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MODELING OF VOID FRACTION AND LIQUID TEMPERATURE PROFILE EVOLUTION IN VERTICAL SUBCOOLED NUCLEATE BOILING FLOW

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ABSTRACT

A three-dimensional bubble-tracking model, which simulates subcooled nucleate boiling in a heated vertical cylindrical tube, is presented. The behavior of the liquid-vapor system results from motion, interaction and heat transfer mechanisms prescribed mostly at the level of individually-tracked vapor bubbles. The model takes into account bubble nucleation and liquid heating caused by wall heat flux, bubble sliding on the tube wall, bubble departure from the tube wall, bubble condensation in the low-temperature tube core region, bubble interaction through wake drift, bubble collisions and coalescence, bubble radial migration towards the tube core region, and turbulent dispersion in the liquid phase. Simulated void fraction and liquid temperature radial profiles on different axial locations of a heated channel are compared with experimental results from other authors

INTRODUCTION

Void fraction and liquid temperature behaviors in subcooled regions of forced convective nucleate boiling flows in vertical channels may be modeled using various approaches with different time and length scales. One-dimensional models (which also include some two-fluid models) with various degrees of empiricism may predict fairly well void fraction and liquid temperature, averaged over the flow cross-section. These kinds of models are used in so-called thermal-hydraulic codes to simulate transients in nuclear power plants (for example, as carried out by Parzer et al., 1995). However, these models cannot predict the evolution of void fraction and liquid temperature radial profiles along heated channels. Also, the development of second-order accurate schemes for two-fluid models, which limit numerical diffusion, is at present mostly confined to adiabatic flows (Tiselj & Petelin, 1998). On the other hand, "sophisticated" models based on local instantaneous description of the flow (such as proposed by Juric & Tryggvason, 1998) are still computationally too demanding to be applied to boiling systems which may have a complex interface structure due to the presence of up to several thousand bubbles. Intermediate-level models include multidimensional two-fluid models (for instance, as proposed by Kurul & Podowski, 1991) and so-called bubble-tracking models (Mortensen & Trapp, 1992, Kljenak & Mavko, 1999), in which vapor is distributed in liquid in the form of individually tracked bubbles.

In the present work, a three-dimensional bubble-tracking model is presented: the model already presented in earlier works (Kljenak & Mavko, 1997, Kljenak, 1998, Kljenak, 1999, Kljenak & Mavko, 1999) was further developed. The model enables the modeling of the axial evolution of void fraction and liquid temperature profiles when a vapor-liquid flow in a vertical cylindrical tube is heated with a known heat flux. The overall behavior of the vapor-liquid system results from motion, interaction and boiling mechanisms prescribed mostly at the level of individual bubbles. Empiricism concerning the structure of the gas-liquid interface is thus included at a "more fundamental" level than in one-dimensional models or multi-dimensional two-fluid models. The model takes into account the following heat transfer phenomena: heating of the liquid, bubble nucleation on the tube wall and bubble condensation in the low-temperature tube core region. Differences in bubble velocities, due to wake drift and liquid velocity gradient, cause bubbles to collide and eventually merge into larger bubbles. Bubble interaction may be disrupted by turbulent dispersion in the liquid phase.

Subcooled nucleate boiling was simulated for experimental conditions from Sekoguchi et al. (1980, 1981), who have observed subcooled and low quality boiling flow of water in cylindrical tubes. The agreement between simulated and measured void fraction and liquid temperature radial profiles is satisfactory.

PHYSICAL MODEL

Bubble Axial Motion and Interaction

Bubbles are modeled as rigid ellipsoids which move upwards in a vertical channel with their symmetry axis always vertical. Larger, spherical cap bubbles, may be created by coalescence and expansion of smaller bubbles.

Bubble instantaneous axial velocity is calculated as the sum of local liquid velocity and bubble relative velocity, which is obtained from a correlation by Peebles and Garber (1953, as cited by Wallis, 1969). The local instantaneous liquid velocity is equal to the sum of a hypothetical undisturbed liquid velocity (a 1/7th power law was assumed) and an eventual increase due to wake drift caused by nearby bubbles. Bubble axial motion was thus simplified, which was necessary because of the long computation times due to the large number of bubbles which are present in boiling systems. The liquid velocity gradient causes bubbles located at different radial coordinates to move with different velocities. The liquid velocity behind bubble i, which is increased due to wake drift, is calculated as:

$$w_l(z,r,t) = w_{l\infty}(z,r,t) + \left[\frac{l_{i,\max}}{2(z_i - z)}\right]^{2\zeta/3} (w_{bi}(t) - w_{l\infty}(z_i, r_i, t)) \quad (1)$$

where ζ is an empirical exponent, set equal to 3.0.

