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Dušenje torzijskih vibracij in raziskava učinkovitosti

A Damper of Torsional Vibrations and an Investigation of Its Efficiency

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V prispevku so prikazani izvirni dušilniki torzijskih vibracij, ki učinkovito delujejo v širokem območju motilnih frekvenc. Raziskali smo tudi nekaj možnih oblikovalskih različic. Osnovni sestavni del dušilnika je rotacijski upogibni obroč. V prispevku raziskujemo gibajoči se sistem na osnovi sistema nelinearnih diferencialnih enačb. Z razdelitvijo gibanja na enakomerno vrtenje in nihanje, razvojem koeficientov enačbe v Taylorjevo potenčno vrsto in izključitvijo ustaljenih delov, dobimo sistem enačb za majhna nihanja. Sistem vsebuje vztrajnostne, upogibne in žiroskopske člene. Izpeljali smo gibalne enačbe in formulirali stabilnostne pogoje za dinamično ravnotežje sistema. Ustrezno pozornost smo namenili tudi drugim možnim nestabilnim oblikam in področjem upogibnega obroča. Analizirali smo lastne frekvence sistema. Oceno učinkovitosti dušilnika glede na različne parametre smo pridobili iz izraza za ekvivalentni vztrajnostni moment in njegovih mejnih vrednosti.

(Ključne besede: absorberji vibracij, obroči krožni, vibracije, stabilnost, ekvivalent vztrajnostnih momentov)

This paper reviews an original torsional vibration damper retaining its efficiency over a wide disturbing frequency band. Some potential design alternatives are considered. The basic structural element of the damper is a rotary flexible ring. The paper investigates the motion system on the basis of a set of nonlinear differential equations. By separating the motion into uniform rotary and oscillatory, expanding the equation coefficients into a Taylor's power series and excluding the static members, a system of equations for insignificant oscillations is derived. The system contains inertia, flexible and gyroscopic terms. The equations of motion are derived and the stability conditions for the system's dynamic balance are formulated. Proper consideration is given to other possible loss-of-stability forms and regions of the flexible ring. An analysis of the system's natural frequencies is made. The efficiency estimation of the damper versus various parameters is effected as a result of the expression of the equivalent inertia moment and its limit values.

(Keywords: torsional vibration dampers, rotary flexible rings, vibrations, stability, equivalent inertia moments)

0 INTRODUCTION

New devices, mechanisms, assemblies and machines should be very efficient. High efficiency can be ensured by power- and speed-related properties.

An increase in transmittable powers and speeds of motion is accompanied by an intensification of vibrations in the systems, and such vibrations frequently exceed their dynamic loads. The level of vibrations becomes one of the key criteria of quality and reliability of machines. Because of this, a limitation of the dynamic overloads of machine assemblies is an urgent problem, directly related to an increase of efficiency, reliability, accuracy and longevity of machines, mechanisms, assemblies and devices.

The authors have worked on damping the torsional vibrations of complicated rotating rotor systems for several decades. They explored various methods and measures, for example, the first of them [1] also carried out theoretical and experimental investigations on the development of effective dampers of torsional vibrations.

There is a variety of designs of dampers of torsional vibrations that can be naturally inserted into the structure of a relevant unit. Seeking a natural arrangement is one of the causes of the abovementioned variety of designs; however, successful designs of vibration dampers are rare.

An advantage of the frictional dampers of torsional vibrations is their capability to preserve their efficiency in a certain frequency range. However, seeking for essential efficiency in such a case leads to a non-proportional increase of sizes, weight, etc., the more so that frictional damping of the vibrations is bound with the elimination of heating energy, wear and the use of special materials.

Many works – both theoretical and experimental – have been devoted to investigations of vibration processes in mechanical rotor systems ([2] to [7]).

The well-known pendulous vibration damper has an excellent feature of self-tuning for one harmonic of the torsional vibrations on any change of the speed of rotation of the system. However, it almost does not affect the adjacent harmonics and other torsional vibrations (for example, ones of random character). In addition, a pendulous vibration damper is completely discussed in transitional modes of motion, in particular during the starting period.

The authors set the task to develop such a dynamic damper of torsional vibrations that would be tunable for a wide range of disturbing harmonics (frequencies), remaining a natural element of the rotating system.

It became clear that the set task may be solved to a certain extent by the use of a vibration damper, based on the rotating elastic ring situated on two pendulous rings in the shape of elastic frames. The torsional vibrations of the system generate transversal bending vibrations of the elastic ring. The damping system includes an elastic ring, special masses that can be additionally fixed on it and elastic frames.

1 THE SCHEMES OF CONSTRUCTIONS OF VIBRATION DAMPERS

The design scheme of the simplest damper of torsional vibrations in the shape of a rotating elastic ring [8] is presented in Fig. 1,a. The key element of such a vibration damper is the ring 2 connected to the principal system 1 with two

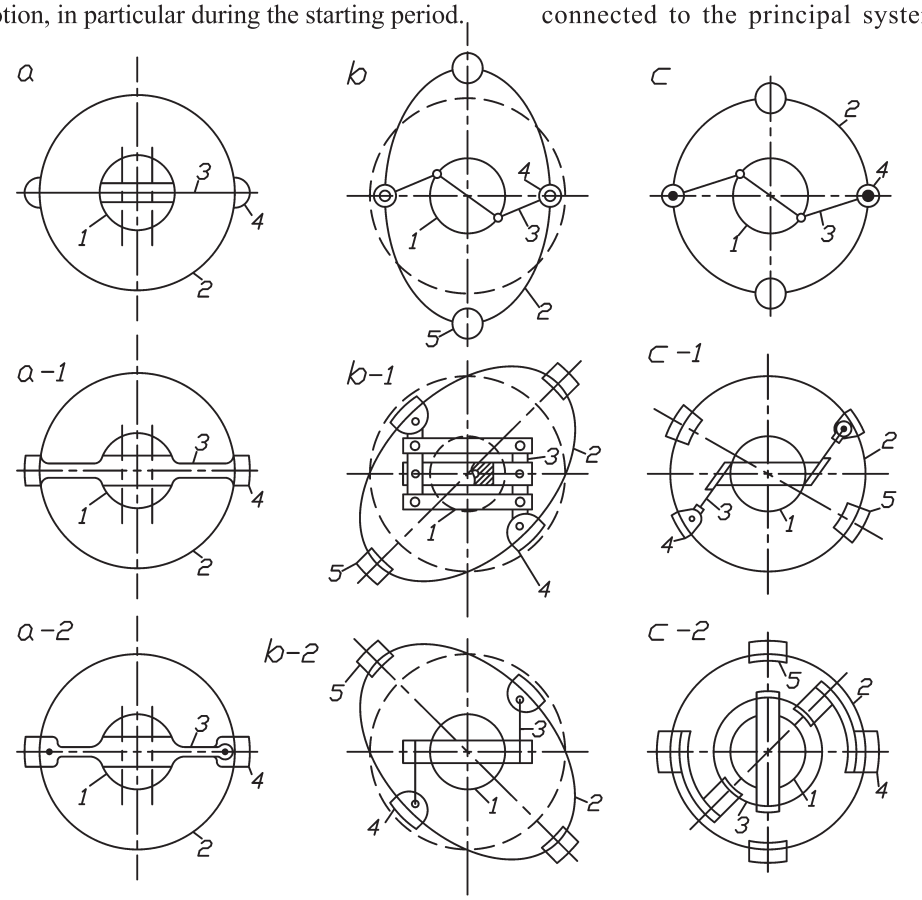

Fig. 1. *Schemes of dampers of torsional vibrations based on an elastic ring*

opposite frames 3. The ring can be equipped with supplemental masses 5 (Fig. 1,b,c), perpendicular to masses 4.

