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Računalniško simuliranje dinamike rotorjev

Computer Simulation of the Dynamics of Rotors

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Dinamika rotorjev se je v zadnjih letih razvila v široko paleto specialnosti. Tudi strokovnjaki težko prepoznajo in povežejo vse vplive in pojave, ki se pojavljajo v praksi. V ta namen je bil razvit programski paket, ki omogoča obravnavo upogibnih nihanj linearnih sistemov in vrednotenje vpliva različnih parametrov na dinamiko rotorjev. Posebna pozornost je bila posvečena žiroskopskemu učinku in modeliranju drsnih ležajev ter njunemu vplivu na lastne frekvence in kritične hitrosti. Numerični del razvitega programskega paketa je zasnovan na metodi končnih elementov. Program je namenjen konstrukterjem za določitev dinamičnega obnašanja rotorja v začetni fazi razvoja.

The dynamics of rotors has in recent years developed into a wide range of specialties. Even for the experts it is difficult to recognize and draw a correlation among all the phenomena that occur in practice. For that purpose a computer program has been developed that enables evaluation of the bending vibrations of linear systems and assessment of the influence of various parameters on rotordynamics. Special attention has been given to the gyroscopic effect and modeling of oil-film bearings and its influence on eigenfrequencies and critical speeds. The numerical part of the developed software program is based on the finite element method. The program is intended to be used as an efficient and easy to use tool in the early stages of design.

0 UVOD

Pojem dinamike rotorjev pokriva zelo široko področje problematike; od uravnoteženja togih rotorjev do obravnave nestabilnosti zaradi nelinearnosti v sistemu. Zaradi kompleksnosti posameznih sklopov se je veda v zadnjih desetletjih razvila v panogo z veliko posebnosti. Tudi strokovnjaki težko prepoznajo in ovrednotijo celotno paleto problemov in povežejo vplive bistvenih parametrov. V nadaljevanju smo obravnavo omejili na določanje lastnih frekvenc in kritičnih hitrosti rotorjev. Glede na dejstvo, da analitične rešitve pokrivajo zelo ozko področje problematike, je bil v preteklosti razvit niz diskretnih metod, ki omogočajo obravnavo problematike dinamike rotorjev. Dandanes se v glavnem uporabljata dve metodi, in sicer metoda končnih elementov (v nadaljevanju MKE) in metoda prenosnih matrik. Kljub vse bolj splošni uporabnosti in razširjenosti metode končnih elementov se metoda prenosnih matrik še vedno uporablja za obravnavo nekaterih specifičnih primerov.

Sodobno zasnovo metode prenosnih matrik, ki temelji na uporabi računalnika, lahko najdemo v [1]. Nekoliko spremenjeno izražanje metode, ki omogoča popis žiroskopskega učinka in splošen opis tirnice gibanja poljubne točke v sistemu, lahko najdemo v [2] in [3]. Metoda omogoča določitev vpliva deformljivosti diska na lastne frekvence in kritične hitrosti [4] kakor tudi obravnavo večlinijskih sistemov (rotorja s podporno konstrukcijo) [5].

0 INTRODUCTION

The term rotordynamics covers a wide range of topics, from the balancing of rigid rotors to evaluation of instability due to non-linearity of the system. Rotordynamics has because of the complexity of individual topics, developed into a wide range of specialties. Even for the experts it is difficult to evaluate whole range of problems and draw the correlations among significant parameters. The following discussion has been restricted to the determination of eigenfrequencies and critical speeds of rotors. Due to the fact that analytical solutions are available only for a limited area of problems a range of discrete methods has been developed that make the evaluation of rotordynamics possible. Nowadays two methods are being broadly used. These are, first the finite element method (FEM in the following) and, second, the transfer matrix method. Despite the fact that FEM is more and more used, the transfer matrix method is still used for evaluation of some specific cases.

The basic principles of the transfer matrix method, based on the use of the computer, can be found in [1]. A somewhat modified formulation of the method that enables evaluation of gyroscopic effect and general description of the path of an arbitrary point in the system can be found in [2] and [3]. The method was extended to include the influence of the disk's flexibility on eigenfrequencies and critical speeds [4] as well as evaluation of multi-line systems (systems with support structure) [5].

Glede na izkušnje z metodo prenosnih matrik [6] se je izkazalo, da je omenjena metoda manj primerna za obravnavo večlinijskih konstrukcij oz. rotorjev, kjer je treba poleg osnovne linije modelirati tudi podporno konstrukcijo. V ta namen razvijamo v Laboratoriju za dinamiko strojev in konstrukcij (LADISK) na Fakulteti za strojništvo v Ljubljani programski paket, ki temelji na MKE in bo v končni obliki omogočal obravnavo upogibnih nihanj rotorjev skupaj s podporno konstrukcijo, vrednotenje različnih parametrov in njihov vpliv na dinamiko rotorjev. Podrobneje je trenutno stanje lastnega razvoja prikazano v [7] in [8].

V literaturi lahko opazimo široko uporabnost metode končnih elementov pri obravnavi dinamike rotorjev. V [9] je podana obširna problematika na podlagi omenjene metode. Metoda omogoča tudi določitev lastnih frekvenc [10] in kritičnih hitrosti [11] rotirajočega zvitega nosilca po Timošenkovi enačbi. Za bolj natančen popis rotorja lahko uporabimo osno simetrične končne elemente [12]. Pri pogonskih agregatih pride zaradi premika pogonske gredi motorja in rotorja do dodatnih vplivov na dinamiko rotorja [13]. Linearna formulacija problema je navadno zadostna, seveda pa metoda končnih elementov omogoča tudi popis nelinearnih nihanj [14]. Pri delu z metodo končnih elementov imamo, za razliko od metode prenosnih matrik, kjer je velikost prenosnih matrik konstantna, opravka z razmeroma velikimi matrikami, zato je pogosto priporočljiva redukcija matrik [12]. Z upoštevanjem žiroskopskega učinka postane dinamična matrika sistema nesimetrična. Za reševanje problema lastnih vrednosti moramo zato uporabljati splošne algoritme [15] in [16], ki pa so časovno zahtevni. Pri obravnavi nekaterih specifičnih primerov se problemu lahko izognemo s preoblikovanjem dinamične matrike v simetrično [17].

1 TEORETIČNO OZADJE

Za popis linearnih upogibnih nihanj oz. za določitev matrične gibalne enačbe (1), je bila uporabljena metoda končnih elementov. Osnove uporabljene metode, kakor tudi načini določitve matrik posameznih elementov, so splošno znane. Med drugim jih lahko najdemo v [9] in [18].

1.1 Problem lastnih vrednosti

Probleme lastnih nihanj lahko splošno popišemo s homogeno gibalno enačbo v obliki:

Considering our own experiences with the transfer matrix method [6] it has become evident that the use of the method is less suitable, since besides the rotor itself the support structure has also to be modeled. For that purpose a computer program, based on the FEM, has been developed in the Laboratory for the dynamics of machines and structures (LADISK) at the Faculty of Mechanical Engineering in Ljubljana, thus providing the means for evaluation of various parameters and their influence on rotordynamics. The details of current development are shown in [7] and [8].

A wide range of use of the FEM in rotordynamics can be found in the literature. A variety of issues is covered and discussed using the FEM in [9]. The eigenfrequencies [10] and critical speeds [11] of the pre-twisted rotor, described by Timoshenko's equation, can be determined using the FEM. For more accurate description of the rotor, the axisymmetric finite elements can be used [12]. Considering coupled systems such as driving machine-rotor the additional influences on rotordynamics can be detected due to misalignment of the driving shaft and rotor [13]. Linear formulation of the problem is usually sufficient, but the FEM can also be used for evaluation of non-linear vibrations [14]. Application of the FEM, in contrast to the transfer matrix method where the size of transfer matrices is constant, implies the use of relatively large matrices. Matrix reduction is therefore recommended [12]. Considering the gyroscopic effect, the dynamic matrix becomes asymmetric. Solving the eigenvalue problem implies the use of time-consuming general algorithms [15] and [16]. The problem can be avoided in some specific cases by using the transformation of the dynamic matrix in symmetric form [17].

1 THEORETICAL BACKGROUND

The FEM has been used to describe flexural vibrations and to determine the matrix equation of motion (1). The basic principles of the method used as well as the mechanism to determine the matrices of distinct elements, are well known. Among others they can be found in [9] and [18].

1.1 The Eigenvalue Problem

The problem of natural vibrations can be generally described by the homogeneous equation of motion in the form:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{B}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0 \qquad (1),$$

kjer so **M** masna, **B** vsota dušilne in žiroskopske matrike ter **K** togostna matrika. Pomike in njihove časovne odvode označimo z $\mathbf{x}$, $\dot{\mathbf{x}}$ in $\ddot{\mathbf{x}}$. Rešitev gibalne enačbe (1) vodi do problema lastnih vrednosti. Kot rezultat dobimo lastne frekvence in pripadajoče

where **M**, **B** and **K** are mass, the sum of damping and gyroscopic matrices and stiffness matrices respectively. Displacements and their time derivatives are denoted by $\mathbf{x}$, $\dot{\mathbf{x}}$ and $\ddot{\mathbf{x}}$. Solution of the equation of motion (1) leads to the eigenvalue problem. As a lastne oblike obravnavanega sistema. Ob zanemarljivem žiroskopskem vplivu in dušenju, pri $\mathbf{B}=0$, lahko zapišemo standarden problem lastnih vrednosti v obliki:

result one can obtain eigenfrequencies and corresponding mode shapes of the system under consideration. With negligible gyroscopic effect and damping, with $\mathbf{B}=0$, we can write the classic eigenvalue problem in the form:

$$\mathbf{A}-\lambda\mathbf{I}=0 \tag{2}$$

kjer pomenita **A** dinamično matriko sistema in **I** enotsko matriko. V primeru, ko sta masna in togostna matrika simetrični, lahko z dekompozicijo ene od matrik na spodnjo in zgornjo trikotno matriko zagotovimo tudi simetričnost dinamične matrike **A**. Takšen problem je razmeroma preprosto rešljiv [15], [16] in [19].

where **A** represents the dynamic matrix of the system and **I** is the identity matrix. In the case of symmetric mass and stiffness matrices, the symmetricity of dynamic matrix **A** can be guaranteed through the decomposition of one of the matrices in the lower and upper triangular matrix. These kinds of problem are relatively easy to solve [15], [16] and [19].

