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MICROSCALE NONEQUILIBRIUM HEAT TRANSFER IN TECHNOLOGIES USING INTENSIVE HEAT SOURCES

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ABSTRACT

Microscale nonequilibrium heat conduction appears in all states of matter including solids, liquids, gases and plasmas where heat is carried by phonons, atoms, molecules, electrons or photons. A unified model of diffusion, wave and parallel conduction is introduced. Internal parameters and conditions of unambiguity describing the nonequilibrium are discussed from the physical point of view. Nonequilibrium heat transfer applications in technologies of microscale heating of materials by energetical photons, electrons and ions are presented.

1. INTRODUCTION

The development of new industrial technologies and urgency of solutions of complicated physical processes in materials require new approaches. It is the question of higher demands on the knowledge of physical substance of the processes and its mathematical description. Closer investigation on the character of heat transfer processes reveals that the usually applied macroscale approach is, in some cases, not sufficient and incorrect. Laws of equilibrium thermodynamics, used as initial basis for theoretical elaboration on the description of physical processes, show their limited applicability in the new technologies. However, the application of laws of nonequilibrium thermodynamics requires changes in the macroscale approach to the processes. The current development of solid state physics and mathematical physics enables introduction of such a microscale approach.

The state-of-the-art theoretical study of non-equilibrium heat transfer processes is actually summarized in several articles. The review articles (Joseph & Preziosi, 1989) and (Joseph & Preziosi, 1990) introduce a list of published works on non-equilibrium heat conduction also called the wave or non-fourier heat conduction. Thermal shocks in relation to the thermal wave theory are discusses in (Tzou, 1992). The main contribution represents explanations of thermal deformation around the thermomechanical wavefront and applications of damage mechanics in prediction of microcracks origination and propagation. Article (Tien & Chen, 1994) contains theoretical explanations on microscale conductive and radiative heat transport and challenges of practical application of microscale heat transfer. The article (Özisik & Tzou, 1994) deals with categorization of publications on the thermal wave theory complemented with examples of thermal shock phenomena, thermal resonance and propagation of heat waves. The review (Duncan & Peterson, 1994) also summarizes development in microscale heat transfer. The first comprehensive publication with the aim of unification the microscale and macroscale theory of heat conduction appears the book (Tzou, 1997) introducing the lagging theory. Various microscale heat transfer processes are described in relation to non-equilibrium thermodynamics. Results of published experimental evidences on non-fourier heat conduction are extensively discussed including measurement of thermal pulses in liquid helium, pulsed laser heating of metals or heat transfer in non-homogeneous materials. Mathematical representation of non-equilibrium thermomechanical processes is shown there together with defect propagation through the material.

It should be noted that almost all the published literature, excluding several rare exceptions, deal with mathematical theory of microscale or nonequilibrium heat transfer. Deeper insight into physics of the processes is often missing. Moreover, development of the theory to industrial applications appears still insufficient from the engineering point of view. It also confirms the fact that the majority of applications including computational systems are based on classical heat conduction theory expressed by the Fourier law of diffusion conduction.

This paper proceeds from the previously published work of (Honner & Kuneš, 1999) where a unified description of two physically different wave diffusion and parallel nonequilibrium heat conduction has been presented. Dimensionless criteria specifying the nonequilibrium state of the material and the heat propagation character have been introduced to find application regions of the individual modes of heat propagation.

The aim of the paper is to demonstrate the applicability of the nonequilibrium heat transfer processes in technologies of material microscale heating by energetical photons, electrons and ions.

2. PHYSICS OF MICROSCALE HEAT TRANSFER

Heat transfer processes in material microstructure despite their difference can be divided into two different groups - wave diffusion heat conduction and parallel heat conduction.

2.1 Wave diffusion heat conduction

Wave heat conduction proceeds from the wave behavior of elementary particles, heat carriers, in the microstructure of materials. Diffusion conduction arises from elastic and inelastic collisions, when the original direction of propagation is lost and the finite speed of heat carriers can be changed. Relaxation time of the material τ_q represents the inelastic collisional process of the thermal wave decay when the energy of the wave is transformed into the internal energy of the material. Thermalization time τ_T represents the effect of elastic collisions, when thermal wave momentum is conserved however the velocity distribution of the energy carriers is produced. Both processes lead to the diffusion character of heat conduction and bring the material to steady state. Values of both times specify the time-range of nonequilibrium wave heat conduction. Their ratio then describes superior heat propagation character.

