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Potenčni zakon kot model zmesi elektrofiltrskega pepela in vode

The Power Law as a Model for an Electrostatic Filter Ash and Water Mixture

Primož Ternik - Jure Marn - Damijan Kanduti - Evgen Dervarič

Prispevek obravnava tok zmesi elektrofiltrskega pepela in vode. Eksperimentalni podatki, pridobljeni v cevnem viskozimetru Premogovnika Velenje d.d., so bili uporabljeni kot osnova za določitev modela potenčnega zakona po metodi najmanjših kvadratov. S tako dobljenim modelom je bila izvedena numerična analiza toka z metodo končnih prostornin, rezultati pa primerjani z eksperimentalnimi. Kljub povprečno 90-odstotnemu ujemanju numeričnih in eksperimentalnih rezultatov bo v prihodnje treba namesto potenčnega zakona uporabiti kak drug, npr. Siskov model, katerega teoretične osnove so prav tako del prispevka.

(Ključne besede: tekočine nenewtonske, modeli reološki, modeliranje, zakoni potenčni)

This paper deals with the flow of an electrostatic filter ash and water mixture. Experimental data obtained from a pipe viscometer in Premogovnik Velenje Ltd. were used as the basis to determine the Power Law model by the least-squares method. With this model a numerical analysis with the finite volume method was performed and the results were compared with experimental data. Despite the 90% agreement between the numerical and experimental results there is still a need to use some other rheological models instead of the Power Law model, for example the Sisko model, the theoretical basis of which is also an integral part of this contribution.

(Keywords: non-Newtonian fluids, rheological models, modelling, Power Law)

0 UVOD

Hidravlični prenos trdnih snovi je učinkovit in ekonomičen način prenosa predvsem, kadar gre za velike količine snovi in večje razdalje. Značilne primere takšnega prenosa srečujemo v celi vrsti industrijskih dejavnosti; od oskrbovanja z vodo do hidravličnega prenosa v različnih vejah rudarstva, kemije in drugih vej.

Pretakanje teh tekočin po cevovodih lahko proučujemo, če poznamo njihove fizikalne lastnosti (gostoto, viskoznost, stisljivost) oziroma vpliv zunanjih dejavnikov (temperatura, tlak) na te lastnosti. Prenosni sistem je določen, ko poznamo fizikalne lastnosti trdne in tekoče faze ter dimenzije cevnih sistemov.

Posebna skupina teh tekočin so suspenzije - tekočine, v katerih bolj ali manj lebdi veliko majhnih,

0 INTRODUCTION

The hydraulic transport of solids is an effective and economical way of moving large amounts of solids over large distances. Typical examples of this kind of transportation can be found in almost all industrial applications, from water supply to hydraulic transport in different branches of mining, chemistry, etc.

The flow of these fluids in pipelines can be analysed if we know their physical properties (density, viscosity, compressibility) and the influence of external factors (temperature, pressure) on these properties. The transport system is determined by the physical properties of the solid and the fluid phase as well as the dimensions of the pipe systems.

A subset of these fluids is the group of suspensions fluids, in which small, solid particles are trdnih delcev. Tako zmes lahko v določenih (tržnih) primerih obravnavamo kot homogeno (nenewtonsko) tekočino ter upoštevamo njene povprečne lastnosti. Najprej določimo ustrezni reološki model, nato pa izberemo ustrezne mešalne in črpalne naprave ter hidravlični prenos morebiti avtomatiziramo.

Za opis reološkega obnašanja pastastih tekočin uporabljamo empirične modele, ki podajajo strižno napetost (ali viskoznost) v odvisnosti od deformacijske hitrosti. Najbolj razširjeni so Ostwald-de Waele oz. potenčni zakon, Binghamov plastični model, Cassonov model, redkeje pa Herschel-Bulkyjev in Siskov model.

Kljub dosegljivosti omenjenih modelov pa zelo redko lahko zanesljivo in dovolj natančno opišemo reološko obnašanje takih tekočin. Pastaste tekočine so namreč zapleteni sistemi, sestavljeni iz trdnih delcev različnih velikosti in oblik ter tekočin, in vprašanje je, ali lahko že sedanji modeli popisujejo njihovo obnašanje pri hidravličnem prenosu. Zato moramo tudi na konstante posameznih modelnih enačb gledati le kot na modelne parametre ne pa kot na "prave" fizikalne lastnosti.

floating. Such a mixture is, in engineering applications, usually treated as a homogeneous (non-Newtonian) fluid with some artificial, average properties of this "fluid". First, we must try to find a suitable rheological model for these fluids, and then select suitable mixing and pumping devices in order to eventually automate the hydraulic transport.

For a rheological description of slurries we use empirical models relating shear stress (or viscosity) and shear rate. The most commonly used are the Ostwald-de Waele or Power Law, the Bingham plastic model, the Casson model, and rarely the Herschel-Bulky and Sisko models.

Despite wide spread use of the above- mentioned models it is quite rare that the rheological behavior of these fluids is described reliably and accurately enough. Slurries are complex systems comprised of liquids and solid particles of different shapes and sizes; there is a question as to whether the existing models can describe their behavior during hydraulic transport. This is why we must take the constants in particular model equations only as model parameters and not as "true" physical properties.

1 FIZIKALNE LASTNOSTI

1 PHYSICAL PROPERTIES

V praksi gostoto suspenzije določimo na podlagi gostote trdnih delcev in tekočine. Gostota homogene suspenzije je tako [1]:

In practice, the density of a suspension is defined in terms of solid particles and fluid densities. The density of a suspension is given by [1]:

$$\rho_m = \frac{100}{\frac{\xi_s}{\rho_s} + \frac{100-\xi_s}{\rho_l}} \qquad (1).$$

Einstein [2] predlaga viskoznost razredčenih suspenzij kot:

Einstein [2] suggests the viscosity of dilute suspensions is:

$$\frac{\eta_m}{\eta_l} = 1 + 2,5\psi \qquad (2),$$

medtem ko Guth in Simha [3] za koncentrirane suspenzije predlagata:

while Guth and Simha [3] suggest for concentrated suspensions:

$$\frac{\eta_m}{\eta_l} = 1 + K_1\psi + K_2\psi^2 + ... \qquad (3),$$

pri čemer sta vrednosti konstant K_1 = 2,5 [2] in K_2 = 14,1 [3].

Še popolnejši model viskoznosti koncentriranih suspenzij podaja Thomas [2] kot:

using the values of K_1 = 2.5 [2] and K_2 = 14.1 [3].

A more complete viscosity model for concentrated suspensions is suggested by Thomas [2] as:

$$\frac{\eta_m}{\eta_l} = 1 + 2,5\psi + 10,05\psi^2 + A\exp\left(B\psi\right) \qquad (4),$$

pri čemer sta A = 0,00273 in B = 16,6.

Naslednjo enačbo za viskoznost koncentriranih suspenzij so predlagali Gay idr. [4]:

using A = 0.00273 and B = 16.6.