Tudi v splošnem primeru, kjer je $\mathbf{B}\neq 0$ in so matrike lahko tudi nesimetrične, moramo enačbo (1) zapisati v obliki (2). Množenje enačbe (1) z $\mathbf{M}^{-1}$ pripelje, ob enakosti $\dot{\mathbf{x}}=\dot{\mathbf{x}}$, do enačb:

In the general case, with $\mathbf{B}\neq 0$, and generally asymmetric matrices, the equation (1) has to be written in the form of (2). Multiplication of the equation (1) by $\mathbf{M}^{-1}$ and using identity $\dot{\mathbf{x}}=\dot{\mathbf{x}}$ leads to the equations:

$$\dot{\mathbf{x}}=\mathbf{I}\dot{\mathbf{x}}$$

$$\ddot{\mathbf{x}}=-\mathbf{M}^{-1}\mathbf{K}\mathbf{x}-\mathbf{M}^{-1}\mathbf{B}\dot{\mathbf{x}}$$

Z uvedbo

Denoting

$$\mathbf{z}=\begin{Bmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{Bmatrix} \quad \begin{matrix}\text{in}\\ \text{and}\end{matrix} \quad \mathbf{C}=\begin{bmatrix}0 & \mathbf{I}\\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{B}\end{bmatrix} \tag{3}$$

lahko tako preoblikovano gibalno enačbo zapišemo v obliki:

one can write the equation of motion in the form:

$$\dot{\mathbf{z}}-\mathbf{C}\mathbf{z}=0 \tag{4}$$

Lastne vrednosti so v splošnem kompleksno konjugirani pari in jih lahko zapišemo v obliki:

Eigenvalues are generally complex conjugate pairs and can thus be written in the form:

$$\lambda_k=\alpha_k\pm i\cdot\omega_k \tag{5}$$

kjer pomenita ω_k k-to lastno frekvenco sistema in a_k koeficient rasti nihanj. Za negativne vrednosti a_k se amplituda nihanj sčasoma zmanjšuje, medtem ko se za pozitivne vrednosti povečuje. To pomeni, da je sistem pri pozitivnih vrednostih koeficienta rasti nihanj nestabilen.

where ω_k represents the k-th eigenfrequency of the system and a_k the coefficient governing the growth of vibrations. For negative values of a_k vibrations decrease with time and for positive values vibrations increase with time. This means that for positive values the system becomes unstable.

Podobno kakor lastne vrednosti so tudi lastni vektorji v splošnem kompleksni. Lastna nihanja celotnega modela lahko popišemo z gibanjem njegovih vozlišč. V uporabljenem matematičnem modelu je gibanje sestavljeno iz pomikov, pravokotnih na os rotorja in ustrezajočih zasukov. Za numerično analizo zadošča zgolj obravnava pomikov, saj lahko zasuke določimo iz upogibnice rotorja.

Similar to the eigenvalues, the mode shapes are generally complex. The natural vibrations of the whole model are described by the motions at certain nodes. In the mathematical model used these motions consist of displacements perpendicular to the rotor axis and the corresponding rotations. For the numerical analysis it is sufficient to evaluate only displacements, because rotations can be calculated from the bent form of the rotor.

Pomike v poljubnem vozlišču lahko zapišemo [9] z enačbo:

The displacement of an arbitrary node can be written [9] by the equation:

$$x_{ik}(t)=C_k e^{\alpha_k t}\left[r_{ik}\sin(\omega_k t+\gamma_k)+s_{ik}\cos(\omega_k t+\gamma_k)\right] \tag{6}$$

kjer so: α_k, ω_k - realna in imaginarna komponenta lastne vrednosti, r_{ik}, s_{ik} - ustrezna realna in imaginarna komponenta lastnega vektorja in C_k, γ_k - konstanti, ki ju določimo iz začetnih pogojev.

Pri obravnavi lastnih nihanj lahko zanemarimo spremembo amplitude nihanj in zapišemo $\alpha_k = 0$. Nadalje lahko zanemarimo konstanti, odvisni od začetnih pogojev, in zapišemo $C_k = 1$ in $\gamma_k = 0$. Tako poenostavljeno enačbo lahko zapišemo v kompleksni obliki kot:

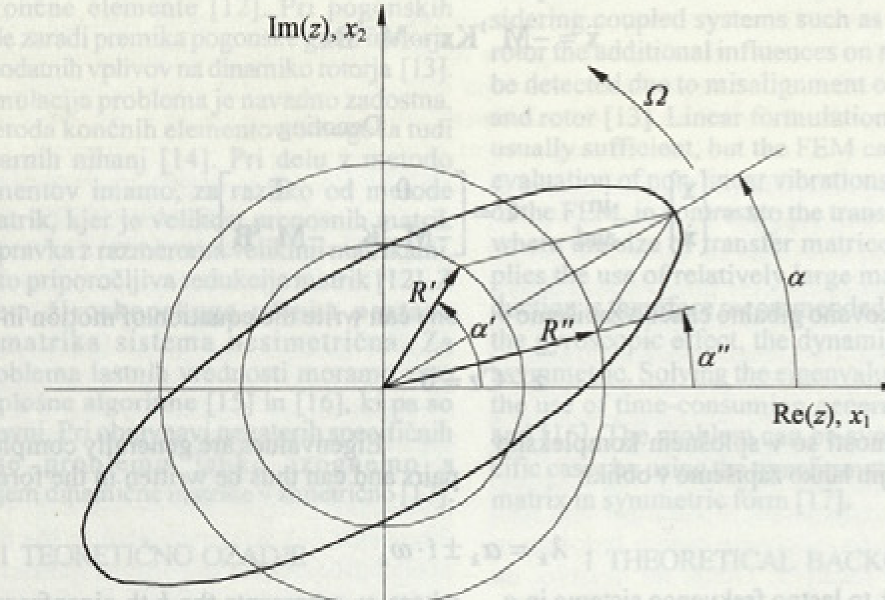
where: α_k, ω_k - are the real and imaginary parts of an eigenvalue, r_{ik}, s_{ik} - are the real and imaginary parts of an eigenvector, and C_k, γ_k - are constants found from initial conditions.

When considering only natural vibrations one can neglect the change in amplitude and put $\alpha_k = 0$. Further we can with no serious restrictions put $C_k = 1$ and $\gamma_k = 0$. In this way the simplified equation has the form:

$$z(t) = R' e^{j(\omega t + \alpha')} + R'' e^{-j(\omega t - \alpha'')} \tag{7}$$

Opazimo lahko, da je eliptična tirnica na sliki 1 sestavljena kot vsota dveh vrtečih se vektorjev, ki imata oba kotno hitrost ω. Vektor dolžine R' rotira sinhrono s smerjo rotacije rotorja, medtem ko drugi vektor, dolžine R'', rotira asinhrono.

It can be seen that the elliptical path in Figure 1 consists of the sum of two rotating vectors, both having angular velocity ω. One vector of length R' rotates in the positive direction while the other of length R'' rotates in the negative direction.



Sl. 1. *Eliptična tirnica*
Fig. 1. *Elliptical path*

Za določitev smeri opletanja vozlišča vpeljemo rotacijsko število:

To find the direction of rotation we introduce the rotation number:

$$u = \frac{R' - R''}{R' + R''} \tag{8}$$

ki določa smer opletanja obravnavanega vozlišča. Glede na rotacijsko število u je opletanje lahko:

$-1 \le u < 0$ asinhrono opletanje,
$u = 0$ gibanje vzdolž premice ali
$0 < u \le 1$ sinhrono opletanje.

which indicates the direction of rotation of the node considered. Considering the rotation number u, the rotation can be:

$-1 \le u < 0$ backward whirl,
$u = 0$ motion takes place along the straight line, or
$0 < u \le 1$ forward whirl.

1.2 Reševanje problema lastnih vrednosti

Za reševanje problema lastnih vrednosti smo testirali in primerjali niz numeričnih metod. Za simetrične matrike smo uporabili Hauseholderjevo metodo. Da bi obdržali simetričnost dinamične matrike **A**, definirane v enačbi (2), smo za masno matriko **M** uporabili razstavitev Choleskega. Za reševanje splošnega problema lastnih vrednosti, definiranega z enačbo (4), program uporablja za določitev lastnih vrednosti HQR (Hessenbergovo) metodo in inverzno iteracijo vektorjev za določitev lastnih oblik. Podrobneje so omenjene metode predstavljene v [15], [16] in [19].