The mathematical description of the wave diffusion heat conduction and the dimensionless parameters specifying the nonequilibrium state of the material and the heat propagation character has been summarized by (Honner & Kuneš, 1999). Further details can be found in (Honner, 1999) or (Tzou, 1997).

Phonon heat transfer. Heat is carried by elastic waves - phonons in solids with more or less periodic arrangement of atoms in the lattice. Phonon propagates in a certain direction with a certain limited velocity. The phonon mean free path is mostly determined by the geometrical scattering, means scattering on geometrical lattice nonperfections, and by collisions with the other phonons.

The physical nature of the phonon-phonon interaction can be explained as the following: The presence of the first phonon causes a periodic elastic deformation of the lattice, which in time and space changes its elastic properties. The second phonon feels the modulation of the elastic properties and is scattered with origination of the third phonon similarly as the second phonon would be scattered by a moving three-dimensional lattice (Kittel, 1976).

There are two basic phonon scattering mechanisms in solids: R-processes (umklapp scattering) and N-processes. Both two are dependent on frequency or energy of phonons. The role of R- and N-processes in creation of thermal resistance of the material and in equilibration of the phonon system is essentially different. R-processes are the processes causing the loss of momentum and thus creating thermal resistance. However they destroy momentum selectively as a function of phonon state (resonance, etc). N-processes are the momentum conserving processes. They do not directly contribute to thermal resistance. However they contribute indirectly because they act as an intermediate to interchange the phonon states.

The processes of the geometrical scattering on the lattice defects have the same effect on phonon heat conduction as the R-processes.

Mathematical theory of phonon heat transfer incorporating the effect of elastic and inelastic collisions of phonons in solids has been developed by (Guyer & Krumhansl, 1966). And further discussed by (Tzou, 1997).

Electron heat transfer. In metals, heat is carried by free electrons as well as by phonons. During the propagation the electron interact with the other electrons and with the vibrating ions arranged in the lattice.

The interaction among the electrons comes from the electrostatic forces. However due to the Pauli exclusion principle and shielding of the coulombic interaction the electron mean free path for the electron-electron collision is relatively long (10^4 Å) compared with the typical distance between two electrons (2 Å). As for the thermal resistance of metals the electron-phonon collisions are the important processes for determination of thermal conductivity values.

Electron-phonon scattering processes redistribute the electron occupation in individual energy levels. During a collision electron is transferred from one state to another state accompanied by a phonon emission or absorption. Such processes can be divided into two groups as elastic and inelastic electron-phonon scattering.

The elastic scattering does not cause the energy exchange between the electron and the lattice but it restores space homogeneity of electrons. This process determines the electron thermal conductivity.

On the other hand, the inelastic scattering causes the energy exchange between the electron and the lattice. If the electron temperature is higher then the lattice temperature, electrons pass from higher energy levels to lower ones and the energy is transferred from electrons to the lattice. If the electron temperature is lower than the lattice temperature, the process works in the opposite direction. This inelastic electron-phonon scattering has no effect on thermal resistance but it acts as an internal heat transfer discussed later in connection with the non-equilibrium parallel heat conduction.

Radiative heat transfer. The radiation is understood as wave transfer of electromagnetic energy by photons with transverse vibration in relation to the direction of their propagation. Photon of a certain wavelength emitted by an atom or molecule propagates straightforward in a certain direction as far as it is absorbed. The direction of the newly emitted photon has no connection with the direction of the absorbed photon. If the photon mean free path is relatively short compared to the considered thermal system dimension, the radiative heat transfer can be effectively described by the diffusion equation satisfying the Fourier law and photon thermal conductivity can be defined.

Molecular heat transfer. Freely propagating atoms or molecules in gases undergo collisions which set the velocity distribution and which create the thermal resistance. The result of the collisions is the diffusion character of heat and mass transfer. However if the mean free path is comparable with dimensions of the considered thermal system, for example at low pressures, the simple diffusion consideration on the heat, mass and momentum transfer does not make a sense. Intensity of molecular transfer in different directions is not equal and the radiative or wave nature of the particle propagation appears dominant.

2.2 Parallel heat conduction

In non-homogeneous materials energy can be carried in several ways or by means of different heat carriers that constitute their own thermal subsystems in the material. Heat conduction in the framework of the subsystem is generally represented by the wave-diffusion heat conduction model. The thermal bond among the subsystems is expressed in the value of the coefficient of internal heat transfer. Two or more subsystems can occur in their own thermal equilibrium characterized by the temperature of the subsystem, however the material in its entirety can occur in nonequilibrium. The state of the material is then characterized by different temperatures of individual subsystems caused by different amount of energy in the subsystems. Similarly to wave heat conduction, relaxation and thermalization times and in addition parallel thermal diffusivity are introduced. Their values express the way of parallel heat propagation and time-range of nonequilibrium state of the material.

