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Računsko reševanje inverznega problema oblikovanja nadzvočne šobe

A Numerical Solution to the Inverse Problem of Supersonic-Nozzle Design

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Računsko oblikovanje nadzvočne šobe je občutljivo glede stabilnosti v področju nadzvočnega toka. Računski model, predstavljen v tem prispevku, se izogiba tej nestabilnosti z uvajanjem analitično določene porazdelitve tlaka na osi osnosimetrične nadzvočne šobe. Parametri toka v inverzno oblikovani šobi so preverjeni s programom FLUENT in prikazujejo enakomerno porazdelitev po prečnih prerezih vzdolž šobe.

(Ključne besede: šobe nadzvočne, oblikovanje šob, modeli računski, problemi inverzni)

The numerical design of a nozzle is sensitive to stability in the region of supersonic flow. In the numerical algorithm presented in this paper the instability is avoided by the introduction of an analytically set pressure distribution on the axis of the axisymmetrical supersonic nozzle. The flow parameters of the inverse designed nozzle are checked by the application code FLUENT and they show a regular distribution on cross-sections along the nozzle.

(Keywords: supersonic nozzle, nozzle design, numerical solutions, inverse problems)

0 UVOD

Računski postopek oblikovanja nadzvočne šobe je še posebej občutljiv glede stabilnosti v področju nadzvočnega toka ([1] do [5]). Čeprav je uporaba računskih metod pogosta v praksi, le redko najdemo ustrezni algoritem v obliki uporabniškega programa za rešitev inverznega problema prenosa toplote in snovi. Problem je inverzen, ker je področje neznano [6].

V rešitvi, prikazani v tem prispevku, je v primeru osnosimetrične šobe določena izvirna analitična porazdelitev tlaka na osi simetrije. Izračun oblike šobe in parametrov toka je izveden po koračnem postopku po [7]. Začetni pogoj je izpeljan posebej. Parametri toka, v tako oblikovani šobi, so preverjeni z uporabo programa FLUENT. Dobljeni rezultati se dobro ujemajo.

0 INTRODUCTION

The numerical algorithm of supersonic-nozzle design is particularly sensitive to stability in the region of supersonic flow ([1] to [5]). Although the application of numerical methods is very common in practice it is very rarely possible to find an appropriate application code for the solution of the inverse heat- and mass-transfer problems. The problem is inverse because the domain is unknown [6].

In the solution presented in this paper, for the case of an axis-symmetrical nozzle, the original analytic pressure distribution on the axes of symmetry is defined. The numerical calculation of the nozzle form and the flow parameters were performed with the marching algorithm according to [7]. The initial condition was derived separately. The flow parameters in the nozzle designed in this way were checked for closed domain with the application code FLUENT and the results obtained correspond very well.

1 OPIS MATEMATIČNEGA MODELA

Poleg kontinuitetnih, gibalnih in energijskih enačb, ki popisujejo tok v šobi, uvedemo funkcijo toka $\bar{\Psi}$ z izrazom:

1 DESCRIPTION OF THE MATHEMATICAL MODEL

In addition to equations of continuity, motion and energy, which describe the flow in the nozzle, the stream function $\bar{\Psi}$ is introduced with the expression:

$$\vec{w} \cdot \nabla \bar{\Psi} = 0$$

oziroma v valjnih koordinatah za osnosimetrični primer:

i.e. in cylindrical coordinates for the axisymmetrical case:

$$\bar{u}\frac{\partial\bar{\Psi}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{\Psi}}{\partial\bar{r}} = 0$$

kjer so: $\bar{x}$ – koordinata na osi simetrije; $\bar{r}$ – radialna koordinata pravokotna na os simetrije, $\bar{u}$, $\bar{v}$ – projekcije hitrosti $\bar{w}$ na osi $\bar{x}$ in $\bar{r}$. Sedaj enačbe toka prikažemo preprosto:

where: $\bar{x}$ is the coordinate in the axis of symmetry; $\bar{r}$ is the radial coordinate perpendicular to the axis of symmetry; $\bar{u}$, $\bar{v}$ are the projections of velocity $\bar{w}$ on the axes $\bar{x}$ and $\bar{r}$. Now the flow equations are expressed in the simple manner:

$$\frac{\partial\bar{p}}{\partial\bar{\Psi}} = -\frac{\gamma}{\bar{r}}\frac{\partial\bar{v}}{\partial\bar{x}} \quad (1)$$

$$\frac{\partial\bar{r}^2}{\partial\bar{\Psi}} = \frac{2}{\bar{\rho}\bar{u}} \quad (2)$$

$$\frac{\partial\bar{r}}{\partial\bar{x}} = -\frac{\bar{v}}{\bar{u}} \quad (3)$$

$$\bar{p} = \bar{\rho}^{\gamma} \quad (4)$$

$$\bar{u} = \left[\frac{\kappa+1}{\kappa-1} - \frac{2}{\kappa-1}\,\bar{p}^{\frac{\kappa-1}{\kappa}} - \bar{v}^2\right]^{\frac{1}{2}} \quad (5),$$

kjer so: $\kappa = c_p/c_v$ – konstantni eksponent izentrope (za zrak κ = 1,4); c_p, c_v – specifična toplota pri konstantnem tlaku oz. prostornini; $\bar{\rho}$, $\bar{p}$ – gostota oz. tlak snovi v šobi.

Geometrijske količine in hitrosti so normirane:

where: $\kappa = c_p/c_v$ is the constant isentropic exponent (for air κ = 1.4); c_p, c_v are the specific heat at constant pressure i.e. at constant volume; $\bar{\rho}$, $\bar{p}$ is the fluid density i.e. the pressure in the nozzle.

The geometrical quantities and velocities are normalised:

$$r = \frac{\bar{r}}{r_*} \quad ; \quad x = \frac{\bar{x}}{r_*} \quad (6)$$

$$u = \frac{\bar{u}}{a_*} \quad ; \quad v = \frac{\bar{v}}{a_*} \quad (7),$$

kjer so s črto zgoraj označene dejanske količine; $a_* = (\kappa p_*/\rho_*)^{1/2}$ – kritična hitrost širjenja nizkotlačnih motenj; p_*, ρ_* – vrednosti tlaka in gostote v kritični točki. Normirana funkcija toka je:

where the overbar denotes the real quantities; $a_* = (\kappa p_*/\rho_*)^{1/2}$ is the critical propagation velocity of the low-pressure disturbance; p_* and ρ_* are the values of pressure and density at the critical position. The normalised stream function is:

$$\Psi = \frac{\bar{\Psi}}{\rho_* a_* r_*^2}. \quad (8)$$

Normiranje prostorninskih koordinat s kritičnim r_* ni nujno. V tem primeru je mogoče spremeniti merilo za posamezne količine.

Normalising of the space coordinates with critical r_* is not necessary. In this case it is possible to change the measure for particular quantities.

2 ALGORITEM REŠEVANJA

Koračni algoritem. Za rešitev gornjega sistema enačb za vsak Ψ ($\Psi = \Psi_j$) je treba določiti neznanke r, p, ρ, u in v (kot brezdimenzijske količine) za vsak x od vstopa do izstopa iz šobe. Področje je odprto v smeri pravokotno na os simetrije, problem je hiperboličen. Očitno je, da je za koračni postopek treba določiti začetne pogoje. Na osi simetrije je $\Psi_0 = 0$, kar ne moremo upoštevati kot začetni pogoj, ki ga bomo določili kasneje.

Če so vrednosti neznank v vozlih pri $\Psi = \Psi_j$ določene, lahko določimo njihove vrednosti pri

2 ALGORITHM OF THE SOLUTION

Marching algorithm. For the solution of the above system of equations for every Ψ($\Psi = \Psi_j$) it is necessary to determine the unknowns r, p, ρ, u and v (as dimensionless quantities) for every x from the nozzle input to the nozzle output. The domain is open in the direction perpendicular to the axis of symmetry, and the problem is hyperbolic. It is evident that for the marching algorithm the initial conditions have to be set. On the axis of symmetry $\psi_0 = 0$, which cannot be taken as an initial condition, and will be determined below.

If the values of the unknowns in nodes where $\Psi = \Psi_j$ have already been determined then their val-

$\Psi = \Psi_{j+1}$ prek diskretiziranih enačb drugega reda natančnosti.

1. Z diskretizacijo enačbe (2) dobimo:

ues on $\Psi = \Psi_{j+1}$ are determined by means of discreted equations of the second order of accuracy as follows.