1.3 Teorija kratkega drsnega ležaja

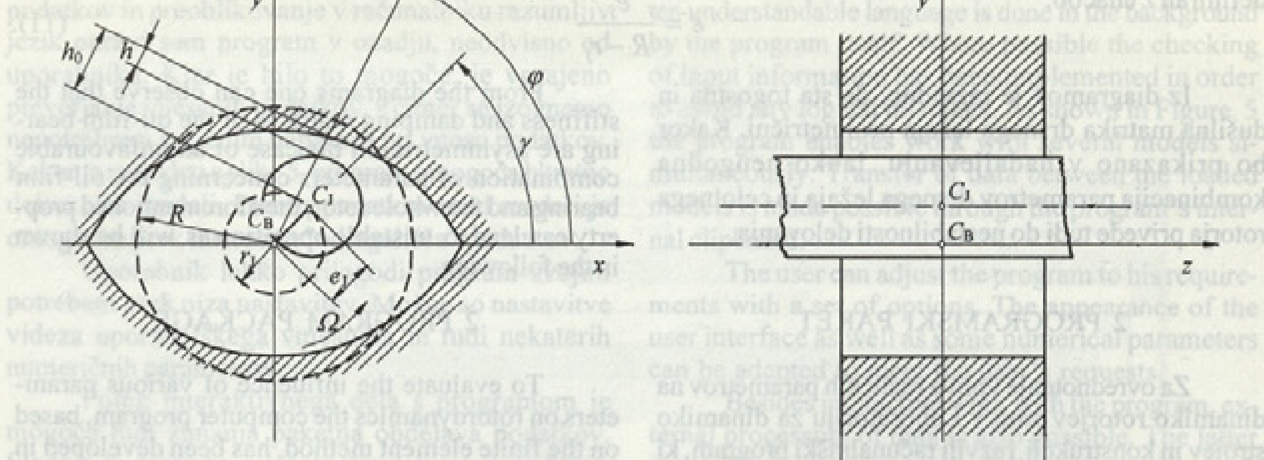
Osnove MKE, kakor tudi načini določevanja matrik posameznih elementov, so splošno znani [9] in [18]. V nadaljevanju je zaradi pomembnega vpliva na dinamično obnašanje celotnega rotorja zato na kratko predstavljen le uporabljeni model drsnega ležaja [9]. Primer takšnega ležaja je prikazan na sliki 2.

1.2 Solving the Eigenvalue Problem

To solve the eigenvalue problem a range of numerical routines has been tested and compared. For symmetric matrices the Hauseholder method has been used. To retain symmetric properties of the dynamic matrix **A** defined in equation (2), Cholesky decomposition of mass matrix **M** has been used. Considering the general eigenvalue problem defined by equation (4) the HQR (Hessenberg) algorithm has been used to calculate eigenvalues, while inverse vector iteration has been applied to obtain eigenvectors. Details of all aforementioned methods can be found in references [15], [16] and [19].

1.3 Oil-Film Bearing

The FEM as well as the mechanism to determine the matrices of distinct elements are well known [9] and [18]. Because of the significant influence on the dynamic behaviour of the entire rotor, the model of the oil-film bearing is shortly discussed in the following. An example is shown in Figure 2.



Sl. 2. *Drsni ležaj*

Fig. 2. *Oil-film bearing*

Tlačne razmere v drsnem ležaju navadno popišemo z Reynoldsovo enačbo [9]. Za primer drsnega ležaja s krožno ležajno blazinico ima Reynoldsova enačba, ob zanemaritvi spremembe tlaka v obodni smeri, obliko:

The pressure conditions in the oil-film bearing are usually described by the Reynolds' equation [9]. For the case of circular bearing, and neglecting the change in pressure in circumferential direction, we can write the Reynolds' equation:

$$\frac{\partial^2 p}{\partial z^2}=\frac{6\eta}{h^3(\varphi,t)}\left[e_J(\Omega-2\dot{\gamma})\sin(\varphi-\gamma)-2\dot{e}_J\cos(\varphi-\gamma)\right] \tag{9},$$

kjer so: η - dinamična viskoznost maziva, Ω - kotna hitrost gredi oz. čepa in p - tlak maziva.

Govorimo o teoriji kratkega drsnega ležaja oz. o linearnem modelu drsnega ležaja. Za statično obremenjen vodoravno podprt rotor lahko togostne in dušilne koeficiente določimo z odvajanjem komponent rezultirajoče sile glede na pomike:

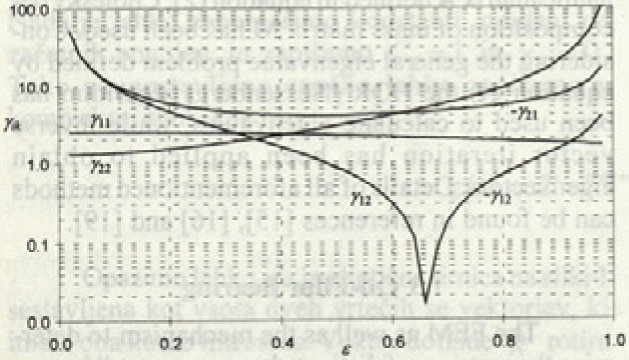
where: η - is the dynamic viscosity of the lubricant, Ω - is the angular velocity of the shaft and p - the pressure of the lubricant.

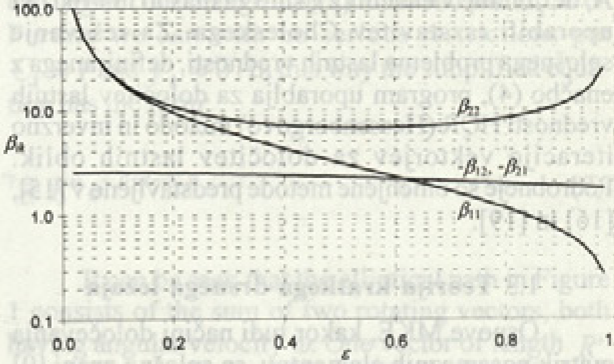
The theory is known as short oil-film bearing theory or linear oil-film bearing model. For a horizontally supported rotor with static load one can obtain stiffness and damping coefficients by differentiating component forces with respect to displacements:

$$k_{ik} = \frac{\partial F_i}{\partial x_k} = \gamma_{ik} \frac{F_0}{\delta}, \qquad d_{ik} = \frac{\partial F_i}{\partial \dot{x}_k} = \beta_{ik} \frac{F_0}{\delta \Omega} \qquad (10),$$

kjer sta γ_{ik} in β_{ik} brezdimenzijska togostna in dušilna koeficienta.

where γ_{ik} and β_{ik} are dimensionless stiffness and damping coefficients respectively.





Sl. 3. *Brezdimenzijski togostni in dušilni koeficienti*

Fig. 3. *Dimensionless stiffness and damping coefficients*

Na opisani način določena brezdimenzijska koeficienta sta prikazana na sliki 3 v odvisnosti od razmernika ekscentričnosti drsnega ležaja. Ta je definiran z enačbo:

Coefficients obtained in the described way are shown in Figure 3 as functions of the eccentricity ratio. The latter is defined by the equation:

$$\varepsilon = \frac{e_j}{R - r_j} \qquad (11).$$

Iz diagramov je razvidno, da sta togostna in dušilna matrika drsnega ležaja nesimetrični. Kakor bo prikazano v nadaljevanju, lahko neugodna kombinacija parametrov drsnega ležaja in celotnega rotorja privede tudi do nestabilnosti delovanja.

From the diagrams one can observe that the stiffness and damping matrices of the oil-film bearing are asymmetric. In the case of an unfavourable combination of parameters concerning the oil-film bearing and the whole rotor, the aforementioned property can lead to unstable operation as will be shown in the following.

2 PROGRAMSKI PAKET

Za ovrednotenje vpliva različnih parametrov na dinamiko rotorjev smo v Laboratoriju za dinamiko strojev in konstrukcij razvili računalniški program, ki temelji na metodi končnih elementov. Namen programa je zapolniti vrzeli komercialnih programov, ki večinoma (z redkimi izjemami) ne zadoščajo posebnim zahtevam obravnave dinamike rotorjev (žiroskopski učinek ter nesimetrične togostne in dušilne matrike).

Izdelani program vključuje numerično vrednotenje matematičnega modela in tudi grafično pripravo podatkov ter grafični prikaz dobljenih rezultatov. Groba programska shema je prikazana na sliki 4.

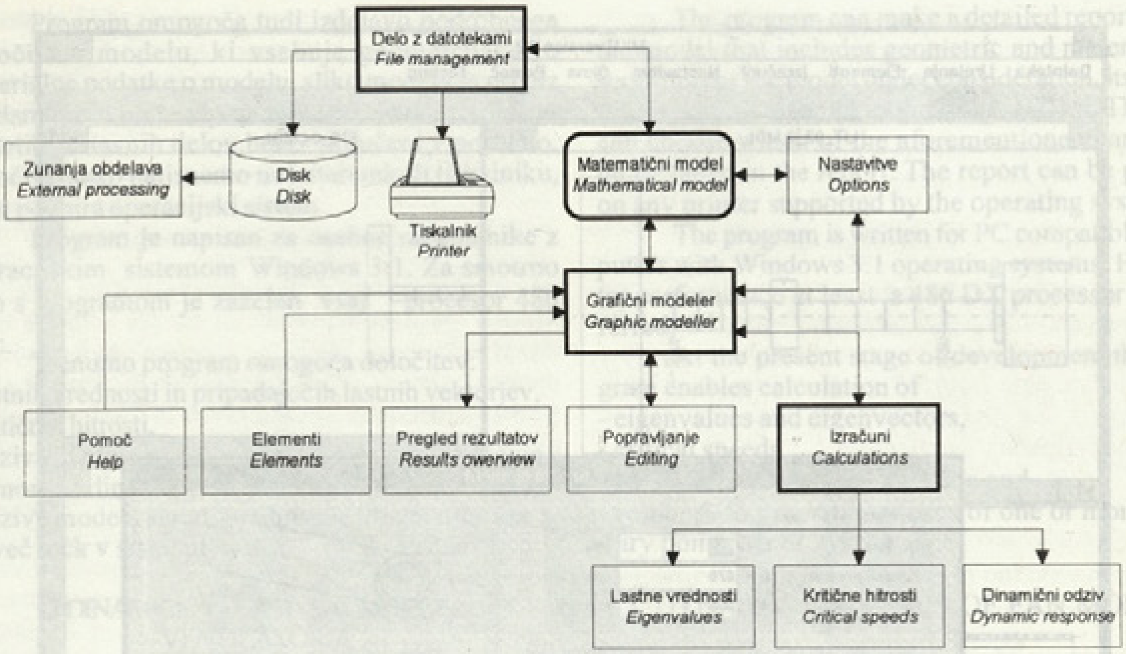
Zaradi poenostavitve dela s programom je bil razvit grafični uporabniški vmesnik, ki omogoča preprosto modeliranje in obdelavo podatkov. Vnos podatkov je omogočen s sistemom dialogov in naborov. Primer vnosa podatkov za gredni element je prikazan na sliki 5. Končni uporabnik tako ne potrebuje nobenega znanja o ukazih za modeliranje matematičnega modela. Interpretacijo vnesenih

2 PROGRAM PACKAGE

To evaluate the influence of various parameters on rotordynamics the computer program, based on the finite element method, has been developed in the Laboratory for dynamics of machines and structures. The program is intended to fill the gaps of general FEM programs, which (with rare exemptions) do not meet the special requirements of rotordynamics (gyroscopic effect and asymmetric stiffness and damping matrices).