Parallel heat propagation manifests itself in the cases of higher differences between thermal properties of the components and in the cases of weak thermal exchange among the components.

The mathematical description of the parallel heat conduction and the dimensionless parameters specifying the nonequilibrium state of the material and the heat propagation character has been summarized by (Honner & Kuneš, 1999). Details can be found also in (Honner, 1999) or (Tzou, 1997).

Electron and phonon in metals. Heat transfer in metals can serve us as a good example for elucidation of the parallel heat conduction. As was discussed in previous chapter on the wave diffusion conduction, heat is carried by electrons as well as by phonons in metals. And additionally there is some energy exchange between the two.

Assume the electron heat transfer is mathematically represented by diffusion heat conduction equation. Also the phonon heat transfer can be expressed by diffusion heat conduction equation. The energy exchange in the inelastic electron-phonon collisions is expressed as a term that couples the two equations together. If one combine the two equations, eq. (1) is the result.

Electrons and heavy particles in plasma. Electrons and heavy particles (atoms, ions, molecules) have got so different properties leading to special features of various plasmas used in advanced physical technologies.

Particles in plasma (ionized gas) undergo collisions making heat transfer similar to gas. As the intensity and efficiency of the energy transfer between electrons and heavy particles is low, plasma is always in nonequilibrium state characterized by different temperatures of electrons and the other particles. Mathematical modelling of heat transfer uses at leats two diffusion heat conduction equations for electrons and heavy particles, where the energy exchange is expressed as the couple term between the two equation.

The nonequilibrium in the other states of matter is something unusual and sometimes even undesirable. On contrast, the nonequilibrium behavior of plasma has found technological applications. Even plasma can not exist in equilibrium. From this point of view it is interesting to look at the values of such thermal properties as relaxation and thermalization time or parallel thermal diffusivity in comparison with the other "equilibrium" materials (Honner & Kuneš, 1999).

Composite and porous materials. Parallel heat transfer can not be observed only at the atomic levels. In a nonhomogeneous materials such as composites or porous materials, that are made of two or more mingled components, parallel heat conduction also takes place. It comes from different heat conduction in components of composites or parallel conduction, convection or radiation in porous materials.

Parallel heat conduction is interesting in biological or loose materials with relatively large relaxation times. However these large values of relaxation time are often incorrectly related to the wave heat conduction processes. Thus it leads to wrong conclusions on the practical applications of the thermal wave theory.

3. MATHEMATICAL MODELLING OF NONEQUILIBRIUM HEAT CONDUCTION

Analyses of microscale heat conduction processes show the possibility to develop the unified mathematical description of equilibrium and nonequilibrium microscale heat conduction. (Tzou, 1997). Definition of nonequilibrium heat conduction equation however appears only as a part of the problem. The same importance must be taken into the definition of nonequilibrium conditions of unambiguity.

3.1 General heat conduction equation

Thermal behavior of materials is represented by four thermo-mechanical properties: thermal diffusivity α [$m^2.s^{-1}$], parallel thermal diffusivity α_P [$m^2.s^{-1}$], relaxation time t_q [s] and thermalization time t_T [s]. Heat conduction equation, the base equation of the model, is proposed in the form

$$-\alpha_P t_T \nabla^4 T + t_T \frac{\partial}{\partial t}\nabla^2 T + \nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t} + \frac{\tau_q}{\alpha}\frac{\partial^2 T}{\partial t^2} \,. \quad (1)$$

Values of individual parameters in eq. (1) express ways of heat propagation and reflect different heat transfer mechanisms in the microstructure of the material. Wave heat conduction (nonequilibrium conduction of the first kind) and parallel heat conduction (nonequilibrium conduction of the second kind) have similar mathematical representation in spite of their total physical difference. Moreover, both of the conduction modes can occur in materials simultaneously. Equilibrium diffusion heat conduction based on the Fourier law appears as a limit case of both nonequilibrium conduction modes. The mentioned ways of heat conduction are observed in all states of matter including solids, liquids, gases and plasmas where heat is carried by phonons, atoms, molecules, electrons or photons.