1. By discretisation of equation (2) is obtained:

$$r_{i,j+1}^{[l]} = \left\{ r_{i,j}^2 + \left[\left(\frac{1}{\rho u} \right)_{i,j} + \left(\frac{1}{\rho u} \right)_{i,j+1}^{[l-1]} \right] \Delta \Psi \right\}^{\frac{1}{2}} \qquad (9),$$

kjer sta: $x = i\,\Delta x$, $[l]$ - številka iteracije.

2. Z diskretizacijo enačbe (1) dobimo:

where: $x = i\,\Delta x$; $[l]$ is the number of the iteration.

2. By discretisation of equation (1) is obtained:

$$p_{i,j+1}^{[l]} = p_{i,j} - \frac{1}{2}\gamma \left[\left(\frac{1}{r}\frac{\partial v}{\partial x} \right)_{i,j} + \left(\frac{1}{r}\frac{\partial v}{\partial x} \right)_{i,j+1}^{[l-1]} \right] \Delta \Psi \qquad (10).$$

3. Z diskretizacijo enačbe (5) dobimo:

3. By discretisation of equation (5) is obtained:

$$v_{i,j+1}^{[l]} = \left[\frac{\frac{\partial r}{\partial x}}{\sqrt{1 + \left(\frac{\partial r}{\partial x} \right)^2}} \left(\frac{\kappa+1}{\kappa-1} - \frac{2}{\kappa-1} p^{\frac{\kappa-1}{\kappa}} \right)^{\frac{1}{2}} \right]^{[l]} \qquad (11).$$

4. Z diskretizacijo enačbe (4) dobimo:

4. By discretisation of equation (4) is obtained:

$$\rho_{i,j+1}^{[l]} = \left(p^{\frac{1}{\gamma}} \right)_{i,j+1}^{[l]} \qquad (12).$$

5. Nazadnje je:

5. Finally:

$$u_{i,j+1}^{[l]} = \left[\frac{\kappa+1}{\kappa-1} - \frac{2}{\kappa-1} p^{\frac{\kappa-1}{\kappa}} - v^2 \right]_{i,j+1}^{[l]} \qquad (13).$$

Algoritem je koračen po koordinati Ψ, vsak $\Psi = \Psi_j$ lahko uporabimo kot obliko kanala šobe. Iterativen postopek je potreben, ker so količine na desni strani z indeksom $i, j+1$ neznane.

Začetni pogoji. Stabilnost rešitve dobimo z izvirno analitično določeno enačbo spremembe tlaka na osi šobe ($\Psi = 0$):

The algorithm is marching along coordinate Ψ, and every $\Psi = \Psi_j$ can be used as a shape of the nozzle channel. The iterative procedure is necessary because the quantities on the right-hand side with index $i, j+1$ are unknown.

*Initial conditions. The s*tability of the solution is obtained by an original analytical set equation of pressure change on the nozzle axis ($\Psi = 0$):

$$p(x,0) = a\,th(-bx) + c\,; \quad -2 \le x \le 3 \qquad (14),$$

kjer so:

where:

$$p = \frac{\bar{p}}{p_*}$$

$$a = \frac{1}{2}(p_{in} - p_{out})$$

$$c = \frac{1}{2}(p_{in} + p_{out})$$

Iz

From

$$p = \left(\frac{\kappa+1}{2 + (\kappa-1) Ma^2} \right)^{\kappa/\kappa-1} \qquad (15)$$

za mejne pogoje $Ma_{in} = 0{,}1$ in $Ma_{out} = 2{,}1$ dobimo:

for boundary conditions $Ma_{in} = 0.1$ and $Ma_{out} = 2.1$ are obtained:

$$p_{in} = 1{,}880 \qquad p_{out} = 0{,}207 \qquad (16).$$

Strmina krivulje za $x = 0$ preko koeficienta $b > 0$ lahko izberemo poljubno. Priporočeno je, da vzamemo $b < 4$. Pri manjšem b je sprememba tlaka v šobi počasnejša. V prejšnjih enačbah indeksi *in* in *out* označujejo vhodni oz. izhodni prerez šobe.

The slope of the curve for $x = 0$ over coefficient $b > 0$ can be chosen arbitrarily. It is recommended that $b < 4$ is taken. For smaller b the pressure change in the nozzle is slower. In the previous equations the subscripts *in* and *out* denote the input, i.e. the output section of the nozzle.

