivosti so v območju visokih valovnih števil, kjer obe manjši ločljivosti (128x72x65 in 128x72x129) kažeta večjo moč kakor NNS največje ločljivosti (256x128x129), kar kaže na nekoliko prepočasno dušenje turbulentnih fluktuacij. Kljub temu v obravnavanih spektrih ni zaslediti kopičenja pri visokih vrednostih valovnih številih. Kopičenje (spekter pri visokih valovnih številih začne naraščati) se pojavi takrat, ko najmanjše skale niso dovolj natančno modelirane in viskozni (temperaturni) disipaciji na viskozni (konduktivni) skali ne uspe spremeniti vse turbulentne kinetične energije v

If a Fourier transformation over the auto-correlation function is made, and the result is appropriately normalized [6], the temperature spectrum at a given distance from the wall y_0 in the streamwise direction x is obtained. Spectra in the spanwise direction z are obtained in a similar way.

Turbulent motion consists of vortices of different dimensions. The largest vortices are defined by the flow geometry, whereas the smallest one is defined by the viscous forces. A high dissipation of the turbulent kinetic energy into the heat is typical for the smallest vortices. Viscous shear stress makes deformation work, which transforms the turbulent kinetic energy into the internal energy of the fluid. In other words, larger vortices represented by lower wave-number modes diffuse into smaller vortices represented by higher wave-number modes in the spectrum diagrams. The energy of the smaller vortices is smaller than the energy of the larger vortices, therefore the spectra decrease with increasing wave-number modes. The comparisons of the velocity (the most important velocity spectra for the velocity component u_x are shown in the figures) and the temperature spectra show the expected results: the velocity spectra decay faster to zero than the temperature spectra at Pr > 1. Approximately the same decay rate is obtained for Pr = 1. A higher Prandtl number means slower decays of low wave-number modes into the high wave-number modes, and thus requires a more detailed resolution to capture all the significant thermal scales.

This is confirmed in Fig. 6, which shows the streamwise and spanwise spectra of the temperature and the velocity fluctuations at y^+=3.7 from the wall. Minor differences are seen in the velocity-fluctuation spectra (Figs. 6a and 6b), whereas these differences are larger in the temperature-fluctuation spectra (Figs. 6c and 6d). The main differences between the numerical simulations at different resolutions are observed at higher wave-number modes, where both lower resolutions (128x72x65 in 128x72x129) show stronger modes than the well-resolved DNS at the higher resolution. This indicates too-slow damping of the turbulent fluctuations. However, there are no “pile-up” phenomena seen at high wave-number modes. The pile-up phenomenon (the spectrum starts to grow at high wave-number modes) appears when the smallest scales are not properly modeled and viscous (temperature) dissipation and viscous (conductive) scale cannot



Sl. 6. *Spektri vzdolž (a, c) in prečno na kanal (b, d) y^+ = 3.7 od grete stene: a, b) temperatura, c, d) hitrost vzdolž toka*

Fig. 6. *Spectra in streamwise (a, c) and spanwise (b, d) directions at y^+ = 3.7 from the heated wall: a, b) temperature, c, d) streamwise velocity*

notranjo energijo. To pomeni, da moramo povečati ločljivost računske mreže, če hočemo zajeti najmanjše (Kolmogorovove) skale.

3 SKLEP

V prvem delu je bila opisana neposredna numerična simulacija (NNS) polno razvitega turbulentnega toka pri Reynoldsovem številu Re = 4580 (Re_τ = 150) in Prandtlovem številu Pr = 0,025 ter Re = 5260 (Re_τ = 170,8) in Prandtlovih številih Pr = 1 in Pr =5,4. Rezultati kažejo, da Prandtlovo število močno vpliva na statistiko turbulentnega polja. Rezultati opravljenih NNS lahko rabijo pri razvoju turbulentih modelov za tokove v bolj zapletenih geometrijskih oblikah in pri višjih vrednostih Reynoldsovih števil.

V drugem delu smo preučevali numerične simulacije treh različnih ločljivosti pri Re_τ = 170,8 in Pr = 5,4. Ločljivost naj bi bila teoretično sorazmerna kvadratnemu korenu Prandtlovega števila [8], vendar rezultati kažejo, da je vsaj pri Pr = 5,4 ta zahteva nekoliko preostra. Naši rezultati (profili povprečnih

change all the turbulent kinetic energy into internal energy. It means that the resolution should be increased in order to capture the smallest (Kolmogorov) scales.

3 CONCLUSION

In the first part of the paper the DNS of the fully-developed turbulent flow at Reynolds number Re = 4580 (Re_τ = 150) and Prandtl number Pr = 0.025, and at Re = 5260 (Re_τ = 170.8) and Prandtl numbers Pr = 1, 5.4, were performed. The results show that the Prandtl number has a strong influence on the turbulent statistics. The obtained results can be used for developing turbulent models for flows in a more complex geometry and at higher Reynolds numbers.

