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ANALYTICAL REPRESENTATION OF THE INTERNAL SHAPE FACTOR OF SHEATHED ELECTRICAL HEATING ELEMENTS

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ABSTRACT

This paper focuses on the study of the internal shape factor of sheathed electrical heating elements. First, the studied geometries are presented and three dimensionless parameters are introduced. The analysis of finite elements simulations leads to the proposition of an analytical expression of the dimensionless internal shape factor. Then a non uniformity factor is proposed and it is shown that this non uniformity is in all cases inferior to 6%, and in most cases inferior to 1%. Then the proposed relation is applied to two examples, one determining the maximum heat flux for a given temperature, the second one determining the maximum temperature of the heating wire for a given heat flux.

INTRODUCTION

Sheathed electrical heating elements are widely used in many processes (liquid heating, gas heating, infrared heating, ...) as they are robust in a large temperature range. When the final user is not directly concerned by the internal temperature distribution, he is looking for a long life of the heating element. This can be achieved by the manufacturer of the element by carefully choosing the diameter of the heating wire. This becomes important when the service temperature of the sheathed element becomes high (about 1,000 °C). The two main techniques that are used to manufacture sheathed electrical heating elements are presented in figure 1.

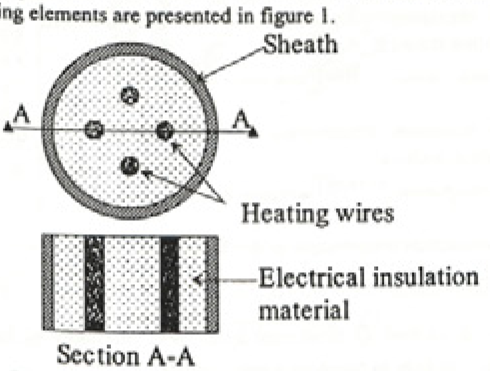

Fig. 1a: sheathed heating element with straight wires

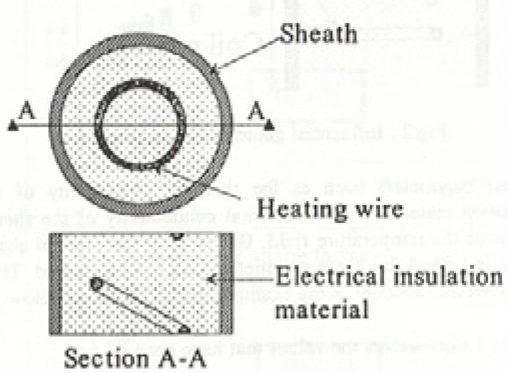

Fig. 1b: sheathed heating element with coiled wires

The most used wires are the coiled wires. So, this study focuses on this type of sheathed elements. Only few papers deal with the temperature distribution in electrical heating elements (Daurelle, 1990) (Thibault, 1991) (Lalot, 1994). This can be due to the fact that this distribution strongly depends on the value of the thermal conductivity of the electrical insulation material. This value depends on the final value of the density of the material, and so depends on the way this material is compacted. It is possible to measure the thermal conductivity by comparing experimental data (e.g. the temperature difference between the axis of the element and its surface for a known heat flux) and computed results. This has been done for few different types of elements and at different locations (straight parts and bends), but the results are confidential. To avoid such confidentiality problems, instead of giving the internal thermal resistance, a dimensionless internal shape factor will be computed. This dimensionless shape factor is defined as the ratio of the actual shape factor to the shape factor of a heating element that would consist of a continuous cylindrical heating part.

In a previous work (Lalot, 1994), it has been shown that the coiled wire may be approximated by a torus. This means that the 3D effect is negligible, and that axi-symmetric numerical simulations can accurately represent the actual phenomenon.

THEORETICAL DEVELOPMENT

Description of the studied geometries

Figure 2 shows the different geometrical parameters that influence the temperature field in the sheathed element.