Določenih pogojev $p(x,0)$ na osi simetrije ne moremo neposredno upoštevati kot začetne pogoje. Vrednost neznank vzdolž tokovnice $\Psi = \Psi_1$ izračunamo s povprečenjem njihovega razvoja v red oblike:

The set conditions $p(x,0)$ on the axis of symmetry $p(x,0)$ cannot be used directly as initial conditions. The values of the unknowns on the stream line $\Psi = \Psi_1$ are calculated by means of their development in series of the form:

$$f\left(x,\Psi\right)=\sum_{n=0}^{N} f_n(x)\Psi^n+\sqrt{\Psi}\sum_{n=0}^{N} f_n'\left(x\right)\Psi^n \qquad (17),$$

kjer je $f(x, \Psi_1) = r, p, v, \rho$ in n na $\Psi = \Psi_1$, ki je blizu osi simetrije.

where $f(x, \Psi_1) = r, p, v, \rho$ and n on $\Psi = \Psi_1$, which is close to the axis of symmetry.

Posamezno odvisnost spremenljivke za osnosimetrično šobo dobimo:
a) Iz enačbe (2) izhaja:

The single dependence of the variable for the axisymmetrical nozzle is obtained as follows.
a) From equation (2):

$$\rho\, u\, r\frac{\partial r}{\partial \Psi}=1 \qquad (18).$$

Za spremenljivke ρ, u, r vzamemo red $f(\rho), f(u), f(r)$:

For variables ρ, u, r the order $f(\rho), f(u), f(r)$ is taken:

$$\frac{\partial r}{\partial \Psi}=\sum_{n=0}^{N} n\, r_n(x)\Psi^{n-1}+\frac{1}{2\sqrt{\Psi}}\sum_{n=0}^{N} r_n^1(0)\Psi^{n-1}+\sqrt{\Psi}\sum_{n=0}^{N} n\, r_n^1\Psi^{n-1} \qquad (19).$$

Po množenju se izenačijo koeficienti z enakimi eksponenti spremenljivke Ψ na levi strani s svojimi dvojniki na desni strani. Dobimo r_n in r_n'.

After multiplication, the coefficients with equal exponents of variable Ψ on the left-hand side are equalised with their counterparts on the right-hand side, and r_n and r_n' are obtained.

b) Iz enačbe (1) dobimo koeficienta p_n in p_n' na podoben način.
c) Iz enačbe (3) izhaja:

b) The coefficients p_n and p_n' are obtained in a similar way from equation (1).
c) From equation (3) follows:

$$v=u\frac{\partial r}{\partial x} \qquad (20),$$

od koder določimo koeficienta v_n in v_n'.
d) Iz enačbe (4) določimo ρ_n in ρ_n'.
e) Nazadnje iz enačbe (5) izhaja:

from where the coefficients v_n and v_n' are determined.
d) From equation (4) ρ_n and ρ_n' are determined.
e) Finally, from equation (5) follows:

$$\rho\, u\frac{\partial u}{\partial \Psi}+\rho\, v\frac{\partial v}{\partial \Psi}=-\frac{1}{\gamma}\frac{\partial p}{\partial \Psi}$$

Na osi simetrije je $\Psi = 0$, $v = 0$, $v_0 = 0$. Iz enačbe (4) $\rho_0 = p_0^{1/\kappa}$ in iz enačbe (5) izhaja:

On the axis of symmetry $\Psi = 0$, $v = 0$, $v_0 = 0$. From equation (4) $\rho_0 = p_0^{1/\kappa}$ and from equation (5) follows:

$$\rho\, u\frac{\partial u}{\partial \Psi}+\rho\, v\frac{\partial v}{\partial \Psi}=-\frac{1}{\gamma}\frac{\partial p}{\partial \Psi}$$

Posebej dobimo: $r_n(x) = 0$, $p_n'(x) = 0$, $v_n(x) = 0$, $\rho_n(x) = 0$ in $u_n' = 0$.
Glede na to so:

Specially obtained are: $r_n(x) = 0$, $p_n'(x) = 0$, $v_n(x) = 0$, $\rho_n(x) = 0$ and $u_n' = 0$.
Accordingly:

$$\begin{aligned} r(x) &= \sqrt{\Psi_1}\sum_{n=0}^{N} r_n'(x)\Psi_1^n + r_0 \\ p(x) &= \sum_{n=0}^{N} p_n(x)\Psi_1^n \\ v(x) &= \sqrt{\Psi_1}\sum_{n=0}^{N} v_n'(x)\Psi_1^n \\ \rho(x) &= \sum_{n=0}^{N} \rho_n(x)\Psi_1^n \\ u(x) &= \sum_{n=0}^{N} u_n(x)\Psi_1^n \end{aligned} \qquad (21).$$

Posamezni koeficienti so:

The particular coefficients are:

$$
\begin{aligned}
r_0' &= \left(\frac{2}{u_0 \rho_0}\right)^{1/2} \\
r_1' &= -\frac{1}{4} r_0' \left(\frac{u_1}{u_0} + \frac{\rho_1}{\rho_0}\right) \\
p_1 &= -\kappa \frac{1}{r_0'} \frac{\partial v_0}{\partial x} \\
p_2 &= -\frac{1}{2} \kappa \frac{1}{r_0'} \frac{\partial v_1'}{\partial x} - \frac{1}{2} p_1 \frac{r_1'}{r_0'} \\
v_0' &= u_0 \frac{\partial r_0'}{\partial x} \\
v_1' &= u_1 \frac{\partial r_0'}{\partial x} + u_0 \frac{\partial r_1'}{\partial x} \\
u_0 &= \left(\frac{\kappa+1}{\kappa-1} - \frac{2}{\kappa-1} p_0^{\kappa-1/\kappa}\right)^{1/2} \\
u_1 &= -\frac{1}{\kappa} \frac{1}{\rho_0 u_0} p_1 - \frac{1}{2} \frac{\left(v_o'\right)^2}{u_0} \\
\rho_0 &= p_0^{1/\kappa} \\
\frac{\rho_1}{\rho_0} &= \frac{p_1}{\kappa p_0}
\end{aligned} \qquad (22).
$$

Izračun koeficientov začnemo z znanim $p(x,0)$ iz enačbe (14).

Koeficienti za dvoizmerno šobo so drugačni, dobimo pa jih na podoben način.

Prehod na realne parametre za določene robne pogoje na vhodu in izhodu je preprost.

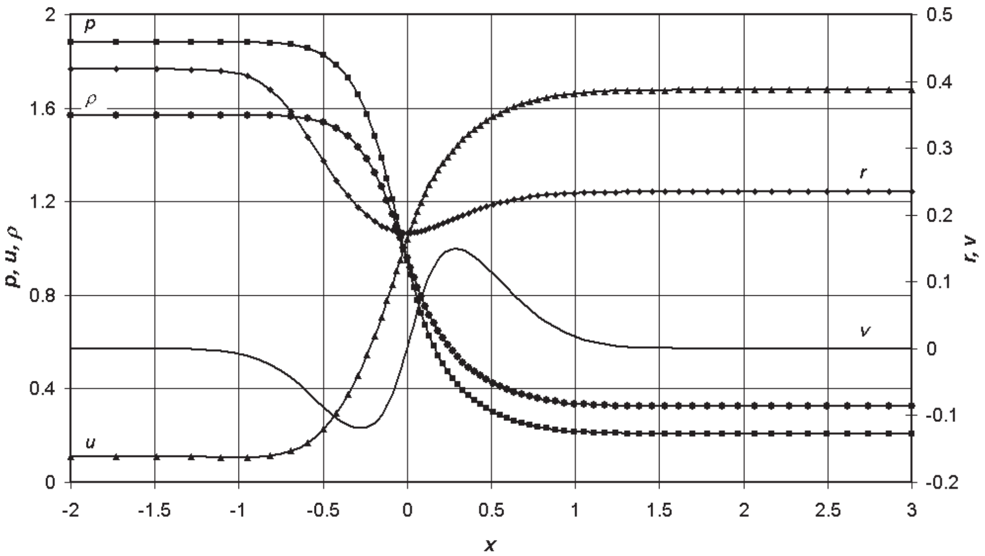
Na opisani način dobljene krivulje Ψ= konst. (Ψ_1=0,001 kot začetni pogoj in z iterativnim postopkom Ψ_2= 0,005, Ψ_3= 0,01 in Ψ_4= 0,015) so prikazane sliki 1. V koordinatnem sistemu *x, r* za Ψ= 0,015 smo izračunali parametre *u, v,* ρ in *r* po enačbah (6) in (7) za tlak *p*, določen po enačbi (14). Prikazani so na sliki 2.