The developed program incorporates numerical processing of the mathematical model as well as graphical pre- and post-processing of the given numerical information. A rough program chart is shown in Figure 4.

To simplify handling of the program, a graphics interface has been created that enables elegant modeling and processing of data. Input of data is supported through the system of dialogues and menues. An example of data input is shown in Figure 5 for the beam element. The user thus does not need any knowledge about the syntax of a modeling language. Interpretation and translation of input data to comput-



Sl. 4. *Programska shema*
Fig. 4. *Program chart*

podatkov in preoblikovanje v računalniku razumljiv jezik opravi sam program v ozadju, neodvisno od uporabnika. Kjer je bilo to mogoče, je vgrajeno preverjanje vnesenih podatkov, s čimer se izognemo nepotrebnim logičnim napakam pri vnosu podatkov. Kakor je razvidno s slike 5, program omogoča hkratno delo z več modeli. Prenos podatkov med modeli je omogočen prek internega "odlagališča" podatkov.

Uporabnik lahko prilagodi program svojim potrebam prek niza nastavitev. Možne so nastavitve videza uporabniškega vmesnika in tudi nekaterih numeričnih parametrov.

Poleg interaktivnega dela s programom je mogoča tudi zunanja paketna obdelava podatkov. Slednja je mogoča s programiranjem v poljubnem programskem jeziku, seveda pod pogojem, da uporabnik pozna obliko zapisa datotek in so mu na voljo potrebne numerične rutine. Zunanja paketna obdelava podatkov ima velikokrat prednost pred interaktivnim delom s programom, vendar od uporabnika terja dosti več znanja o samem programu in tudi o splošni numerični analizi.

Za namen prikaza numeričnih rezultatov je bil narejen grafični "post-procesor". Na sliki 6 je predstavljen primer prikaza izračunanih lastnih frekvenc in pripadajočih lastnih oblik modela.

Kakor smo že omenili, program omogoča tudi zunanjo obdelavo podatkov. Ta je omogočena tudi prek izvoza podatkov in rezultatov v obliki ASCII. Tako zapisane podatke lahko uporabljamo z večino komercialnih programskih paketov. Omogočen je izvoz končnih rezultatov (lastne vrednosti in kritične hitrosti) pa tudi posameznih matrik v enačbi (1).

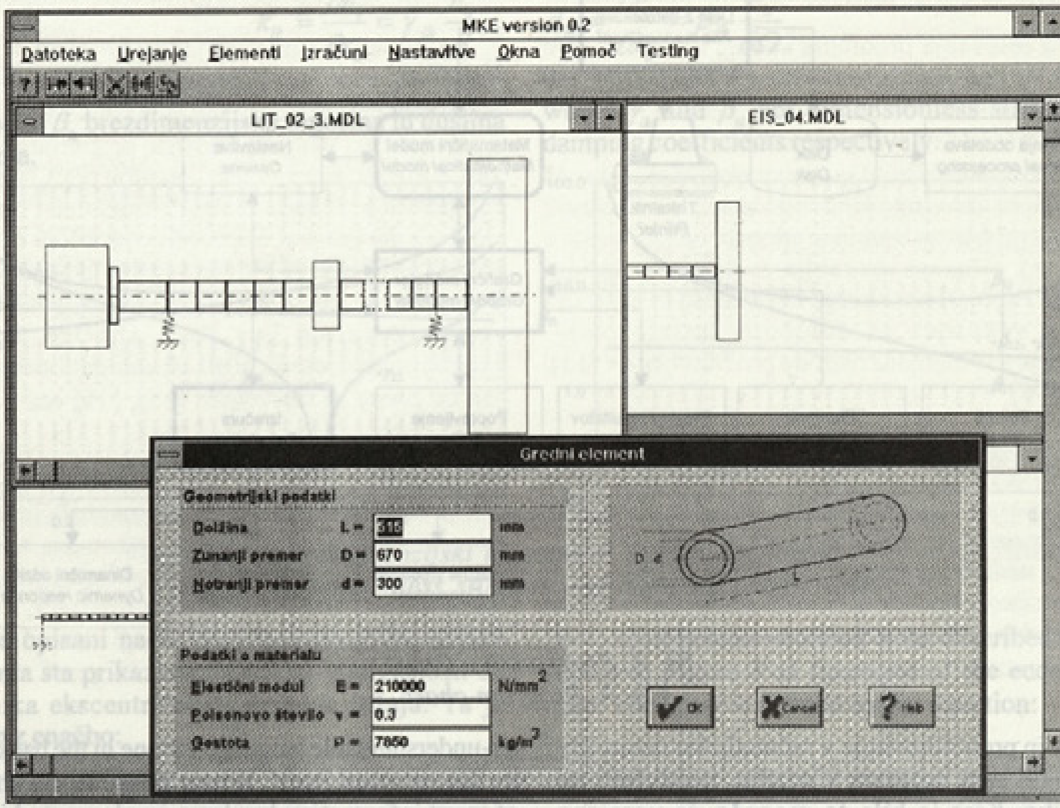
ter-understandable language is done in the background by the program itself. Where possible the checking of input information has been implemented in order to avoid any logical mistakes. As shown in Figure 5 the program enables work with several models simultaneously. Transfer of data between the loaded models is made possible through the program's internal clipboard.

The user can adjust the program to his requirements with a set of options. The appearance of the user interface as well as some numerical parameters can be adapted to meet the user's requests.

Besides interactive work with the program, external processing of data is also possible. The latter is possible by programming in arbitrary computer language, assuming the file format used by the program and numerical routines are available to the user. That kind of work is sometimes preferred to an interactive approach but it requires detailed knowledge about the program structure and general numerical analysis.

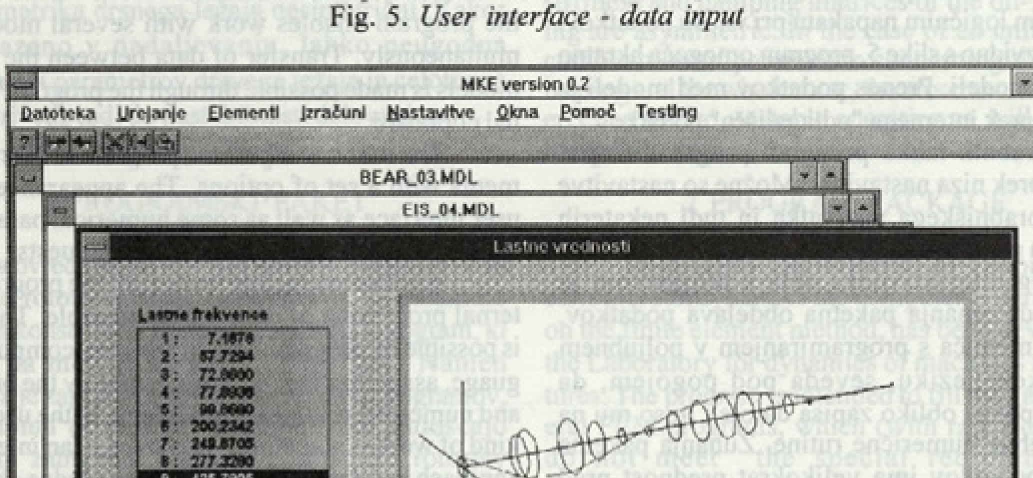
To review the calculations a postprocessor has been developed which enables graphic representation of results. An example is shown in Figure 6 where calculated eigenfrequencies and corresponding mode shapes are shown.

As mentioned before, the program supports further processing of numerical results. This has been made possible through the export of data in the ASCII file format. The results, written in that form, can be accessed and further processed by almost any commercial computer program. Export of results (eigenvalues and critical speeds) as well as individual matrices in equation (1) is also possible.



Sl. 5. *Uporabniški vmesnik - vnos podatkov*

Fig. 5. *User interface - data input*



Sl. 6. *Uporabniški vmesnik - prikaz rezultatov*

Sl. 6. *User interface - data output*

Program omogoča tudi izdelavo podrobnega poročila o modelu, ki vsebuje geometrijske in materialne podatke o modelu, sliko modela in prikaz izdelanih izračunov. Uporabnik lahko izbira, kateri od naštetih sestavnih delov bodo vključeni v poročilo. Poročilo lahko natisnemo na kateremkoli tiskalniku, ki ga podpira operacijski sistem.

Program je napisan za osebne računalnike z operacijskim sistemom Windows 3.1. Za smotrno delo s programom je zaželen vsaj procesor 486 DX.

Trenutno program omogoča določitev:
- lastnih vrednosti in pripadajočih lastnih vektorjev,
- kritičnih hitrosti,
- odziva modela v ustaljenem stanju zaradi zunanje harmonske dinamične obremenitve in
- odziva modela zaradi predpisane kinematike ene ali več točk v sistemu.