1.1 The principle of operation of a vibration damper on the base of an elastic ring

In the case of the absence of rotation or ideally uniform rotation, the axial line of the ring is an ideal circle in the limits of stability. Torsional vibrations of the principal system cause bending of the elastic frames 3 and periodic compression of the ring 2 in the transversal direction. The ring 2, because of its elasticity and centrifugal mass, can efficiently damp the torsional vibrations of the shaft 1 upon certain parameters across a wide range of harmonics.

The technical realization of the vibration damper according to this scheme is presented in Fig. 1, a-1, a-2. In Fig. 1, a-2, the elastic frames are connected to the ring with swivel clamps.

If two supplemental masses of a particular size are fixed to the ring, symmetrically with the axis of rotation in the plane perpendicular to the plane of the elastic frames (see Fig., 1,b), the ring is extended into an ellipse-shaped body on the rotation. In many cases, such an extended ring exhibits improved vibration-damping properties. The bent centrifugal pendulums 3 can be stabilized by a swivel parallelogram (Fig. 1, b-1) or replaced with symmetric elastic frames 3, tilted by a certain angle with respect to the radius (Fig. 1, b-2).

In many cases the efficiency of the vibration damper can be increased by fixing the elastic ring on two symmetrically tilted pendulums (see Fig. 1, c). The tilted pendulums can be elastic frames (Fig. 1, c-1) or elastically fixed tilted pendulums (Fig. 1, c-2). The vibration damper presented in Fig. 1, c-2 includes one more elastic rings of small diameter 3 in the middle, clamped at two opposite points. The ring is an elastic swivel.

1.2 Investigation of the operation of the vibration damper

Let us start the investigation of the operation of the vibration dampers (see Fig. 1) with a calculation of the potential energy of the deformed ring and the kinetic energy of the system. The potential energy of a half-ring as an elastic body, deformed by the impact of concentrated forces (Fig. 2), can be found from the following expression according to [8]:

$$\Pi_{1/2} = a_B P^2 + b_B PQ + a_B Q^2 \quad (1),$$

where a_B *and* b_B are coefficients, and P and Q are fictitious forces.

Let us find the coefficients a_B and b_B. For this purpose, the shifts of the ring in the direction of the forces P and Q should be found. In accordance with [8]:

$$u_{1\,Q=0} = \frac{\partial \Pi_{1/2}}{\partial P} = 2a_B P + b_B Q = 2a_B P = -\left(\frac{1}{\pi} - \frac{\pi}{8}\right)\frac{PR^3}{EI}$$

$$u_{2\,Q=0} = \frac{\partial \Pi_{1/2}}{\partial Q} = b_B P + 2a_B Q = b_B P = -\left(\frac{1}{\pi} - \frac{1}{4}\right)\frac{PR^3}{EI} \quad (2).$$

Thus:

$$a_B = \frac{1}{4}\left(\frac{\pi}{4} - \frac{2}{\pi}\right)\frac{R^3}{EI}, \quad b_B = -\frac{1}{2}\left(\frac{2}{\pi} - \frac{1}{2}\right)\frac{R^3}{EI} \quad (3).$$

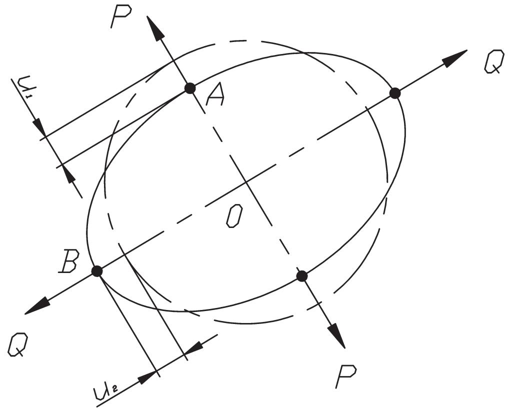

Fig. 2. *For determining the potential energy of a deformed elastic ring*

Let us consider that the shift of point A consists of two components, i.e.:

$$u_1 = u^{\Delta} - ku_0 \quad (4),$$

where u^{Δ} is the shift of point A caused by the force *P*, u_0 is the shift of the point B caused by the force *Q*:

$$k = -\frac{b_B}{2a_B} = \frac{2(4-\pi)}{\pi^2 - 8} \quad (5),$$

correspondingly:

$$u_2 = u_0 - ku^{\Delta} \quad (6).$$

On the basis of Equations (4) and (6), we find u_2:

$$u_2 = u_0\left(1-k^2\right) - ku_1 \quad (7).$$

Let us find the potential energy of the deformed ring as a function of the shifts u_1 and u_2. From the above expressions (2) and (3), the forces *P* and *Q* are expressed as follows:

$$P = \frac{u_1 + ku_0}{2a_B} \quad (8)$$

$$Q = \frac{u_0}{2a_B} \quad (9).$$

After the insertion of the expressions of the forces from (8) and (9) into the expression (1), we find the full potential energy of the elastic ring from the forces *P* and *Q* as follows:

$$\Pi = 2\Pi_{1/2} = \frac{1}{2a_B}u_1^2 + \frac{4a_B^2 - b_B^2}{8a_B^3}u_0^2 = \frac{1}{2}C_{n_1}u_1^2 + \frac{1}{2}C_{n_2}u_0^2 \quad (10),$$

where:

$$C_{n_1} = \frac{1}{a_B} \quad \text{and} \quad C_{n_2} = \frac{4a_B^2 - b_B^2}{4a_B^3} \quad (11).$$

The total potential energy of the system (Π_Σ) consists of the potential energy of the elastic ring (Π_k), the potential energy of other deformable elements of the vibration damper (Π_i) and the potential energy of the torsion of the shaft (Π_v), i.e.:

$$\Pi_\Sigma = \Pi_k + \Pi_i + \Pi_v \quad (12),$$

where:

$$\Pi_k = f\left(\varphi_1 - \varphi_2, u_0\right), \quad \Pi_i = f\left(\varphi_1 - \varphi_2\right) = \frac{1}{2}C\left(\varphi_1 - \varphi_2\right)^2$$
$$\Pi_v = f\left(\varphi_1\right) \quad (13).$$

In order to calculate the kinetic energy, let us consider the system to be a system with three degrees of freedom. Fig. 3 presents the estimated scheme of the vibration damper, where φ_1 and φ_2 are independent of the angular coordinates, *R* is the initial radius of the elastic ring, *m* is the reduced mass of the part of the ring with the relevant concentrated mass, m_0 is the reduced mass of the part of the ring with the supplemental mass, ρ is the distance of the mass *m* from the axis of rotation, ρ_0 is the distance of the mass m_0 from the axis of rotation, ρ and ρ_0 in the general case are a function of the rotational deformation of the vibration damper. If the radial shift of the mass $m(u_1)$ is a function of (φ_1 - φ_2), the radial shift of the mass m_0 also includes the independent component u_0 that is the third generalized coordinate.