A necessary condition for a trailing bubble to be influenced by a leading bubble through wake drift is that bubbles overlap in the lateral direction by more than a certain critical fraction, called *minimum relative overlapping* (a similar approach was also proposed by Mortensen & Trapp, 1992). Bubbles which do overlap and collide axially for whatever reason may remain sticking for some time, move along together with the upper bubble's velocity and eventually merge or separate. If bubbles do not overlap more than the critical fraction, the motion of the trailing bubble is not affected by the leading bubble and bubbles behave as if they would not overlap at all. This rule was prescribed to approximate the influence of bubble agitation which occurs in real bubbly flow and allows bubbles to overtake one another. The drawback of this approach is that bubbles may temporarily spatially overlap, which is not physically realistic.

Bubble Lateral Migration

Bubble distribution over the tube cross-section supposedly results from the interaction of different phenomena, such as liquid turbulence, transverse lift force and bubble interaction. As most of the experimental and theoretical work on void fraction distribution in bubbly flow deals with adiabatic air-water flow (for instance, as reported by Liu, 1993, Žun et al., 1993, or Ohnuki & Akimoto, 1998), state-of-the-art findings were not applied to the present model. Rather, a simple approach was adopted, based on experimental observations of nucleate boiling by various authors (for instance, by Bibeau & Salcudean, 1994): after detaching from the tube wall, bubbles tend to migrate towards the low-temperature tube core region (if their motion is not restricted by other bubbles), where they eventually condense.

In earlier work (Kljenak & Mavko, 1997; Kljenak, 1998), the process of bubble detachment from the tube wall was considered together with the process of bubble lateral migration. To achieve bubble detachment from the wall without excessive migration towards the tube core region, the probability of lateral migration was prescribed as proportional to the liquid velocity gradient. In the present work, the process of bubble lateral migration is considered as being distinct from the process of bubble detachment. When bubbles migrate radially, the lift force, which is related to the liquid velocity gradient, represents a restraining force. The probability of lateral migration p_m, which should thus increase with decreasing velocity gradient, is calculated as:

$$p_m = 1.0 - k_m \left[\left|\frac{\partial w_l}{\partial y}\right|\right]^{1/2} \quad (2)$$

where k_m is an empirical coefficient. Bubble lateral migration is attempted every time a bubble moves a distance equal to its maximum vertical chord length in the axial direction. The migration is simulated as a radial movement consisting of individual steps of a quarter of the bubble width.

Turbulent Dispersion

The relative motion between bubbles is mainly influenced by the eddy motion of the length scale of bubble size (Prince and Blanch, 1990). Approaches which consider the structure of the liquid turbulence, for instance based on discrete vortex simulation (Sene et al., 1994, Yang & Thomas, 1994), are at present too complex to be applied on the scale of bubbly flow over an entire tube, with several thousand bubbles present. A much more simple approach was thus adopted in the present model: turbulent dispersion in the liquid phase, which may affect wake drift or sticking bubbles which have just collided, is modeled as a succession of random events with a prescribed probability of outcome. Each event has two possible outcomes: turbulent dispersion either does or does not occur. A larger probability of dispersion simulates a higher turbulence intensity.

In the proposed model, disruption of wake drift is related to the turbulence length scale: wake drift behind a bubble may be sporadically interrupted and eventually resumed later, after the bubble has traveled a distance equal to *D/20* in the axial direction (see Žun et al., 1993). However, disturbance of sticking bubbles is connected to the event of bubble axial collision: after a collision, the outcome of turbulent dispersion simulation determines whether bubbles will remain sticking until eventual merging or separation, or will not influence each other. This approach is based on the observed behavior of bubbles within clusters in the experiments of Mao & Core (1993) and Stewart (1995).