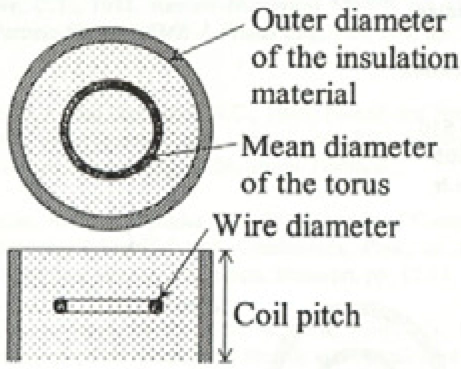

Fig.2 : Influential geometrical parameters

Other parameters such as the thermal conductivity of the insulation material and the thermal conductivity of the sheath influence the temperature field. But as only the internal shape factor is looked for, these parameters will not be studied. They are taken into account in the example that is presented below.

Figure 3 summarizes the values that have been studied.

Outer diameter of the insulation material (mm)						7				
Mean diameter of the torus (mm)					2	4	6	8	10	
Wire diameter (mm)	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Dimensionless coil pitch (wire diameter)			4		8		12		16	20

Fig. 3 : Numerical values of the geometrical parameters

Combining all the values of the geometrical parameters makes a set of 250 geometries. All these geometries have been simulated using the finite element software Systus+ (SYSTUS International, 1999). The geometries have been prepared using a Visual Basic program. Figure 4 shows an example of a geometry and the corresponding temperature field.

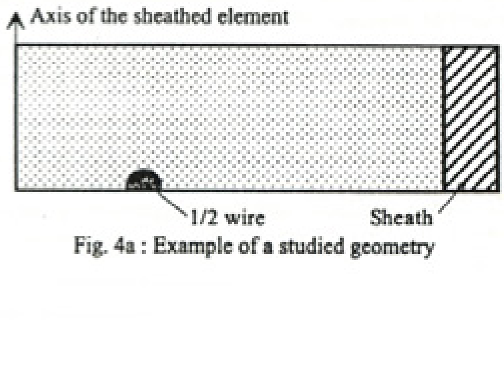

Fig. 4a : Example of a studied geometry

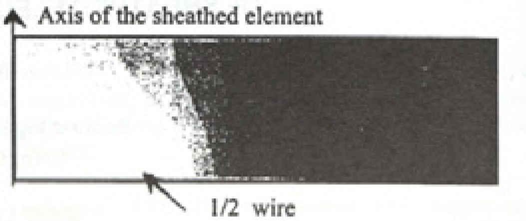

Fig. 4b : Example of a temperature field in a sheathed element

It is legitimate to think that the dimensionless shape factor will not directly depend on the parameters, but will only depend on geometrical ratios. This explains that only one outer diameter has been studied.

It is possible to introduce 3 dimensionless geometrical parameters:

$$G1 = \frac{\textit{outer diameter of the insulation material}}{\textit{mean diameter of the torus}} = \frac{D_i}{D_t},$$

$$G2 = \frac{\textit{outer diameter of the insulation material}}{\textit{wire diameter}} = \frac{D_i}{D_w},$$

$$G3 = \textit{dimensionless coil pitch} = \frac{\textit{coil pitch}}{\textit{wire diameter}} = \frac{p}{D_w}.$$

The aim of this study is to give an analytical expression of the internal shape factor as a function of these 3 dimensionless geometrical parameters.

Analysis of the simulations

As it has been previously said, 250 simulations have been carried out. For all these simulations, 4 temperatures have been extracted from the results:

- the mean temperature on the outer diameter of the insulation material

$$T_{mean\ outer\ diameter} = \frac{1}{n_{nodes\ outer\ diameter}} \sum T_{node\ outer\ diameter},$$

- the maximum temperature on the outer diameter of the insulation material

$$T_{max\ outer\ diameter} = Max(T_{node\ outer\ diameter}),$$

- the minimum temperature on the outer diameter of the insulation material

$$T_{min\ outer\ diameter} = Min(T_{node\ outer\ diameter}),$$

- the maximum temperature of the wire.

$$T_{max\ wire} = Max(T_{node\ wire})$$

Knowing the heat Q dissipated by the wire (normally by Joule effect), it is easy to compute a shape factor:

$$S=\frac{Q}{k_i\left(T_{max\ wire}-T_{mean\ outer\ diameter}\right)}.\qquad(1)$$

The mean temperature on the outer diameter has been chosen because it is the one that is easy to calculate during service. Usually, the convection coefficient between the sheathed element and the fluid to be heated is known. Knowing the temperature of the fluid and the amount of heat per unit surface to be transferred, it is easy to calculate the mean surface temperature of the element. Knowing the thickness of the sheath and its thermal conductivity, it is then possible to calculate the mean temperature of the outer diameter of the insulation material.