The kinetic energy of the system, taking into account the existence of two couples of masses, is

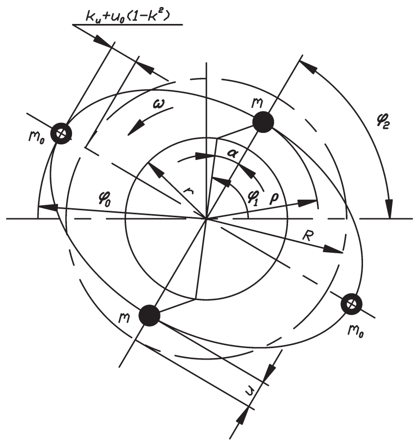

Fig. 3. *The estimation scheme of the vibration damper*

found from the following expression:

$$T = mv^2 + m_0 v_1^2 + \frac{1}{2} I_1 \dot{\varphi}_1^2 \qquad (14),$$

where:

$$v^2 = \rho^2 \dot{\varphi}_2^2 + \left[\frac{d\rho}{d(\varphi_1 - \varphi_2)} \right]^2 (\dot{\varphi}_1 - \dot{\varphi}_2)^2$$

$$v_1^2 = \rho_0^2 \dot{\varphi}_2^2 + \dot{\rho}_0^2$$

I_1 is moment of inertia of the rotated mass; the super point means differentiation with respect to time.

In order to simplify the estimated dependences, we consider that the radius of the string clamping (r) equals a half of the radius of the elastic ring (R).

Then, the following is concluded from Fig. 3:

$$\begin{gathered} \rho = R\cos(\varphi_1 - \varphi_2) \\ \rho_0 = R + 2kr\left[1 - \cos(\varphi_1 - \varphi_2)\right] + u_0\left(1 - k^2\right) \\ \dot{\rho}_0 = 2kr\sin(\varphi_1 - \varphi_2)(\dot{\varphi}_1 - \dot{\varphi}_2) + \dot{u}_0\left(1 - k^2\right) \end{gathered} \qquad (15).$$

Finally, we find the following expression for the kinetic energy:

$$\begin{aligned} T = &\left\{ \frac{1}{2} I_1 + m\left[\frac{d\rho}{d(\varphi_1 - \varphi_2)} \right]^2 + m_0 \left[\frac{d\rho_0}{d(\varphi_1 - \varphi_2)} \right]^2 \right\} \dot{\varphi}_1^2 - \\ &-2\left\{ m\left[\frac{d\rho}{d(\varphi_1 - \varphi_2)} \right]^2 + m_0 \left[\frac{d\rho_0}{d(\varphi_1 - \varphi_2)} \right]^2 \right\} \dot{\varphi}_1 \dot{\varphi}_2 + \\ &+\left\{ m\left[\left[\frac{d\rho}{d(\varphi_1 - \varphi_2)} \right]^2 + \rho^2 \right] + m_0 \left[\left[\frac{d\rho_0}{d(\varphi_1 - \varphi_2)} \right]^2 + \rho_0^2 \right] \right\} \dot{\varphi}_2^2 \end{aligned} \qquad (16).$$

After relevant transformations, we find the kinetic energy as a homogenous quadratic form of the generalized speeds [8]:

$$\begin{aligned} T = &\frac{1}{2} A_{11} \dot{\varphi}_1^2 + A_{12} \dot{\varphi}_1 \dot{\varphi}_2 + \frac{1}{2} A_{22} \dot{\varphi}_2^2 + \\ &+ \frac{1}{2} A_{33} \dot{u}_0^2 + A_{13} \dot{\varphi}_1 \dot{u}_0 + A_{23} \dot{\varphi}_2 \dot{u}_0 \end{aligned} \qquad (17),$$

where the values of the relevant coefficients are as follows:

$$\begin{gathered} A_{11} = I_1 + 8mr^2 \sin^2(\varphi_1 - \varphi_2) + 8m_0 k^2 r^2 \sin^2(\varphi_1 - \varphi_2) \\ A_{12} = -\left[8mr^2 \sin^2(\varphi_1 - \varphi_2) + 8m_0 k^2 r^2 \sin^2(\varphi_1 - \varphi_2) \right] \\ A_{22} = 8mr^2 + 2m_0 \left\{ R + 2kr\left[1 - \cos(\varphi_1 - \varphi_2)\right] + u_0\left(1 - k^2\right) \right\}^2 + \\ +8m_0 k^2 r^2 \sin^2(\varphi_1 - \varphi_2) \\ A_{33} = 2m_0\left(1 - k^2\right)^2 \\ A_{13} = 4m_0 kr \sin(\varphi_1 - \varphi_2)\left(1 - k^2\right) \\ A_{23} = -4m_0 kr \sin(\varphi_1 - \varphi_2)\left(1 - k^2\right) \end{gathered} \qquad (18).$$

For the formation of differential equations of motion we will use Lagrange equations of the second order [9]:

$$\begin{gathered} \frac{d}{dt}\frac{\partial T}{\partial \dot{\varphi}_i} - \frac{\partial T}{\partial \varphi_i} = -\frac{\partial \Pi}{\partial \varphi_i} + M_i \quad (i = 1, 2) \\ \frac{d}{dt}\frac{\partial T}{\partial \dot{u}} - \frac{\partial T}{\partial u} = -\frac{\partial \Pi}{\partial \dot{u}} \end{gathered} \qquad (19),$$

where M_i is the generalized moment of potential forces.