Despite contrary experimental evidence (for example, as reported by Serizawa et al., 1975), intensities of turbulent dispersion were assumed constant over the tube cross-section to keep the model relatively simple. Also, breakup of bubbles was not considered at present. The main difficulty with assumptions concerning interaction between bubbles resides in the lack of quantitative experimental evidence on the behavior of bubbles within clusters in turbulent bubbly flow.

Bubble Coalescence

After axial collision, bubbles which overlap more than the minimum relative overlapping either remain sticking together or do not affect each other, depending on the outcome of turbulent dispersion simulation (which is carried out immediately after each collision). Bubbles merge after sticking together for a certain time interval (so-called "rest time"). Coalescence of bubbles occurs instantly only if an edge of the leading bubble is close to the tube wall whereas the trailing bubble is farther away, as the impact between bubbles is presumably stronger due to larger velocity differences. Bubble lateral collisions do not result in coalescence, as it was assumed that the impact of

the collision is not strong enough to lead to rupture of the vapor-liquid interface.

Liquid Temperature

The temperature of the tube wall is determined from a correlation by Shah (1977, as cited by Kandlikar, 1998) :

$$q'' = \left[230(m^{\circ}h_{lg})^{-0.5} h_l (T_w - T_s)\right]^2 \tag{3}$$

where the single-phase heat transfer coefficient h_l is calculated from the Dittus-Boelter correlation (Collier, 1981), and h_{lg} indicates the difference between vapor and liquid specific enthalpies at saturation conditions.

The liquid temperature T is assumed to obey the following law (Sekoguchi et al., 1981):

$$\frac{T_w - T}{T_w - T_c} = \left[\frac{y}{R}\right]^{1/m} \tag{4}$$

where the factor m was set equal to 4.0. The liquid temperature profile at different axial locations along the observed tube region is obtained from steady-state values of the thermodynamic equilibrium quality, with the local void fraction profile taken into account.

Heat Flux Partitioning

An approach based on the model proposed by Zeitoun & Shoukri (1997) is used to determine respective parts of the wall heat flux consumed for bubble nucleation on tube walls and heating of the liquid. The wall heat flux is given as :

$$q'' = C_l h_l (T_w - < T_l >) + q''_p + q''_g \tag{5}$$

The first term on the r.h.s. of eq. (5) represents heat transfer due to single-phase forced convection. Although the factor C_l accounts for the portion of the heating surface not covered by bubbles, its value was assumed to be 1.0, as in the work of Zeitoun & Shoukri (1997). The single-phase heat transfer coefficient h_l is again calculated from the Dittus-Boelter correlation (Collier, 1981). The term q''_p denotes the energy transfer to the liquid due to the agitation of the thermal boundary layer caused by bubble growth - collapse cycle, which is also referred as the "pumping" component. The term q''_g denotes the heat flux consumed for bubble nucleation. The ratio between the pumping and nucleation components is called the pumping factor ε and is calculated according to the principles stated by Zeitoun & Shoukri (1997) as :

$$\varepsilon = \frac{\frac{\pi}{4} a^2 \delta \rho_l c_{pl}}{\frac{\pi}{6} a^2 b \rho_v h_{lg}} \left(\frac{T_w - < T_l >}{2}\right) \tag{6}$$

where the thermal boundary layer thickness δ is calculated from:

$$\delta = \frac{k_l (T_w - < T_l >)}{q''} \tag{7}$$

Bubble Nucleation

Bubbles are nucleated with a constant frequency at fixed nucleation sites, which are randomly distributed over the wall surface, and than grow instantly to so-called departure size, which is constant at each nucleation site. The nucleation site density, which varies along the flow, is determined from a balance between vapor generation rate through nucleation, bubble departure sizes and nucleation frequencies. The bubble departure diameter is assumed to obey a Gaussian distribution. The frequency of bubble nucleation at individual sites is assumed constant and calculated from the following correlation by Zuber (1963, from Collier, 1981):

$$f = \frac{0.59}{d_d}\left[\frac{\sigma g(\rho_l - \rho_g)}{\rho_l^2}\right]^{1/4} \tag{8}$$

Bubble Sliding and Detachment

After nucleation, bubbles first slide on the tube wall and than tend to detach and migrate towards the tube core region if their motion is not restricted by other bubbles. According to Van Helden et al. (1995), bubbles on the tube wall are subjected to the lift force, surface tension force, corrected buoyancy force, expansion force, drag force, and temperature drop force. In their experiments on subcooled nucleate boiling at low pressure, which were performed in a vertical annulus, Bibeau & Salcudean (1994) observed that the bubble sliding distance on the heated surface decreases with increasing void fraction. In the present work, it was assumed that the decrease of the sliding distance is related to the decrease of the distance between nucleation sites: the proliferation of bubble nucleation sites increases the disturbance of the liquid layer near the wall, thus creating more favorable conditions for bubble detachment.