The maximum temperature of the wire has been chosen because this is this temperature that determines if there is any danger that the wire melts.

Then, this shape factor is compared to the shape factor of a sheathed element that would consists of a continuous heating cylinder:

$$S_0=\frac{2\pi p}{Ln\left(\frac{D_i}{D_t}\right)}=\frac{2\pi p}{Ln(G1)}.\qquad(2)$$

The comparison is made through the calculation of the dimensionless shape factor defined as follows:

$$\overline{S}=\frac{S}{S_0}=\frac{Q\,Ln(G1)}{2\pi k_i\,p\left(T_{max\ wire}-T_{mean\ outer\ diameter}\right)}.\qquad(3)$$

The first idea is to look for a dependence of the dimensionless shape factor on the ratio of the diameters $G1$ (Fig. 5). It can clearly be seen that there is no straight dependence and that the dimensionless shape factor surely depends more on the other dimensionless parameters.

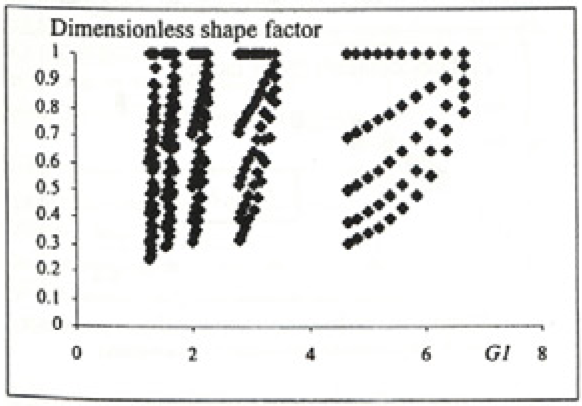

Fig. 5 : Evolution of the dimensionless shape factor versus the dimensionless parameter $G1$

Five distinct zones can be seen in Figure 5, so the next idea is to plot the evolution of the dimensionless shape factor versus the dimensionless parameter $G2$ (Figure 6) for the five values of the dimensionless parameter G3. For clarity reasons, the results are presented separately.

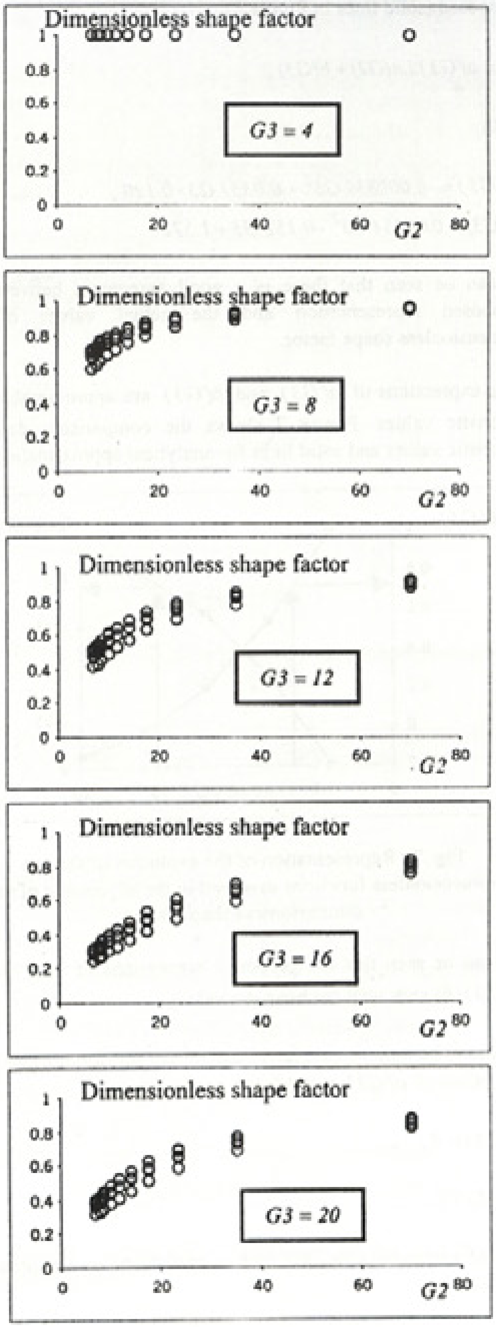

Fig. 6 : Evolution of the dimensionless shape factor versus the dimensionless parameter $G2$ for five values of $G3$

It can be seen that there is a large difference between the actual shape factor and the one that could be calculated assuming that the heating element is a continuous cylinder.