The calculation of the coefficients starts with the known $p(x,0)$ from equation (14).

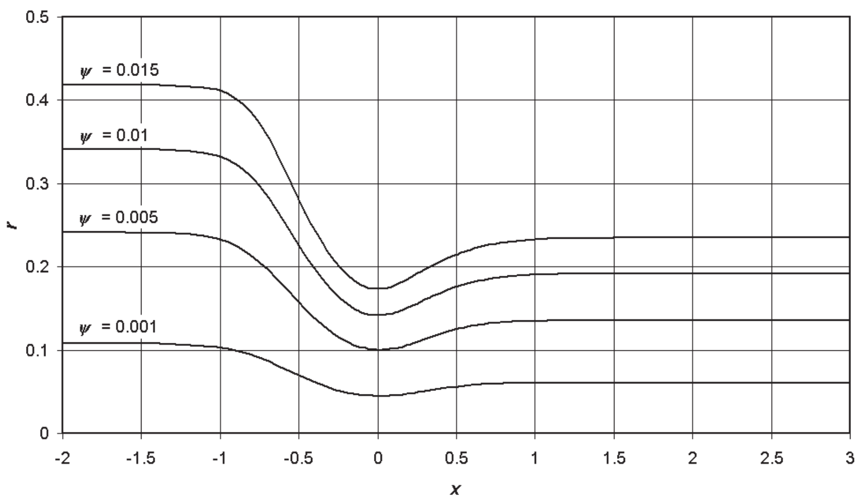
The coefficients for a two-dimensional nozzle are different, and they are obtained in a similar way.

The transition to the real parameters for the set boundary conditions on the input and output is simple.

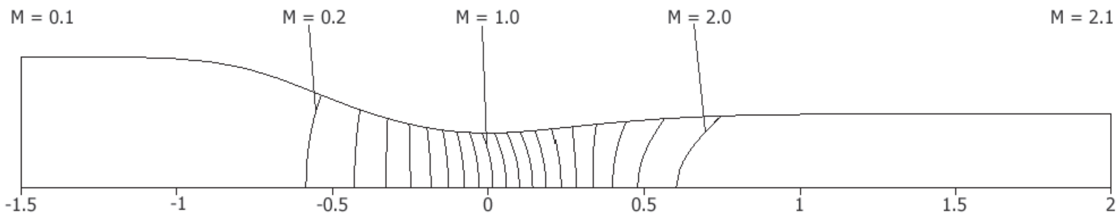
In the described manner the obtained curves Ψ = const. (Ψ_1= 0.001 as initial condition and by the iterative procedure Ψ_2=0.005, Ψ_3=0.01 and Ψ_4=0.015) are presented in Figure 1. In the coordinate system *x, r* for Ψ=0.015 the parameters *u, v,* ρ and *r* were calculated by equations (6) and (7) for the set pressure *p* according to equation (14) and they are presented in Figure 2.



Sl. 1. *Polmeri šobe za* Ψ *= konst., dobljeni z opisano metodo*
Fig. 1. *The nozzle radii for* Ψ *= const. obtained by the described method*



Sl. 2. *Normirani parametri u, v, r in r za $\Psi = 0{,}015$*
Fig. 2. *The normalised parameters u, v, ρ and r for $\Psi = 0{,}015$*



Sl.3. *Profil šobe in krivulje enakih hitrosti (Machovih števil)*
Fig. 3. *The profile of the nozzle and the curves of constant velocities (i.e. of Mach numbers)*

Na sliki 3 so za določene mejne pogoje in začetni pogoj prikazani profil šobe in krivulje enakih hitrosti (oz. Machovih števil).

S slike 2 vidimo, da so vse spremenljivke nespremenljive za $-2 \leq x < -1{,}5$ in $2 < x \leq 3$ in da ta del šobe ni potreben (ravni del povzroči turbulentno mejno plast). Zato je na sliki 3 prikazana prostorska oblika šobe za $-1{,}5 \leq x \leq 2$.

Za določene mejne pogoje in dobljeno obliko $r = r(x)$ za $\Psi = 0{,}015$ smo izvedli izračun z najbolj znanim programom FLUENT. Kot rezultat smo dobili normirane (bezdimenzijske) vrednosti spremenljivk p, u, v in ρ kot funkcije x.