Next, the equation for the viscosity of concentrated suspensions is suggested by Gay et al.[4]:

$$\frac{\eta_m}{\eta_l} = \exp\left[2,5 + \left(\frac{\psi}{\psi_{\max} - \psi}\right)^n\right]\frac{\psi}{\psi_{\max}} \qquad (5),$$

kjer je Ψ_{max} največji dosegljiv prostorninski delež trdnih delcev.

Specifično toploto suspenzij so Thomas [5] ter Orr in Dalla Valle [6] podali na temelju specifičnih toplot čistih sestavin kot:

where Ψ_{max} is the maximum volumetric concentration.

Thomas [5] as well as Orr and Dalla Valle [6] gave the specific heat of suspensions on the basis of the specific heats of the pure components as:

$$\left(c_p\right)_m = \frac{\left(c_p\right)_s \xi_s + \left(c_p\right)_l \xi_l}{100} \qquad (6).$$

Glede toplotnih prevodnosti sta Orr in Dalla Valle [6] predlagala metodo, s katero lahko preprečimo usedanje delcev; to je dodajanje majhnih količin agarja in s tem tvorbo gela. Vrednost tako izmerjene toplotne prevodnosti nato popravimo za velikost vpliva agarja na toplotno prevodnost. Na ta način izmerjene toplotne prevodnosti se dobro ujemajo z vrednostmi toplotnih prevodnosti, izračunanih po naslednji enačbi [5]:

Regarding thermal conductivities, Orr and Dalla Valle [6] proposed a method by which particle settling could be eliminated by adding small quantities of agar to the suspension, thus forming a gel. The value of the thermal conductivity measured in this way is then corrected for the effects of the agar. The thermal conductivities of slurries measured by this gelling technique show good agreement with those calculated by means of the following equation [5]:

$$k_m = k_l \left[\frac{2k_l + k_s - 2\psi\left(k_l - k_s\right)}{2k_l + k_s + \psi\left(k_l - k_s\right)} \right] \qquad (7).$$

2 NENEWTONSKE TEKOČINE

2 NON-NEWTONIAN FLUIDS

Suspenzije in še cela vrsta drugih tekočin, ki jih srečujemo v različnih panogah industrije, se ne podrejajo Newtonovem zakonu viskoznega tečenja:

Suspensions as well as many other fluids that can be met in many industrial applications do not obey Newton’s law of viscous flow:

$$\tau = \eta\dot{\gamma} \qquad (8).$$

V takih primerih viskoznost ni več konstantna veličina, ampak je odvisna od deformacijske hitrosti in strižne napetosti, viskozno obnašanje pa skušamo popisati z dejansko (navidezno) viskoznostjo:

In these cases, viscosity is no longer a constant quantity, but depends on shear rate and shear stress, while viscous behavior is described with the effective (apparent) viscosity:

$$\eta_{eff} = \frac{\tau}{\dot{\gamma}} \qquad (9).$$

Pri tem delimo nenewtonske tekočine v:

Časovno neodvisne, pri katerih je strižna napetost neodvisna od časa in je odvisna le od velikosti deformacijske hitrosti:

Non-Newtonian fluids can be classified as:

Time-independent fluids with a shear stress independent of time and only a function of the shear rate:

$$\tau = f\left(\dot{\gamma}\right) \qquad (10).$$

Primer so Binghamove plastične tekočine, pri katerih je obnašanje opisano z enačbo:

For example: Bingham plastic fluids described by:

$$\tau - \tau_0 = \eta \frac{dv}{dy} \qquad (11)$$

in

and

$$\tau \rangle \tau_0$$

Psevdoplastične, za katere je značilno padanje viskoznosti z večanjem deformacijske hitrosti. Najbolj razširjen je Ostwald-de Waele oziroma potenčni zakon:

Pseudoplastic fluids, featuring a characteristic decrease of the viscosity with an increase in the shear rate. The most widely used is the Power Law:

$$\tau = K\dot{\gamma}^n \qquad (12)$$

za

for

$n\langle 1$

Pri tem sta K in n konstanti za posamezno tekočino.

Psevdoplastične tekočine izkazujejo tri izrazita področja (sl. 1); spodnje in zgornje Newtonovo področje ter področje spremenljive viskoznosti (psevdoplastično področje). Viskoznost v spodnjem Newtonovem področju je η_0, v zgornjem Newtonovem področju pa η_∞. V področju spremenljive viskoznosti je pogosto uporabljena enačba potenčnega zakona (12) za odvisnost strižne napetosti in deformacijske hitrosti.

Za suspenzije je primerno uporabiti Siskov model:

Where K and n are constants of a particular fluid.

Pseudoplastic fluids exhibit three distinct regions (Fig. 1): the lower and upper Newtonian regions as well as the region of variable viscosity (pseudoplastic region). The viscosity in the lower Newtonian region is η_0 and in the upper Newtonian region is η_∞. In the region of variable viscosity the Power Law (12) is often used to correlate the shear stress and the shear rate.

For suspensions it is reasonable to use the Sisko model:

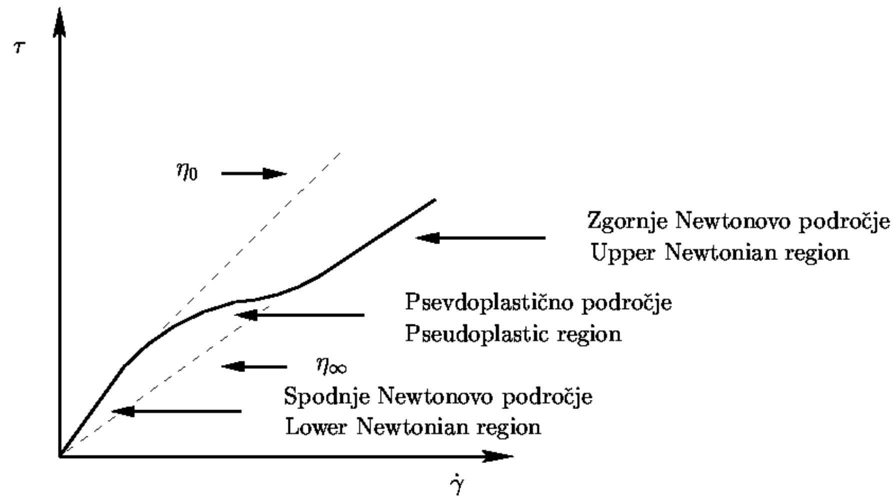
$$\tau = \dot{\gamma}\left(\eta_\infty + K_s \tau^{n_s - 1}\right) \quad (13),$$

kjer sta K_s in n_s parametra, ki jih določimo eksperimentalno.

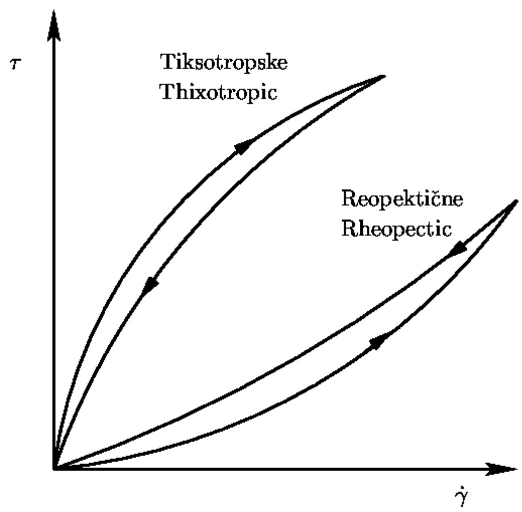
Za dilatantne tekočine so povečanje viskoznosti s povečanjem deformacijske hitrosti

with K_s and n_s experimentally determined.