After the differentiation, we find the equations of motion of the system, neglecting the frictional forces:

$$\begin{gathered} A_{11}\ddot{\varphi}_1 + A_{12}\ddot{\varphi}_2 + A_{13}\ddot{u}_0 + \frac{1}{2}\frac{\partial A_{11}}{\partial(\varphi_1 - \varphi_2)}(\dot{\varphi}_1 - \dot{\varphi}_2)^2 - \\ -\frac{\partial w_1}{\partial(\varphi_1 - \varphi_2)}\dot{\varphi}_2^2 \left[\frac{\partial A_{13}}{\partial(\varphi_1 - \varphi_2)} + \frac{\partial A_{23}}{\partial(\varphi_1 - \varphi_2)} \right] \dot{\varphi}_2 \dot{u} + \\ + \frac{\partial \Pi}{\partial(\varphi_1 - \varphi_2)} + \frac{\partial \Pi_v}{\partial \varphi_1} = M_1 \\ A_{12}\ddot{\varphi}_1 + A_{22}\ddot{\varphi}_2 + A_{23}\ddot{u}_0 - \frac{1}{2}\frac{\partial A_{22}}{\partial(\varphi_1 - \varphi_2)}(\dot{\varphi}_1 - \dot{\varphi}_2)^2 + \\ + \frac{\partial w_1}{\partial(\varphi_1 - \varphi_2)}\dot{\varphi}_1^2 + \frac{\partial A_{22}}{\partial u_0}\dot{\varphi}_2 \dot{u}_0 + \\ + \left[\frac{\partial A_{13}}{\partial(\varphi_1 - \varphi_2)} + \frac{\partial A_{23}}{\partial(\varphi_1 - \varphi_2)} \right] \dot{\varphi}_1 \dot{u}_0 - \frac{\partial \Pi}{\partial(\varphi_1 - \varphi_2)} = 0 \\ A_{13}\ddot{\varphi}_1 + A_{23}\ddot{\varphi}_2 + A_{33}\ddot{u}_0 + \frac{\partial A_{13}}{\partial(\varphi_1 - \varphi_2)}\dot{\varphi}_1^2 - \\ - \left[\frac{\partial A_{13}}{\partial(\varphi_1 - \varphi_2)} + \frac{\partial A_{23}}{\partial(\varphi_1 - \varphi_2)} \right] \dot{\varphi}_1 \dot{\varphi}_2 - \\ - \left[\frac{\partial A_{23}}{\partial(\varphi_1 - \varphi_2)} + \frac{1}{2}\frac{\partial A_{22}}{\partial u_0} \right] \dot{\varphi}_2^2 + \frac{\partial \Pi_k}{\partial u_0} = 0 \end{gathered} \qquad (20),$$

here $w_1 = \frac{1}{2} A_{11} + A_{12} + \frac{1}{2} A_{22}$.

After the disintegration of the coefficients of the equations (20) into a Taylor‘s series, we find according to [9]:

$$u_0 = u_0^c + v$$

(u_0^c is the statistic component, v is a small varying value), where each equation is divided into two parts, corresponding to stationary and vibrating motion.

1.3 Investigation of the stability of the dynamic balance

Taking into account a certain identity of two first equations of the system (20), its is described by two equations only:

$$\Pi^{"} - w^{"}\omega^2 = 0$$
$$\Pi^{xx} - \frac{1}{2}a_{22}^{xx}\omega^2 = 0 \qquad (21).$$

Let us check the stability of the positiveness of the matrix determinant:

$$\Delta = \begin{vmatrix} \Pi'' - w''\omega^2 & \Pi'^{x} - w'^{x}\omega^2 \\ \Pi'^{x} - w'^{x}\omega^2 & \Pi^{xx} - \frac{1}{2}a_{22}^{xx}\omega^2 \end{vmatrix} \rangle \; 0 \qquad (22),$$

thus:

$$\left(\Pi'' - w''\omega^2\right)\left(\Pi^{xx} - \frac{1}{2}a_{22}\omega^2\right) - \left(\Pi'^{x} - w'^{x}\omega^2\right)^2 \rangle \; 0$$

where:

$$w' = \frac{1}{2}a'_{11} + a'_{12} + \frac{1}{2}a'_{22}$$
$$w'' = \frac{1}{2}a''_{11} + a''_{12} + \frac{1}{2}a''_{22}$$
$$' = \frac{d}{d\left(\varphi_1 - \varphi_2\right)}, \quad '' = \frac{d^2}{d\left(\varphi_1 - \varphi_2\right)^2}$$
$$^{x} = \frac{d}{du_0^{c}}, \quad ^{xx} = \frac{d^2}{d\left(u_0^{c}\right)^2}, \quad ^{\prime x} = \frac{d^2}{d\left(\varphi_1 - \varphi_2\right)du_0^{c}}$$

The system will be stable if $\Delta > 0$ and

$$\Pi'' - w''\omega^2 \langle \; 0 \qquad (23).$$

With an experimental investigation of the various schemes of dynamic vibration dampers on the basis of an elastic ring, some other forms of loss of stability were obtained: 1) because of an excessive increase in the ring‘s radius for its extension, 2) because of a symmetrical deflection of the ring from the axis of rotation, 3) because of non-symmetrical sideways deflections, etc. On the basis of the experimental data, an analytic investigation on the stability of dynamic balances of the elastic ring was carried out at the preset static deformation, and some peculiarities were cleared up.

The criterion of the stability of the dynamic balance shall be considered as the existence of the maximum of kinetic potential in the preset position (point). In such a case, if the kinetic energy itself is equal to the maximum, the stability shall be considered natural and the position of the balance will not depend on the mode of the speed.

If for the point under investigation the kinetic energy is equal to the minimum or at least does not depend on the disturbance under discussion, the forced stability will only be possible at this point, i.e., we will consider that rigid forced stabilization is possible due to elastic elements. In other cases, we will ensure forced stability is possible on a certain shift of the point of dynamic balance. The size of such a shift depends, among other factors, on the mode of the speed. However, such a shift usually is bound with the appearance of a certain instability that is not allowed in a vibration damping system.

Let us discuss various cases:

1. Stability of an ideally symmetric concentric ring in the case of its uniform rotation

Let us suppose that the concentricity of the ring is ensured in any case and no static bending exists. The kinetic energy of the ring is:

$$T = \pi R\gamma_n\omega^2\left(R + \frac{\Delta l}{2\pi}\right)^2 \qquad (24),$$

where R is the initial radius of bending of the elastic ring, γ_n is the mass of the unit of length of the ring, ω is the average rotational frequency of the system, Δl is the absolute elongation of the ring.

The potential energy of the ring is:

$$\Pi = \frac{1}{4}\frac{EF}{\pi R}\left(\Delta l\right)^2 \qquad (25),$$

where F is the area of the cross-section of the elastic ring, E is the module of longitudinal elasticity.

Let us consider that a stable extension of the ring corresponds to the maximum kinetic potential, i.e., the following condition should be satisfied:

$$\frac{\partial\left(T - \Pi\right)}{\partial\left(\Delta l\right)} = 0, \qquad \frac{\partial^2\left(T - \Pi\right)}{\partial\left(\Delta l\right)^2} \langle \; 0 \qquad (26).$$

In such a case, the extension of the ring will be as follows:

$$\Delta l = \frac{2\pi R^3\gamma_n\omega^2}{EF - \gamma_n R^2\omega^2} \qquad (27)$$

and the limit value of the angular speed will be:

$$\omega^2 \langle \; \frac{EF}{\gamma_n R^2} \qquad (28).$$

The condition (28) identifies the limit of the zone of a stable extension of the ring.