Relativna razlika najvplivnejše spremenljivke $p(x)$, izračunana z uporabo programa FLUENT po določeni enačbi (14), kaže zelo majhno odstopanje spremembe tlaka. Z naslednjim primerom je prikazano, da te razlike ne vplivajo na rezultate toka v kritičnem prerezu in na izhodu iz šobe. S tem je pokazano, da je predstavljena metoda natančna kakor FLUENT. Dejansko natančnost obeh izračunov je treba preveriti z natančnimi meritvami.

Primer. Izberemo: $\dot{m} = 0{,}5$ kg/s; $\overline{T}_{tot} = 300$ K; $\overline{p}_{out} = 101\,300$ Pa; $Ma_{in} = 0{,}1$; $Ma_{out} = 2{,}1$; $\gamma = 1{,}4$; $R = 287$ J/kgK.

Z osnovnim izračunom za izentropni tok dobimo:

Na vstopnem prerezu:

$\overline{T}_{tot,in} = 299{,}4\ \text{K} = \overline{T}_{tot}$; $\overline{p}_{tot,in} = 926\,400\ \text{Pa} = \overline{p}_{tot}$;

Finally, for set boundary and initial conditions the profile of the nozzle and the curves of constant velocities (i.e. of Mach numbers) are given in Fig. 3.

From Figure 2 it is evident that all variables are constant for $-2 \leq x < -1.5$ and $2 < x \leq 3$ and this part of the nozzle is unnecessary (the flat part generates the turbulent boundary layer). Therefore, the three-dimensional shape of the nozzle is presented in Figure 3 for $-1.5 \leq x \leq 2$.

For the set boundary conditions and the obtained shape $r = r(x)$ for $\Psi = 0.015$ the calculation with the best-known application code FLUENT is performed. As a result, the normalised (non-dimensional) values of the variables p, u, v and ρ are obtained as a function of x.

The relative difference of the most influential variable $p(x)$ from the calculation with FLUENT according to the set equation (14) shows a very small deviation of the pressure change. The example below illustrates that these differences do not influence the results of flow in the critical section and on the nozzle outlet. This shows that the proposed method is at the FLUENT accuracy level. The real accuracy of both calculations needs to be checked with a precise measurement.

Example. Known: $\dot{m} = 0.5$ kg/s; $\overline{T}_{tot} = 300$ K; $\overline{p}_{out} = 101\,300$ Pa; $Ma_{in} = 0.1$; $Ma_{out} = 2.1$; $\gamma = 1.4$; $R = 287$ J/kgK.

By elementary calculation for isentropic flow the following values are obtained.

At the input section:

$\overline{T}_{tot,in} = 299.4\ \text{K} = \overline{T}_{tot}$; $\overline{p}_{tot,in} = 926\,400\ \text{Pa} = \overline{p}_{tot}$;

V kritičnem prerezu:
$\bar{T}_{throt} = 250$ K; $\bar{p}_{throt} = 489\,400$ Pa; $\bar{\rho}_{throt} = 6{,}821$ kg/m^3;
$\bar{u}_{throt} = 316\,938$ m/s; $\bar{r}_{throt} = 8{,}58$ mm;
Na izstopnem prerezu:
$\bar{T}_{out} = 159{,}4$ K; $\bar{u}_{out} = 531{,}46$ m/s;

Za $\Psi = 0{,}815$ iz $r_* = \frac{\bar{r}_{throt}}{r_{min}} = \frac{0{,}00858}{1{,}1735} = 0{,}05$ sta vstopni in izstopni polmer:
$\bar{r}_{in} = r_{in} r_* = 0{,}418 \cdot 0{,}05 = 20{,}9$ mm;
$\bar{r}_{out} = r_{out} r_* = 0{,}207 \cdot 0{,}05 = 10{,}35$ mm,
in dolžina šobe:
$\bar{L} = (1{,}5 + 2)\, 0{,}05 = 175$ mm.

