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An Improved Form for Natural Convection Heat Transfer Correlations

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Abstract

Natural convection heat transfer correlations are usually in the form Nusselt{Rayleigh}, and occasionally in the form Nusselt{Rayleigh*}. Both forms are inconvenient because they oftentimes require indirect solution:

- When Nusselt{Rayleigh} correlations are used to calculate heat flux, solution is simple and direct. But when they are used to calculate boundary layer temperature difference, solution must be indirect—i.e. must be based on an indirect method such as iteration or trial-and-error.
- When Nusselt{Rayleigh*} correlations are used to calculate boundary layer temperature difference, solution is direct. But when they are used to calculate heat flux, solution must be indirect.

This manuscript describes an improved form for natural convection heat transfer correlations. The improved form allows direct solution for heat flux *and* for boundary layer temperature difference.

Included in this manuscript are graphical and analytical correlations in the improved form obtained by transforming Nu{Ra} correlations from the literature.

Introduction

In forced convection, the heat flux (q) is essentially proportional to the boundary layer temperature difference (ΔT). Therefore the heat transfer coefficient (h) is a *constant* coefficient—ie its value is independent of ΔT and q. Because h is a constant coefficient, correlations in the usual form Nu{Re,Pr} can be solved directly for q *and* for ΔT.

In natural convection, q is a nonlinear function of ΔT, and therefore h is a *variable* coefficient—ie its value is dependent on ΔT (or equally on q). Because h is a variable coefficient, correlations in the form Nu{Ra} cannot be solved directly for ΔT. They must be solved using an indirect method such as iteration or trial-and-error. Similarly, correlations in the form Nu{Ra*} cannot be solved directly for q, but must be solved using an indirect method.

This manuscript describes an improved form for natural convection heat transfer correlations. The improved form allows direct solution for q *and* for ΔT.

Applications in which Nu{Ra} and Nu{Ra*} correlations are solved directly

Nu{Ra} correlations are solved directly if the value of q is to be calculated. The solution is obtained as follows:

- Note that Nu{Ra} is $(hD/k)\{c_p\rho^2 g\beta\Delta T D^3/\mu k\}$.
- Calculate the value of $(c_p\rho^2 g\beta\Delta T D^3/\mu k)$ from the given information.
- Calculate the value of (hD/k) from the given Nu{Ra} correlation and the calculated value of $(c_p\rho^2 g\beta\Delta T D^3/\mu k)$.
- Calculate the value of h from the calculated value of (hD/k) and the given information.
- Calculate q from the calculated value of h, the given value of ΔT, and $q = h\Delta T$.

Nu{Ra*} correlations are solved directly if the value of ΔT is to be determined. The solution is obtained as follows:

- Note that Nu{Ra*} is $(hD/k)\{qc_p\rho^2 g\beta D^4/\mu k^2\}$.
- Calculate the value of $(qc_p\rho^2 g\beta D^4/\mu k^2)$ from the given information.

b	unspecified function of fluid flow rate, fluid properties, and geometry	Nu	Nusselt number hD/k (dimensionless)
Bu	symbol arbitrarily assigned to $qc_p\rho^2g\beta D^4/\mu k^2$; identical to NuRa and Ra* (dimensionless)	Pr	Prandtl number $c_p\mu/k$ (dimensionless)
c_p	specific heat, J/kgK	q	heat flux, W/m^2
D	diameter, m	Ra	Rayleigh number $c_p\rho^2g\beta\Delta TD^3/\mu k$ (dimensionless)
G	mass flow rate, kg/s	Ra*	modified Rayleigh number, RaNu (dimensionless)
g	gravity constant, m/s^2	Re	Reynolds number DG/μ (dimensionless)
h	heat transfer coefficient, W/m^{2}K	T	temperature, K
k	thermal conductivity, W/mK	β	temperature coefficient of volume expansion, K^{-1}
L	length, m	μ	dynamic viscosity, kg/m s
		ρ	density, kg/m^3

- Calculate the value of (hD/k) from the calculated value of $\{qc_p\rho^2g\beta D^4/\mu k^2\}$ and the given Nu{Ra*} correlation.
- Calculate the value of h from the calculated value of (hD/k) and the given information.
- Calculate ΔT from the calculated value of h, the given value of q, and ΔT = q/h.