2. Symmetrical longitudinal extension of the ring

As our investigations showed, the maximum efficiency of the vibration damping is achieved when $z > 1$ (in Fig. 3, $z = m_0/m$) and the ring operates as extended. If we consider that the ring is deformed according to the scheme provided in Fig. 3, the element and a more precise investigation provides that the angle $\alpha = 0$, if $z < 1$, and $\alpha > 0$, if $z > 1$, i.e., the deformation, identified with the torsion angle α, when $z > 1$, has a stable fixed value, dependent on the rigidity of the elastic ring and other elements.

It is notable that the bending rigidity of the ring itself does not affect the limits of the zones of stability ($z = 1$), it only defines (together with other parameters) the value of the static deformation.

3. Symmetrical sideways deflection of the ring

The element investigation of stability of the positions of dynamic balance upon the condition of the extremity of the kinetic potential allowed us to make the following conclusions:

a) zero deformation of the ring is possible only on $z < 1/3$ (if the ring is fixed on swivels of pendulum frames);
b) if $z = 1/3$, a certain statistical deformation of the ring is set, depending on the mode of the speeds and the rigidity of the rings;
c) if $z > 1/3$, the ring does not achieve a natural balance, so a symmetric deflection may transform into a non-symmetrical one.

Some other possible disturbances of the ring were discussed, and the conditions of balance were explored as well.

2 INVESTIGATION OF THE EFFICIENCY OF OPERATION OF THE VIBRATION DAMPER

Let us divide the motion along the cyclic coordinates φ_1, φ_2 into a uniform rotational motion and vibrations around it, using the expressions:

$$\begin{aligned} \varphi_1 &= \alpha_1 + \omega t + \beta_1 \\ \varphi_2 &= \alpha_2 + \omega t + \beta_2 \end{aligned} \tag{29}$$

and introduce the transformation:

$$u = u_0{}^c + v \tag{30}$$

where α_1 and α_2 are the phases of motion of the driving and driven links, respectively, ω is the average frequency of rotation, t is the time, β_1 and β_2 are small deflections of the coordinates φ_1 and φ_2 from the state of uniform rotational motion, $u_0{}^c$ is the static component, and v is a small variable value.

After the disintegration of the coefficients of the non-linear equations of motion into a Taylor's series and the elimination of the static parts of the equations that identify the dynamic balance, the linearized equations of small free vibrations around the position of stationary motion will be as follows:

$$\begin{aligned}
&a_{11}\ddot{\beta}_1 + a_{12}\ddot{\beta}_2 + a_{13}\ddot{v} - 2w'\omega\dot{\beta}_2 - \omega\left(a'_{13} + a'_{23}\right)\dot{v} - \\
&-\left(w''\omega^2 - \Pi'' - \Pi''_v\right)\beta_1 + \left(w''\omega^2 - \Pi''\right)\beta_2 - \frac{1}{2}a'^{x}_{22}\,\omega^2 v = 0 \\
&a_{11}\ddot{\beta}_1 + a_{22}\ddot{\beta}_2 + a_{23}\ddot{v} + 2w'\omega\dot{\beta}_1 + \omega\left(a'_{13} + a'_{23} + a^{x}_{22}\right)\dot{v} + \\
&+\left(w''\omega^2 - \Pi''\right)\beta_1 - \left(w''\omega^2 - \Pi''\right)\beta_2 + \frac{1}{2}a'^{x}_{22}\,\omega^2 v = 0 \\
&a_{13}\ddot{\beta}_1 + a_{22}\ddot{\beta}_2 + a_{33}\ddot{v} + \omega\left(a'_{13} + a'_{23}\right)\dot{\beta}_1 - \omega\left(a'_{13} + a'_{23} + a^{x}_{22}\right)\beta_2 - \\
&-\frac{1}{2}a'^{x}_{22}\,\omega^2\beta_2 + \left(\Pi^{xx}_k - \frac{1}{2}a'^{x}_{22}\,\omega^2\right)v = 0
\end{aligned} \tag{31}$$

The linearized equations of motion (31) include inertial, gyroscopic and quasi-elastic members. Thus, the dynamic link of the vibration damper is a rather complicated link between rotating objects, and in the general case it cannot be reduced to the usual (linear or non-linear) elasticity.

2.1 Solution of the system of equations

The natural frequencies of the system with the vibration damper can be found on the basis of the characteristic determinant:

$$\begin{vmatrix}
a_{11}\lambda^2 - w''\omega^2 + \Pi''_v & a_{12}\lambda^2 - 2w'\omega\lambda + w''\omega^2 - \Pi'' & a_{13}\lambda^2 - \frac{1}{2}a'^{x}_{22}\,\omega^2 \\
a_{12}\lambda^2 + 2w'\omega\lambda + w''\omega^2 - \Pi'' & a_{22}\lambda^2 - w''\omega^2 + \Pi'' & a_{23}\lambda^2 + a^{x}_{22}\omega\lambda + \frac{1}{2}a'^{x}_{22}\,\omega^2 \\
a_{13}\lambda^2 - \frac{1}{2}a'^{x}_{22}\,\omega^2 & a_{23}\lambda^2 - a^{x}_{22}\omega\lambda + \frac{1}{2}a'^{x}_{22}\,\omega^2 & a_{33}\lambda^2 + \Pi^{xx} - \frac{1}{2}a^{xx}_{22}\omega^2
\end{vmatrix} = 0 \tag{32}$$

where $\lambda = i\omega p^c$, p^c is one of the natural frequencies. This determinant (32) provides three natural frequencies.

For a determination of the natural frequencies of a vibration damping system only, the below reduced equation (33) can be used; this equation is the condition of frequency tuning for the vibration damper as well:

$$\begin{aligned}
&\left(a_{22}a_{33} - a_{23}{}^2\right)\lambda^4 + \\
&+\left[a_{22}\left(\Pi^{xx} - \frac{1}{2}a_{22}{}^{xx}\omega^2\right) - a_{33}\left(w''\omega^2 - \Pi''\right) + \left(a_{22}{}'^{x} - a_{22}{}'^{x}\,a_{23}\right)\omega^2\right]\lambda^2 - \\
&-\left[\left(w''\omega^2 - \Pi''\right)\left(\Pi^{xx} - \frac{1}{2}a^{xx}{}_{22}\omega^2\right) + \frac{1}{4}a_{22}{}'^{x2}\,\omega^4\right] = 0
\end{aligned} \tag{33}$$

For an assessment of the impact of the vibration damper on the vibration of the system, we find an expression of the equivalent moment of inertia I_e, the value of fictitious mass, rigidly connected with the principal system. The vibration damping effect of the mass with respect to the principal system is equivalent to the relevant effect of the supplemental vibration damping unit.