To analytically represent the results the following equation is proposed (solid lines in Figure 8):

$$\bar{S} = a(G3)\,Ln(G2) + b(G3)\,, \qquad (4)$$

with:

$$a(G3) = -0.000839\,G3^2 + 0.0337\,G3 - 0.120\,,$$
$$b(G3) = 0.00354\,G3^2 - 0.157\,G3 + 1.572\,.$$

It can be seen that there is a good agreement between the proposed representation and the actual values of the dimensionless shape factor.

The expressions of $a(G3)$ and $b(G3)$ are approximations of heuristic values. Figure 7 shows the comparison: dots for heuristic values and solid lines for analytical approximations.

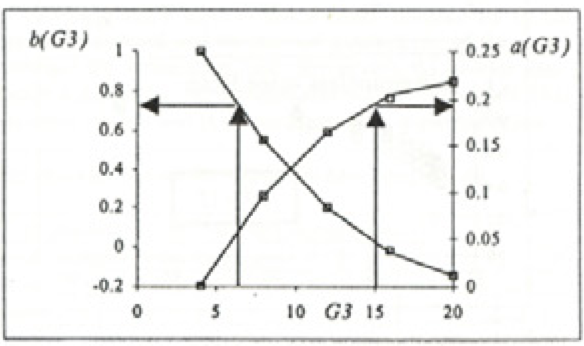

Fig. 7 : Representation of the evolution of the two dimensionless functions involved in the expression of the dimensionless shape factor

It can be seen that the proposed expressions of $a(G3)$ and $b(G3)$ fit very well the heuristic values.

It has to be noticed that for values of $G3$ that are inferior to 4, the values of $a(G3)$ and $b(G3)$ are constant:

$$a(G3) = 0\,,$$

$$b(G3) = 1\,.$$

In that particular case, the coil is equivalent to a continuous cylinder.

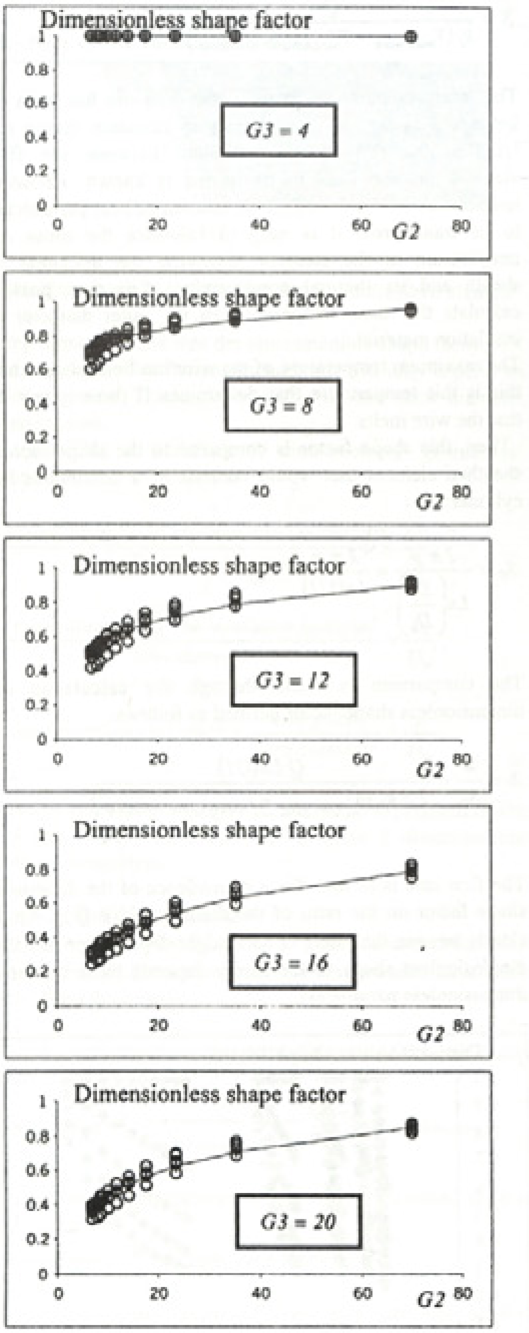

Fig. 8 : Comparison of the actual dimensionless shape factor with the proposed analytical representation

