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Izboljšan algoritem za simulacijo turbine avtomobilskega turbopolnilnika

A Novel Algorithm for the Simulation of an Automotive Turbocharger Turbine

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Predstavljen je inovativni algoritem za simulacijo delovanja turbine turbopolnilnika z metodo karakteristik, ki omogoča simulacijo spremenljivega izstopnega oz. vstopnega tlaka na robnem elementu in spremenljivih fizikalnih lastnosti plina. Teorija metode karakteristik sloni na predpostavki idealnega plina z nespremenljivimi fizikalnimi lastnostmi – lastnosti plina se ne spreminjajo s temperaturo in koncentracijo. Zato so, za doseganje natančnejših in resničnejših rezultatov, izpeljane izpopolnjene enačbe za popis robnega elementa, ki upoštevajo spremenljive fizikalne lastnosti plina in omogočajo opazovanje sestave plinov. Izpopolnjene enačbe in inovativen algoritem upravljanja z njimi zagotavljajo mnogo boljšo ohranitev mase pri toku plina skozi robni element ob le nekoliko večji časovni zahtevnosti.

(Ključne besede: polnilniki turbinski, pogoji robni, metode karakteristik, lastnosti plina, lastnosti fizikalne)

A new, innovative algorithm for the simulation of a turbocharger turbine based on the method of characteristics is introduced. This algorithm makes it possible to consider the variable inlet and outlet pressures as well as the variable gas properties. The theory of the method of characteristics is derived for ideal and perfect gases – the physical properties of the gas do not change with a variation in temperature and composition. New equations for the simulation of the boundary element suitable for considering the variation in gas properties and the tracking of the gas concentration are therefore derived. The improved equations and the improved algorithm ensure a much better conservation of mass flux for only a slightly increased computational time.

(Keywords: turbocharger turbine, boundary conditions, method of characteristics, gas properties, variable physical properties)

0 UVOD

Računalniške simulacije so postale neizogibni člen v razvoju motorjev z notranjim zgorevanjem. Z uporabo simulacij se bistveno skrajšajo čas in stroški razvoja, saj omogočajo razmeroma natančno vnaprejšnjo določitev karakteristik motorja in s tem definicijo osnovne motorske konfiguracije že pred izdelavo prototipov. Simulacijski model virtualnega motorja bi naj omogočal razmeroma natančno simulacijo delovanja motorja v ustaljenem in prehodnem režimu. Zapletenost matematičnega opisa postopkov v motorju in z njo povezana časovna zahtevnost ustvarja pogoje za izbiro simulacijske metode. Zmogljivosti sedanjih računalnikov ne omogočajo uporabe trirazsežnih metod za simulacijo prehodnega režima delovanja motorja [1], zato

0 INTRODUCTION

Computer simulations have became an indispensable stage in the development process of internal combustion engines. Thus, it is possible to determine engine characteristics in advance, before a prototype is build. This means that the application of computer simulations shortens the engine development time and reduces costs. A universal model of the virtual engine should be capable of simulating processes in a real engine based on physical principles. The complexity of the mathematical model and the computational times determine the choice of the simulation method. At present, multidimensional models have several limitations when modelling of the whole engine system is considered [1]; they are unable to model the transient operation of turbocharged engines. Therefore, a delovanje celotnega motorja in delovanje motorja v prehodnem režimu najpogosteje simuliramo s kombinacijo ničrazsežnih, enorazsežnih in navideznorazsežnih metod, ki zagotavljajo dober kompromis med natančnostjo rezultatov in časovno zahtevnostjo ([1] in [2]).

Tok plina v polnilnih in predvsem v izpušnih zbiralnikih motorjev z notranjim zgorevanjem je izrazito neustaljen, kar pomeni, da je za popis pojavov treba uporabiti modele, ki so zmožni opisati tokovno dinamiko. Običajno se za simulacijo postopkov v polnilnih in izpušnih zbiralnikih uporabljajo enorazsežne Eulerjeve enačbe, s katerimi z zadostno natančnostjo popišemo dogajanje v zbiralnikih ([1] in [3]). Enorazsežne enačbe najpogosteje rešujemo z numeričnimi shemami, ki temeljijo na diskretni integralski obliki ohranitvenih enačb ([1] in [2]). Diferencialne enačbe so torej zapisane v ohranitveni obliki in so primerne tudi za popis tlačnih valov z veliko amplitudo. Zavedati pa se je treba, da je z metodami, ki temeljijo na diskretni integralski obliki ohranitvenih enačb, mogoče simulirati le preproste robne elemente, kot so: odprti in zaprti konec ter ventil ([1], [4] in [5]). Uporaba metode karakteristik je torej neizogibna za simulacijo robnih elementov, kot so: turbina, kompresor, cevni spoj, hladilnik polnilnega zraka itn. [1]. Metoda karakteristik je široko uporabljena za simulacije polnilnih in izpušnih zbiralnikov, a ima dve znani pomanjkljivosti [1]: metoda ni ohranitvena in ne popiše natančno tlačnih valov z veliko amplitudo.