The periodic component of the reactive torsional moment is:

$$M_p = -\left(a_{11} - I_1\right)\ddot{\beta}_1 - a_{12}\ddot{\beta}_2 - a_{13}\ddot{v} + 2w'\omega\dot{\beta}_2 + \omega\left(a_{13}' + a_{23}'\right)\dot{v} - \\ -\left(w''\omega^2 - \Pi''\right)\left(\beta_1 - \beta_2\right) + \frac{1}{2}a_{22}{}'^{x}\omega^2 v \quad (34)$$

and the equivalent moment of inertia is:

$$I_e = -\frac{M_p}{\ddot{\beta}_1} \quad (35).$$

After insertion of the values:

$$\beta_i = A_i \sin pt + B_i \cos pt \qquad i=1,2$$
$$v = A_3 \sin pt + B_3 \cos pt$$

as well as their fluxions $\dot{\beta}_i$, $\ddot{\beta}_i$, $\dot{v}$, $\ddot{v}$, and ratios A_2 / A_1, B_2 / B_1 and so on into (35), we find the following expression for the equivalent moment of inertia:

$$I_e = \frac{\left[\left(a_{11}-I_1\right)\left(a_{22}a_{23}-a_{23}{}^2\right)-2a_{12}a_{23}{}^2-a_{22}a_{23}{}^2-a_{12}{}^2a_{33}\right]p^4+}{\left(a_{22}a_{33}-a_{23}{}^2\right)p^4-\left[a_{22}\left(\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\right)-a_{33}\left(w''\omega^2-\Pi'\right)-a_{22}{}'^{x}a_{23}\omega^2+\right]}$$

$$\frac{+\left\{\left(2w-I_1\right)\left[a_{33}\left(w''\alpha\omega^2-\Pi''\right)-a_{13}a_{22}{}'^{x}\omega^2\right]-\left[\left(a_{11}-I_1\right)a_{22}-a_{12}{}^2\right]\right\}}{+a_{22}{}^{x2}\omega^2\ \left.\right]p^2-\left(\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\right)\left(w''\omega^2-\Pi''\right)+\frac{1}{4}a_{22}{}'^{x2}\omega^4}+ \quad (36).$$

$$+\left[\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\right]+4w'\omega^2\left(a_{13}a_{22}{}^{x}-w'a_{23}\right)-\left(a_{11}-I_1\right)a_{22}{}^{x2}\omega^2p^2+$$

$$+\left\{\begin{matrix}2w'\omega^2\left[2w'\left(\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\right)+a_{22}{}'^{x}a_{22}{}^{x}\omega^2\right]-\left(w''\omega^2-\Pi''\right)a_{22}{}^{x2}\omega^2- \\ -\left(2w-I_1\right)\left(\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\right)\left(w''\omega^2-\Pi''\right)-\left(2w-I_1\right)\frac{1}{4}a_{22}{}'^{x2}\iota\omega^4\end{matrix}\right\}$$

The limit values of the equivalent moment of inertia for low- frequency disturbances will be found from the following expression:

$$\lim_{p\to 0} I_e = \frac{2w'\omega^2\left[2w'\left(\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\right)+a_{22}{}'^{x}a_{22}{}^{xx}\omega^2\right]-\left(w''\omega^2-\Pi''\right)a_{22}{}^{xx2}\omega^2}{-\left(\Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\varepsilon\omega^2\right)\left(w''\omega^2-\Pi''\right)-\frac{1}{4}a_{22}{}'^{x2}\omega^4}- \quad (37)$$

$$-\left(2w-I_1\right)\left(\left(\ \Pi^{xx}-\frac{1}{2}a_{22}{}^{xx}\omega^2\ \right)\left(w''\omega^2\right)\left(w''\omega^2-\Pi''\right)-\left(2w-I_1\right)\frac{1}{4}a_{22}{}'^{x}\alpha\omega^4\right)$$

and for high-frequency disturbances:

$$\lim_{p\to\infty} I_e = \frac{\left(a_{11}-I_1\right)\left(a_{22}a_{33}-a_{23}{}^2\right)-2a_{12}a_{23}{}^2-a_{22}a_{23}-a_{12}{}^2a_{33}}{a_{22}a_{33}-a_{23}{}^2} \quad (38).$$

In our case, the coefficients will be expressed as follows:

$$a_{11} = I_1 + 2mR^2\sin^2\alpha\left(1+zk^2\right)$$
$$a_{12} = -2mR^2\sin^2\alpha\left(1+zk^2\right)$$
$$a_{22} = 2mR^2\left(1+z\right)+4mR^2zk\left(1-\cos\alpha\right)\left(1+k\right)+$$
$$+2mu_0z\left(1-k^2\right)\left\{u_0\left(1-k^2\right)+2R\left[1+k\left(1-\cos\alpha\right)\right]\right\}$$
$$a_{13} = 2mzkR\left(1-k^2\right)\sin\alpha \quad (39),$$
$$a_{23} = -2mzkR\left(1-k^2\right)\sin\alpha$$
$$a_{33} = 2mz\left(1-k^2\right)^2$$

where $R = 2r$, $m_0 = mz$, α_1 - $\alpha_2 = \alpha$, and z is the ratio of the centrifugal masses m_0 and m. Other coefficients will be expressed as follows:

$$a_{11}' = 2mR^2\sin 2\alpha\left(1+zk^2\right)$$
$$a_{22}' = 4mzkR\left(1+k\right)\sin\alpha\left[R+u_0\left(1-k\right)\right]$$
$$a_{12}' = -2mR^2\sin 2\alpha\left(1+zk^2\right)$$
$$w'' = 2mzkR\left(1+k\right)\cos\alpha\left[R+u_0\left(1-k\right)\right]-2mR^2\cos 2\alpha\left(1+zk^2\right)$$
$$a_{11}'' = 4mR^2\cos 2\alpha\left(1+zk^2\right)$$
$$a_{22}'' = 4mzkR\left(1+k\right)\cos\alpha\left[R+u_0\left(1-k\right)\right]$$
$$a_{12}'' = -4mR^2\cos 2\alpha\left(1+zk^2\right) \quad (40).$$
$$a_{22}{}'^{x} = 4mzkR\left(1-k^2\right)\sin\alpha$$
$$a_{22}{}^{xx} = 4mz\left(1-k^2\right)^2$$
$$a_{22}{}^{x} = 4mz\left(1-k^2\right)\left\{kR\left(1-\cos\alpha\right)+\left[R+u_0\left(1-k^2\right)\right]\right\}$$
$$\Pi'' = C_{n1}R^2\left(\cos\alpha-\cos 2\alpha\right)$$
$$\Pi^{xx} = C_{n2} = \frac{8\pi^2\left(\pi^3-20\pi+32\right)}{\left(\pi^2-8\right)^3}\frac{EI}{R^3}$$
$$C_{n1} = \frac{16\pi}{\pi^2-8}\frac{EI}{R^3}$$