It is often assumed that the heat flux is constant at the surface of an electrical heating element. The simulations that have been carried out to compute the internal shape factor allow the calculation of a non uniformity factor. It is proposed to define this non uniformity factor as follows:

$$\delta = \frac{T_{max\,outer\,diameter} - T_{min\,outer\,diameter}}{T_{max\,wire} - T_{mean\,outer\,diameter}}.$$

This factor is null when the wire can be assimilated to a continuous cylinder, and should increase when the wire diameter is small compared to the pitch of the coil. So, it is informative to plot the evolution of this factor versus the dimensionless internal shape factor (Fig. 9).

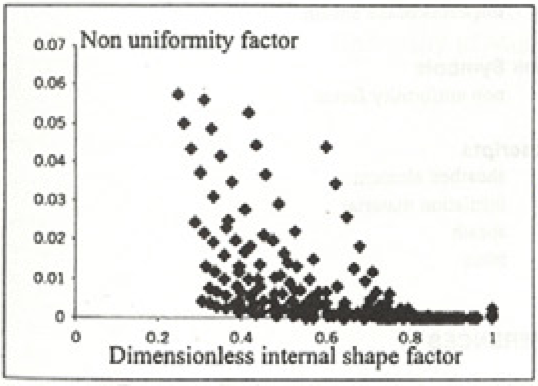

Fig. 9 : Evolution of the non uniformity factor versus the dimensionless internal shape factor

It can be seen that the non uniformity is limited to 6%, and that in most cases, it is inferior to 1%.

Now that the analytical expression of the dimensionless shape factor is established, it is possible to present examples.

APPLICATIONS

High temperature air heater

In this example, it will be supposed that air has to be heated at 1,000°C and that the maximum wire temperature is 1,200°C. The application of the proposed equations will lead to the calculation of the maximum heat q that it will be possible to deliver per unit surface of sheathed heating element.

It is necessary to assume other numerical values:

- convection coefficient: $h = 100\,W/m^2K$,
- element outer diameter: $D_e = 16\,mm$,
- sheath thickness: $t = 1\,mm$,
- sheath material thermal conductivity: $k_s = 15\,W/mK$,
- insulation material thermal conductivity: $k_i = 10\,W/mK$.

In that case, the dissipated heat flux may be written as follows:

$$q = \frac{T_{max\,wire} - T_{air}}{\frac{\pi D_e p}{S k_i} + \frac{D_e}{2 k_s} Ln\left(\frac{D_e}{D_e - 2t}\right) + \frac{1}{h}}. \qquad (5)$$

Introducing equations (2) and (4) in equation (5), leads to the following expression:

$$q = \frac{T_{max\,wire} - T_{air}}{\frac{D_e}{2 k_i} Ln(G1) \frac{1}{a(G3) Ln(G2) + b(G3)} + \frac{D_e}{2 k_s} Ln\left(\frac{D_e}{D_i}\right) + \frac{1}{h}}.$$

Then it can be said that the lowest value has to be chosen for $G1$, but technological constraints lead to the following relation: $D_{t\,max} = D_i - 2 D_w$. From Figure 8, it can be concluded that the lowest value of $G3$ has to be chosen. Then it is possible to plot the evolution of q versus the wire diameter (Figure 10).

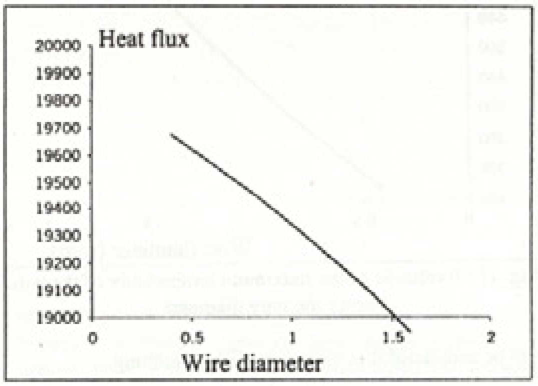

Fig. 10 : Evolution of the heat flux per unit surface versus the wire diameter

It can be concluded that the heat flux does not depend a lot on the wire diameter. Taking a wire diameter inferior to 1.5 mm, the safe value would be $q = 19{,}000\,W/m^2$.