Propagation character depends on the ratio of parallel and standard thermal diffusivity

$$K_\alpha = \frac{\alpha_P}{\alpha} \quad (2)$$

and has two limits. In the case $K_\alpha \to 0$ ($\alpha_P \to 0$), heat conduction in the material is carried mostly by one kind of energy carrier. In the opposite case when $K_\alpha \to 1$ ($\alpha_P \to \alpha$), heat is carried almost exclusively in parallel mode. In the material with two different components, for example electrons and phonons in metals, it means thermal capacity of the first component is much higher than heat capacity of the second one. On the contrary, thermal conductivity of the first one has to be much lower than conductivity of the second one.

The second parameter of propagation character appears the ratio of thermalization and relaxation time of the material

$$K_t = \frac{t_T}{t_q} = \frac{Fo_T}{Fo_q}, \quad (3)$$

Its effect is shown in **Fig.1**. The value $K_t = 1$ indicates diffusion conduction. When $K_t < 1$, the model represents wave conduction transiting to diffusion in later times. When $K_t > 1$, the model represents parallel conduction. Values of wave or parallel relaxation times define time-ranges of the corresponding nonequilibrium heat conduction.

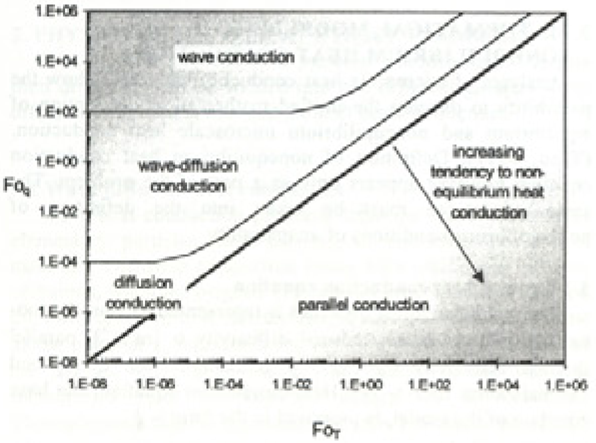

Fig.1: Regions of application for parallel, wave, wave-diffusion and diffusion heat conduction

3.2 Internal Sources

In the case of nonequilibrium heat transfer with internal sources, the base heat conduction equation (1) is expanded by higher order terms of internal sources. These additional terms have their own physical sense and they considerably influence the heat transfer process being modelled representing the nonequilibrium effects of internal sources sometimes also called apparent heating (Tzou, 1997).

Wave conduction. The wave nonequilibrium internal source term comes from the assumption of the modified Fourier law. It's importance increases in the same situations when the importance of the wave term in the base heat conduction equation increases (Honner & Kuneš, 1999).

Parallel conduction. Heat conduction equations expressed in temperatures of both material components are equal without the presence of the nonequilibrium higher order terms of internal sources. Having the equal equations, the resultant computed temperature of all components must be identical. Thus the parallel nonequilibrium can not be produced only by the effect of conventional internal sources terms. The presence of nonequilibrium higher order terms in the heat conduction equation differs according to the assumed component temperature so it disturbs the symmetry of the thermal system being modelled and the nonequilibrium heat transfer process with different component temperatures can be simulated.

3.3 Material properties

The relaxation time of either wave diffusion or parallel heat conduction is the most important material property, because it specifies time-ranges of the nonequilibrium propagation. Values of the relaxation time are summarized for various materials and ways of heat propagation in (Honner & Kuneš, 1999).

3.4 Initial conditions

Initial conditions represent the initial thermal state at the beginning of the process being modelled. If we consider only the equilibrium diffusion heat conduction the definition of temperature distribution is sufficient. However modelling of the nonequilibrium process of either wave or parallel nature requires specifying one more initial condition - the initial time rate of temperature change. This second condition is used to define the initial nonequilibrium state.

Wave conduction. Physically the second initial condition expresses the ratio of equilibrium and nonequilibrium temperature at the beginning of the process. The nonequilibrium temperature represents the kinetic energy of the thermal wave travelling throughout the material and therefore it is known as kinetic temperature. The equilibrium temperature is related to the diffusion conduction and satisfies the Fourier law of heat flux according to the temperature gradient.

Parallel conduction. In the case of parallel conduction the initial time rate of temperature change is connected with the temperature distribution in the individual material components. Temperatures of the material internal components need not be equal and the second initial condition thus defines the initial nonequilibrium state of the material.