The program can make a detailed report about the model that includes geometric and material descriptions of the model under consideration, its drawing as well as all calculations carried out. The user can choose which of the aforementioned parts will be included in the report. The report can be printed on any printer supported by the operating system.

The program is written for PC compatible computers with Windows 3.1 operating systems. For better performance at least a 486 DX processor is preferred.

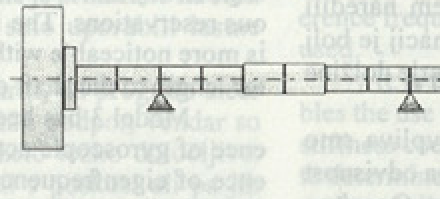
At the present stage of development the program enables calculation of
- eigenvalues and eigenvectors,
- critical speeds,
- response due to dynamic force and
- response to given kinematics of one or more arbitrary points of the system.

3 DINAMIČNA ANALIZA MODELA VENTILATORJA

Za analizo posameznih vplivov na dinamiko rotorjev, predvsem žiroskopskega učinka in vpliva drsnih ležajev oz. za predstavitve programskih zmogljivosti, smo oblikovali model ventilatorja. Osnovni model je prikazan na sliki 7. Ventilator ima celotno maso 436 kg in dolžino 1280 mm. Opazovano območje obratovanja ventilatorja je od 0 do 1000 rad/s.

3 DYNAMIC ANALYSIS OF FAN MODEL

In order to analyse the distinct influences on rotordynamics, mainly the gyroscopic effect and the influence of the oil-film bearing, and to demonstrate the program capabilities a ventilator model has been built. The basic model is shown in Figure 7. The ventilator has a total mass of 436 kg and a length of 1280 mm. The range of operation in interest is from 0 to 1000 rad/s.



Sl. 7. *Model ventilatorja*
Sl. 7. *Ventilator model*

Za vrednotenje posameznih vplivov na lastne frekvence in kritične hitrosti smo primerjali nekaj variacij osnovnega modela. Osnovni geometrijski in materialni podatki so enaki za vse obravnavane modele. Izvedba podprtja in opis upoštevanih dodatnih parametrov je za vse modele zbran v preglednici 1.

To identify distinct influences on eigenfrequencies and critical speeds, several variations of the basic model were considered. Geometric and material properties are the same for all the tested models. Description of the support conditions and additional model parameters are given in Table 1.

Preglednica 1: *Opis modelnih parametrov*
Table 1: *Description of model parameters*

model *model*	1	2	3	4	5	6	7
togi ležaji *rigid bearings*	×	×	×				
elastični ležaji *elastic bearings*				×	×		
drsni ležaji *oil-film bearings*						×	×
strižne deformacije *share deflection*		×	×	×	×	×	×
žiroskopski efekt *gyroscopic effect*			×		×		×

Matematični model sestavlja 13 elementov. Modeli z elastičnimi podporami in drsnimi ležaji (modeli 4-7) imajo tako 14 x 4 = 56 prostostnih stopenj. Modeli s togo izvedbo podprtja (modeli 1-3) imajo na mestu podprtja onemogočene pomike v vseh smereh in imajo tako 52 prostostnih stopenj.

3.1 Rotor s togim podprtjem

Za modela 1 in 2 smo predpostavili togo podprtje rotorja. Model 1 predstavlja osnovni model rotorja brez dodatnih vplivov, medtem ko smo lastne frekvence modela 2 določili z upoštevanjem strižnih deformacij. Za oba modela so prve štiri lastne frekvence podane v preglednici 2.

The numerical model consists of 13 elements. Models with elastic supports and oil-film bearings (models 4-7) have therefore 14 x 4 = 56 degrees of freedom. Rigidly supported models (models 1-3) have at the support point disabled motions in all directions and have therefore 52 degrees of freedom.

3.1 Rotor with rigid bearings

For models 1 and 2 rigid bearings were assumed. Model 1 represents the basic model with no additional influences, while the eigenvalues for model 2 were calculated considering shear deflection. The first four eigenfrequencies are given in Table 2 for both models.

Preglednica. 2: *Lastne frekvence modelov 1 in 2*
Table 2: *Eigenfrequencies of models 1 and 2*

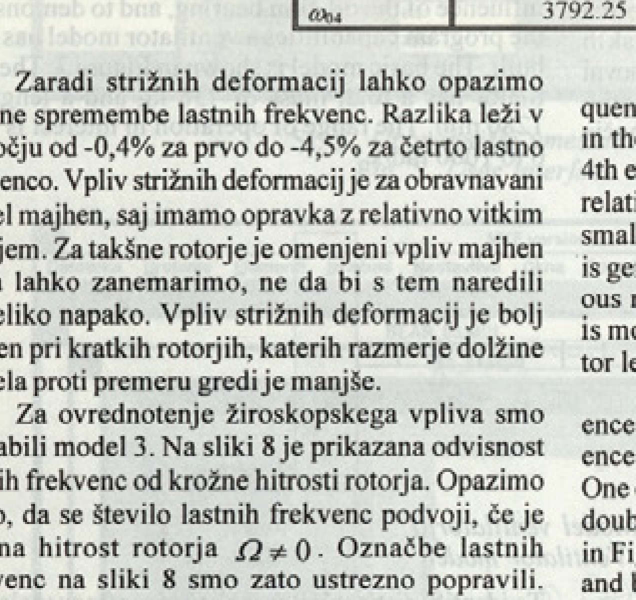
	Model 1	Model 2
ω_{01}	137.02	136.43
ω_{02}	299.43	297.18
ω_{03}	2533.64	2467.20
ω_{04}	3792.25	3619.60

Zaradi strižnih deformacij lahko opazimo majhne spremembe lastnih frekvenc. Razlika leži v območju od -0,4% za prvo do -4,5% za četrto lastno frekvenco. Vpliv strižnih deformacij je za obravnavani model majhen, saj imamo opravka z relativno vitkim rotorjem. Za takšne rotorje je omenjeni vpliv majhen in ga lahko zanemarimo, ne da bi s tem naredili preveliko napako. Vpliv strižnih deformacij je bolj opazen pri kratkih rotorjih, katerih razmerje dolžine modela proti premeru gredi je manjše.

Za ovrednotenje žiroskopskega vpliva smo uporabili model 3. Na sliki 8 je prikazana odvisnost lastnih frekvenc od krožne hitrosti rotorja. Opazimo lahko, da se število lastnih frekvenc podvoji, če je krožna hitrost rotorja $\Omega \neq 0$. Označbe lastnih frekvenc na sliki 8 smo zato ustrezno popravili. Opazimo lahko sinhrono in asinhrono opletanje rotorja.

One can observe small changes in natural frequencies due to shear deflection. The difference lies in the range from -0.4% for the 1st to -4.5% for the 4th eigenfrequency. Due to the fact that the model is relatively slender, the influence of shear deflection is small. For such rotors the aforementioned influence is generally small and can be neglected with no serious reservations. The influence of shear deflection is more noticeable with rotors where the ratio of rotor length to the shaft diameter becomes smaller.

Model 3 has been used to estimate the influence of gyroscopic action. In Figure 8 the dependence of eigenfrequencies on rotor speed is shown. One can observe that the number of eigenfrequencies doubles with the shaft speed $\Omega \neq 0$, so the notation in Figure 8 has been adjusted accordingly. Forward and backward whirl of rotor can also be noticed.



Sl. 8. *Lastne frekvence modela 3*
Sl. 8. *Eigenfrequencies of model 3*

Preglednica 3: *Kritične hitrosti modela 3*
Table 3: *Critical speeds of model 3*

$\Omega_{cr,1}$ = 94.42 rad/s

$\Omega_{cr,2}$ = 168.02 rad/s

$\Omega_{cr,3}$ = 249.60 rad/s

$\Omega_{cr,4}$ = 321.57 rad/s

Kakor je razvidno s slike 8, je vpliv žiroskopskega vpliva velik in ga nikakor ne smemo zanemariti. Sklep velja splošno za rotorje z rotacijskimi telesi, katerih masni vztrajnostni momenti so veliki v primerjavi z masnimi vztrajnostnimi momenti same gredi modela. Opravili smo tudi primerjavo lastnih frekvenc za samo gred modela 3. V tem primeru lahko vpliv žiroskopskega vpliva zanemarimo, saj je razlika lastnih frekvenc, z upoštevanjem žiroskopskega vpliva in brez njega, manjša od 1 odstotka.

Pri $\Omega = \Omega_S$ lahko opazimo zanimiv pojav. Za vrednosti krožne frekvence $\Omega > \Omega_S$ je namreč tretja lastna frekvenca modela manjša od druge lastne frekvence. Obe frekvenci ohranita svojo smer opletanja; t. j. sinhrono opletanje za 2. in asinhrono opletanje za 3. lastno frekvenco. Omenjeni pojav je opazen pri modelih z velikimi rotacijskimi telesi, katerih masni vztrajnostni momenti so veliki v primerjavi z masnimi vztrajnostnimi momenti same gredi rotorja. V opazovanem območju lahko opazimo štiri kritične hitrosti, ki so označene na sliki 8 in prikazane v preglednici 3.

3.2 Rotor z elastičnim podprtjem

Za določitev vpliva elastičnosti podpor na lastne frekvence in kritične hitrosti rotorja smo uporabili modela 4 in 5. Uporabili smo poenostavljen model izotropnih podpor. Obe podpori sta modelirani z enakimi vzmetnimi konstantami. Model 4 smo uporabili za oceno vpliva togosti ležajev na prve štiri lastne frekvence. Razmerje lastnih frekvenc modela 4 in referenčnih lastnih frekvenc je prikazano na sliki 9. Kot referenčno vrednost smo uporabili lastne frekvence modela 2.

