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Določitev tornega količnika v brazdah z napetostno funkcijo

Determination of the Friction Coefficient of Groove Forms Using the Stress-Function Method

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Največja sila trenja, ki lahko nastane v brazdah, je funkcija dejanskega tornega količnika med vrvjo in brazdo. Torni količnik je predstavljen na različne načina za vsako vrsto oblike brazde. V prispevku je prikazan obrazec za porazdelitev tlaka na stični površini okrogle in/ali brazde v obliki črke U. Z metodo napetostne funkcije smo določili tudi torni količnik brazd.

(Ključne besede: žleb na obodu koluta, torni količnik, napetostne funkcije)

The maximum traction that can be developed in sheave grooves is a function of the actual coefficient of friction between the rope and the groove. The coefficient of friction is presented in different ways for every type of groove form. In this paper an expression for the pressure distribution on the contact surface of round and/or U-shaped grooves is obtained, and the friction coefficient of the groove forms is determined by the stress-function method.

(Keywords: rope groove, friction coefficient, stress function method)

0 INTRODUCTION

Driving sheaves are widely employed to transmit power to the ropes that drive elevators, cable cars, funiculars, etc. The groove form favourably increases the effective coefficient of friction between the rope and the groove at the expense of increasing the pressure and wear on the groove surface. The trade off between the traction produced and the pressure causing the abrasion of the groove may be best explained by the concept of shape factors for the coefficient of friction of U-shaped grooves. During normal operations sheave and drum grooves are under constant pressure.

The groove form affects the magnitude of the tractive force on the driving sheaves for power transmission. The contact area between the rope and the groove is smaller with U-shaped grooves than with round grooves since the rope loses contact with the groove where the undercut is machined. Thus an undercut groove provides a tighter gripping action due to an increased groove pressure, and its traction capability is greater than that of a round groove. However, a round groove has a longer rope life and a lower level of noise because of the lower groove pressure at high speeds. When the problem of insufficient traction arises with a round groove it should be noted that it can be overcome by increasing the angle of wrap, by changing the groove form to a U-shaped groove with an appropriate shape or by using a material with a higher coefficient of friction [1].

The U-shaped groove sheave, found predominantly in older installations, is the sheave of choice for optimum rope life. Its large size, when compared with the drive sheave diameters in newer installations, in combination with its supportive grooves minimizes the amount of abrasion and fatigue. The support given to the rope by the groove is illustrated in Fig.1. The groove cradles the rope, resulting in low groove pressures that allow the wires and strands to move about freely while the rope is

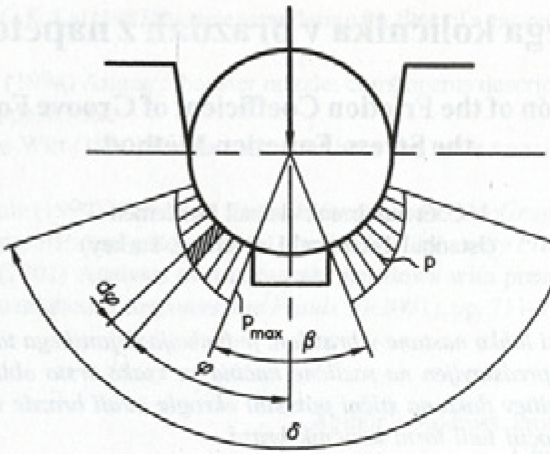

Fig.1. *The U-shaped groove form*

operating. Also important to the U-shaped groove's success in achieving excellent rope life is the relative diameter of the sheave required to maintain traction. In general, an undercut U groove, a modern type of groove, increases the traction by increasing the groove pressures. The beauty of these groove types is that the diameter of the sheave utilizing this modern groove design can be reduced.

The maximum traction that can be developed in the sheave grooves is a function of the effective coefficient of friction between the rope and the groove and the angle of contact that the rope makes with the circumference of the sheave (known as the angle of wrap). The groove form can favourably increase the effective coefficient of friction between the rope and the groove since the radial force due to the rope tension produces greater normal and frictional forces acting along the area of contact given by the shape of the groove [2].