Bubble detachment from the heated wall is modeled probabilistically, the probability of detachment being either constant or increasing linearly with decreasing average distance between nucleation sites. Bubble detachment is attempted every time a bubble moves a distance equal to its maximum vertical chord length in the axial direction. The detachment is simulated as a radial movement towards the tube center-line of a quarter of the bubble width.

Bubble Condensation

Bubbles which move laterally into the low-temperature tube core region collapse. The mechanisms governing bubble condensation in real flow are complex and depend on various parameters (Zeitoun et al., 1995). In the present model, it was assumed that bubbles condense if the difference between the liquid temperature corresponding to the bubble center and the saturation temperature exceeds a certain value ΔT_{cond}, which was adjusted to obtain a reasonable agreement between simulation and experimental results.

NUMERICAL MODEL

Bubble Axial and Lateral Motion

Bubble axial motion was simulated with a simple discrete time-step method, neglecting inertial effects:

$$z_i(t + \Delta t) = z_i(t) + w_{bi}(t) \cdot \Delta t \tag{9}$$

As bubbles in the proposed model undergo significant accelerations only briefly before axial collision with a leading bubble or after radial migration towards the tube center-line, the added mass effect was not taken into account. After each axial displacement during a time step, bubbles which overlapped more than the minimum relative overlapping and whose interaction was not prevented by turbulent dispersion were adjusted if they overlapped in the axial direction. Adjustments started at the tube entrance, and upper bubbles were adjusted with respect to lower bubbles.

Bubbles' cross-sectional coordinates assumed discrete values which correspond to points located on concentric circles centered on the tube axis (Fig. 1). The distance between neighboring points must be of the order of a fraction of the smallest bubbles' width. Bubble lateral movements were modeled as instantaneous jumps to other points which occur between successive time steps.

Volume Fraction and Energy Coupling

The vertical tube was divided in the axial direction into control volumes. The length of a control volume was of the same order of magnitude as the tube diameter. The total simulation time was divided into *averaging time intervals* during which passages of bubbles through the upper boundaries of control volumes were recorded. At the end of each interval, the liquid velocity profile in each volume was corrected to satisfy the condition (Kowe et al., 1988):

$$A_c j_l = \int_A w_{l\infty}(1-\alpha)dA_c + C_m w_{bl} \int_A \alpha dA_c \qquad (10)$$

where α denotes the time-averaged local void fraction obtained from recordings of bubble passages at the control volume upper boundary and integrals were calculated over the tube cross-section. The coefficient of added mass C_m was set equal to 0.5. In the same way, the liquid temperature profile was set so that the thermodynamic equilibrium quality at the control volume upper boundary assumed the steady-state value. These liquid velocity and temperature profiles were then used in the calculations during the next averaging time interval. Within each control volume, liquid velocities and temperatures were calculated by linear interpolation between values which correspond to the control volume lower and upper boundaries.

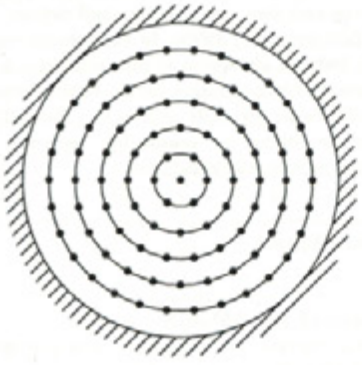

Figure 1. Schematic representation of bubble centers' discrete cross-sectional coordinates.

Bubble Coalescence, Nucleation and Condensation

Mergers between bubbles, bubble condensations and bubble nucleations were modeled as instantaneous events which occur between time steps. After each averaging time interval, the part of the heat flux which is consumed for bubble nucleation was determined again for each control volume, as described earlier. Nucleation sites on the tube wall were than generated randomly and used during the next averaging time interval. The number of nucleation sites was determined from bubble departure diameters and corresponding nucleation frequencies.