Pri tem moramo poudariti, da program sicer omogoča modeliranje elastičnih podpor, vendar so vrednosti za togost ležajev zelo težko določljive. Običajno jih moramo določiti s poskusi ali pa jih ocenimo na podlagi izkušenj.

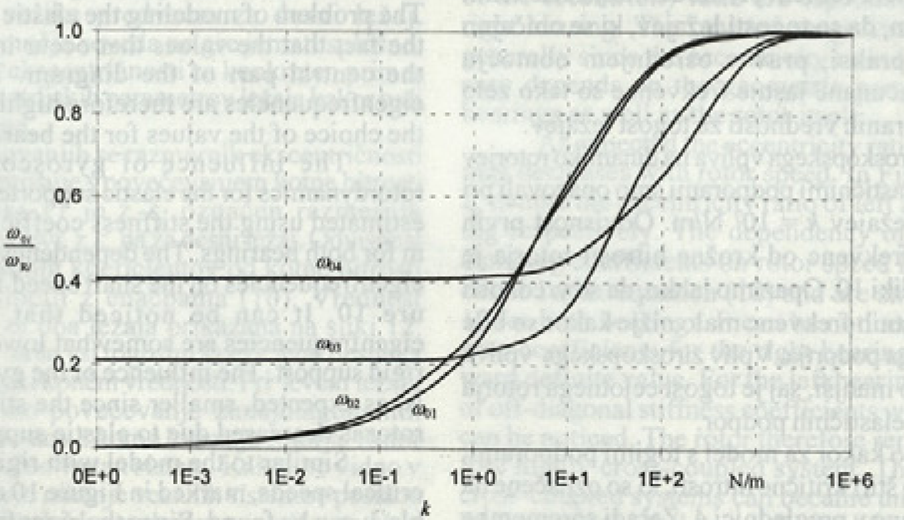
As can be seen from the graph, the influence of gyroscopic action can not be neglected. This is generally the case with rotors having rotating bodies with mass moments of inertia relatively large compared to that one of the shaft itself. Calculations have also been done for the shaft of model 3 alone. In that case, the influence of gyroscopic action can be neglected since the changes of eigenfrequencies in the whole range of operation are less than 1%.

At $\Omega = \Omega_S$ an interesting phenomenon can be observed. For $\Omega > \Omega_S$ the 3rd eigenfrequency is smaller than the 2nd. Both retain their direction of rotation: that is, forward whirl for the 2nd and backward whirl for the 3rd eigenfrequency. The phenomenon can be observed with rotors having rotating bodies with mass moments of inertia relatively large compared that of the shaft itself. Four critical speeds, marked in Figure 8 and listed in Table 3, can be found in the observed range of shaft speed.

3.2 Rotor with flexible bearings

To study the influence of bearing flexibility on eigenfrequencies and critical speeds models 4 and 5 have been considered. Bearings were assumed to be isotropic. Both bearings have the same stiffness coefficients. Model 4 has been used to assess the influence of bearing stiffness on the first four eigenfrequencies. The ratios of eigenfrequencies over reference frequencies are shown in Figure 9. As reference frequencies, the values of model 2 have been used.

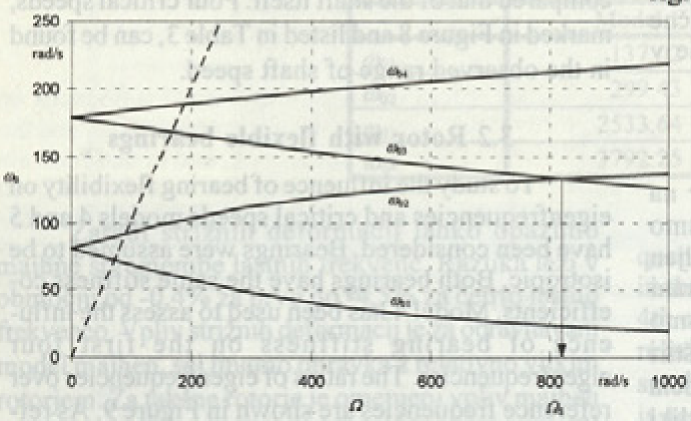
It is to be emphasized that the program enables the use of elastic supports but the values for the stiffness coefficients of the bearings are still difficult to determine. Usually they have to be determined experimentally or based on experience.



Sl. 9. *Lastne frekvence modela 4*
Fig. 9. *Eigenfreqencies of model 4*

Povečanje togosti ležajev vpliva na povečanje lastnih frekvenc, ki limitirajo k vrednostim za model s togimi podporami (model 2). Zmanjšanje togosti ležajev pa se kaže z nižjimi vrednostmi lastnih frekvenc, ki limitirajo k vrednostim nepodprtega modela. Za zelo majhne togosti ležajev se prvi lastni frekvenci približata vrednosti nič. Pri teh vrednostih celoten rotor niha kot togo telo.

Na sliki 9 so lahko razvidna 3 ločena področja. V območju na začetku diagrama, kjer je togost ležajev v območju $0 \leq k \leq 10^{-2}$ N/m, so spremembe lastnih frekvenc, v odvisnosti od spremembe togosti podprtja, zelo majhne. Za 3. in 4. lastno frekvenco je to območje še večje $0 \leq k \leq 1$ N/m. Podobno lahko sklepamo za območje na desnem robu diagrama, kjer se vrednosti lastnih frekvenc ujemajo z vrednostmi togo podprtega modela.



Sl. 10. *Lastne frekvence modela 5*

Fig. 10. *Eigenfrequencies of model 5*

V srednjem delu diagrama, kjer je togost ležajev v območju $1 \leq k \leq 10^2$ N/m, so lastne frekvence zelo občutljive na spremembo togosti ležajev. Že majhna sprememba togosti lahko pomeni veliko spremembo lastnih frekvenc. Problem pri modeliranju elastičnih podpor je v tem, da so togosti ležajev, ki se običajno pojavljajo v praksi, prav v osrednjem območju diagrama. Izračunane lastne frekvence so tako zelo odvisne od izbranih vrednosti za togost ležajev.

Vpliv žiroskopskega vpliva na dinamiko rotorjev pri modelu z elastičnimi podporami smo opazovali pri togosti obeh ležajev $k = 10^7$ N/m. Odvisnost prvih štirih lastnih frekvenc od krožne hitrosti rotorja je prikazana na sliki 10. Opazimo lahko, da so vrednosti prvih štirih lastnih frekvenc malo nižje kakor so bile za primer togega podprtja. Vpliv žiroskopskega vpliva je pričakovano manjši, saj je togost celotnega rotorja manjša zaradi elastičnih podpor.

Podobno kakor za model s togimi podporami, lahko opazimo štiri kritične hitrosti, ki so označene na sliki 10 in zbrane v preglednici 4. Zaradi spremembe lastnih frekvenc pa so ustrezno nižje tudi kritične hitrosti.

The increase of bearing stiffness results in increased eigenfrequencies which in the limit approach the values of the model with rigid bearings (model 2). On the other hand the decrease of bearing stiffness results in decreased eigenfrequencies which in the limit approach the values of the free rotor with respect to support conditions. For very small values of bearing stiffness coefficients of the bearings, the first two eigenfrequencies approach zero. At that value the entire rotor vibrates as a rigid body.

Three distinct areas can be seen in Figure 9. In the initial part of the diagram where the bearing stiffness lies in the range $0 \leq k \leq 10^{-2}$ N/m the changes of eigenfrequencies are small with regard to the change of the bearing stiffness. The area is even wider for the 3rd and 4th eigenfrequency and amounts to $0 \leq k \leq 1$ N/m. The same can be said for the area at the right side of the diagram where the values correspond to the eigenfrequencies of the rigidly supported rotor.

Preglednica 4: *Kritične hitrosti modela*
Table 4: *Critical speeds of model 5*

$\Omega_{cr,1}$	=	72.25 rad/s
$\Omega_{cr,2}$	=	92.13 rad/s
$\Omega_{cr,3}$	=	168.34 rad/s
$\Omega_{cr,4}$	=	188.35 rad/s

In the central part of the diagram where the bearing stiffness lies in the range $1 \leq k \leq 10^2$ N/m, the eigenfrequencies are very sensitive to the change of bearing stiffness. Even the small change of the stiffness results in a large change of eigenfrequencies. The problem of modeling the elastic supports lies in the fact that the values that occur in practice lie in the central part of the diagram. The calculated eigenfrequencies are therefore highly dependent on the choice of the values for the bearing stiffness.

The influence of gyroscopic action on rotordynamics for the elastic supported rotor has been estimated using the stiffness coefficients $k = 10^7$ N/m for both bearings. The dependency of the first four eigenfrequencies on the shaft speed is shown in Figure 10. It can be noticed that the first four eigenfrequencies are somewhat lower than those of rigid support. The influence of the gyroscopic action is, as expected, smaller since the stiffness of entire rotor is decreased due to elastic supports.

Similar to the model with rigid bearings four critical speeds, marked in Figure 10 and listed in Table 4, can be found. Since the eigenfrequencies have changed, the critical speeds are correspondingly lower.