Applications in which Nu{Ra} and Nu{Ra*} correlations cannot be solved directly

Nu{Ra} correlations cannot be solved directly for ΔT because neither $(c_p\rho^2g\beta\Delta TD^3/\mu k)$ nor (hD/k) can be calculated if ΔT is not included in the given information. An indirect solution such as the following is required:

- Note that Nu{Ra} is $(hD/k)\{c_p\rho^2g\beta\Delta TD^3/\mu k\}$.
- Select ΔT_1, an initial estimate of ΔT.
- Calculate $\{c_p\rho^2g\beta\Delta T_1D^3/\mu k\}$ from ΔT_1 and the given information.
- Calculate $(hD/k)_1$ from the given correlation and the calculated value of $\{c_p\rho^2g\beta\Delta T_1D^3/\mu k\}$.
- Calculate h_1 from $(hD/k)_1$ and the given information.
- Calculate ΔT_2 from the calculated value of h_1, the given value of q, and Equation (1).

$$\Delta T_2 = q/h_1 \tag{1}$$

- Iterate until convergence is obtained.
- If the solution diverges, select a different iteration scheme. Or select a different indirect method, such as trial-and-error.

Nu{Ra*} correlations cannot be solved directly for q because neither (hd/k) nor $(qc_p\rho^2g\beta D^4/\mu k^2)$ can be calculated if q is not included in the given information. An indirect solution analogous to the above is required.

The underlying problem with Nu{Ra} and Nu{Ra*}

The problem with Nu{Ra} may be seen by noting that

$$Nu \propto h \tag{2}$$

$$Ra \propto \Delta T \tag{3}$$

Relations (2) and (3) indicate that Nu{Ra} correlations are in the form

$$h = b\,f\{\Delta T\} \tag{4}$$

where b is a function of fluid properties, fluid flow rate, and geometry. The underlying problem with Nu{Ra} correlations is that they are in the form of Eq. (4), a form that allows direct solution for h if ΔT is given, but does not allow direct solution for h if q is given.

The problem with Nu{Ra*} may be seen by noting that

$$Ra^* \propto q \tag{5}$$

Relations (2) and (5) indicate that Nu{Ra*} correlations are in the form

$$h = b\,f\{q\} \tag{6}$$

The underlying problem with Nu{Ra*} correlations is that they are in the form of Eq. (6), a form that allows direct solution for h if q is given, but does not allow direct solution for h if ΔT is given.

An improved correlation form that allows direct solution for *both* q and ΔT

Equation (7) is in a form that allows direct solution for *both* q and ΔT because the left side is dependent on q but independent of ΔT, and the right side is dependent on ΔT but independent of q.

$$q = b\,f\{\Delta T\} \tag{7}$$

A dimensionless correlation in the form of Eq. (7) requires the following:

- A dimensionless group that is dependent on ΔT and independent of q.
- A dimensionless group that is dependent on q and independent of ΔT.

Ra satisfies the first requirement. The second requirement is satisfied by the dimensionless group $qc_p\rho^2g\beta D^4/\mu k^2$. Let us arbitrarily assign the symbol Bu to this dimensionless group. (Note that Bu is identical to Ra*, the product of Ra and Nu. Also note that $q\rho^2 g\beta D^4/\mu^2 k$ also satisfies the second requirement.)

Since Bu is dependent on q and independent of ΔT, and since Ra is dependent on ΔT and independent of q, Bu{Ra} correlations are in the form of Eq. (7).

Transforming graphical Nu{Ra} and Nu{Ra*} correlations to the form of Eq. (7)

Graphical Nu{Ra} correlations are transformed to the form of Eq. (7) in the following manner :

- List the Nu,Ra coordinates on a spreadsheet.
- Multiply each Nu coordinate by the corresponding Ra coordinate to obtain Bu,Ra coordinates.
- Prepare a Bu{Ra} graphical correlation by plotting the Bu,Ra coordinates.

Figure 1 is a Bu{Ra} chart obtained by transforming a Nu{Ra} chart that appears in McAdams[1] and also in Kreith and Bohn [2]. Table 1 contains the spreadsheet calculations for the transformation. The Nu,Ra coordinates in Table 1 are those listed on the Nu{Ra} chart in Kreith and Bohn [2].

Graphical Nu{Ra*} charts are transformed to the form of Eq. (7) as follows:

- List the Nu,Ra* coordinates on a spreadsheet.
- Divide each Ra* coordinate by the corresponding Nu coordinate to obtain Ra coordinates.
- Multiply each Nu coordinate by the corresponding Ra coordinate to obtain Bu coordinates.
- Prepare a Bu{Ra} graphical correlation by plotting the Bu,Ra coordinates.

Transforming analytical Nu{Ra} and Nu{Ra*}correlations to dimensionless correlations in the form of Eq. (7)

Analytical Nu{Ra} correlations are transformed to dimensionless correlations in the form of Eq. (7) by multiplying both sides of Nu{Ra} correlations by Ra or Gr. Multiplying by Ra results in dimensionless Bu{Ra} correlations. Table 2 lists several Bu{Ra} correlations obtained by transforming Nu{Ra} correlations from the literature.