3.5 Boundary conditions

Boundary conditions represent the heat transfer processes between the surface of the considered region and the surrounding environment. The conditions have to be in accordance with the description of processes inside the thermal system being modelled. Therefore, they have to reflect also the nonequilibrium wave or parallel heat propagation character. The application of only the simple diffusion boundary condition appears wrong.

Similarly to the initial nonequilibrium condition the second additional nonequilibrium boundary condition has to be introduced or the first equilibrium boundary condition has to be expanded by higher order terms to express nonequilibrium behavior as well.

Wave conduction. Boundary condition of the first kind specifies the surface temperature. Combined condition for wave-diffusion heat transfer has been derived by (Kronberg at al.,1998) as

$$T(0,t)+\frac{2-\mathrm{a}}{\mathrm{a}}\cdot\frac{q(0,t)}{c_p\rho u}=T_w(t)\,, \tag{4}$$

where a is the accommodation coefficient. The derivation is based on the concept of heat transfer to the rarefied gas, where a jump temperature change appears on the distance of the molecule mean free path. The comparison of various wave-diffusion problems solution assuming diffusion and wave boundary conditions is discussed in (Kronberg at al.,1998) in order to show why the new boundary condition eq.(4) seems to be more realistic in modelling of thermal wave processes.

Differences in boundary conditions of the second, third and fourth kind comes from the relation of the modified Fourier law, where heat flux is not related only to the temperature gradient but is additionally related also to the temporal change of the heat flux. Details can be found in numerous literature on thermal waves, for example in (Tzou, 1997) or (Özisik & Tzou, 1994).

Parallel conduction. In the parallel conduction process modelling it is necessary to define boundary conditions for each material component. These conditions can differ in values as well as in the overall heat transfer character. It means the amount of energy supplied to the individual material components can differ in relation to their heat capacity. The effect of such a non-symmetric boundary conditions is responsible for creation of the nonequilibrium state of the material which is characterized by the different temperatures of the material components.

Description of heat transfer in a glow discharge plasma near the cathode can serve us as an example of such a boundary condition producing the nonequilibrium. Energy is supplied to the plasma from the cathode by the electron heat flux meanwhile the heavy particles transmit their energy from the plasma to the cathode being cooled.

4. APPLICATIONS IN TECHNOLOGIES USING INTENSIVE HEAT SOURCES

Advanced physical technologies utilize intensive heat sources such as concentrated beams of photons, electrons or ions for controlled thermal affection of materials. Micro-treatment of material surfaces, films or coatings deposition by thermal spraying or vacuum physical processes, welding, machining, cutting, drilling, etc. are the industrial applications of them.

The objective of modelling the microscale heat transfer processes in the new technologies is especially to control location and depth of the heat affected zone, where the required technological processes occur. Differences in these processes ranges from internal structure changes of the treated material as far as its evaporation.

Material heating, as **Fig.2** shows, usually consists of three microscale processes: excitation (absorption), conduction, and relaxation. These indicate interactions among energy carriers in the microstructure of the material.

Excitation of the energy carriers occurs in the absorption region, where impacting particles transmit their energy to the heat carriers of the material. Excited carriers then spread the energy out of the absorption region and at the same time, they redistribute the energy to other heat carriers (relaxation).

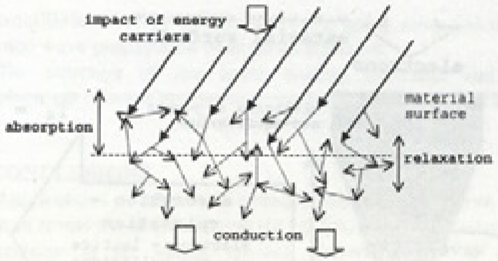

Fig.2: External heating of materials - process schema

The depth of the relaxation region is defined by diffusivity of excited carriers and by the time needed for the carriers to reach thermal equilibrium. The time is called relaxation time of the material. For a certain amount of absorbed energy, the higher relaxation region, which is the result of higher diffusivity or longer relaxation time, is heated up to a lower equilibrium temperature. On the contrary, a smaller relaxation region caused by more intensive collisional process leads to a higher equilibrium temperature.

4.1 Laser beam heating

Laser beam heating of metals can be well used to show occurence of both wave diffusion and parallel nonequilibrium heat transfer. There are two types of heat carriers in metals - electrons and phonons. Electrons interact directly with the impacting photons and then transmit the absorbed energy to the lattice vibration by means of electron-phonon collisions. Radiation is thus transformed into the internal material energy as a multistep process. Involvement of the wave and parallel heat conduction in this process is shown in **Fig.3.**

Electrons are excited from the low energy levels to the levels of higher energy in collisions with photons and the electron distribution is carried away from the initial equilibrium at a certain temperature. It takes several femtoseconds to establish new electron equilibrium however at higher temperature. In this time interval, which is done by the relaxation time of wave heat conduction, the electron heat wave propagation from the absorption region takes place.