High heat flux water heater

In this example, it will be supposed that water has to be heated at 100°C and the heat flux should be $q = 1{,}000\,kW/m^2$. The application of the proposed equations will lead to the calculation of the maximum temperature of the wire. Here again, it is necessary to assume other numerical values:

- convection coefficient: $h = 10{,}000\,W/m^2K$,
- element outer diameter: $D_e = 16\,mm$,
- sheath thickness: $t = 1\,mm$,
- sheath material thermal conductivity: $k_s = 15\,W/mK$,
- insulation material thermal conductivity: $k_i = 10\,W/mK$.

In that case, the maximum temperature of the wire may be written as follows:

$$T_{max\,wire} = q\left(\frac{\pi D_e p}{S k_i} + \frac{D_e}{2 k_s} Ln\left(\frac{D_e}{D_e - 2t}\right) + \frac{1}{h}\right) + T_{water}. \qquad (6)$$

Introducing equations (2) and (4) in equation (6), leads to the following expression:

$$T_{max\ wire} = q\left(\frac{D_e}{2k_i}\frac{Ln(G1)}{a(G3)Ln(G2)+b(G3)} + \frac{D_e}{2k_s}Ln\left(\frac{D_e}{D_e-2t}\right)+\frac{1}{h}\right)+T_{water}\,.$$

The same technological constraints lead to a maximum temperature of about 650°C (Figure 11).

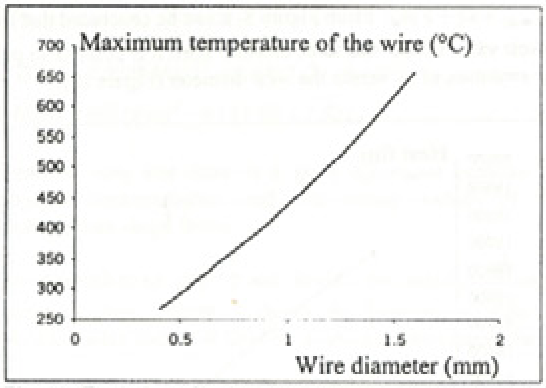

Fig. 11 : Evolution of the maximum temperature of the wire versus the wire diameter

It can be concluded that there is no risk of melting.

CONCLUSIONS

Simulations of the heat transfer in a sheathed electrical heating element has led to an analytical expression of the internal shape factor. This allows the prediction of the maximum temperature of the heating wire for a given heat flux, or the maximum heat flux for a given wire temperature. These simulations have allowed the calculation of a non uniformity factor. It has been shown that this non uniformity factor is, in most cases, inferior to 1%. Future studies will address straight wires sheathed elements.

ACKNOWLEDGMENTS

This study was partially sponsored by SYSTUS International and by VULCANIC (major French manufacturer of electrical heating elements).

NOMENCLATURE

a	function in equation (4)	
b	function in equation (4)	
D	diameter	[m]
$G1$	ratio of the outer diameter of the insulation material to the mean diameter of the torus, D_i / D_t	
$G2$	ratio of the outer diameter of the insulation material to the diameter of the wire, D_i / D_w	
$G3$	ratio of the coil pitch to the diameter of the wire, p / D_w	
h	convection coefficient	[W/m²K]
k	thermal conductivity	[W/mK]
p	pitch of the coiled wire	[m]
Q	heat rate	[W]
q	heat flux	[W/m²]
S	shape factor	[m]
S_0	reference shape factor (for a continuous heating cylinder)	[m]
$\overline{S}$	dimensionless shape factor	
t	thickness of the sheath	

Greek Symbols

δ	non uniformity factor

Subscripts

e	sheathed element
i	insulation material
s	sheath
t	torus

REFERENCES

Daurelle, J.-V., 1990, *Internship report*, VULCANIC

Thibault, P., 1991, *Internship report*, VULCANIC

Lalot, S., 1994, *Etude d'un Réchauffeur Electrique pour Fluides Corrosifs*, Thèse de doctorat, Université de Valenciennes et du Hainaut Cambrésis

SYSTUS International, 1999, Systus+ user's manual