3.3 Rotor z drsnimi ležaji

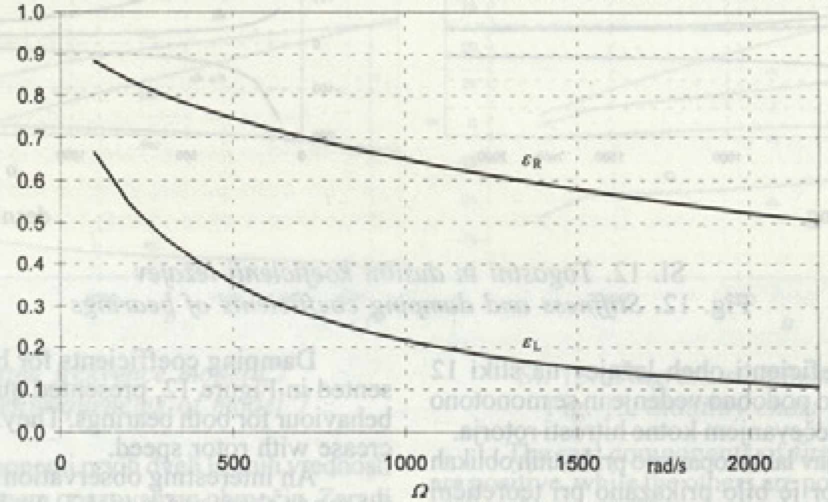
Za ovrednotenje vpliva drsnih ležajev na dinamiko rotorjev smo obravnavali modela 6 in 7. Uporabljena je bila teorija kratkih drsnih ležajev, podrobneje predstavljena v poglavju o rotorjih z drsnimi ležaji. Predpostavljena dinamična viskoznost maziva je $\eta = 0{,}01$ Ns/m^2 za oba ležaja. Glede na statično obremenitev znaša tlak v ležajih 140 kPa za levi in 1,25 MPa za desni ležaj.

Pri obravnavi drsnih ležajev, katerih razmerje dolžine in premera je manjše od 1, se vrednosti togostnih in dušilnih koeficientov, določenih s teorijo kratkega drsnega ležaja, le malo razlikujejo od realnih vrednosti. Napaka se povečuje z naraščanjem razmernika ekscentričnosti. Rezultati, dobljeni pri zelo velikih vrednostih razmernika ekscentričnosti, $\varepsilon \approx 1$, so zato dvomljivi.

3.3 Rotor with oil-film bearings

The models 6 and 7 have been used to study the influence of the oil-film bearings on rotordynamics. The short oil-film bearing theory has been used as described in the chapter on about rotors with oil-film bearings. The dynamic viscosity of the lubricant was $\eta = 0{,}01$ Ns/m^2 for both bearings. With regard to the static load the bearing pressures are 140 kPa and 1.25 MPa for left and right bearing respectively.

Considering the oil-film bearings with a ratio of the bearing length to the bearing diameter smaller than 1, the values of stiffness and damping coefficients determined by the short bearing theory differ only slightly from the real ones. The difference increases with the eccentricity ratio. Results obtained at very high values of eccentricity ratio, $\varepsilon \approx 1$, should therefore be considered as doubtful.



Sl. 11. *Razmernik ekscentričnosti ležajev*

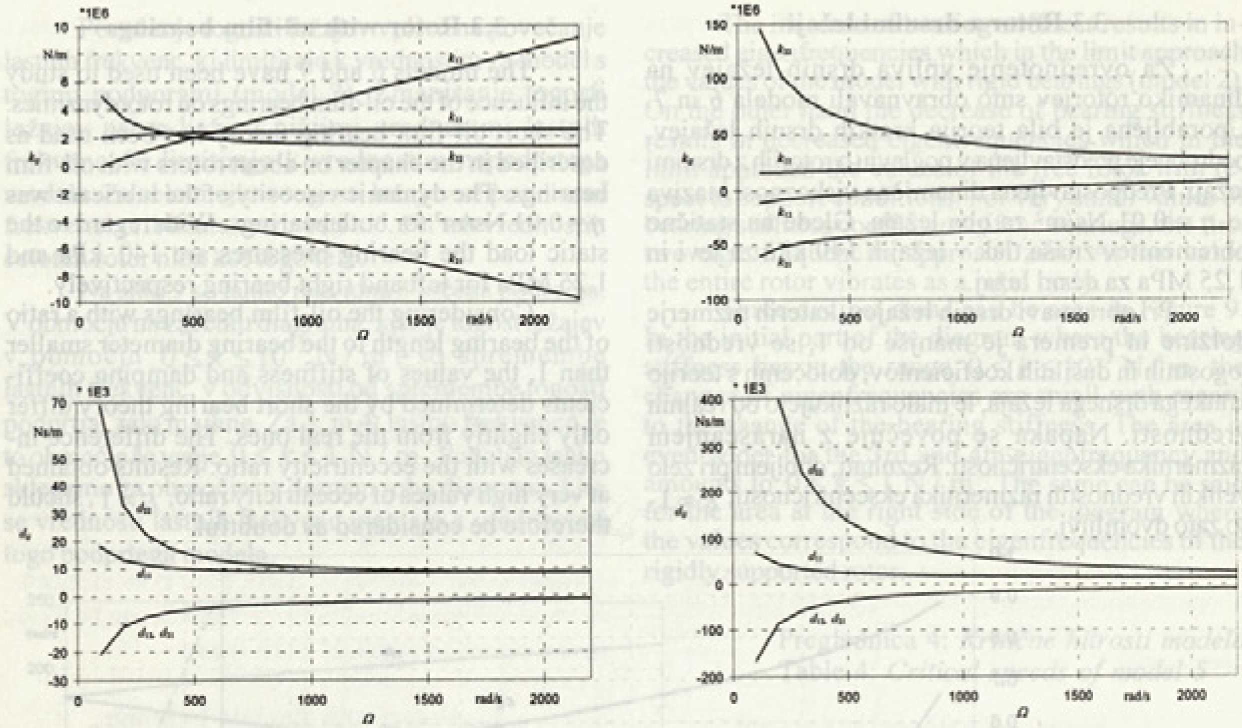
Fig. 11. *Eccentricity ratio of bearings*

Ker se razmernik ekscentričnosti hitro zmanjšuje s povečevanjem krožne hitrosti rotorja Ω, je v nadaljevanju pri obravnavi izpuščeno območje krožne hitrosti $0 \le \Omega \le 200$ rad/s. Kakor je razvidno s slike 11, so vrednosti razmernika ekscentričnosti v tem območju velike predvsem za desni ležaj. Izpuščenega območja seveda ne gre jemati splošno, saj je razmernik ekscentričnosti za konkreten primer odvisen od geometrijskih parametrov ležaja kakor tudi od rotorja samega.

Po pričakovanjih se razmernik ekscentričnosti obeh ležajev zmanjšuje s povečevanjem kotne hitrosti rotorja. Na sliki 11 je z ε_L označen razmernik ekscentričnosti za levi, z ε_R pa za desni ležaj. Odvisnost togostnih in dušilnih koeficientov od kotne hitrosti rotorja smo določili z enačbama (10). Vrednost koeficientov je za oba ležaja prikazana na sliki 12. Opazimo lahko, da se vsi togostni koeficienti desnega ležaja stekajo k neki končni vrednosti. Pri levem ležaju pa lahko opazimo povečevanje zunajdiagonalnih togostnih koeficientov s kotno hitrostjo rotorja. Rotor je zaradi tega močno vezan sistem. Kakor bo vidno v nadaljevanju, lahko nihanje vezanih sistemov postane tudi nestabilno. To lahko razložimo z večjo statično obremenitvijo desnega ležaja, zaradi česar je tlak v ležaju večji.

Since the eccentricity ratio decreases rapidly with shaft speed Ω, the range of the shaft speed $0 \le \Omega \le 200$ rad/s is therefore not evaluated in all the following. As can be seen in Figure 11 the values of the eccentricity ratio are especially high for the right bearing. The omitted range can not be taken generally since the eccentricity ratio for the specific case depends on the geometric parameters of the bearings as well as the rotor itself.

As expected the eccentricity ratio of both bearings decreases with rotor speed. In Figure 11 ε_L and ε_R denote the eccentricity ratio of left and right bearing respectively. The dependency of stiffness and damping coefficients on rotor speed have been calculated after equation (10) and are shown in Figure 12 for both bearings. It can be noticed that all stiffness coefficients for the right bearing converge toward definite value. For the left bearing the increase of off-diagonal stiffness coefficients with rotor speed can be noticed. The rotor therefore represents therefore highly cross-coupled system. The vibration of cross-coupled systems can became unstable, as will be shown in the following. Such behavior can be explained by the higher static load for the right bearing, which also results in increased bearing pressure.



levi ležaj - left bearing desni ležaj - right bearing

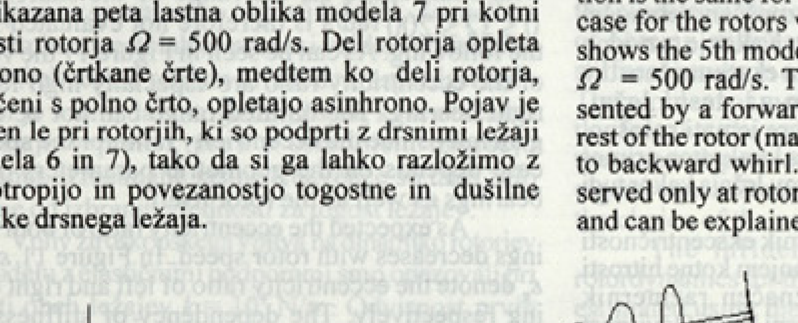
Sl. 12. *Togostni in dušilni koeficienti ležajev*
Fig. 12. *Stiffness and damping coefficients of bearings*

Dušilni koeficienti obeh ležajev na sliki 12 kažejo kvalitativno podobno vedenje in se monotono zmanjšujejo s povečevanjem kotne hitrosti rotorja.

Zanimiv pojav lahko opazimo pri lastnih oblikah za model 7. Kakor je bilo prikazano pri teoretični obravnavi dinamike rotorjev, povzroči žiroskopski vpliv bodisi sinhrono ali pa asinhrono opletanje rotorja. V večini primerov je rotacija celotnega rotorja enaka, kar pa ne velja za rotorje z drsnimi ležaji. Na sliki 13 je prikazana peta lastna oblika modela 7 pri kotni hitrosti rotorja $\Omega = 500$ rad/s. Del rotorja opleta sinhrono (črtkane črte), medtem ko deli rotorja, označeni s polno črto, opletajo asinhrono. Pojav je opazen le pri rotorjih, ki so podprti z drsnimi ležaji (modela 6 in 7), tako da si ga lahko razložimo z anizotropijo in povezanostjo togostne in dušilne matrike drsnega ležaja.