Analytical Nu{Ra*} correlations are transformed to dimensionless correlations in the form of Eq. (7) in the following manner:

- Substitute RaNu for Ra* to obtain Nu{RaNu} correlation.
- Separate Ra and Nu to obtain Nu{Ra} correlation.
- Multiply both sides of the Nu{Ra} correlation by Ra to obtain a Bu{Ra} correlation in the form of Eq. (7).

Improving the reading precision of Bu{Ra} charts

Figure 1 illustrates that Bu{Ra} charts of reasonable size cannot be read with acceptable precision because of the very large range in Bu. However, the precision of charts of reasonable size can be made acceptable by plotting (log Bu – log Ra) vs (log Bu or log Ra) as in Figure 2. (Table 1 lists the calculations that underlie Figure 2.) Figure 2 is read in the following manner:

- From the given information, calculate log Ra or log Bu. This value establishes the horizontal coordinate.
- If log Ra was calculated in the first step, determine (log Bu − log Ra) from the curve marked Ra. Determine log Bu from log Bu = log Ra + (log Bu – log Ra).
- If log Bu was calculated in the first step, determine (log Bu – log Ra) from the curve marked Bu. Determine log Ra from log Ra = log Bu − (log Bu – log Ra).

The impact of film temperature

The fluid properties in natural convection heat transfer correlations are usually evaluated at the film temperature—i.e. at the average temperature in the boundary layer. Since the film temperature usually cannot be determined from the given information, an initial estimate of film temperature must be made, and verified by the subsequent analysis. Thus the use of Nu{Ra} and Bu{Ra} correlations generally involves iteration on the film temperature.

However, the effect is usually so small that the first estimate of film temperature yields a result of sufficient accuracy, and no iteration on film temperature is required.

Selecting a name for the group ($qc_p\rho^2 g\beta D^4/\mu k^2$)

The group ($qc_p\rho^2 g\beta D^4/\mu k^2$) is often assigned the name “modified Rayleigh number” and the symbol Ra*. Since Bu and Ra* are identical, Bu{Ra} correlations are also Ra*{Ra} correlations. Since Ra*{Ra} seems poor terminology, it would be desirable to assign a different name and symbol to the group ($qc_p\rho^2 g\beta D^4/\mu k^2$).

Lienhard and Lienhard [3] discuss the name and symbol usually assigned to ($qc_p\rho^2 g\beta D^4/\mu k^2$):

> *To avoid iterating, we need to eliminate ΔT from the Rayleigh number. We can do this by introducing a modified Rayleigh number, Ra*, defined as Ra* = RaNu.*

In the application discussed herein, a modified Nu is created by multiplying Nu by Ra. Based on the reasoning that leads to Ra*, the group ($qc_p\rho^2 g\beta D^4/\mu k^2$) as used herein would be a “modified Nusselt number”, symbol Nu*.

It would be misleading to use “modified Nusselt number” and Nu* for the group ($qc_p\rho^2 g\beta D^4/\mu k^2$). Nusselt number is closely identified with h, and the group ($qc_p\rho^2 g\beta D^4/\mu k^2$) does not contain h.

It therefore seems advisable to assign a new name and symbol to the group ($qc_p\rho^2 g\beta D^4/\mu k^2$).

Conclusions

- Bu{Ra} should replace both Nu{Ra} and Nu{Ra*} because Bu{Ra} allows direct solution for *both* q and ΔT, whereas Nu{Ra} and Nu{Ra*} do not.

- Graphical and analytical Nu{Ra} and Nu{Ra*} correlations are readily transformed to Bu{Ra} correlations.
- The group ($qc_p\rho^2 g\beta D^4/\mu k^2$) should be assigned a new name and a new symbol.

References

[1] McAdams, W.H., (1954), *Heat Transmission*, p. 176, McGraw-Hill, New York

[2] Kreith, F. and Bohn, M.S., (1986), *Principles of Heat Transfer*, p. 251, Harper and Row, New York

[3] Lienhard, J.H. IV, and Lienhard, J.H. V, (2003), *A Heat Transfer Textbook*, version 1.21, p. 424, Phlogiston Press, Cambridge