Airy introduced his stress function as a device for solving certain problems in linear elastostatics for homogenous isotropic bodies. There are many published studies on the Airy stress function applied to solid mechanics ([3] to [10]). In this paper, to obtain the pressure distribution on the contact surface of undercut groves, the Airy stress-function method was used. The effects of the groove geometry and the angle of wrap on the traction were investigated and tabulated with the ratio of the forces for different angle values by Imrak and Ozkirim [1].

In this paper the application of the Airy stress-function method for determining the shape factors for the coefficient of friction for both round and undercut grooves is presented. The effect of the changes in the groove angle and the undercutting angle on the coefficient of friction and the traction are also studied.

1 BASIC EQUATIONS

Due to the existence of axial symmetry related to the geometry and specific pressure distribution along the boundary of a U-shaped groove it is preferable to employ polar coordinates rather than the Cartesian system and to assume that the stress condition is one of plane stress. The geometry and the loading of an undercut groove are illustrated in Fig. 1. The angle of the outer normal lines of the contact area δ may have a maximum value 180°; the angle of the undercutting β must not be greater than 105°, as shown in Fig. 1.

Due to the normal force and the symmetrical loading in the traction drive, only the plane stress is employed. In the plane-stress state the following relations are valid as long as the mass forces are negligible. By plugging in the equations associated with Airy's stress function into the equilibrium condition we can illustrate that the functions do indeed satisfy the equilibrium. The Airy function is chosen so as to satisfy the equilibrium equations automatically. The equation of compatibility, which means that the body must be physically pieced together in terms of Airy's stress function, is [11]:

$$\nabla^2\phi = \left(\frac{\partial^2}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2}{\partial\varphi^2}+\frac{1}{r}\frac{\partial}{\partial r}\right)\left(\frac{\partial^2\phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}\right)=0 \qquad (1)$$

where ∇^2 is the Laplace operator. The Airy stress function is developed and used to solve classic two-dimensional problems fundamental to stress analysis. The Airy stress-function approach works best for problems where a solid is subjected to prescribed tractions on its boundary, rather than prescribed displacements [8].

Using symmetrical straining we obtain $\partial/\partial r = d/dr$. Therefore, Eq.(1) becomes:

$$\left(\frac{d^2}{dr^2}+\frac{1}{r}\frac{d}{dr}\right)\left(\frac{d^2\phi}{dr^2}+\frac{1}{r}\frac{d\phi}{dr}\right)=0 \qquad (2).$$

Thus, the solution reduces to finding a solution of the differential equation of compatibility that satisfies the boundary conditions of the problem.

To obtain a solution of these two equations, one can write an arbitrary function $\phi = \phi(r,\varphi)$. This arbitrary function is called the Airy stress function [12]. In the case of plane stress and in the event that the body forces are negligible the differential equations of equilibrium in polar coordinates are as follows:

$$\sigma_r=\frac{\partial^2\phi}{\partial t^2} \qquad \sigma_t=\frac{\partial^2\phi}{\partial r^2} \qquad \tau=-\frac{\partial^2\phi}{\partial r\partial t} \qquad (3)$$

and the boundary conditions are $\sigma_r = p$, $\sigma_t = 0$, and $\tau = 0$. $dt = r\sin d\varphi \approx r\,d\varphi$ with respect to the radial derivative. If the second derivative is evaluated in the tangential direction, we advance in the r direction by dr, then the angular change is $d\varphi$. In the case of plane stress and in the event that the body forces are negligible the differential equations of equilibrium in polar coordinates are as follows [11]:

$$\begin{aligned}&\frac{\partial\sigma_r}{\partial r}+\frac{1}{r}\frac{\partial\tau}{\partial\varphi}+\frac{\sigma_r-\sigma_\varphi}{r}=0\\&\frac{1}{r}\frac{\partial\sigma_\varphi}{\partial\varphi}+\frac{\partial\tau}{\partial r}+\frac{2\tau}{r}=0\end{aligned} \qquad (4).$$

The usual method for solving these equations is by introducing a single new function $\phi = \phi(r,\varphi)$, commonly known as Airy's stress function, which satisfies Eqs. (4) and is related to the stresses as follows:

$$\begin{aligned}&\sigma_r=\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}+\frac{1}{r}\frac{\partial\phi}{\partial r} \qquad \sigma_\varphi=\frac{\partial^2\phi}{\partial r^2}\\&\tau=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\varphi}\right)=\frac{1}{r^2}\frac{\partial\phi}{\partial\varphi}-\frac{1}{r}\frac{\partial^2\phi}{\partial r\partial\varphi}\end{aligned} \qquad (5).$$