Dilatant fluids are fluids where the increasing viscosity with increasing shear rate is associated



Sl. 1. *Popolni strižni diagram za psevdoplastične tekočine*
Fig. 1. *Complete shear diagram for pseudoplastic fluids*



Sl. 2. *Časovno odvisne nenewtonske tekoči*ne
Fig. 2. *Time-dependent non-Newtonian fluids*

povezovali s povečanjem prostornine oz. dilatantnim učinkom. Tudi za dilatantne tekočine velja potenčni zakon, le da je v tem primeru $n > 1$. Pojem dilatantnosti je vpeljal Reynolds.

Časovno odvisne (slika 2), med katere prištevamo tiksotropske in reopektične, odvisno od tega, ali se strižna napetost povečuje ali zmanjšuje s časom pri dani deformacijski hitrosti in temperaturi.

Tiksotropske tekočine izkazujejo zmanjševanje strižne napetosti pri konstantni deformacijski hitrosti. Do tega pojava pride zaradi postopne porušitve dela tridimenzionalne strukture v snovi in strižna napetost se prične približevati ravnotežni vrednosti (v odvisnosti od strukturnih lastnosti tekočine in velikosti deformacijske hitrosti).

Splošno je znano, da so notranji mehanizmi, odgovorni za tiksotropsko obnašanje, podobni tistim, ki povzročajo psevdoplastično obnašanje, vendar tiksotropski procesi potekajo mnogo počasneje od psevdoplastičnih.

Za reopektične tekočine je značilno povečevanje viskoznosti (oz. strižne napetosti) s časom. Reopektično obnašanje pripisujejo strjevanju (sesedanju) delcev, ki se približujejo drug drugemu zaradi rahlega gibanja tekočine oz. delovanja strižne napetosti.

with an increase in volume, or a dilatant effect. The Power Law can also be used for dilatant fluids with $n > 1$. It is interesting to note that the term dilatancy was introduced by Reynolds.

Time-dependent fluids (Figure 2) include thixotropic and rheopectic fluids, depend on an increasing or decreasing of the shear stress with time at a given shear rate and temperature, respectively.

Thixotropic fluids exhibit a decrease in shear stress with time at a constant shear rate. This is generally caused by the gradual destruction of some three-dimensional structure in the material, and the shear stress approaches an equilibrium value (depending on the structural properties of the fluid and on the magnitude of the applied shear rate).

It is generally believed that the internal mechanisms responsible for thixotropic behavior are similar to those that cause pseudoplastic behavior, but they differ in respect to time scale.

Rheopectic fluids exhibit an increase in viscosity or in shear stress with time. Rheopexy is attributed to the coagulation (flocculation) of particles when they are forced into close proximity with each other by the gentle movement of the fluid or by the acting shear stress.

3 REOLOŠKE MERITVE

3 RHEOLOGICAL MEASUREMENTS

Newtonov zakon viskoznega tečenja:

Newton's law of viscous flow:

$$F/A = \tau_{yx} = -\eta\left(\frac{dv_x}{d_y}\right) \tag{14}$$

oziroma

or

$$\tau = \eta\left|\frac{dv}{dy}\right| \tag{15}$$

je splošno veljaven tako za laminarne kakor tudi turbulentne tokove, če je v trenutna hitrost.

Določanje reoloških lastnosti običajno izvajamo z laminarnim tokom v kapilarnem (cevnem) viskozimetru, laminarnim tokom med dvema vzporednima ploščama, laminarnim tokom v Couettovem viskozimetru in viskozimetrom s padajočo kroglico.

Viskozimeter s padajočo kroglico meri viskoznost pri zelo majhnih deformacijskih hitrostih in ga lahko uporabimo za določitev η_0 (viskoznosti pri $\dot{\gamma}$=0). V preostalih primerih lahko velikost deformacijske hitrosti dokaj preprosto spreminjamo; zato se največkrat izvajajo eksperimenti s kapilarnimi in/ali rotacijskimi viskozimetri za določanje viskoznosti oz. strižne napetosti v odvisnosti od deformacijske hitrosti.

3.1 Kapilarni viskozimeter

V kapilarnem viskozimetru (sl. 3) spreminjamo masni pretok s spreminjanjem tlaka. Običajno uporabljamo cevi različnih premerov, tako

is generally valid for laminar as well as for turbulent flows as long as v stands for the local velocity.

Determinations of the rheological properties are usually conducted under conditions of: laminar flow in a capillary (pipe) viscometer, laminar flow between parallel plates, laminar flow in a Couette viscometer and a falling-ball viscometer.

The falling-ball viscometer measures viscosity at very low shear rates and can be used to find η_0 (the viscosity at $\dot{\gamma}$=0). In other experiments, the rate of shear is easily varied; therefore capillary and/or rotational experiments are used to determine the viscosity or shear stress as a function of the shear rate.

3.1 Capillary viscometer

For a capillary viscometer (Fig. 3), the flow rate is varied by varying the pressure. Usually, tubes of differing diameters are used along with a pressure da skupaj s spreminjanjem tlaka dobimo zelo široko območje deformacijskih hitrosti. Pri ceveh moramo biti zelo pazljivi, da bo notranjost čim bolj gladka, da bi ohranili laminarni tok v cevi.

Pri teh viskozimetrih dobimo povezavo med deformacijsko hitrostjo in strižno napetostjo iz meritev tlačnega gradienta in prostorninskega toka tekočine skozi cev, pri čemer predpostavljamo, da je tok ustaljen in laminaren, lastnosti tekočine so neodvisne od časa, hitrost tekočine nima radialnih ali obodnih komponent, tekočina je nestisljiva, tlak ne vpliva na viskoznost tekočine, meritev se izvaja v izotermnih razmerah (T = konst).

Za laminarni tok newtonske tekočine v ravni cevi oz. Hagen-Poiseuillev tok (sl. 4) velja gibalna enačba v polarnih koordinatah:

which gives a wide range of shear rates. The tubes are carefully chosen to ensure the inside is as smooth as possible in order to maintain laminar flow.

For these types of viscometers the relationship between shear rate and shear stress is obtained from measurements of the pressure gradient and the volumetric flow rate of the fluid through the pipe. Here it is assumed that the flow is steady and laminar, the fluid properties are independent of time, the fluid velocity has no radial or tangential components, the fluid is incompressible, the fluid viscosity is not influenced by the pressure, the measurement is conducted under isothermal conditions (T = const.).