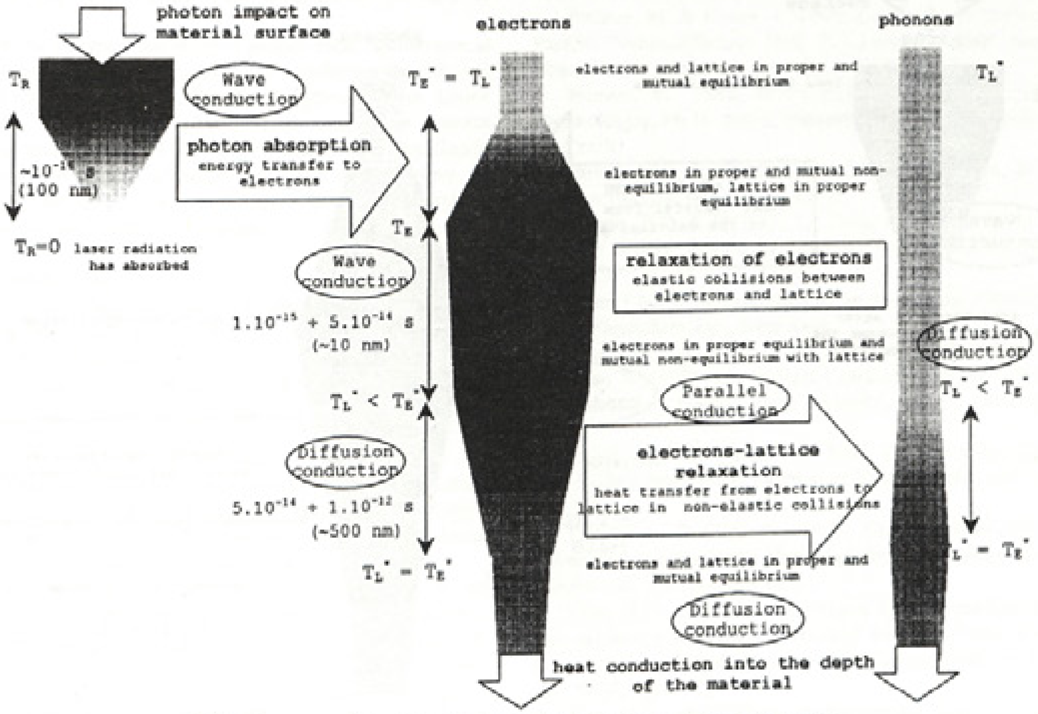

Fig.3: Structure of heat transfer processes during laser heating of metals

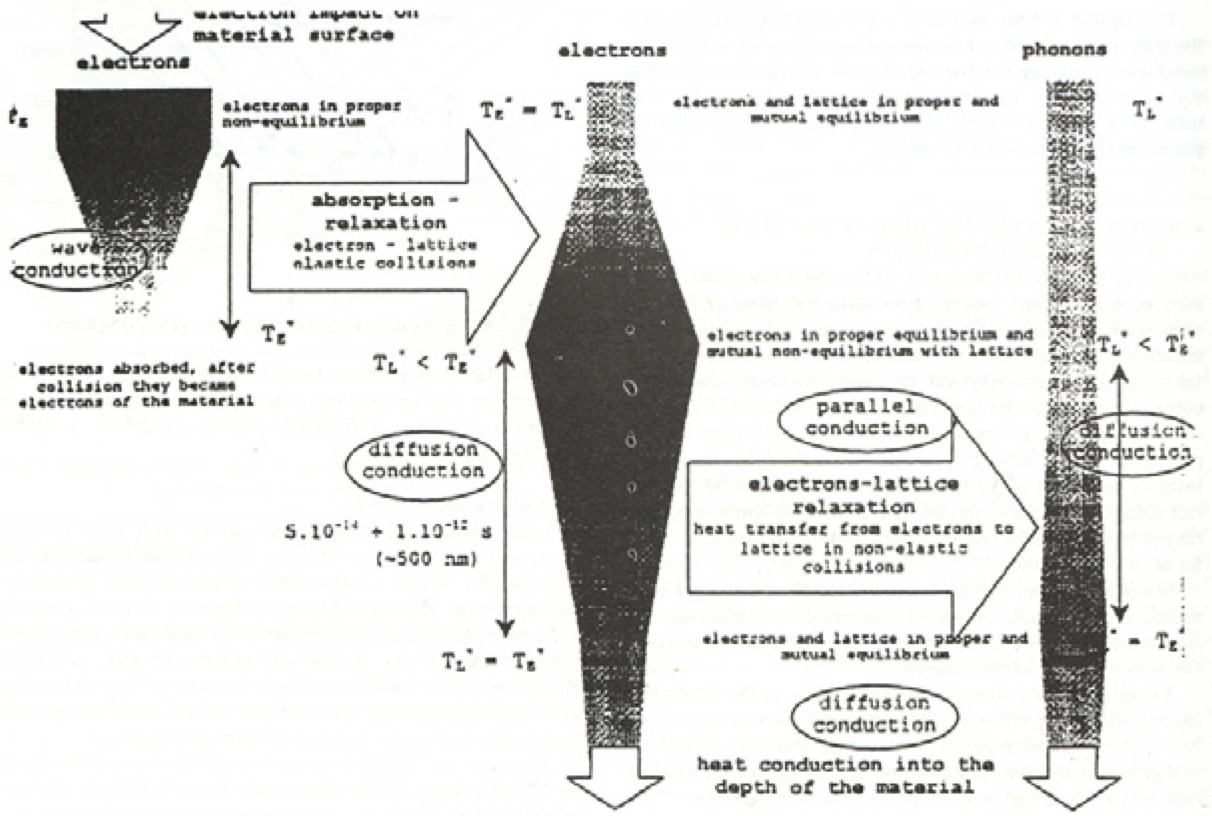

Fig.4: Structure of heat transfer processes during electron beam heating of metals