Nadaljnja pomanjkljivost metode karakteristik je tudi dejstvo, da sloni na predpostavkah o idealnem plinu z nespremenljivimi fizikalnimi lastnostmi (= popolni plin); lastnosti plina se ne spreminjajo s temperaturo in koncentracijo. Prva predpostavka ne pomeni hude omejitve za natančnost metode, saj enačba idealnega plina z zadostno natančnostjo popiše obnašanje plina v opazovanem območju temperatur in tlakov, ki se pojavljajo v polnilnem in izpušnem zbiralniku ([1], [6] in [7]). Mnogo večja ovira pa je predpostavka o popolnem plinu, ki postavlja nespremenljive vrednosti c_p, c_v, κ in R, saj se temperatura in sestava delovnega sredstva bistveno spremenita pri prehodu skozi dele motorja, kakor so kompresor, valj, turbine itn. Za doseganje natančnih rezultatov je torej treba upoštevati spremembe fizikalnih lastnosti plinov, to so c_p, c_v, κ in R. Različne fizikalne lastnosti plinov v polnilnem in izpušnem zbiralniku najpreprosteje upoštevamo z vpeljavo različnih, a nespremenljivih fizikalnih lastnosti plina

combination of zero-, one- and quasi-dimensional is commonly applied when highly accurate simulation data are required, since it ensures a good compromise between the accuracy of the results and the computational requirements ([1] and [2]).

The design of the engine's intake and exhaust manifolds is dependent upon being able to calculate the unsteady flows of the compressible gases flowing through the engine. It is common to use one-dimensional equations to simulate flows in engine manifolds ([1] and [3]). One dimensional equations are commonly solved with schemes based on the discrete integral form of conservation equations. The schemes are, therefore, capable of dealing with shock waves and guarantee the preservation of the integral properties of the governing equations. However, it should be noted that only simple boundaries, e.g., the open and closed ends of the pipe, and outflow through the valve ([1], [4] and [5]). However, if there is a flow through the boundary and interaction of the wave with the boundary is complex, e.g., turbine, compressor, pipe junctions, and intercooler, then such a formulation is not readily possible, and the method of characteristics is still the most appropriate way to incorporate the boundaries into wave-action simulations [1]. However, the method of characteristics has two major defects [1]: it is not conservative, and it cannot cope with large pressure waves.

A further restriction met by the application of the method of characteristics is the fact that it is ideally suited to ideal and perfect gases, i.e., the gas properties do not change with variations in the temperature and concentration. It is clear that for engine intake and exhaust flows there is no need to consider the effects of intermolecular forces [1]. Therefore, the ideal-gas equation adequately represents gas behaviour in the observed range of temperatures and pressures ([1], [6] and [7]). However, the perfect-gas assumption, which implies constant values of c_p, c_v, κ and R, introduces a serious limitation to the use of the method of characteristics, since the temperature and concentration changes are substantial when the working medium passes through the engine components, e.g., compressor, cylinder, and turbine. It is therefore necessary to account for changes in the gas properties: c_p, c_v, κ and R. Hence, the simplest approach to allow for the changes in temperature and composition that occurred in the gas as it passes through the engine is to assume different but constant gas properties in the inlet and exhaust manifolds. Obviously, this simple approximation creates some problems, for example, simulation of the transient v polnilnem in izpušnem zbiralniku. Omenjeni postopek pa naleti na omejitve pri simulaciji širokega razpona obremenitev in vrtilnih frekvenc motorja, saj se pri tem močno spreminja temperatura izpušnih plinov, pri simulaciji vračanja izpušnih plinov, pri simulaciji povratnega toka ostankov zgorevanja v polnilne kanale in pri simulaciji turbine in kompresorja, saj se pri prehodu skozi robni element spremeni temperatura. Drugi, zahtevnejši, a natančnejši postopek simulacije robnih pogojev, ki bo uporabljen v tem prispevku, sloni na časovno in prostorsko spremenljivih lastnostih plina ob upoštevanju adiabatne odvisnosti, ki je pogoj za pregledno izpeljavo enačb metode karakteristik. Primernost slednje poenostavitve je razložena v nadaljevanju, obširnejša analiza pa je predstavljena v [8].

1 VODILNE ENAČBE

Procese v zbiralnikih motorjev običajno modeliramo z enorazsežnimi neviskoznostnimi enačbami ([1] in [2]), v katerih pa upoštevamo trenje s stenami, ki omogoča upoštevanje površinskih sil, ki delujejo na nadzorno prostornino. Neviskoznostna predpostavka zahteva, da je plin dovolj redek, da je dovoljeno zanemariti medmolekulske vplive. Bulaty in Niessner [3] sta pokazala, da so v ceveh zbiralnikov motorjev z notranjim zgorevanjem členi, ki so nastali zaradi viskoznih sil in vzdolžne toplotne prevodnosti, nekaj redov velikosti manjši od členov, ki jih prinaša trenje s stenami in prestop toplote iz plina na stene zbiralnika. Enačbe, izpeljane v nadaljevanju, veljajo tudi za cevi z vzdolžno spremenljivim prerezom. Pri tem se je treba zavedati omejitve, da so enačbe veljavne le za zadovoljivo zmerne spremembe prereza, pri katerih ne prihaja do ločitve toka, ki prav tako omejuje uporabo enorazsežnih enačb v ceveh z majhnim krivinskim polmerom.

Vodilne enačbe enorazsežnega neviskoznega stisljivega toka plina v ceveh s spremenljivim prerezom in upoštevanjem trenja s stenami ter prestopa toplote na stene, razširjene z ohranitvenimi zakoni koncentracij posameznih komponent, zapišemo v obliki ([1] in [2]):

operating regime of a diesel engine, where the temperature and composition of the exhaust gases changes substantially throughout the transient, the simulation of the exhaust-gas recirculation, the simulation of the backflow of the combustion products into the intake ports, and the simulation of the turbocharger turbine and compressor, where there is an abrupt change in temperature when it passes through the boundary element. A more accurate and also more complex approach is based on the spatially and temporally variable gas properties. This approach will be followed here. It relies on the validity of the isentropic relation, which is indispensable in the transparent derivation of the equations of the method of characteristics. This simplification is briefly analysed in the next section, and an extensive analysis can also be found in [8].