The momentum equation in polar coordinates for the laminar flow of non-Newtonian fluids in straight pipes or Hagen-Poiseuille flow (Fig. 4) is:

$$\frac{dp}{dz}=\frac{\eta}{r}\frac{d}{dr}\left[r\frac{dv_z(r)}{dr}\right] \tag{16}$$

katere dvakratno integriranje ob upoštevanju robnih pogojev:

and its integration with appropriate boundary conditions:

$$r=0\rightarrow\frac{dv_z(r)}{dr}=0$$

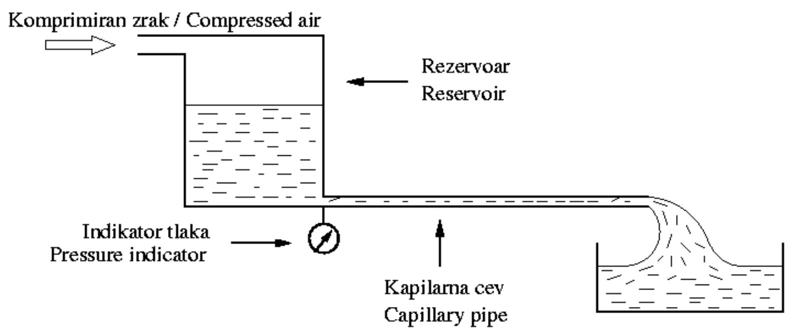
in

and

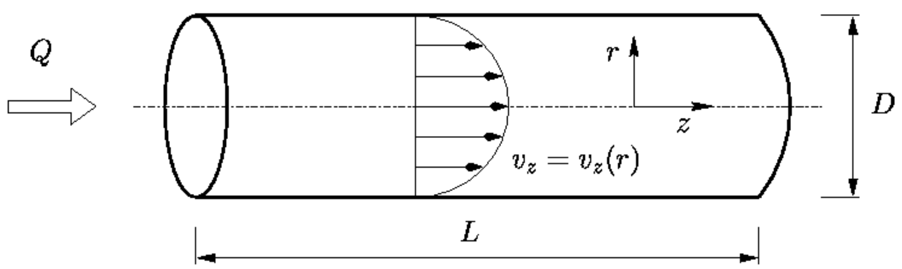
$$r=R\rightarrow v_z(r)=0$$

daje hitrostni profil:

gives the velocity profile:



Sl. 3. *Vodoravni kapilarni viskozimeter*
Fig. 3. *Horizontal capillary viscometer*



Sl. 4. *Hagen-Poiseuillev tok*
Fig. 4. *Hagen-Poiseuille flow*

$$v_z(r)=\frac{\Delta p}{4\eta L}R^2\left[1-\left(\frac{r}{R}\right)^2\right] \quad (17).$$

Največja hitrost $v_{z,max}$ se pojavi v sredini cevi ($r = 0$) in enačbo (17) zapišemo kot:

The maximum velocity occurs at the pipe center ($r = 0$) and Equation (17) can be rewritten as:

$$v_z(r)=v_{z,\max}\left[1-\left(\frac{r}{R}\right)^2\right] \quad (18).$$

Povprečno hitrost izračunamo kot [8]:

The average velocity is computed [8]:

$$\bar{v}_z=\frac{\int_S v_z(r)dS}{\int_S dS} \quad (19),$$

in dobimo:

which gives:

$$\bar{v}_z=\frac{R^2\Delta p}{8\eta L} \quad (20).$$

Z uporabo enačbe (20) zapišemo prostorninski pretok kot:

Using equation (20) the volumetric flow rate is defined as:

$$Q=\bar{v}_z S=\frac{R^2\Delta p}{8\eta L}\pi r^2=\frac{\pi R^4\Delta p}{8\eta L} \quad (21).$$

Iz ravnotežja sil na tekočinskem elementu nestisljive tekočine:

The force balance on an incompressible fluid element:

$$\tau_{rz}A_r=S\Delta p \quad (22)$$

zapišemo strižno napetost na steni:

gives wall shear stress:

$$\tau_R=\frac{R\Delta p}{2L}=\eta\frac{4\bar{v}_z}{R}=\eta\frac{8\bar{v}_z}{D}=\eta\frac{4Q}{\pi R^3} \quad (23).$$

Enačba (23) velja tudi za nenewtonske tekočine, vendar mora tok tekočine biti (ostati) homogen.

Deformacijska hitrost na steni je za tok v cevi:

Equation (23) is valid for non-Newtonian fluids as long as the fluid flow remains laminar.

The wall shear rate for pipe flow is:

$$\dot{\gamma}_{r=R}=\left(\frac{dv_z}{dr}\right)_{r=R}=\frac{4Q}{\pi R^3}=\frac{4\bar{v}_z}{R}=\frac{8\bar{v}_z}{D} \quad (24).$$

Iz definicije prostorninskega pretoka (s hitrostjo v_z) in definicije hitrosti (s hitrostnim gradientom dv_z/dr) lahko zapišemo naslednjo odvisnost [7]:

From the definition of the volumetric flow rate (in terms of the velocity v_z) and the definition of the velocity (in terms of the velocity gradient dv_z/dr) the following relation can be obtained [7]:

$$\frac{4Q}{\pi R^3}=\frac{8\bar{v}_z}{D}=\frac{4}{\tau_R^3}\int_0^{\tau_R}\tau^2\dot{\gamma}d\tau \quad (25)$$

katere dvakratno odvajanje [9] daje zvezo za deformacijsko hitrost laminarnega toka tekočine v ceveh:

and differentiating it twice [9] gives us a relation for the shear rate in laminar flow in pipes:

$$\dot{\gamma}_{r=R}=\left(\frac{dv_z}{dr}\right)_{r=R}=\frac{3}{4}\left(\frac{8\bar{v}_z}{D}\right)+\frac{\tau_R}{4}\left(\frac{d\left(8\bar{v}_z/D\right)}{d\tau_R}\right) \quad (26).$$

Enačba (26) je splošna enačba za tok vseh tekočin v ceveh; edina predpostavka je, da na steni cevi ni zdrsa. Zapišemo jo lahko tudi v naslednji obliki:

Equation (26) is a general equation for all fluids in pipe flow, the only assumption is that no slip occurs at the wall. It can be rearranged to:

$$\dot{\gamma}_R=\left(\frac{d\bar{v}_z}{dr}\right)_R=\frac{3n`+1}{4n`}\left(\frac{8\bar{v}_z}{D}\right) \quad (27),$$

kjer je:

where

$$n`=\frac{d\left(\ln\tau_R\right)}{d\left[\ln\left(8\bar{v}_z/D\right)\right]} \quad (28).$$

Za nenewtonske tekočine lahko enačbo (23) zapišemo v naslednji obliki:

For non-Newtonian fluids, equation (23) can be rewritten in the following form:

$$\tau_R = K`\left(\frac{8\bar{v}_z}{D}\right)^{n`} \quad (29),$$

kjer sta *K'* in *n'* parametra, odvisna od posamezne snovi. Za newtonske tekočine je *n'* = 1 in *n'* = η.

Izraz (8$\bar{v}$ /D) je vrednost deformacijske hitrosti na steni le za newtonske tekočine. Za laminarni tok nenewtonske tekočine, ki jo lahko opišemo s potenčnim zakonom, je deformacijska hitrost na steni podana z naslednjo enačbo:

where *K'* and *n'* are parameters dependent on the particular fluid. For Newtonian fluids *n'* = 1 and *n'* = η.