One can assume the stress function $\phi(r,\varphi) = C F_r r \varphi \sin\varphi$, where F_r denotes the radial force, the distribution C is a constant, r is the radius of the rope and φ is the angle. It can be easily verified that the stress function satisfies the equation of compatibility. Thus, it represents the true stress function. For equilibrium, the stress distribution obtained from Eqs.(5) is:

$$\sigma_r=\frac{1}{r^2}\frac{\partial^2\phi}{\partial\varphi^2}+\frac{1}{r}\frac{\partial\phi}{\partial r}=\frac{2CF_r}{r}\cos\varphi \qquad \sigma_\varphi=\frac{\partial^2\phi}{\partial r^2}=0$$

$$\tau=-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\phi}{\partial\varphi}\right)=\frac{1}{r^2}\frac{\partial\phi}{\partial\varphi}-\frac{1}{r}\frac{\partial^2\phi}{\partial r\partial\varphi}=0 \qquad (6).$$

The radial force distribution F_r per unit length along the circumference of the sheave, induced by the tangential rope tension S, is:

$$F_r=\frac{dN(=Sd\varphi)}{\frac{D}{2}d\varphi}=\frac{2S}{D} \qquad (7),$$

where D is the pitch diameter of the sheave. The boundary conditions along the area of contact, for $r = d/2$, in the radial plane at an angle of α are expressed by:

$$-dN=-F_r\frac{D}{2}d\alpha=2\int_{\beta/2}^{\delta/2}\sigma_r\cos\varphi\, dA \qquad (8),$$

where dA represents an infinitesimal contact area with the dimensions $(D/2)d\alpha$ along the arc of the wrap of the rope on the sheave and $(d/2)d\varphi$ in the radial plane of the sheave, where d is the rope diameter, hence $dA = d\,D\,d\varphi\,d\alpha/4$.

Substituting dA and Eq.(6) into Eq. (8), we obtain:

$$-F_r\frac{D}{2}d\alpha=2\,D\,d\alpha\,C\eta\int_{\beta/2}^{\delta/2}\cos^2\varphi\, d\varphi \qquad (9),$$

and then Eq.(9) reduces to:

$$C\int_{\beta/2}^{\delta/2}\cos^2\varphi\, d\varphi=-\frac{1}{4} \qquad (10).$$

The constant C can now be determined by integrating Eq.(10) and solving it for C so as to fulfil the last of the boundary conditions:

$$C=\frac{-1}{\delta-\beta+\sin\delta-\sin\beta} \qquad (11).$$

By putting Eq.(11) and Eq. (6) into Eq. (7), the field of stress existing in the radial plane within the sheave at an angle of α becomes:

$$p=|\sigma_r|=\frac{8S\cos\varphi}{Dd(\delta-\beta+\sin\delta-\sin\beta)} \qquad (12),$$

where S is the rope tension at the point on the arc of the rope with an angle α.

2 SHAPE FACTOR FOR THE FRICTION COEFFICIENT

The drive traction force is initiated by the friction between the ropes and the sheave grooves in traction. The maximum traction that can be developed in the sheave groove is a function of the

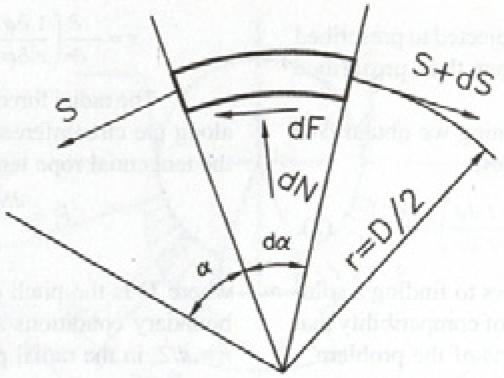

Fig.2. *The free body diagram of an indefinitely small element of the rope*

coefficient of friction between the rope and the groove and the angle of contact that the rope makes with the circumference of the sheave.