1 GOVERNING EQUATIONS

It is common to use one-dimensional conservation laws when modelling the engine manifold flows ([1] and [2]). Hyperbolic differential equations are essentially inviscid, although the pipe-wall friction factor, which enables the representation of the surface force on the control volume, is included. The validity of this simplification was confirmed by Bulaty in Niessner [15]. They have evaluated the magnitude of terms representing the internal viscosity of the fluid in the momentum and energy equations, and also included a term representing diffusion in the energy equation. From [15] it is apparent that the term representing the internal stress caused by the fluid viscosity is negligible in comparison with that which models the effects due to the pipe-wall friction. Furthermore, the term which models the convective heat transfer in the radial direction is many orders of magnitude greater than the terms for viscosity and longitudinal heat conduction [15]. The flow is said to be quasi-one-dimensional, if there is a gradual cross-sectional-area variation the fluid properties are approximately uniform across any cross-section and can be taken as functions of x and t only, i.e., there is no flow separation.

The governing equations of quasi-one-dimensional inviscid compressible flow including wall friction and heat transfer, combined with the species continuity equations for non-reacting chemical species can be written in the form ([1] and [2]):

$$\frac{\partial \mathbf{W}}{\partial t}+\frac{\partial \mathbf{F}(\mathbf{W})}{\partial x}+\mathbf{C}(\mathbf{W})=0 \qquad (1),$$

kjer so

where

$$\mathbf{W}=\begin{bmatrix}\rho F\\ \rho uF\\ \rho e_0 F\\ \rho F\mathbf{Y}\end{bmatrix},\ \mathbf{F}(\mathbf{W})=\begin{bmatrix}\rho uF\\ \rho\left(u^2+p\right)F\\ \rho uh_0 F\\ \rho uF\mathbf{Y}\end{bmatrix},\ \mathbf{C}(\mathbf{W})=\begin{bmatrix}0\\ -p\dfrac{dF}{dx}\\ 0\\ 0\end{bmatrix}+\begin{bmatrix}0\\ \rho GF\\ -\rho qF\\ 0\end{bmatrix},\ \mathbf{Y}=\begin{bmatrix}Y_1\\ \vdots\\ Y_{n-1}\end{bmatrix} \tag{2}$$

in and

$$e_0=e+\frac{1}{2}u^2,\ h_0=e_0+\frac{p}{\rho},\ G=\frac{1}{2}u|u|f\frac{4}{D},\ q=\frac{2fu}{D}c_p\left(T_w-T_g\right) \tag{3}.$$

Enačbi (1) in (2) sta izpeljani s predpostavko, da je difuzija zanemarljiva, kar pomeni, da se koncentracije posameznih komponent prenašajo s konvekcijo. Predpostavka je upravičena za tokove z velikim Reynoldsovim številom, kar je značilno za tokove v zbiralnikih motorjev z notranjim zgorevanjem. Opis algoritma izračuna temperature in tlaka iz vektorja stanja ***W*** ter konstrukcije vektorjev ***F*** in ***C*** ni predmet tega prispevka in ga najdemo v [2].

Eq. (1) and (2) were derived under the assumption that the effects of diffusion are negligible and that species concentration is advected by the flow. This simplification is realistic for the flows with high Reynolds numbers, as is usually the case in the manifolds of internal combustion engines. The algorithm for deriving the temperature and pressure from ***W*** and construction of vectors ***F*** and ***C*** is beyond the scope of this paper and can be found in [2].

2 IZPELJAVA ENAČB METODE KARAKTERISTIK

Izpeljava enačb metode karakteristik je podrobno predstavljena v [1] in [4] in je na tem mestu zaradi prostorske omejitve ne bomo ponavljali. Osnovne enačbe metode karakteristik izpeljemo iz prvih treh komponent vektorjev ***W***, ***F*** in ***C*** (en. (1) in (2)) zapisanih v neohranitveni obliki ([1], [2] in [4]). V nadaljevanju bodo izpeljane enačbe metode karakteristik za simulacijo delovanja turbine turbopolnilnika, ki omogočajo upoštevanje spremenljivih fizikalnih lastnosti plinov.

Izpeljava enačb metode karakteristik sloni na specifični toploti pri stalni prostornini in nespremenljivem tlaku, podanih z enačbama:

2 METHOD OF CHARACTERISTICS

The derivation of the equations of the method of characteristics is presented in detail in [1] and [4] and will be only briefly revised in this paper. The equations of the method of characteristics are derived from the first three components of ***W***, ***F*** and ***C*** (Eq. (1) and (2)) written in non-conservation-law form ([1], [2] and [4]). The boundary conditions of the turbocharger turbine enabling the simulation of variable gas properties and the variable inlet and outlet pressures are derived in this section.

The derivation of the equations of the method of characteristics is based on the following relations for specific heats:

$$c_v=\frac{R}{\kappa-1}\quad \text{in/and}\quad c_p=\frac{\kappa R}{\kappa-1} \tag{4}.$$

Z uporabo omenjenega pristopa na plinih s spremenljivimi fizikalnimi lastnostmi zagrešimo dve napaki. Prva napaka je posledica dejstva, da notranjo energijo in hitrost zvoka računamo z enako trenutno vrednostjo κ, s čimer izračunamo pravilno vrednost hitrosti zvoka, a napačno vrednost notranje energije, ki bi jo bilo treba izračunati s povprečno vrednostjo κ ([1] in [2]). Druga napaka je posledica dejstva, da za izračun adiabatnega razpenjanja iz tlaka p na primerjalni tlak p_{ref}, $a_A=a\left(p_{ref}/p\right)^{(\kappa-1)/2\kappa}$ - uporabimo κ, izračunan pri trenutni temperaturi plina, njegova vrednost pa ni nujno ustrezna v celotnem odseku razpenjanja. Kakor je bilo že povedano, se omenjenim nedoslednostim v splošnem ni mogoče