In the second part the numerical simulations of three different resolutions at Re_τ = 170.8 and Pr = 5.4 were performed. Theoretically, the resolution for the DNS should be proportional to the square root of the Prandtl number [8]; however, results show that this requirement is too stringent in the case of Pr = 5.4. Our results at Pr = 5.4 (mean temperature profiles, hitrosti, fluktuacije, toplotni pretoki) pri najslabši ločljivosti kažejo, da je ločljivost, ki zadošča za simulacijo NNS hitrostnega polja dovolj velika tudi za simulacijo temperaturnega polja pri Pr = 5,4. Majhne razlike so vidne v spektrih fluktuacij temperatur v območju višjih valovnih števil, kjer obe računski mreži z nižjima ločljivostma kažeta počasnejšo spremembo turbulentne kinetične energije v notranjo kot NNS največje ločljivosti.

fluctuations, heat fluxes) for the lowest resolution show that the resolution, which is sufficient for the DNS of the velocity field, is also sufficient for the simulation of the thermal field at Pr = 5.4. Small differences in the temperature spectra are seen at high wave-number modes, where both low resolutions show a slower conversion of the turbulent kinetic energy into internal energy compared with the DNS of the highest resolution.

4 SPREMENLJIVKE
4 NOMENCLATURE

spekter	E	spectrum
polovična višina kanala	h	channel half height
enotski vektor v smeri x (1,0,0)	$\vec{1}_x$	unit vector in x direction (1,0,0)
valovno število	k	wave number
vzdolžna in prečna dolžina kanala	L_1, L_3	streamwise and spanwise length of box
tlak	p	pressure
Prandtlovo število	Pr	Prandtl number
toplotni tok iz stene na tekočino	q_w	wall-to-fluid heat flux
Reynoldsovo število	Re	Reynolds number
Reynoldsovo število raztrosa	Re_τ	friction Reynolds number
avtokorelacijska funkcija	R	auto-correlation function
čas	t	time
temperatura	T	temperature
smer vzdolž, prečno, pravokotno na kanal	x, y, z	streamwise, spanwise, wall normal distance
komponente hitrosti v smereh x, y in z	u, v, w	velocity components in x, y and z directions
raztrosna hitrost	u_τ	dissipative velocity
toplotna difuzivnost	α	thermal diffusivity
brezrazsežna temperaturna razlika	θ	dimensionless temperature difference
toplotna prevodnost	λ	thermal conductivity
kinematična viskoznost	γ	kinematic viscosity
gostota	ρ	density
tekočina	$()_f$	fluid
stena	$()_w$	wall
disipacija	$()_\tau$	dissipation
normalizirano z u_τ, T_τ, v	$()^+$	normalized by u_τ, T_τ, v

5 LITERATURA
5 REFERENCES

[1] Gavrilakis, S., H.M. Tsai, P.R. Voke, D.C. Leslie (1986) Direct and large eddy simulation of turbulence, *Notes on Numerical Fluid Mechanics* Vol. 15, ed. U. Schumann, R. Friedrich, Vieweg, Braunschweig, D.B.R., 105.

[2] Kasagi, N., Y. Tomita, A. Kuroda (1992) Direct numerical simulation of passive scalar field in a turbulent channel flow, *Jornal of Heat Transfer -Transactions of ASME*, Vol. 114, 598-606.

[3] Kawamura, H., K. Ohsaka, H. Abe, K. Yamamoto (1998) DNS of turbulent heat transfer in channel flow with low to medium-high Prandtl number fluid, *International Journal of Heat and Fluid Flow*, Vol. 19, 482-491.

[4] Kawamura, H., H. Abe, Y. Matsuo (1999) DNS of turbulent heat transfer in channel flow with respect to Reynolds and Prandtl number effects, *International Journal of Heat and Fluid Flow,* Vol. 20, 196-207.

[5] Kim, J., P. Moin, R.D. Moser (1987) Turbulence statistics in fully developed channel flow at low Reynolds number, *J. Fluid Mech*., Vol. 130, 133-166.

[6] Kim, J., P. Moin (1989) Transport of passive scalars in a turbulent channel flow, Turbulent shear flows VI, *Springer-Verlag*, Berlin.

[7] Na, Y., T.J. Hanratty (2000) Limiting behavior of turbulent scalar transport close to a wall, *International Journal of Heat and Mass Transfer*, Vol. 43, 1749-1758.
[8] Tennekes, H., J.L. Lumley (1972) A first course in turbulence, *MIT Press*, Cambridge, MA.
[9] Tiselj, I., E. Pogrebnyak, C. Li, A. Mosyak, G. Hetsroni (2001) Effect of wall boundary condition on scalar transfer in a fully developed turbulent flume, *Physics of Fluids,* 13 (4), 1028-1039.
[10] Kader, B.A. (1981) Temperature and concentration profiles in fully turbulent boundary layers, *Int. J. Heat Mass Transf.,* 24, 1541.
[11] Tiselj, I., R. Bergant, B. Mavko, I. Bajsic, G. Hetsroni (2001) DNS of turbulent heat transfer in channel flow with heat conduction in the solid wall, *J. Heat Transf.*, 123, 849-857.

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