In the critical section:
$\bar{T}_{throt} = 250$ K; $\bar{p}_{throt} = 489\,400$ Pa; $\bar{\rho}_{throt} = 6.821$ kg/m^3;
$\bar{u}_{throt} = 316\,938$ m/s; $\bar{r}_{throt} = 8.58$ mm;
At the outlet section:
$\bar{T}_{out} = 159.4$ K; $\bar{u}_{out} = 531.46$ m/s;

For $\Psi = 0.815$ from $r_* = \frac{\bar{r}_{throt}}{r_{min}} = \frac{0.00858}{1.1735} = 0{,}05$ the inlet and outlet radii are:
$\bar{r}_{in} = r_{in} r_* = 0.418 \cdot 0.05 = 20{,}9$ mm;
$\bar{r}_{out} = r_{out} r_* = 0.207 \cdot 0.05 = 10{,}35$ mm,
and the nozzle length:
$\bar{L} = (1.5 + 2)\, 0.05 = 175$ mm.

3 SKLEP

V tem prispevku je prikazana stabilna inverzna računska metoda za oblikovanje nadzvočne šobe z uporabo izvirnega analitičnega izraza za padec tlaka na osi šobe. Na slikah je vidna pravilnost spremembe značilnih parametrov vzdolž šobe. Z uvedbo analitičnega izraza za spremembo tlaka lahko z izbiro koeficienta *b* v enačbi (14) oblikujemo tako daljše kakor krajše šobe. Raziskava je uporabna tudi za dvoizmerne in krožne šobe. Rezultati se dobro ujemajo z izračunom z uporabo programa FLUENT za izbrano obliko šobe.

3 CONCLUSION

This paper presents a stable numerical design method for a supersonic nozzle by the introduction of an original analytical expression for the pressure drop on the axis of the nozzle. The figures show the regularity of the change of the characteristic parameters along the nozzle. Furthermore, by introducing an analytical expression for the pressure distribution it is easily possible, by selecting a coefficient *b* in equation (14), to design both a longer and a shorter nozzle. The research is also applicable to two-dimensional and annular nozzles. These results agree well with the calculation using the application code FLUENT for the obtained shape of the nozzle.

4 OZNAKE
4 NOMENCLATURE

približna kritična hitrost	a_*	m/s	approximate critical velocity
dejanska oz. normirana dolžina šobe	$\bar{L}$, L	m	real i.e. normalised length of nozzle
Machovo število	Ma		Mach number,
masni pretok	$\dot{m}$	kg/s	mass flow rate
dejanski oz. normirani tlak	$\bar{p}$, p	Pa	real, i.e. normalised pressure
posamična plinska konstanta, za zrak 260 J/kgK	R		unit gas constant, for air 260 J/kgK
dejanska oz. normirana radialna koordinata	$\bar{r}$, r	m	real, i.e. normalised radial coordinate
kritični polmer	r_*	m	critical radius
dejanska oz. normirana temperatura	$\bar{T}$, T	K	real, i.e. normalised temperature
dejanska oz. normirana komponenta hitrosti $\vec{w}$ v smeri osi šobe	$\bar{u}$, u	m/s	real, i.e normalised component of velocity $\vec{w}$ in direction of the axis of the nozzle
dejanska oz. normirana komponenta hitrosti $\vec{w}$ v smeri pravokotno na os šobe	$\bar{v}$, v	m/s	real, i.e. normalised component of velocity $\vec{w}$ perpendicular to the axis of the nozzle
hitrost	$\vec{w}$	m/s	velocity
dejanska oz. normirana koordinata vzdolž osi simetrije šobe	$\bar{x}$, x	m	real, i.e. normalised coordinate along the axis of symmetry of the nozzle
dejanska oz. normirana koordinata pravokotno na os šobe	$\bar{y}$, y	m	real, i.e. normalised coordinate perpendicular to the axis of the nozzle
koeficient izentrope, za zrak 1,4	κ		isentropic exponent, for air 1,4
dejanska oz. normirana gostota	$\bar{\rho}$, ρ	kg/m^3	real, i.e. normalised density
dejanska oz. normirana funkcija tokovnice	$\bar{\Psi}$, Ψ	kg/s	real, i.e. normalised stream function

Indeksi:		**Subscripts:**
na vstopnem prerezu šobe	*in*	at input section of nozzle
na izstopnem prerezu šobe	*out*	at output section of nozzle
skupna vrednost	*tot*	total value
v vratu šobe	*trot*	in throat of nozzle
na osi simetrije šobe	0	on axis of symmetry

5 LITERATURA
5 REFERENCES

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