The application of this approach postulates that the same instantaneous value of the ratio of specific heats was used to characterize the energy level and the wave propagation speed. Therefore, the calculated value of the speed of sound is correct, since it is defined by the instantaneous value of the ratio of specific heats, whereas the calculated value of the internal energy is not correct, since it should be calculated with the averaged value of κ ([1] and [2]). Another shortcoming of the above approach is that the adiabatic expansion from the pressure p to pressure p_{ref}, i.e., $a_A=a\left(p_{ref}/p\right)^{(\kappa-1)/2\kappa}$, is calculated with κ evaluated at the instantaneous gas temperature, and its value is not necessarily appropriate for the whole range of the izogniti, saj enačbe metode karakteristik in pripadajočih robnih pogojev niso izpeljane za splošne lastnosti plinov. V nadaljevanju bo pokazano, kako z uporabo nekoliko spremenjenih enačb metode karakteristik in robnih pogojev napake, vpeljane z omenjeno nedoslednostjo, bistveno zmanjšamo. Vpeljani napaki je treba oceniti tudi z vidika namena simulacije, ki naj zagotavlja primeren kompromis med natančnostjo rezultatov in računskimi časi ter napakami vstopnih podatkov, kakor so karakteristike turbine, kompresorja itn. in napako, vpeljano s predpostavko navidezne ustaljenosti.

Za simulacijo delovanja turbine je običajno, da je hitrost U pozitivna za tok v smeri od vstopa do izstopa iz turbine. Iz [1] in [4] povzamemo osnovne enačbe za simulacijo delovanja turbine z metodo karakteristik:

expansion interval. It is generally not possible to circumvent the before-mentioned deficiencies, as was discussed previously. In spite of these deficiencies it will be shown in the paper that it is possible to diminish these shortcomings with the application of the modified equations. It is also very important to estimate the introduced errors of the method regarding the purpose of the method, i.e., it should ensure a good compromise between the accuracy of the results and the computational requirements, on the one hand, and the errors of the input data, e.g., the turbine and compressor characteristics, and the quasi-stationary assumption, on the other.

It is convenient to define U as positive for the flow from the turbine inlet to the outlet. The basic equations for the turbine simulation are taken from [1] and [4]:

$$\lambda^*_{in,n} = A^*_n\left(1+\frac{\kappa_n-1}{2}M_n\right),\ \lambda^*_{out,n} = A^*_n\left(1-\frac{\kappa_n-1}{2}M_n\right),\ n=1,2 \qquad (5),$$

kjer je / where

$$M=\frac{u}{a},\ W^*=\frac{W}{A_A},\ Z=\frac{z}{a_{ref}},\ a_A = a\left(\frac{p_{ref}}{p}\right)^{\frac{\kappa-1}{2\kappa}},\ Z=U,A,A_A,\ z=u,a,a_A \qquad (6)$$

in W poljubna spremenljivka ter Z brezrazsežna spremenljivka; indeks 1 pomeni vstop v turbino in indeks 2 izstop iz turbine.

V enačbah (5) in (6) upoštevamo različne vrednosti fizikalnih veličin za $n = 1,2$, ki se spreminjajo v vsakem integracijskem koraku v odvisnosti od temperature in koncentracije.

Enačbo za masni tok $\dot m = \rho u F$ prepišemo v obliko:

and W is an arbitrary variable and Z is a non-dimensional variable; the index 1 denotes the turbine inlet and index 2 the turbine outlet.

In Equations (5) and (6) different time-dependent gas properties are considered for $n = 1,2$, which change in any integration step as functions of temperature and concentration.

The mass flow rate through the turbine is given by $\dot m = \rho u F$, and can be rewritten as:

$$\dot m = \frac{u_n}{a_n}\frac{p_n}{\sqrt{T_n}}\sqrt{\frac{\kappa_n}{R}}F_n,\ n=1,2 \qquad (7).$$

Z vpeljavo novih spremenljivk: / Introducing new variables:

$$G_n=\frac{\dot m\sqrt{T_n}}{p_n},\ n=1,2 \quad \text{in/and} \quad C_n=\frac{\kappa_n-1}{2}\sqrt{\frac{\kappa_n}{R_n}}F_n,\ n=1,2 \qquad (8)$$

enačbe (5) prepišemo v obliko: / Equation (5) can be rewritten as:

$$\lambda^*_{in,n}=A^*_n\left(1+C_nG_n\right),\ n=1,2 \quad \text{in/and} \quad \lambda^*_{out,n}=A^*_n\left(1-C_nG_n\right),\ n=1,2 \qquad (9).$$

Iz enačbe (8) izhaja zveza: / From Eq. (8) it follows that:

$$G_2=\frac{p_1}{p_2}\sqrt{\frac{T_2}{T_1}}G_1 \qquad (10).$$

Razmerja (9), ki vsebujejo karakteristike turbine, je treba povezati v zveze $\lambda^*_{in,1}/\lambda^*_{in,2}$, $\lambda^*_{out,1}/\lambda^*_{in,1}$ in $\lambda^*_{out,1}/\lambda^*_{out,2}$. Enačbe (5) do (10) so analogne

Equations (9) contain the turbine characteristics, and it is necessary to establish the following functional relations $\lambda^*_{in,1}/\lambda^*_{in,2}$, $\lambda^*_{out,1}/\lambda^*_{in,1}$

enačbam za nespremenljive fizikalne lastnosti plinov, a v predstavljeni simulaciji upoštevamo časovno spremenljive fizikalne parametre, ki so različni za $n = 1,2$, kakor je nakazano v enačbah. Enačbe, izpeljane v nadaljevanju, so zaradi upoštevanja spremenljivih fizikalnih lastnosti zahtevnejše od primerjalnih enačb, izpeljanih za nespremenljive lastnosti plinov.