[4] Holman, J.P. (1981), *Heat Transfer*, p. 275, McGraw-Hill, New York

Table 1 Generation of coordinates used in Figures 1 and 2

Nu[1]	Ra[1]	Bu = NuRa	log Ra	log Bu	(log(Bu) – log(Ra))
0.49	**1.E-04**	4.90E-05	-4	-4.31	-0.31
0.55	**1.E-03**	5.50E-04	-3	-3.26	-0.26
0.66	**1.E-02**	6.61E-03	-2	-2.18	-0.18
0.84	**1.E-01**	8.41E-02	-1	-1.08	-0.08
1.08	**1.E+00**	1.08E+00	0	0.03	0.03
1.51	**1.E+01**	1.51E+01	1	1.18	0.18
2.11	**1.E+02**	2.11E+02	2	2.32	0.32
3.16	**1.E+03**	3.16E+03	3	3.50	0.50
5.37	**1.E+04**	5.37E+04	4	4.73	0.73
9.33	**1.E+05**	9.33E+05	5	5.97	0.97
16.2	**1.E+06**	1.62E+07	6	7.21	1.21
28.8	**1.E+07**	2.88E+08	7	8.46	1.46
51.3	**1.E+08**	5.13E+09	8	9.71	1.71
93.3	**1.E+09**	9.33E+10	9	10.97	1.97

[1] **From Kreith & Bohn [2], p. 251**

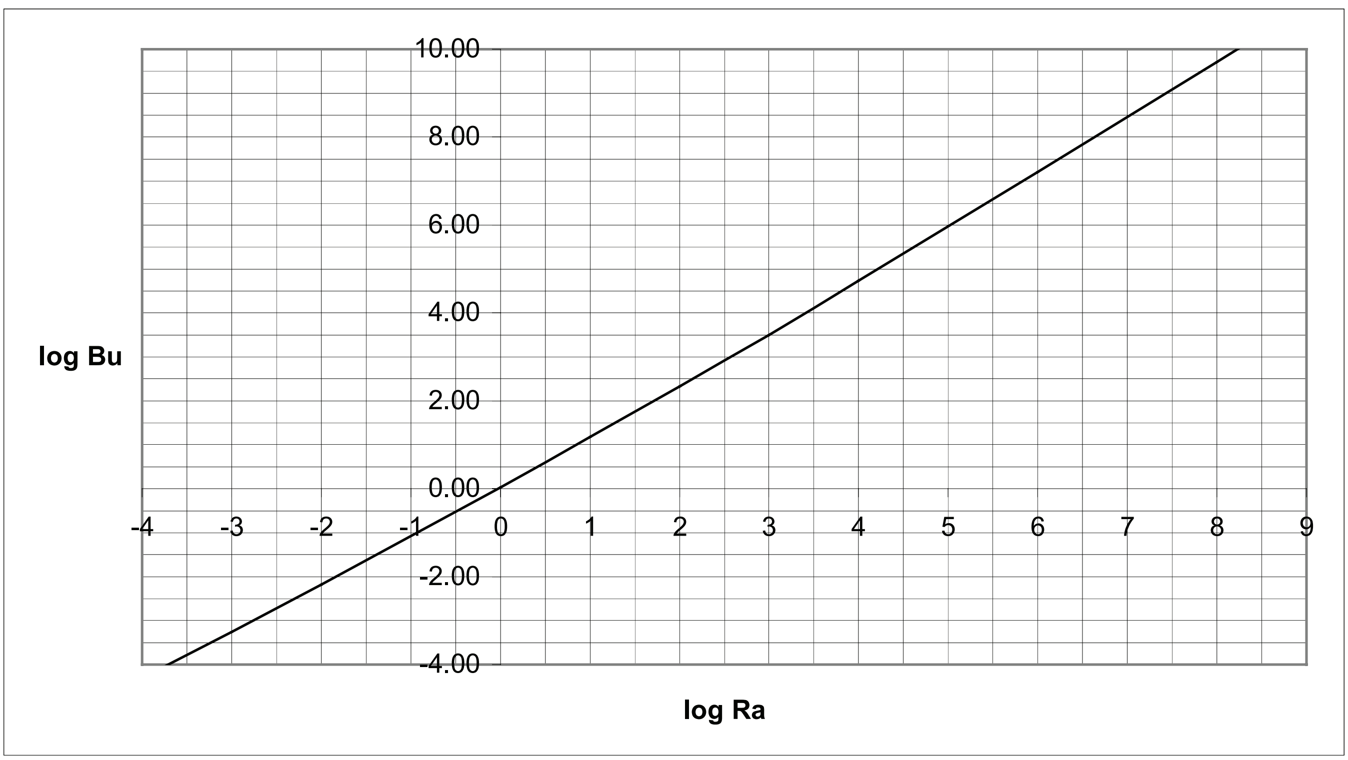

Figure 1 Natural convection heat transfer from horizontal cylinders

Table 2 Transformation of literature correlations

Literature Correlation	Dimensionless transformation	Dimensioned transformation
Nu = .10 Ra^1/3 Holman [4] p 275	Bu = .10 Ra^4/3	q = .10 (ΔTk/D)Ra^1/3
Nu = .0210Ra^2/5 Kreith&Bohn [2] p 253	Bu = .0210 Ra^7/5	q = .021(ΔTk/L)Ra^2/5
Nu = .58 Ra^1/5 Lienhard & Lienhard [3] p 423	Bu = .58 Ra^6/5	q = .58 (ΔTk/L)Ra^1/5