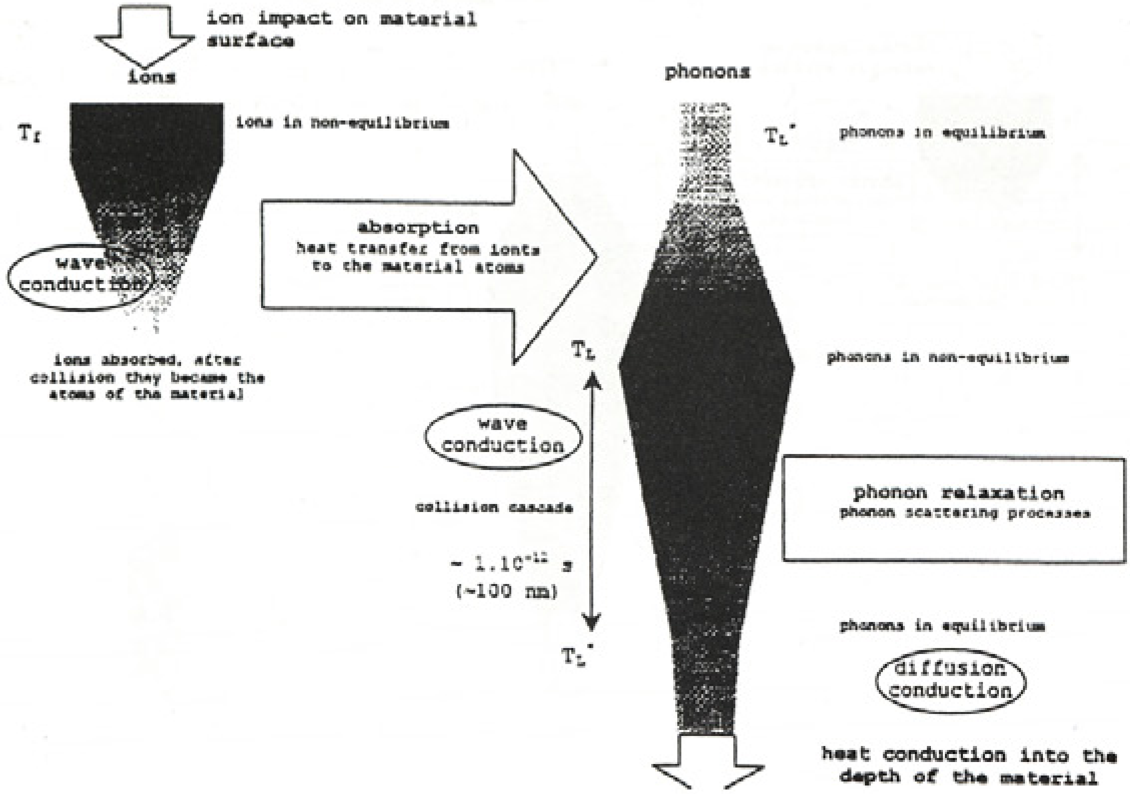

Fig.5: Structure of heat transfer processes during ion beam heating of materials

Electron gas equilibrium is reached when the collisional processes effectivelly take place and the electron propagation character becames diffusive. Electron mass is much lower than atom/ion mass therefore energy transfer from electrons to the lattice appears ineffective in comparison with redistribution of electrons in energy states. Thermal equilibrium is thus set first in the electron system. The nonequilibrium between electrons and phonons maintains because of different lattice and electron temperatures and parallel heat conduction takes place.

Excited electrons deliver the absorbed energy out of the absorption region and also out of the electron relaxation region, because the energy transfer process from electrons to the lattice is not strong enough to convert all the absorbed energy to the lattice heat in this region. The region of electron-phonon relaxation and adjustment of complete local thermal equilibrium with equal electron and lattice temperatures is determined by the relaxation time of parallel heat conduction.

Higher thermal affected zone of the material however with lower lattice temperature is the result of non-equilibrium process consideration compared with the results of only diffusion heat transfer process consideration in metals.

4.2 Electron beam heating

Involvement of the wave and parallel heat conduction in electron beam heating of metals is shown in **Fig. 4.** Impacting electrons penetrate under the material surface, where they are scattered by material electrons and ions arranged in the crystal lattice. Electron-electron collisions with high efficiency of energy transfer are not so important as the electron-phonon collisions. Reasons have been discussed on laser heating above.

So the electron-lattice collisions play the key role in the microscale internal heat transfer. The efficiency of energy transfer is low in such collisions but electrons are almost elastically scattered from their original direction. This process can be assumed as the absorption and the electrons can be later considered as the proper material electrons. This process can be also assumed as the relaxation because the electrons come to the equilibrium and the original wave propagation changes to diffusion.

Despite the electron has reached proper local equilibrium at some temperature T_E^*, the mutual electron-lattice equilibrium has not been achieved yet, because of much lower lattice temperature. Therefore the parallel heat conduction process occurs. The inelastic part of the electron-lattice collisions finally brings metal to the equilibrium at equal temperatures of electrons and the lattice and pure diffusion heat conduction than delivers heat to the depth of the material.