Iz enačb (9), (10) in (6) izhaja:

and $\lambda^*_{out,1} / \lambda^*_{out,2}$. Equations (5) to (10) are analogous to those for the simulation of a turbine with constant gas properties, although in the presented analysis different time-dependent gas properties are considered for $n = 1,2$, as was already indicated in the equations. Furthermore, a set of more complex equations capable of considering variable gas properties is derived to properly represent the turbine characteristics.

Combining Equations (9), (10) and (6) leads to:

$$\frac{\lambda^*_{in,1}}{\lambda^*_{in,2}} = \left(\frac{p_1}{p_{ref}}\right)^{\frac{\kappa_1-1}{2\kappa_1}} \left(\frac{p_{ref}}{p_2}\right)^{\frac{\kappa_2-1}{2\kappa_2}} \left(\frac{1+C_1G_1}{1-C_2\frac{p_1}{p_2}\sqrt{\frac{T_2}{T_1}}G_1}\right) \tag{11}$$

$$\frac{\lambda^*_{out,2}}{\lambda^*_{out,1}} = \left(\frac{p_2}{p_{ref}}\right)^{\frac{\kappa_2-1}{2\kappa_2}} \left(\frac{p_{ref}}{p_1}\right)^{\frac{\kappa_1-1}{2\kappa_1}} \left(\frac{1+C_2\frac{p_1}{p_2}\sqrt{\frac{T_2}{T_1}}G_1}{1-C_1G_1}\right) \tag{12}$$

$$\frac{\lambda^*_{out,1}}{\lambda^*_{in,1}} = \frac{1-C_1G_1}{1+C_1G_1} \tag{13}$$

$$\frac{A_{A_2}}{A_{A_1}} = \sqrt{\frac{T_{A_2}}{T_{A_1}}} = \left(\frac{T_{A_2}}{T_2}\frac{T_2}{T_1}\frac{T_1}{T_{A_1}}\right)^{\frac{1}{2}} \quad \text{in/and} \quad \frac{A_{A_2}}{A_{A_1}} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} \left(\frac{p_1}{p_{ref}}\right)^{\frac{\kappa_1-1}{\kappa_1}} \left(\frac{p_{ref}}{p_2}\right)^{\frac{\kappa_2-1}{\kappa_2}} \tag{14}$$

ter

as well as

$$\frac{T_2}{T_1} = \frac{T_2}{T_{01}}\frac{T_{01}}{T_1} \quad \text{in/and} \quad \frac{T_2}{T_{01}} = 1-\eta_{TS}\left(1-\left(\frac{p_2}{p_{ref}}\right)^{\frac{\kappa_2-1}{\kappa_2}}\left(\frac{p_{ref}}{p_1}\right)^{\frac{\kappa_1-1}{\kappa_1}}\right) \tag{15}$$

Uporaba enačb (11), (12), (14) in (15) omogoča mnogo boljšo ohranitev mase pri pretoku skozi robni element kakor uporaba Bensonovih enačbe [4] z upoštevanjem spremenljivih lastnosti plinov, pri slednjih upoštevamo časovno spremenljive lastnosti plinov, a uporabljamo enake fizikalne stalnice na vstopu in izstopu iz robnega elementa.

Izpeljati je treba tudi nov algoritem za določitev karakteristik turbine, ki jih zaradi spremenljivih fizikalnih lastnosti plina ne moremo izračunati vnaprej, kakor je to predlagal Benson [4], ampak jih računamo v vsakem integracijskem koraku. Benson [4] predlaga, da iz podatkov pretočne karakteristike in izkoristka turbine naredimo tabelirano bazo podatkov $\lambda^*_{in,1} / \lambda^*_{in,2}$, G_1, p_2/p_1 ter T_2/T_1 za različne vrtilne frekvence turbopolnilnika $N/\sqrt{T_{01}}$, kjer G_1 in p_2/p_1 povzamemo iz pretočne karakteristike turbine, T_2/T_1 pa določimo iz izkoristka turbine (en. (15)). G_1, p_2/p_1 in T_2/T_1 določimo z linearno interpolacijo parametrov:

The application of Eq. (11), (12), (14) and (15) leads to the reduction of the error in the mass flow rate, when flowing through the turbine, by one order of magnitude compared to the application of the Benson's equations [4] to the gases with variable gas properties, i.e., the equations do not consider different gas properties for $n = 1,2$.

Benson's algorithm [4] is not appropriate for representing the turbine characteristics for flows with variable gas properties, since turbine characteristics must not be calculated in advance and should, thus, be calculated during each integration step. Benson [4] proposed that the turbine performance data are processed to obtain the tabulated database in the form $\lambda^*_{in,1} / \lambda^*_{in,2}$, G_1, p_2/p_1 and T_2/T_1 for different speed parameters $N/\sqrt{T_{01}}$, where G_1 and p_2/p_1 are determined from the turbine flow characteristics and T_2/T_1 from eq. (15). Then, G_1, p_2/p_1 and T_2/T_1 are determined by linear interpolation for:

$$x = \frac{\lambda_{in,1}^{*}}{\lambda_{in,2}^{*}} \tag{16}$$

in $N/\sqrt{T_{01}}$ ter nato vstavimo v enačbe (11) do (15).

Opisani postopek ni primeren za simulacijo delovanja turbine z upoštevanjem spremenljivih lastnosti plinov, saj je vnaprej pripravljena baza podatkov izračunana z vnaprej določeno vrednostjo κ in R, kar pri simulaciji delovanja motorja v prehodnem režimu, kjer se razmernik zraka in temperatura izpušnih plinov močno spreminjata, privede do napačne razlage karakteristik turbine ali kompresorja.