Damping coefficients for both bearings, presented in Figure 12, presented qualitatively similar behaviour for both bearings. They monotonically decrease with rotor speed.

An interesting observation can be made from the mode shapes of model 7. As has been presented in the theoretical consideration of rotordynamics the gyroscopic action causes either forward or backward whirl of the rotor. In most cases the direction of rotation is the same for the entire rotor, but this is not the case for the rotors with oil-film bearings. Figure 13 shows the 5th mode shape of model 7 at rotor speed $\Omega = 500$ rad/s. The parts of the rotor are represented by a forward whirl (dashed lines) while the rest of the rotor (marked with solid lines) is subjected to backward whirl. This phenomenon has been observed only at rotors, supported by oil-film bearings and can be explained through their anisotrophy.

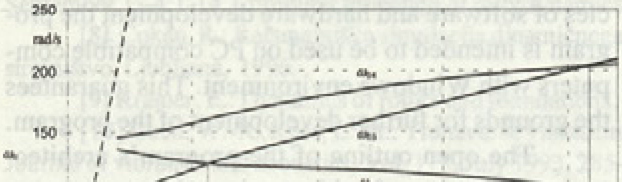


Sl. 13. *Peta lastna oblika modela 7 pri $\Omega = 500$ rad/s*
Fig. 13. *Fifth mode shape of model 7 at $\Omega = 500$ rad/s*

Togostni koeficienti desnega ležaja modela 7 so večji, od tistih, ki so bili uporabljeni pri modelu 5, medtem ko so togostni koeficienti za levi ležaj

Stiffness coefficients for the right bearing of model 7 are relatively large compared to those used for model 5, while the values for the left bearing are primerljivi. Lastne frekvence modela z drsnimi ležaji, slika 14, so nekoliko nižje od lastnih frekvenc modela s homogenimi elastičnimi podporami. To lahko razložimo s povezano togostno in dušilno matriko modela 7. Prav tako je močno izražen žiroskopski vpliv, saj se lastne frekvence močno spreminjajo s povečanjem krožne hitrosti rotorja.

Lastne vrednosti se vedno pojavljajo v kompleksno konjugiranih parih (5). Pri modelih, pri katerih ni dušenja, imajo lastne vrednosti le kompleksno komponento. Če se v modelu pojavi dušenje, dobimo pri lastnih vrednostih tudi realne komponento. Na sliki 15 so prikazane resnične komponente lastnih vrednosti modela z drsnimi ležaji.



Sl.14. *Lastne frekvence modela 7*
Fig.14. *Eigenfrequencies of model 7*

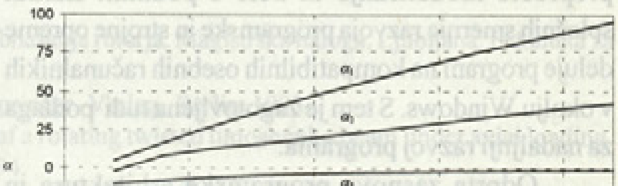
Realni komponenti prvih dveh lastnih vrednosti sta pozitivni v celotnem opazovanem območju. Zaradi tega je obratovanje rotorja nestabilno, saj amplituda nihanj s časom narašča. Za zagotovitev stabilnosti obratovanja bi bilo treba spremeniti geometrijske parametre drsnih ležajev.

4 SKLEP

Namen prispevka je bil predstaviti problematiko dinamike rotorjev in ovrednotiti vpliv posameznih parametrov na dinamično obnašanje rotorja. Analitične rešitve s področja dinamike rotorjev so žal omejene na zelo preproste modele. Pri obravnavi realnih sistemov smo tako prisiljeni uporabljati diskretne metode in numerično reševanje problemov. V pričujočem članku predstavljeni programski paket naj bi zapolnil praznino med komercialnimi programi, ki temeljijo na metodi končnih elementov in ne omogočajo obravnave nekaterih, za dinamiko rotorjev specifičnih pojavov (npr. nesimetrična togostna matrika, žiroskopski vpliv). Program je bil v celoti razvit v Laboratoriju za dinamiko strojev in konstrukcij na Fakulteti za strojništvo v Ljubljani in pomeni uporabno orodje, namenjeno konstrukterju v začetni fazi razvoja rotorjev.

comparable. The eigenfrequencies, shown in Figure 14, are however somewhat lower than those of the model with homogeneous flexible bearings. This can be explained through the influence of cross-coupling. Influence of gyroscopic action is also clearly manifested since the eigenfrequencies change greatly with increasing shaft speed.

Eigenfrequencies always appear as complex conjugate pairs (5). With models with no damping we have only an imaginary component. The presence of damping in the model reveals itself as the real component of the eigenfrequency. Figure 15 shows the real components of the eigenvalues for the model with oil-film bearings.



Sl. 15. *Diagram stabilnosti modela 7*
Fig. 15. *Stability chart of model 7*

The real components of first two eigenvalues are positive, while the others are positive in the whole range of interest. This can be the cause of instability of the system, since the amplitude of vibration increases with time. To achieve stability of operation in such cases the bearing geometric parameters should be changed.

4 CONCLUSION

The purpose of the paper was to present the problems of rotordynamics and to estimate the influence of individual parameters on the dynamic behaviour of the rotor. Analytical solutions in the domain of rotordynamics are unfortunately limited to elementary models. Considering realistic systems, we were forced to use discrete methods and numerical solutions of problems. The program presented in this paper is intended to fill the gaps of general FEM programs, which (with rare exceptions) do not meet the special requirements of rotordynamics (gyroscopic effect and asymmetric stiffness and damping matrices). The program has been entirely developed in the Laboratory for dynamics of machines and structures at the Faculty for Mechanical Engineering in Ljubljana, and provides an useful tool for engineers in the early stage of rotor design.

Razvoj takšnega programa je obsežen projekt, saj obravnavana tematika pokriva široko področje problematike. Poleg problemov, neposredno povezanih z dinamiko rotorjev, je v program vključen modul za vrednotenje diskretnega modela z metodo končnih elementov. Velik del programske kode je namenjen tudi numeričnim metodam za določanje lastnih vrednosti. Poleg Housholderjeve metode za reševanje simetričnih problemov je obravnavi splošnih problemov namenjena Hessenbergova metoda.

Velika pozornost je bila posvečena razvoju uporabniškega vmesnika, ki uporabniku omogoča preprosto modeliranje in delo s podatki. Zaradi splošnih smernic razvoja programske in strojne opreme deluje program na kompatibilnih osebnih računalnikih v okolju Windows. S tem je zagotovljena tudi podlaga za nadaljnji razvoj programa.

Odprta zasnova programske arhitekture in uporaba objektnega programiranja zagotavljata preprosto nadgradnjo programa. Dodajanje novih elementov terja le minimalne popravke v programu, ki že zdaj obsega prek 30.000 programskih vrstic, obsežno knjižnico z elementi uporabniškega vmesnika in obširno pomoč.

Metoda končnih elementov, uporabljena za popis diskretnega matematičnega modela, se je pokazala kot zelo učinkovita. Predstavljeni rezultati kažejo zelo dobro ujemanje s preverjenimi numeričnimi in eksperimentalnimi rezultati (Litostroj, Ljubljana).

Na podlagi predstavljenih rezultatov lahko sklenemo, da vpliva žiroskopskega učinka pri obravnavi dinamike rotorjev nikakor ne moremo zanemariti. To velja še posebej za rotorje z vrtečimi se telesi, katerih glavni masni vztrajnostni momenti so veliki v primerjavi z masnimi vztrajnostnimi momenti gredi. Podobno velja tudi za vpliv drsnih ležajev, ki lahko ob neugodnih konstrukcijskih parametrih povzročijo celo nestabilnost modela.

Programski paket ne pokriva celotnega področja dinamike rotorjev. Vendar so opravljena testiranja in zelo dobro ujemanje z referenčnimi rezultati, skupaj z odprto zasnovano programsko arhitekturo, dobra osnova za nadaljnje delo.

The development of such a program is an extensive project, since the issues discussed cover wide range of problems. Besides the problems directly associated with rotordynamics the program also incorporates a module for evaluation of a discrete model through the finite element method. A large part of the program code is also used by numerical methods for determination of eigenfrequencies. In addition to the Hauseholder method for evaluation of symmetric problems the Hessenberg method has been used for solving the general problems.

Special attention has been given to development of the user interface that enables easy modeling and data processing. Considering the general tendencies of software and hardware development the program is intended to be used on PC compatible computers with Windows environment. This guarantees the grounds for further development of the program.

The open outline of the program's architecture and the use of object-oriented programming assures easy further development of the program. Adding new elements requires only minimum corrections in the program that already contains more than 30,000 program lines, and an extensive library with elements of the user interface and thorough support.

The finite element method, used to describe the discrete mathematical model, has proved to be very effective. The results presented show very good agreement with verified numerical and experimental results (Litostroj, Ljubljana).

On the grounds of the results presented it can be concluded that the influence of gyroscopic action cannot be neglected when considering the rotordynamics. This is in particular valid for rotors which have rotating bodies with mass moments of inertia relatively large compared to that of the shaft itself. The influence of the oil-film bearings on the rotordynamics has been also found important. Unfavorable design parameters may even cause instability of the rotor.

The program does not cover the whole range of the rotordynamics phenomena. The performed tests, and the very good agreement with reference results represent, together with the open outline of the program's architecture, good basic grounds for further development.

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