Term (8$\bar{v}$ /D) is the value for the wall shear rate for Newtonian fluids only. For laminar flow of a non-Newtonian fluid, which can be described by the Power Law, the wall shear rate is given by the following equation:

$$\dot{\gamma}_R = \frac{3n+1}{4n}\left(\frac{8v}{D}\right) \quad (30),$$

medtem ko je strižna napetost na steni:

and the wall shear stress is:

$$\tau_R = K\left(\frac{3n+1}{4n}\right)^n \left(\frac{8\bar{v}}{D}\right)^n \quad (31).$$

Podatki, ki jih dobimo iz kapilarnega (cevnega) viskozimetra, so običajno predstavljeni v diagramu log($D\Delta p/4L$)-log(8$\bar{v}$ /D) (sl. 5).

Lokalna vrednost strmine krivulje tečenja je:

Data obtained from a capillary (pipe) viscometer are usually represented in a log($D\Delta p/4L$)-log(8$\bar{v}$ /D) plot, (Fig. 5).

The local value of the slope to the flow curve is defined as:

$$n` = \frac{d\left(\log \tau_R\right)}{d\left[\log\left(8\bar{v}/D\right)\right]} \quad (32).$$

V področju deformacijskih hitrosti, kjer je *n'* = konst (krivulja toka je premica, kar označuje potenčni zakon) velja:

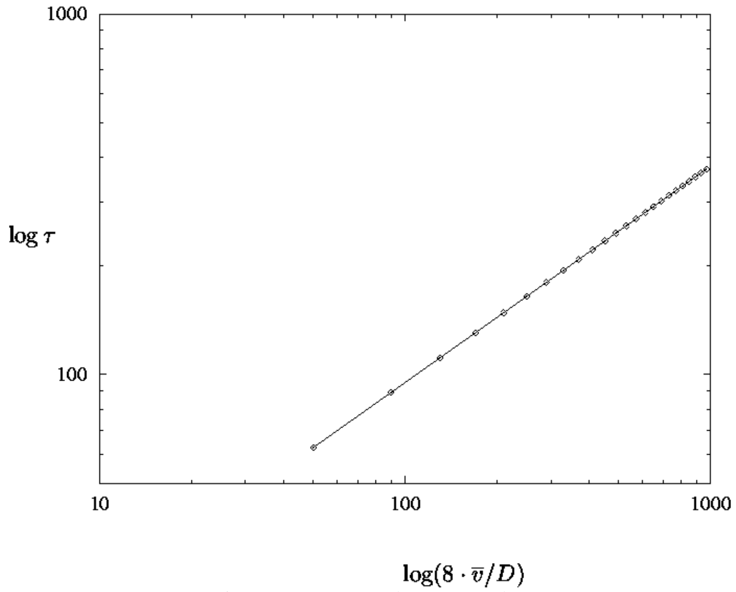
Over segments of the shear rate range where *n'* = const. (flow curve is a straight line, which signifies local Power Law behavior) stands:

$$\tau_R = K`\left(\frac{8\bar{v}}{D}\right)^{n`} \quad (33)$$

oziroma

or

$$n` = n \quad (34)$$



Sl. 5. *Logaritemski strižni diagram*
Fig. 5. *Logarithmic shear diagram*

in

and

$$K` = K\left(\frac{3n+1}{4n}\right)^n \quad (35).$$

4 PRESKUS

4 EXPERIMENT

4.1 Meritve s kapilarnim viskozimetrom

4.1 Capillary viscometer measurements

S kapilarnim viskozimetrom (sl. 6) smo izvedli preskus oziroma meritve tlaka p_m in masnega pretoka $\dot{m}$ za mešanico vode in elektrofiltrskega pepela (63% pepela, 37% vode).

Pri tem je tlačni padec:

An experiment with a capillary viscometer (Fig. 6), that is measurements of pressure p_m and mass flow rate $\dot{m}$, for water and an electrostatic filter ash mixture (63% of ash, 37% of water) was performed.

The pressure drop is:

$$\Delta p = p_m + \rho_m g h_1 - 8\zeta\left(\frac{\dot{m}^2}{\pi^2 \rho_m d^4}\right) \quad (36),$$

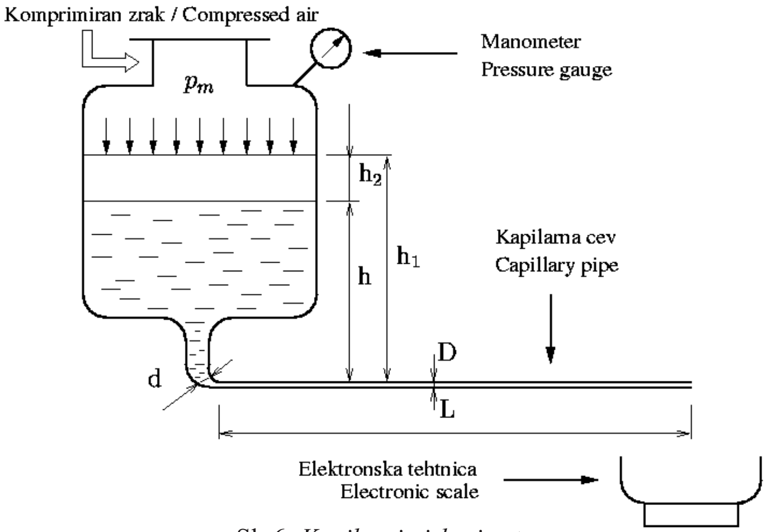
povprečna hitrost:

the average velocity:

$$\bar{v} = \frac{4\dot{m}}{\pi D^2 \rho_m} \quad (37)$$

in značilnosti mešanice ter viskozimetra:

and characteristics of the mixture and the viscometer:



Sl. 6. *Kapilarni viskozimeter*
Fig. 6. *Capillary viscometer*

Preglednica 1. *Rezultati meritev*
Table 1. *Results of measurements*

p_m [Pa]	$\dot{m}$ [kg/s]	Δp [Pa]	$\bar{v}$ [m/s]	$(8\bar{v}/D)$ [1/s]	τ_R [Pa]
482000	0,013	51493,5093	0,1165	95,0612	126,1591
672000	0,020	70490,4226	0,1792	146,2481	172,7015
113000	0,060	116247,6626	0,5375	438,7442	284,8068
135400	0,065	138639,3111	0,5823	475,3062	339,6663
164800	0,096	167972,6188	0,8599	701,9908	411,5329
197500	0,115	200619,0486	1,0301	840,9264	491,5167
227900	0,143	230922,5179	1,2810	1045,6737	565,7602
267500	0,163	270440,7395	1,4601	1191,9218	662,5798
308500	0,209	311212,0804	1,8722	1528,2924	762,4696
353100	0,249	355567,2795	2,2305	1820,7885	871,1398
396000	0,279	398255,6176	2,4992	2040,1607	975,7263

$$\rho_m = 1480\,\mathrm{kg/m^3},\ d = 0{,}021\mathrm{m},\ D = 0{,}0098\mathrm{m},\ L = 1\mathrm{m},\ \zeta = 4{,}745,\ h_1 = 0{,}227\mathrm{m}$$

Kot model za reološki opis nenewtonskega obnašanja omenjene zmesi smo izbrali potenčni zakon (31). Tako smo iz izmerjenih vrednosti Δp in $\dot{m}$ izračunali strižno napetost:

As a model for the description of the rheological behavior of this mixture the Power Law (31) was chosen. On the basis of measured values of Δp and $\dot{m}$ the wall shear stress:

$$\tau_R = \frac{\Delta p D}{4L} \qquad (38)$$

in psevdo (navidezno) deformacijsko hitrost ($8\bar{v}$ /D) na steni cevi (pregl. 1).

and pseudo (apparent) wall shear rate ($8\bar{v}$ /D) were determined (Table 1).