Algoritem določanja karakteristik turbine za spremenljive lastnosti plinov temelji na zamisli, da v vsakem časovnem koraku izračunamo Riemannove spremenljivke na dveh podanih karakteristikah vrtilne frekvence turbine, ki sta v neposredni okolici glede na trenutno vrtilno frekvenco turbopolnilnika, s trenutno vrednostjo κ in R, in jih nato linearno interpoliramo glede na trenutno vrtilno frekvenco turbopolnilnika. Nesmiselno in časovno nespremenljivo bi bilo računati Riemannove spremenljivke vzdolž celotne karakteristike robnega elementa, zato jih izračunamo le v podanih točkah, ki sta v neposredni okolici glede na parameter x, kakor je to prikazano na sliki 1. Zato uvedemo spremenljivke:

and $N/\sqrt{T_{01}}$, and further inserted into Eq. (11) to (15).

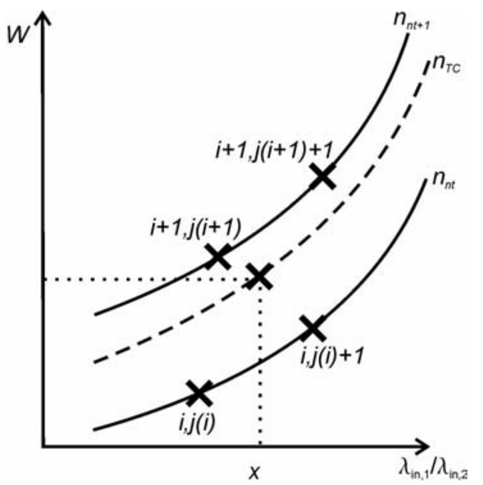
However, Benson's algorithm [4] is not appropriate for representing turbine characteristics for the flows with variable gas properties, since tabulated data prepared in advance are unable to consider variations in κ and R, and therefore distort the interpretation of the turbine characteristics for a wide operating range of the engine; the temperature and composition change significantly with the change in the operating regime of the engine.

Therefore, a new algorithm that is able to consider variable gas properties should be developed. It is obvious that Riemann variables must be determined in each time step with an instantaneous value of κ and R on two turbine speed characteristics lying in the vicinity of the instantaneous turbocharger speed. The values corresponding to the instantaneous turbocharger speed are then obtained by linear interpolation. The determination of the whole turbine map in each time step would be inappropriate, therefore only the nearest neighbours, considering parameter x, of the turbine operating point are processed, as presented in Fig. 1. Let us define:

$$\lambda_{in,1}^{*}(i), \lambda_{out,1}^{*}(i), A_{A_1}(i), \lambda_{in,2}^{*}(i), \lambda_{out,2}^{*}(i), A_{A_2}(i); i = nt, nt+1 \tag{17},$$

kjer indeks 1 pomeni vrednosti pred turbino in indeks 2 vrednosti za turbino. Interpolirane vrednosti nato izračunamo kot:

where index 1 denotes the turbine inlet and index 2 denotes the turbine outlet. The interpolated values are thus determined as



Sl. 1. *Shematski prikaz približka poljubne veličine iz začetnih podatkov*
Fig. 1. *Approximation of parameters from the input data*

$$W = \left(1-k_{nt}\right) W\left(nt\right) + k_{nt} W\left(nt+1\right) \quad \text{kjer je/where} \quad k_{nt} = \frac{n_{TC} - n_{nt}}{n_{nt+1} - n_{nt}} \tag{18}$$

W pomeni poljubno Riemannovo spremenljivko iz enačbe (17), indeksa nt in nt +1 pa podani karakteristiki robnega elementa, ki sta najbliže vrtilni frekvenci turbopolnilnika (TC) (sl. 1).

Vrednosti Riemannovih spremenljivk določimo iz parametrov:

W is an arbitrary Riemann variable from Eq. (17) and the indexes nt and nt +1 are turbine speed characteristics lying in the vicinity of the instantaneous turbocharger speed (TC) (Fig. 1).

Riemann variables are determined from the parameters:

$$G_1(i,k), \left(\frac{p_2}{p_1}\right)_{(i,k)}, \left(\frac{T_2}{T_1}\right)_{(i,k)} ; i = nt, nt+1, k = j(i), j(i)+1 \tag{19}$$

ki jih interpoliramo med točkama (i,j(i)) in (i,j(i)+1) glede na vrednost parametra x (en. (16)).

Za parametre G_1, p_2/p_1 ter T_2/T_1, ki jih uporabimo v enačbah (11) do (14), sledi:

interpolated between (i, j(i)) and (i, j(i)+1) for the parameter x (Eq. (16)).

For parameters G_1, p_2/p_1 and T_2/T_1 applied in Equations (11) to (14) it follows:

$$W = \left(1-k_{ni}\right) W\left(i, j\left(i\right)\right) + k_{ni} W\left(i, j\left(i\right)+1\right) \quad \text{kjer je/where} \quad k_{ni} = \frac{x - x_{dat}\left(i, j\left(i\right)\right)}{x_{dat}\left(i, j\left(i\right)+1\right) - x_{dat}\left(i, j\left(i\right)\right)} \tag{20}$$

in

and

$$x_{dat}\left(i,k\right) = \frac{\lambda^*_{in,1}(i,k)}{\lambda^*_{in,2}(i,k)} ; i = nt, nt+1, k = j(i), j(i)+1 \tag{21}$$

W pa pomeni poljubno spremenljivko. Z enačbami (11) do (14) tako pridelamo neznane Riemannove spremenljivke, predstavljene v en. (17). Diagram poteka za izračun enorazsežnih karakteristik turbine z zgornjimi enačbami je v [2].