4.3 Ion beam heating

Impact of accelerated ions (ion bombardment) produces outstanding microscale processes in the affected material such as sputtering of target atoms, secondary electron emission, ion implantation, structural changes, etc. These processes that are accompanied by heat transfer have found numerous applications in advanced physical technologies.

Accelerated ions deliver nonequilibrium energy to the material surface as momentum transfer. The energy is converted into heat - lattice vibrations. Depend on mass, atomic number and energy of the bombarding ions and target atoms several types of ion-surface interactions can occur.

Modelling of such processes is usually simplified to the sequence of binary collisions. The first collision occurs between the impacting ion and the target atom on the ion trajectory. Later target atoms collide among others and transmit the energy by means of phonons deeper to the material. In the micro-thermomechanics therminology the process can be described as travelling thermal wave with their decay. The original nonequilibrium energy transfer process is going away and shock thermal wave propagation changes to diffusion.

The structure of ion beam heating processes and the involvement of wave and parallel conduction is shown in **Fig.5.**

5. CONCLUSIONS

Applications of advanced technologies using intensive heat sources reveal the usual macroscale equilibrium heat conduction description is no longer correct. Several kinds of non-equilibrium heat conduction have to be taken into account. Despite the physical difference of heat transfer processes in material microstructures, the general macroscale model can be developed for both equilibrium and nonequilibrium conduction.

The first part of the paper has been devoted to the microscale heat transfer processes that occur in various materials. Wave diffusion heat conduction takes place in phonon, electron, photon and molecular heat transfer. Parallel conduction can be found in metals, plasmas and composite and porous materials.

The second part of the paper is dealing with macroscale mathematical modelling of microscale heat transfer processes. The general heat conduction equation has been introduced to represent equilibrium and both nonequilibrium heat conductions. Special attention in discussions of internal sources and initial and boundary conditions definition is focussed on the modelling of nonequilibrium heat transfer.

The third part of the paper shows applications of the discussed microscale nonequilibrium heat conduction in technologies using phonon, electron and ion beam heat sources for heat treatment of materials.

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