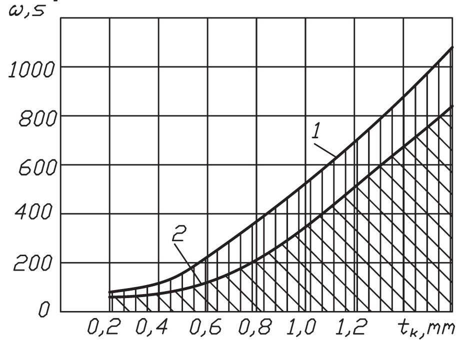

Fig.4. *Dependence of the critical frequency of rotation (ω) of the damper on the thickness of the elastic ring (t_k), when b = 20 mm, z = 2,0, m = 0.03 kg, where the curve 1 corresponds to R = 80 mm and the curve 2 to R = 100 mm (the zones of stability are shaded)*

From the equations of dynamic balance we find the average angle of torsion α_1-α_2 of the vibration damper and u_0^c is the static component of the independent variable u_0:

$$\alpha_1-\alpha_2=\arccos\frac{C_{n1}C_{n2}-2C_{n2}mzk\left(1+k\right)\omega^2-2C_{n1}mz\left(1-k^2\right)^2\omega^2}{C_{n1}C_{n2}-2m\omega^2\left[C_{n2}\left(zk^2+1\right)+z\left(1-k^2\right)^2\left(C_{n1}-2m\omega^2\right)\right]} \quad (41)$$

$$u_0{}^c=\frac{2mzR\omega^2\left[C_{n1}-2m\omega^2\left(1+k\right)\right]\left(1-k\right)^2}{C_{n1}C_{n2}-2m\omega^2\left[C_{n2}\left(zk^2+1\right)+z\left(1-k^2\right)^2\left(C_{n1}-2m\omega^2\right)\right]} \quad (42).$$

The complete expression of the equivalent moment of inertia (36) of the vibration damper under discussion is not presented here because of its length. The equivalent moment of inertia in the high-frequency zone of disturbances is expressed as follows:

$$\lim_{p\to\infty} I_e=\frac{C_1}{C_2}$$

$$C_1=2mR^2\sin^2\alpha\left\{\begin{array}{l}2Ru_0z\left(1-k_0{}^2\right)\left[k\left(1-\cos\alpha\right)+1\right]+\\+R^2\left[\begin{array}{l}\cos^2\alpha+z\left(1-k^2\sin^2\alpha\right)+\\+2zk\left(1-\cos\alpha\right)\left(1-k\right)\end{array}\right]+zu_0\left(1-k^2\right)^2\end{array}\right\}$$

$$C_2=-\left\{\begin{array}{l}R^2\left(1+z\right)+2R^2zk\left(1-\cos\alpha\right)\left(1+k\right)+\\+zu_0\left(1-k^2\right)\left[u_0\left(1-k^2\right)+2R\left(1+k\right)\left(1-\cos\alpha\right)\right]-zk^2R^2\sin^2\alpha\end{array}\right\} \quad (43).$$

As specific quantitative calculations showed, it is sufficient to describe the deformed ring in many constructions of vibration dampers with only two generalized coordinates, i.e., it can be considered that a transversal compression of the ring is proportional (in some cases equal) to its longitudinal extension.

In such a case, the expression for kinetic energy is reduced to the well-known quadratic trinomial [11], and its coefficients are as follows:

$$\begin{array}{l}A_{11}=I_1+8mr^2\sin^2\left(\varphi_1-\varphi_2\right)+8m_0k^2r^2\sin^2\left(\varphi_1-\varphi_2\right)\\A_{12}=-\left[8mr^2\sin^2\left(\varphi_1-\varphi_2\right)+8m_0k^2r^2\sin^2\left(\varphi_1-\varphi_2\right)\right]\\A_{22}=8mr^2+2m_0\left\{R+2kr\left[1-\cos\left(\varphi_1-\varphi_2\right)\right]\right\}^2+8m_0k^2r^2\sin^2\left(\varphi_1-\varphi_2\right)\end{array} \quad (44).$$

Correspondingly, the potential energy will be equal to:

$$\Pi_c=\Pi_k+\Pi_v \quad (45),$$

where $\Pi_k=\frac{1}{2}C_{n1}\left(2r\right)^2\left[1-\cos\left(\varphi_1-\varphi_2\right)\right]^2$ is the potential energy of the elastic ring; Π_v is the potential energy of the shaft torsion that is a function of φ_1.

Based on the methods described in [12], we find linearized equations for the small torsional vibrations that may be used for an assessment of the stability of dynamic balance and the efficiency of the vibration

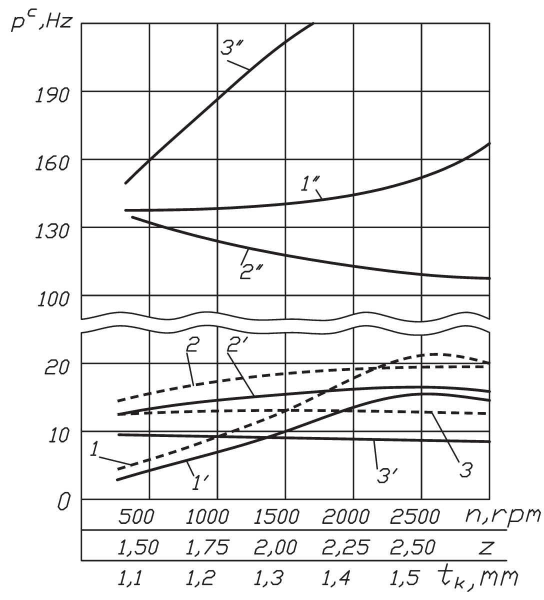

Fig.5. *Dependence of the natural frequencies of vibration damper (p^c) on its structural parameters and the number of revolutions, when the radius of the elastic ring R = 80 mm, m = 0.03 kg, n = 1500 rpm, z = 1.25, t_k = 1.1 mm, b = 20 mm (the solid lines are obtained by using the equations (33) and the dotted ones by using the simplified calculation (48)), where the curves 1, 1', 1'' – p^c = f(n), 2, 2', 2'' - p^c = f(z), 3, 3', 3'' – p^c = f(t_k), the index ' corresponds to the first frequency and the index " – to the second frequency (in the formula (33))*

damper. To a certain extent, such cases are described in [7] to [9] as well.

In this case, the stable positions of the dynamic balance of the ring are described by one of the following conditions:

a) if $z < 1$, $\alpha_1 - \alpha_2 = 0$,

$$\text{b) if } z > 1,\ \alpha_1 - \alpha_2 = \arccos \frac{C_{n1} - 4zm\omega^2}{C_{n1} - 2(1+z)m^2} \quad (46).$$

For the case b), the minimum cross-section of the ring (its axial moment of inertia) will be found from the following inequality:

$$I \geq \frac{\pi^2 - 8}{16\pi} \frac{(3z+1)m\omega^2}{E} \quad (47).$$

3 RESULTS

The possible combinations of the parameters are illustrated in Fig.4.