Koncentracijo v robni točki izračunamo enako kakor karakteristiko poti, kar je podrobno opisano v [2].

where W is again an arbitrary variable. The Riemann variables used in Eq. (17) are determined from eq. (11) to (14). The flow chart for determining the turbine characteristics with the presented equations can be found in [2].

The concentration at the boundaries is determined in a similar way to the path characteristics; extensively described in [2].

3 REZULTATI

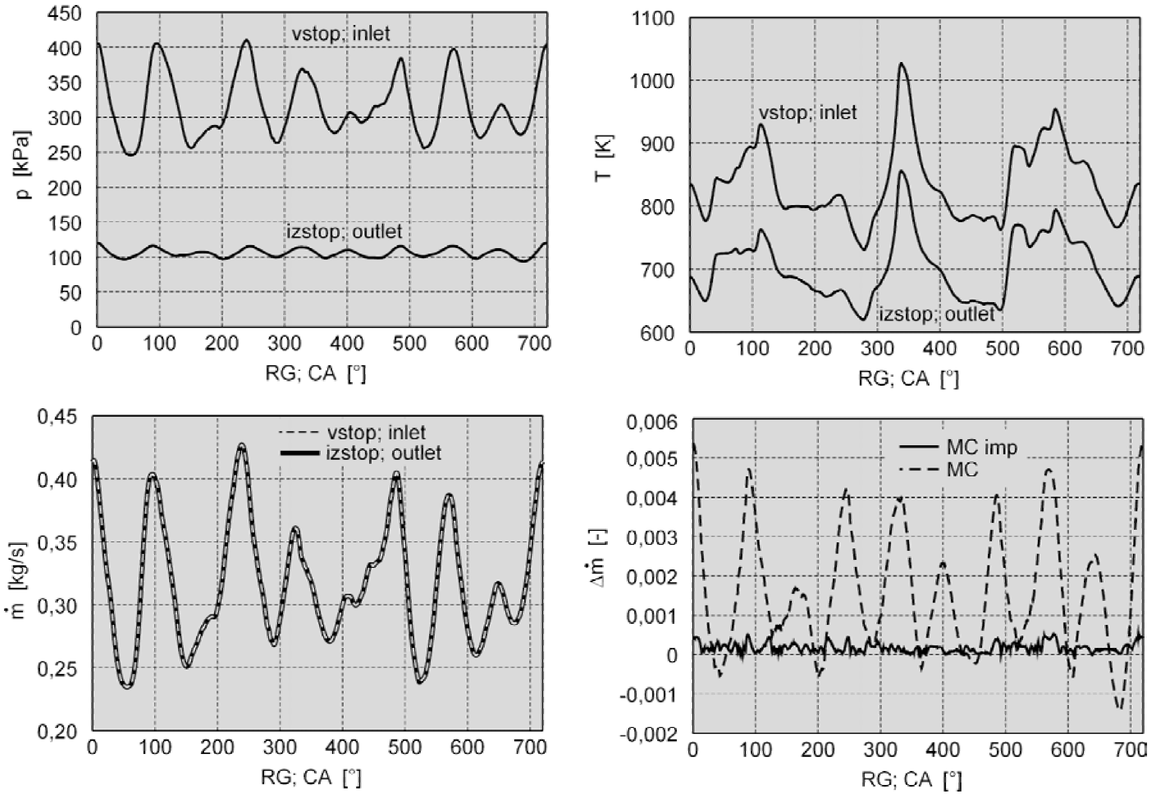
V nadaljevanju so prikazani parametri turbine, ki so izračunani s lastnim simulacijskim programom za numerično modeliranje delovanja tlačno polnjenega motorja, v katerega je vključen tudi predstavljeni algoritem za simulacijo turbine. Simulacijski program, napisan v programskem jeziku FORTRAN, je podrobno predstavljen v [2] in vsebuje dva algoritma za simulacijo delovanja turbine. Prvi algoritem (MC) sloni na enačbah, izpeljanih v [4], kjer upoštevamo časovno spremenljive lastnosti plinov, ne upoštevamo pa različnih fizikalnih stalnic na vstopu in izstopu iz robnega elementa. Drugi algoritem (MCimp), ki omogoča upoštevanje različnih fizikalnih stalnic na vstopu in izstopu iz robnega elementa, pa sloni na enačbah (5) do (21).

Na sliki 2 so prikazani parametri turbine za simulacijo delovanja šestvaljnega motorja STEYR 236, opremljenega s kompresorjem Holset H1E-8264AX in turbino Holset J12S5 pri 4300 min^{-1} in

3 RESULTS

The numerical results of the turbine parameters are presented to assess the validity of the turbine simulation model. Therefore, the turbine model is implemented in the author’s general engine thermodynamics and fluid mechanics simulation program of a turbocharged engine. A FORTRAN simulation program is described in detail in Ref. [2] and includes two algorithms for the turbine simulation. The first algorithm (MC) is based on Benson’s equations [4], where time-dependent upstream parameters are used in the equations. Whereas the second algorithm (MCimp) based on Eq. (5) to (21) makes it possible to consider different gas properties at the turbine’s inlet and outlet.