4.2 Določanje modelnih parametrov

V nadaljevanju so določene konstante potenčnega zakona na temelju izvedenih meritev z metodo najmanjših kvadratov. Osnovno enačbo:

4.2 Model parameters determination

On the basis of experiments performed, the constants for the Power Law model were derived with a least-squares method. Log is applied to the equation:

$$\tau_R = K\left(\frac{3n+1}{4n}\right)^n \left(\frac{8\bar{v}}{D}\right)^n \qquad (39)$$

logaritmiramo in dobimo naslednjo obliko:

to arrive at:

$$\ln \tau_R = \ln\left[K\left(\frac{3n+1}{4n}\right)^n\right] + n\ln\left(\frac{8\bar{v}}{D}\right) \qquad (40).$$

Če vpeljemo:

With introduction of

$$Y = \ln \tau_R, \qquad (41),$$

$$X = \ln\left(\frac{8\bar{v}}{D}\right) \qquad (42)$$

in

and

$$A = \ln\left[K\left(\frac{3n+1}{4n}\right)^n\right] \qquad (43),$$

lahko enačbo (40) zapišemo v obliki linearne funkcije:

Equation (40) can be rewritten as a linear function:

$$Y = nX + A \qquad (44).$$

Z uporabo metode najmanjših kvadratov izračunamo vrednosti parametra K in n za podane s preskusi določene vrednosti τ_R in ($8\bar{v}$ /D) kot:

The values of parameter K and n for experimentally determined values of τ_R and ($8\bar{v}$ /D) are computed with the least-squares method as:

$$n = \frac{m\sum\left[\ln(8\bar{v}/D)_i \ln(\tau_R)_i\right] - \sum\ln(8\bar{v}/D)_i \sum\ln(\tau_R)_i}{m\sum\left[\ln(8\bar{v}/D)_i\right]^2 - \left[\sum\ln(8\bar{v}/D)_i\right]^2} \qquad (45)$$

in

and

$$K = \frac{\exp A}{\left(\frac{3n+1}{4n}\right)^n} \qquad (46),$$

za

for

$$A = \left[\frac{\sum\ln(\tau_R)_i \sum\left[\ln(8\bar{v}/D)_i\right]^2 - \sum\ln(8\bar{v}/D)_i \sum\left[\ln(8\bar{v}/D)_i \ln(\tau_R)_i\right]}{m\sum\left[\ln(8\bar{v}/D)_i\right]^2 - \left[\sum\ln(8\bar{v}/D)_i\right]^2}\right] \qquad (47),$$

Pri tem je m število eksperimentalnih točk in i = 1, 2,...,m.
Tako dobimo:

where m is the number of experimental points and i = 1, 2,...,m.
Hence:

$$n=\frac{11\cdot 441,2291-71,5716\cdot 66,8101}{11\cdot 475,6109\cdot 71,5716^{2}}=0,657$$

$$A=\frac{66,8101\cdot 475,6109-71,5716\cdot 441,2291}{11\cdot 475,6109-71,5716^{2}}=1,7957$$

$$K=\frac{\exp 1,7957}{\left(\frac{3\cdot 0,657+1}{4\cdot 0,657}\right)^{0,657}}=5,559$$

in zapišemo potenčni zakon (39) z vrednostmi parametrov K in n:

and the Power Law (39) for determined values of K and n is:

$$\tau_R=5,559\left(\frac{3\,0,657+1}{4\,0,657}\right)^{0,657}\left(\frac{8\bar{v}}{D}\right)^{0,657} \quad (48).$$

Preglednica 2 prikazuje vrednosti posameznih koeficientov za izračun parametrov potenčnega zakona z metodo najmanjših kvadratov.

Table 2 shows values of the individual coefficients used to determine the parameters for the Power Law with the least-squares method.

4.3 Numerično simuliranje - CFX 4.3

S tako določenimi parametri potenčnega zakona (48) smo s programskim paketom CFX 4.3 simulirali tok zmesi vode in elektrofiltrskega pepela skozi cev kapilarnega viskozimetra. Pri tem smo cev diskretizirali s petblokovno strukturo, da smo zagotovili ortogonalnost elementov sl. 7. Slika 8 prikazuje blokovno strukturo celotnega računskega območja cevi.

Kot robne pogoje smo predpisali tlak na vstopu in izstopu oziroma ustrezni tlačni padec v cevi.

4.3 Numerical simulation - CFX 4.3

With the derived parameters for the Power Law (48), a capillary flow simulation for the electrostatic filter ash and water mixture was performed with the CFX 4.3 numerical package. The pipe was described with a five-block structure to ensure the orthogonallity of the elements, see Figure 7. Figure 8 shows the block structure of the whole computational domain of a pipe.

As boundary conditions, the pressure at the inlet and outlet or the corresponding pressure drop was prescribed.

4.4 Rezultati

Slika 9 prikazuje rezultate numeričnega simuliranja toka skozi cev za predpisan tlačni padec Δp = 51494 Pa, kot dobljen iz preskusa za $\dot{m}$ = 0,013 kg/s.

Enako so bile izvedene primerjave za vse točke iz preglednice 1, rezultate prikazuje slika 10.

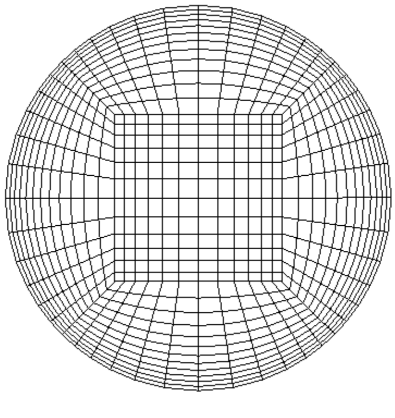
4.4 Results

Figure 9 shows the results of a numerical simulation for a pipe flow for a prescribed pressure drop Δp = 51494 Pa as obtained from the experiment for $\dot{m}$ = 0.013 kg/s.

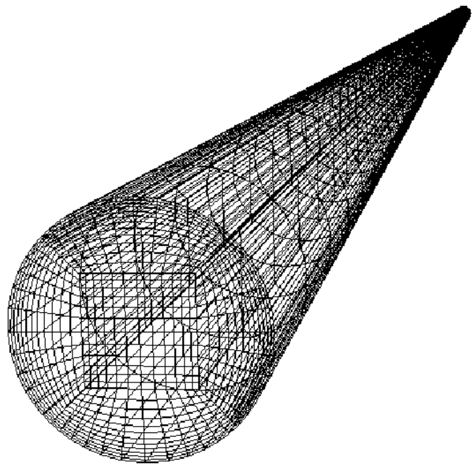
In the same way the comparison was performed for all the results from Table 1, the results are shown in Figure 10.