The frequency of resonance tuning of the vibration damper is:

$$p^c = \omega \sqrt{\frac{\frac{C_{n1}}{2m\omega^2}\left(\cos\alpha - \cos 2\alpha - \left[zk(1+k)\cos\alpha - (1+zk^2)\cos 2\alpha\right]\right)}{1+z+2zk(1-\cos\alpha)(1+k)}} \quad (48).$$

Fig. 5 illustrates some curves of natural frequencies.

The expression for the equivalent moment of inertia will be as follows:

$$I_e = 2mR^2 \frac{(A - B\sin^2\alpha)\left[B\sin^2\alpha\left(\frac{p}{\omega}\right)^2 + C\cos\alpha - B\cos 2\alpha - DE\right] - 4\sin^2\alpha(C - B\cos\alpha)^2}{A\left(\frac{p}{\omega}\right)^2 + C\cos\alpha - B\cos 2\alpha - DE} \quad (49),$$

where:

$$A = 1 + z + 2zk(1+k)(1-\cos\alpha)$$

$$B = 1 + zk^2$$

$$C = zk(1+k)$$

$$D = \frac{C_{n1}}{2m\omega^2}$$

$$E = \cos\alpha - \cos 2\alpha$$

The dependence of equivalent moment of inertia structural and performance parameters is presented in Fig.6. The limit value I_e in the low-frequency zone:

$$\lim_{p\to 0} I_e = 2mR^2\left[A - B\sin^2\alpha - \frac{8m\sin^2\alpha(C - B\cos\alpha)^2\omega^2}{2m(C\cos\alpha - B\cos 2\alpha)\omega^2 - C_{n1}E}\right] \quad (50),$$

in the high frequency zone:

$$\lim_{p\to\infty} I_e = 2BmR^2\sin^2\alpha\left(1 - \frac{B\sin^2\alpha}{A}\right) \quad (51).$$

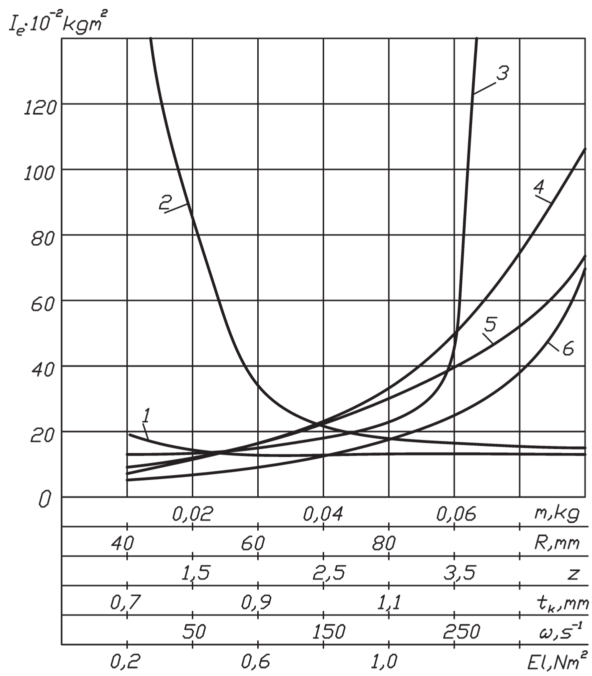

Fig. 6. *Equivalent moment of inertia of the damper, where 1 – $I_e = f(m)$, 2 – $I_e = f(R)$, 3 – $I_e = f(z)$, 4 – $I_e = f(t_k)$, 5 – $I_e = f(\omega)$, and 6 – $I_e = f(EI)$*

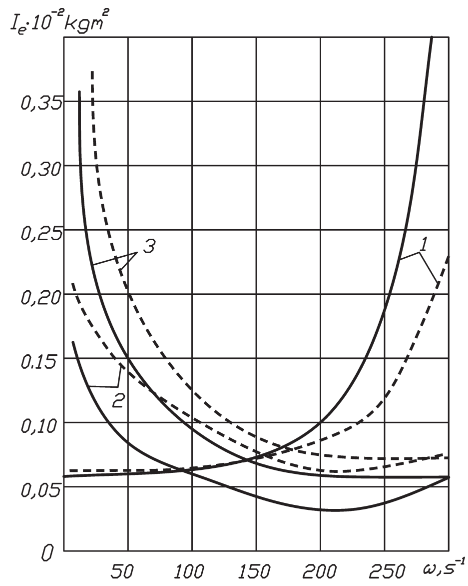

Fig.7. *Comparison of the efficiency of the damper for various estimated models (the dotted lines correspond to the approximate calculation), if R = 80 mm, m = 0.03 kg, z = 2.0, t_k = 1.2 mm, b = 20 mm, where 1 -* $\frac{p}{\omega} = 0{,}1,\ 2 - \frac{p}{\omega} = 1{,}0,\ 3 - \frac{p}{\omega} = 10.$

The comparison of the values of the equivalent moment of inertia, found from the equation (36) and the simplified calculation (49), is provided in Fig. 7.

4 CONCLUSIONS

The general results of the investigation are provided in the presented schemes of vibration dampers:

- the schemes a, a-1, a-2 (Fig. 1) are distinguished by natural stability, if $z < 1/3$. In such a case, the frames 3 may be designed in the shape of strings that only resist extension,
- if $z > 1/3$, a rigid forced stabilization can be achieved because of the elasticity of the frames 3,
- the vibration dampers, showed in the schemes b and c, can be stabilized by the introduction of a relevant elastic element, resistant to the deflection of the frames 3,
- the remaining schemes are distinguished by rigid stability and preserve their strict symmetry on the relevant rigidity of the elastic elements. The side stabilization of the extended elastic ring can be ensured by a connection of the deflected centrifugal pendulums 3 with the swivel parallelogram (see Fig. 1, b-1) or the replacement of centrifugal pendulums with symmetric elastic frames 3, situated at a certain angle with respect to the radius (see Fig. 1, b-2),
- in many cases the efficiency of a vibration damper may be increased by fixing the elastic ring on two symmetrically deflected centrifugal pendulums (Fig. 1, c). In such a case, the vibration damper is not a stable system; it is inclined to a non-symmetric sideways "deflection". The stabilization of the ring can be ensured if the deflected pendulums are elastic frames (see Fig. 1, c-1) or elastically fixed pendulums (see Fig. 1, c-2).
- the analytical investigation of the efficiency of the vibration damper, applying special sets of programmes, allows us to state that a vibration damper can be sufficiently precisely presented as a vibrating system with two degrees of freedom for most practically important ranges,
- in many cases the simplified calculation ensures sufficient accuracy,
- with an increase in the radius of the elastic ring, the critical frequencies of rotation of the vibration damper decrease,
- the value of the equivalent moment of inertia is mostly affected by the thickness of the elastic ring,
- the efficiency of a vibration damper increases with an increase in its natural frequencies.

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