Fig. 2 represents the turbine parameters while simulating the STEYR 236 6-cylinder engine, equipped with the Holset H1E-8264AX compressor and the Holset J12S5 turbine at 4300 rpm and 100% load. The



Sl. 2. *Parametri turbine pri 4300 min^{-1} in 100% obremenitvi*
Fig. 2. *Turbine parameters at 4300 rpm and 100% load*

100-odstotni obremenitvi. Prikazani so tlak, temperatura in masni tok za vstop in izstop iz turbine, ki so izračunani z algoritmom MCimp, ter primerjava napake ohranitve masnega toka med vstopom in izstopom iz turbine $\Delta\dot{m} = \left(\dot{m}_1 - \dot{m}_2\right)/\dot{m}_2$ za metodi MC in MCimp. S slike je razvidno, da je napaka ohranitve mase za metodo MCimp manjša približno za en red velikosti v primerjavi z metodo MC, kar pomeni bistveno izboljšanje v ohranitvi mase. Napaka v ohranitvi mase za simulirane turbine znaša za metodo MC do približno 0,55%, pri ustaljenih testih drugih turbin pa se je napaka pri uporabi metode MC v nekaterih primerih povzpela tudi do 3%, kar je nedopustno z vidika natančnosti simulacije. Tudi v slednjih primerih je bila napaka v ohranitvi mase z metodo MCimp manjša za približno en red velikosti.

pressure, temperature and mass flow rate are calculated with the subroutine MCimp; it should be noted that these results do not significantly deviate from those calculated with subroutine MC. However, there is a great difference between subroutine MCimp and MC when the conservation of the mass flow rate is considered. The error in the conservation of the mass flow rate, $\Delta\dot{m} = \left(\dot{m}_1 - \dot{m}_2\right)/\dot{m}_2$, for the subroutine MC is approximately one order of magnitude larger than that of the subroutine MCimp. Thus, the error in $\Delta\dot{m}$ is up to around 0.55%. However, it should be noted that $\Delta\dot{m}$ for the subroutine MC is up to 3% in some turbine steady-state tests, which is intolerable for accurate simulations, whereas the reduction of $\Delta\dot{m}$ by one order of magnitude when applying subroutine MCimp significantly improved the accuracy of the simulation.

4 SKLEP

V prispevku so izpeljane enačbe za simulacijo delovanja turbine turbopolnilnika, ki omogočajo upoštevanje spremenljivih fizikalnih lastnosti plina. Predstavljene enačbe in inovativen algoritem

4 CONCLUSION

In this paper extended equations suitable for simulating a turbocharger turbine for flows with variable gas properties are presented. These equations are solved with an innovative algorithm upravljanja z njimi zagotavljajo mnogo boljšo ohranitev mase pri pretoku skozi robni element. Iz rezultatov in drugih testnih primerov lahko povzamemo, da predstavljeni algoritem zmanjša napako v ohranitvi mase za približno en red velikosti v primerjavi z algoritmom, ki sloni na enačbah predstavljenih v literaturi ob upoštevanju spreminjajočih se fizikalnih lastnosti plina. Napaka v ohranitvi mase metode MCimp je običajno pod 0,1%, le v določenih testnih primerih je dosegla 0,3%. Slednja napaka zagotavlja resničnost simulacije, saj je to napako potrebno oceniti glede na druge napake vstopnih podatkov in uporabljenih modelov. Časovna zahtevnost inovativnega algoritma je le nekoliko večja od algoritma za nespremenljive lastnosti plinov, predstavljenega v literaturi, kljub sprotnemu izračunu karakteristik, ki upošteva trenutne vrednosti fizikalnih parametrov na vstopu in izstopu iz turbine.

that is capable of considering variations in the gas properties for every time step. The combination of the extended equations and the innovative algorithm ensures a much better conservation of the mass flow rate when flowing through the turbine in comparison with the commonly applied equations, modified in such terms that variations in the gas properties are considered. The error in $\Delta\dot{m}$ of the method MCimp is usually less than 0.1%, and reached a maximum value of 0.3% in some test cases. The new method significantly improved the accuracy of the simulation and indicates a step towards more accurate one-dimensional techniques. It should also be noted that the computational time of the new algorithm is only slightly longer than that of the basic Benson algorithm [4] despite the simultaneous calculation of turbine characteristics with instant values of the parameters.

5 OZNAČBE
5 SYMBOLS

brezrazsežna hitrost zvoka	A	-	non-dimensional speed of sound
brezrazsežna raven entropije	A_A	-	non-dimensional entropy level
hitrost zvoka	a	m/s	speed of sound
hitrost zvoka pri primerjalnem tlaku – raven entropije; $a_A = a\left(p_{ref}/p\right)^{(\kappa-1)/2\kappa}$	a_A	m/s	speed of sound at reference pressure - entropy level; $a_A = a\left(p_{ref}/p\right)^{(\kappa-1)/2\kappa}$
primerjalna hitrost zvoka	a_{ref}	m/s	reference speed of sound
specifična toplota pri nespremenljivem tlaku	c_p	J/kgK	specific heat at constant pressure
specifična toplota pri nespremenljivi prostornini	c_v	J/kgK	specific heat at constant volume
primerjalni hidravlični premer	D	m	equivalent hydraulic diameter
specifična notranja energija	e	J/kg	specific internal energy
prerez cevi	F	m^2	cross-sectional area
koeficient trenja s stenami	f	-	wall-friction coefficient
specifična entalpija	h	J/kg	specific enthalpy
tlak	p	Pa	pressure
toplotni tok na enoto mase	q	W/kg	rate of heat transfer per unit mass
plinska stalnica	R	J/kgK	specific gas constant
temperatura	T	K	temperature
čas	t	s	time
brezrazsežna hitrost plina	U	-	non-dimensional gas velocity
hitrost plina	u	m/s	gas velocity
poljubna spremenljivka	W	-	arbitrary variable
masni delež komponente i	Y_i	-	mass fraction of component i
celotni statični izkoristek turbine	η_{TS}	-	total static-turbine efficiency
eksponent adiabate	κ	-	ratio of specific heats
Riemannova spremenljivka	λ	-	Riemann variable
gostota	ρ	kg/m^3	density

INDEKSI		SUBSCRIPTS
plin	g	gas
stena	w	wall
zastojni	0	stagnation

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