Preglednica 2. *Metoda najmanjših kvadratov*
Table 2. *Least-squares method*

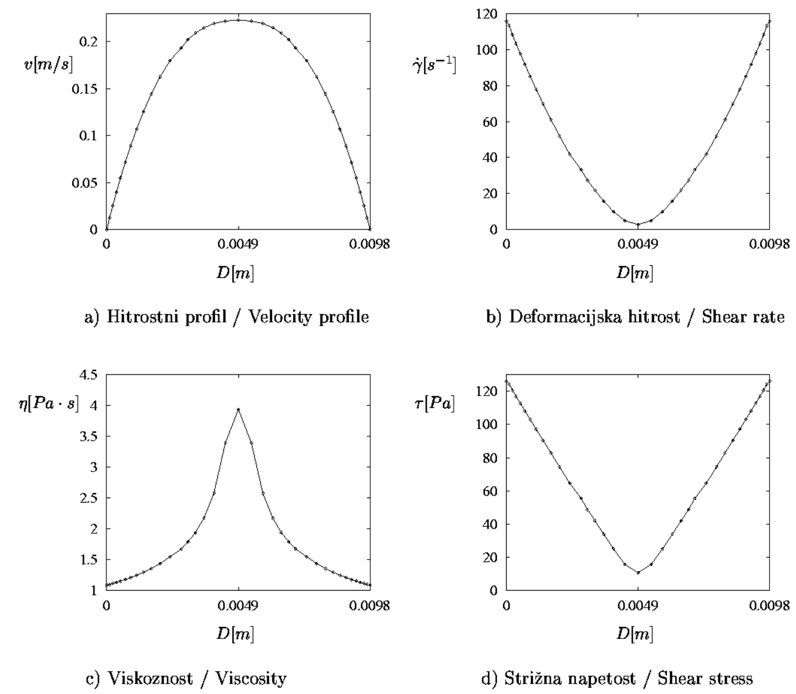
i	$(\tau_R)_i$	$(8\bar{v}/D)_i$	$\ln(\tau_R)_i$	$\ln(8\bar{v}/D)_i$	$\ln(\tau_R)_i \ln(8\bar{v}/D)_i$	$[\ln(8\bar{v}/D)_i]^2$
1	126,1591	95,0612	4,8375	4,5545	22,0327	20,7437
2	172,7015	146,2481	5,1516	4,9853	25,6821	24,8533
3	284,8068	438,7442	5,6518	6,0839	34,3795	37,0019
4	339,6663	475,3062	5,8280	6,1640	35,9233	37,9944
5	411,5329	701,9908	6,0199	6,5539	39,4539	42,9539
6	491,5167	840,9264	6,1975	6,7345	41,7371	45,3535
7	565,7602	1045,6737	6,3382	6,9524	44,0656	48,3361
8	662,5798	1191,9218	6,4961	7,0833	46,0143	50,1735
9	762,4696	1528,2924	6,6366	7,3319	48,6587	53,7569
10	871,1398	1820,7885	6,7698	7,5070	50,8211	56,3554
11	975,7263	2040,1606	6,8832	7,6208	52,4552	58,0763
$\sum$			66,8101	71,5716	441,2291	475,6109



Sl. 7. *5-blokovna struktura*
Fig. 7. *5-block structure*



Sl. 8. *Blokovna struktura računskega območja*
Fig. 8. *Block structure of a computational domain*



Sl. 9. *Rezultati numeričnega simuliranja*
Fig. 9. *Numerical simulation results*

Na natančnost numeričnega simuliranja lahko sklepamo s slike 11, ki prikazuje primerjavo numerično izračunanega hitrostnega profila in teoretičnega hitrostnega profila, izračunanega po naslednji enačbi:

The accuracy of the numerical simulation can be concluded from Figure 11, which shows a comparison of the numerically obtained velocity profile and the theoretical velocity profile, computed from the following equation:

$$v(r)=\left[\frac{\Delta p}{2LK}\right]^{1/n}\left[\frac{n}{n+1}\right]\left[R^{(n+1)/n}-r^{(n+1)/n}\right] \quad (49),$$

za eksperimentalno vrednost τ_R = 126,1591 Pa.

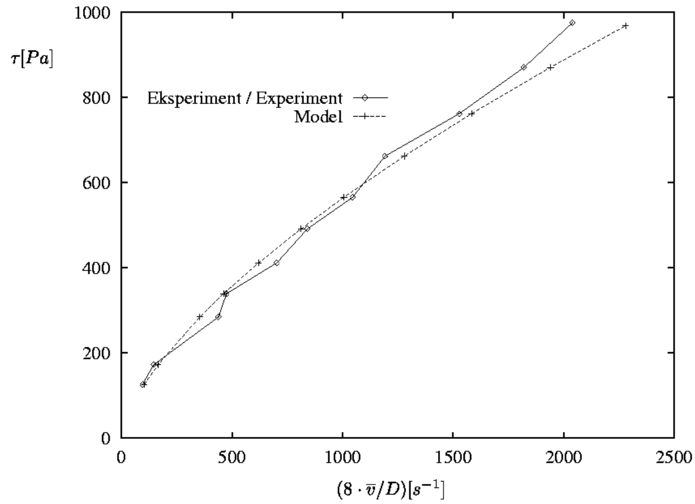
Preglednica 3 prikazuje primerjavo rezultatov numeričnega simuliranja in preskusa ter posamezna odstopanja rezultatov.

Iz prikazanih rezultatov je razvidno zelo dobro ujemanje eksperimentalno in numerično dobljenih rezultatov, predvsem v primeru strižnih napetosti. Vzrok

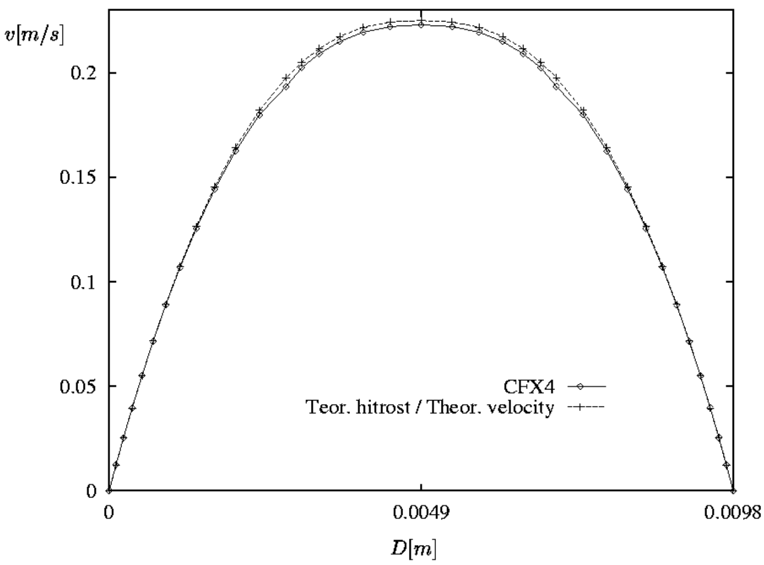
for the experimental value τ_R = 126.1591 Pa.