The groove form favourably increases the actual coefficient of friction between the rope and the sheave. Considering the equilibrium condition of an indefinitely small element of the rope shown in Fig.2 when the rope is about to slide, the elementary tangential friction force dF developed by the radial force dN can be obtained as follows:

$$dF = 2 \int_{0}^{\delta/2} \mu_a \, p \, dA \qquad (13).$$

Substituting dA, Eq. (12) and into Eq. (13), we obtain:

$$\mu_{eff} S \, d\alpha = \frac{4 \mu_a S}{(\delta - \beta + \sin\delta - \sin\beta)} \int_{\beta/2}^{\delta/2} \cos\varphi \, d\varphi \, d\alpha \qquad (14).$$

After the rearrangement and integration the final expression becomes:

$$\mu_{eff} = 4 \, \mu_a \, \frac{\sin\frac{\delta}{2} - \sin\frac{\beta}{2}}{(\delta - \beta + \sin\delta - \sin\beta)} \qquad (15).$$

The shape factor for the coefficient of friction can be defined as:

$$a = 4 \, \frac{\sin\frac{\delta}{2} - \sin\frac{\beta}{2}}{(\delta - \beta + \sin\delta - \sin\beta)} \qquad (16).$$

The shape factors for the coefficient of friction are plotted in Fig. 3. The figure shows how it changes with the changes in the angle of the outer normal lines of the contact area, δ and the angle of undercutting, β. The shape factor for the coefficient of friction gets its maximum value, i.e., $a = 4/\pi$, when the angle δ becomes 180° and the groove is round.

3 CONCLUSIONS

The groove form favourably increases the effective coefficient of friction between the rope and the groove since the radial force due to the rope

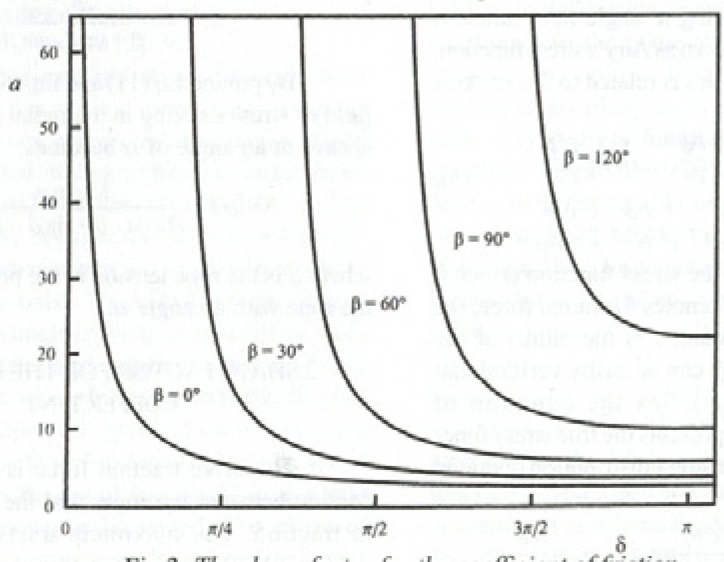

Fig.3. *The shape factor for the coefficient of friction*

tension produces greater normal and frictional forces acting along the area of contact given by the shape of the groove. Therefore, this work introduces the concept of shape factors for the coefficient of friction for U-shaped grooves and also derives it by means of the Airy stress-function method.

From a careful analysis it can easily be seen that when the angle of the groove decreases the traction improves, but also so does the specific pressure and resultant wear of both the grooves and the ropes. The round groove gives lower traction but a longer rope life, lower specific pressure and lower degree of noise than the undercut. When it is essential to use round grooves the traction capability can still be improved by using non-metallic groove liners with a high coefficient of friction. It is also advisable that the angle of the undercutting should be under 90°, and must not be greater than 105° whenever undercut grooves are in use.

4 SYMBOLS

a	shape factor for the coefficient of friction
A	contact area
d	rope diameter
D	pitch diameter of the sheave
F	tangential friction force
F_r	radial force distribution
N	radial force
S	rope tension
α	angle of wrap
β	angle of undercutting
ϕ	Airy's stress function
δ	angle of the outer normal lines of the contact area
μ_{eff}	effective coefficient of friction
μ_a	coefficient of friction

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