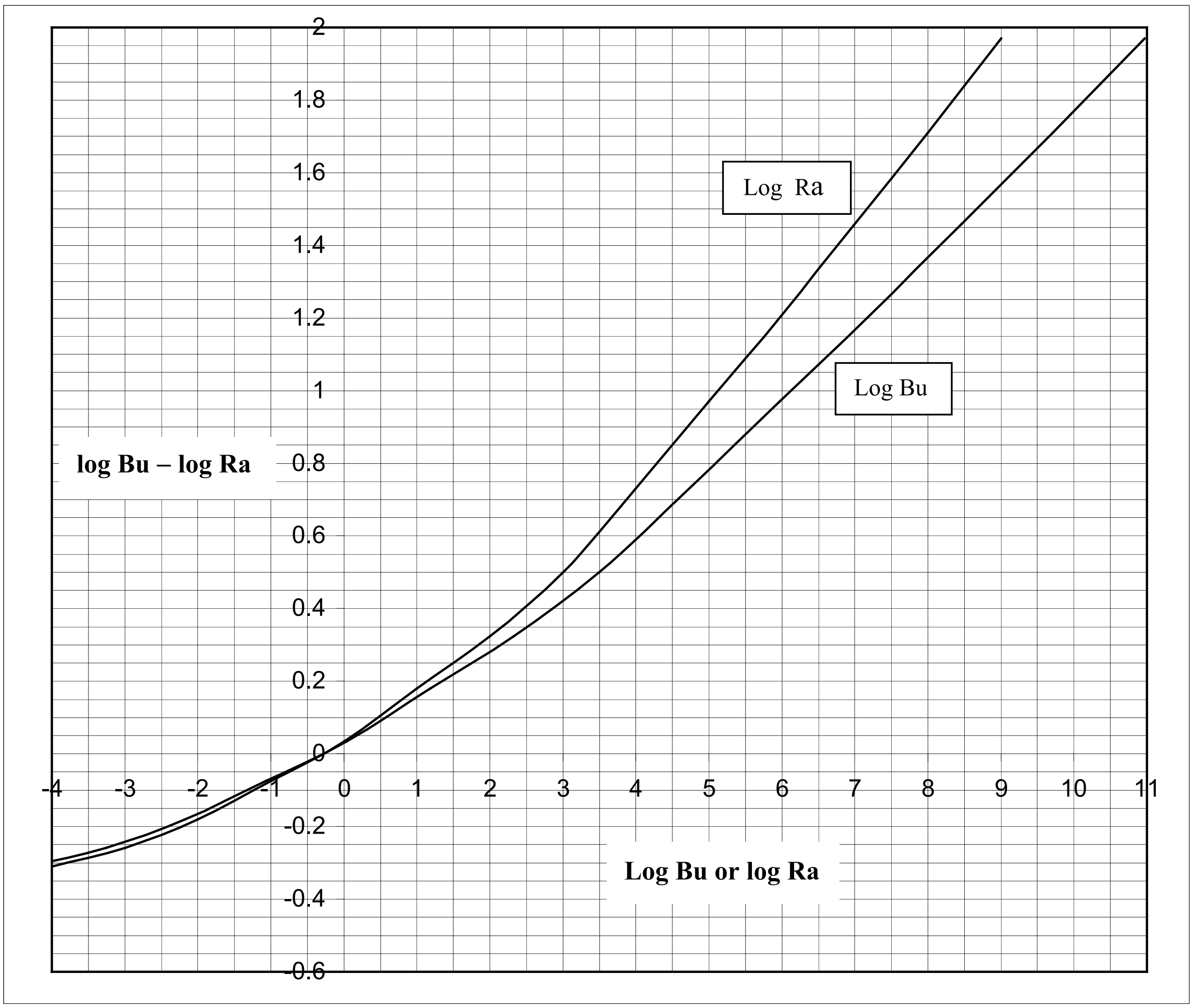

Figure 2 Natural convection heat transfer from horizontal cylinders