Table 3 shows the results of a numerical simulation and the results obtained from the experiment as well as the particular deviations of the results.

Good agreement between the experimentally and numerically obtained results is shown, in particular for the shear stresses. The reason for some



Sl. 10. *Primerjava modela in preskusa*
Fig. 10. *Comparison of model and experiment*



Sl. 11. *Primerjava hitrostnih profilov*
Fig. 11. *Comparison of velocity profiles*

Preglednica 3. *Primerjava rezultatov*
Table 3. *Comparison of results*

τ_R [Pa]	τ_R − CFX [Pa]	$\Delta\tau_R$ [%]	$(8\bar{v}/D)$ [1/s]	$(8\bar{v}/D)$ − CFX [1/s]	$\Delta(8\bar{v}/D)$ [%]
126,159	126,192	0,026	95,061	102,495	7,253
172,702	172,743	0,024	146,248	165,298	11,525
284,807	284,883	0,027	438,807	353,965	-23,969
339,666	339,752	0,025	475,306	462,798	-2,703
411,533	411,629	0,023	701,991	619,792	-13,262
491,517	491,641	0,025	840,926	812,199	-3,537
565,760	565,894	0,024	1045,674	1006,101	-3,933
662,580	662,742	0,025	1191,922	1279,587	6,851
762,470	762,654	0,024	1528,292	1584,490	3,547
871,140	871,362	0,026	1820,789	1940,770	6,182
975,726	968,879	-0,707	2040,161	2280,849	10,553

za nekoliko večja odstopanja deformacijskih hitrosti je najverjetneje v samem preskusu oziroma meritvah masnega pretoka kar potrjuje tudi primerjava (ujemanje) med numerično in teoretično dobljenim hitrostnim profilom. Verjetno tudi sam potenčni zakon, kot dvoparametrični reološki model, ne popisuje dovolj dobro odvisnost strižne napetosti od deformacijske hitrosti za zapletenejše sisteme, kar pastasti materiali vsekakor so. Tako bo v prihodnje primerneje uporabiti troparametrični oziroma Siskov model.

larger deviations in the shear rates is most likely in the experiment or in the mass flow rate measurements, which is also confirmed by the comparison with the numerically and theoretically obtained velocity profiles. Also, the Power Law, as a two-parameter rheological model, lacks a description of the dependency between the shear stress and the shear rate for more complex systems, which paste fluids most certainly are. So, in the future it will be more appropriate to use Sisko's three parametric model.

5 SKLEP

Prispevek obravnava modeliranje pastastih tekočin z uporabo potenčnega zakona. Na podlagi eksperimentalnih podatkov (meritve s kapilarnim viskozimetrom) sta bila izpeljana parametra potenčnega zakona (K in n) za opis reološkega obnašanja zmesi vode in elektrofiltrskega pepela. Natančnost oz. primernost izpeljanega modela smo testirali z numeričnim simuliranjem zmesi v kapilarni cevi viskozimetra. Rezultati simuliranja (predvsem strižna napetost na steni cevi) se zelo dobro ujemajo z eksperimentalnimi rezultati, v primeru deformacijskih hitrosti na steni pa prihaja do odstopanja (ki se spreminja ne le po velikosti, temveč tudi po predznaku). Vzrok je lahko v ne dovolj natančnih meritvah masnega pretoka, na temelju katerega se izračuna povprečna hitrost oziroma deformacijska hitrost, ali pa v neprimernosti potenčnega zakona za reološki opis pastaste zmesi.

5 CONCLUSION

This contribution deals with the modelling of slurries using the Power Law. With experimentally obtained data (capillary viscometer measurements) the parameters for the Power Law (K and n) were derived in order to describe the rheological behavior of the electrostatic filter ash and the water mixture. The accuracy, as well as adequacy, of the derived model was tested with a numerical simulation of a mixture flow in a pipe. The simulation results (first, of the wall shear stress) show good agreement with the experimental results but in the case of wall shear rates there are some deviations, with changes not only in value but also in sign. The reason may be in insufficiently accurate measurements of the mass flow rate, which helps us to calculate the average velocity or shear rate as well as in the unsuitability of the Power Law for a rheological description of the paste mixture.

6 OZNAČBE
6 SYMBOLS

plašč tekočinskega elementa	A_r	m^2	area upon which shear stress acts
premer cevi	D	m	pipe diameter
konsistenčni količnik tekočine	K	Pa s	consistency factor of Power Law fluid
dolžina cevi	L	m	pipe length
prostorniski pretok	Q	m^3/s	volume rate of flow
polmer cevi	R	m	radius of pipe
prerez cevi	S	m^2	pipe cross sectional area
specifična toplota	c_p	J/kgK	specific heat
notranji premer izstopnega priključka	d	m	internal diameter of connection
višina zmesi v zalogovniku	h_1	m	height of the mixture in reservoir
količnik prevoda toplote	k	W/mK	thermal conductivity
masni pretok	$\dot{m}$	kg/s	mass flow rate
indeks toka	n	-	Power Law index
tlak	p	Pa	pressure
tlačni padec	Δp	Pa	pressure drop
polarne koordinate	r, z	-	cylindrical coordinates
kartezične koordinate	x, y	-	cartesian coordinates
hitrost	v	m/s	velocity
povprečna hitrost	$\bar{v}$	m/s	average velocity
deformacijska hitrost	$\dot{\gamma}$	s^{-1}	shear rate
deformacijska hitrost na steni cevi	$\dot{\gamma}_R$	s^{-1}	wall shear rate
koeficient izgub (zožitve...)	ζ	-	coefficient of losses (contractions…)
dinamična viskoznost	η	Pa s	dynamic viscosity

navidezna (efektivna) dinamična viskoznost	η_{eff}	Pa s	apparent (effective) dynamic viscosity
spodnja mejna dinamična viskoznost	η_0	Pa s	low-shear limiting dynamic viscosity
zgornja mejna dinamična viskoznost	η_∞	Pa s	high-shear limiting dynamic viscosity
masni delež	ξ	%	weight percent solids
gostota	ρ	kg/m^3	density
strižna napetost	τ	Pa	shear stress
strižna napetost tečenja	τ_0	Pa	yield stress
strižna napetost na steni cevi	τ_R	Pa	wall shear stress
prostorninski delež trdnih delcev	ψ	%	volume concentration of solids

Indeksi		**Subscripts**
tekočina	l	liquid
zmes	m	suspension
trdni delci	s	solids

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Naslova avtorjev: Primož Ternik
prof.dr. Jure Marn
Univerza v Mariboru
Fakulteta za strojništvo
Smetanova 17
2000 Maribor

Damijan Kanduti
dr. Evgen Dervarič
Premogovnik Velenje
Partizanska 78
3320 Velenje

Authors Addresses: Primož Ternik
Prof.Dr. Jure Marn
University of Maribor
Faculty of Mechanical Eng.
Smetanova 17
2000 Maribor, Slovenia

Damijan Kanduti
Dr. Evgen Dervarič
Coal Mines Velenje
Partizanska 78
3320 Velenje, Slovenia

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