RESULTS AND DISCUSSION

Experimental Conditions

There is very little published experimental data on the evolution of local parameters in forced convective boiling in vertical cylindrical tubes. Some results on subcooled and low quality boiling flow are reported by Sekoguchi et al. (1980, 1981). Two experimental runs (referred as "1" and "2" in the present work) were selected to assess the capability of the present model to simulate subcooled nucleate boiling. Experimental conditions for both runs are presented in Table 1.

Simulation Results

The flow was simulated over distances z/D equal to 35 (run 1) and 30 (run 2). Bubbles were generated randomly at the beginning of the considered tube region to help obtaining a void fraction profile similar to the experimental one.

Figures 2 and 3 depict experimental and simulated time-averaged void fraction radial profiles at different axial locations along the flow. The proposed model simulates quite well the gradual increase of the void fraction peak and the widening of the layer near the wall, in which bubbles are present. The main discrepancy is that the void fraction peak moves too quickly away from the tube wall, especially in run 1 (Fig. 2). As the simulated process is a complex phenomenon, caused by interacting and mutually competing processes of wake drift, differing bubble velocities due to liquid velocity gradient, bubble lateral motion, bubble collisions and coalescence, turbulent dispersion, bubble nucleation, bubble sliding on tube walls and bubble condensation, the overall agreement between simulations and experiments indicates that the basic mechanisms which govern void fraction profile development in subcooled nucleate boiling have been adequately taken into account.

One of the most influential parameters in bubbly flow is allegedly bubble size which was not measured and therefore had to be assumed. In run 1, bubble departure diameters were assumed to obey a Gaussian distribution between values 0.4 mm and 2.0 mm, whereas in run 2, the limiting values were 0.4 and 1.0 mm. Both ranges are roughly in accordance with experimental results from Bibeau & Salcudean (1994). Bubbles already present at the initial axial location were supposed to assume the maximum departure size.

Table 1. Experimental conditions of Sekoguchi et al. (1981).

Run	D [mm]	P [atm]	q'' [kW/m^2]	m° [kg/m^2·s]
1	15.78	2	189.6	695
2	13.55	4	232.6	500

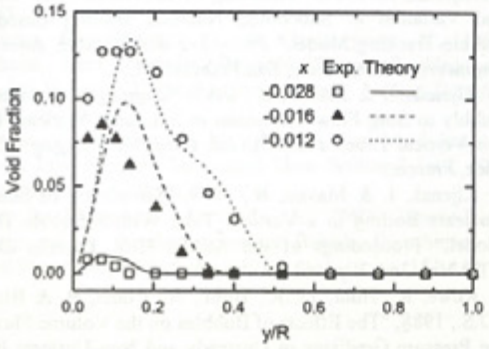

Figure 2. Simulated and experimental void fraction radial profiles (run 1 from Sekoguchi et al., 1981).

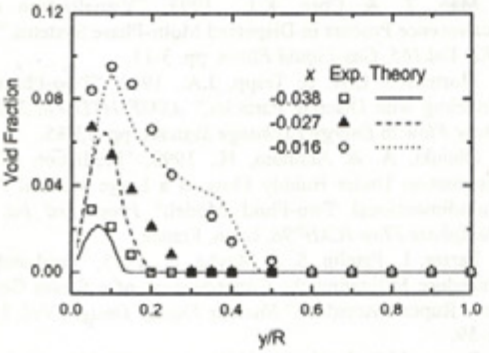

Figure 3. Simulated and experimental void fraction radial profiles (run 2 from Sekoguchi et al., 1981).

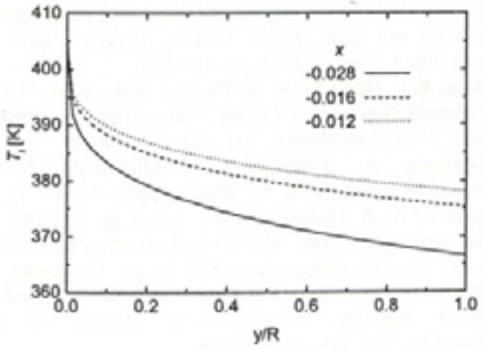

Figure 4. Simulated liquid temperature radial profiles (run 1).

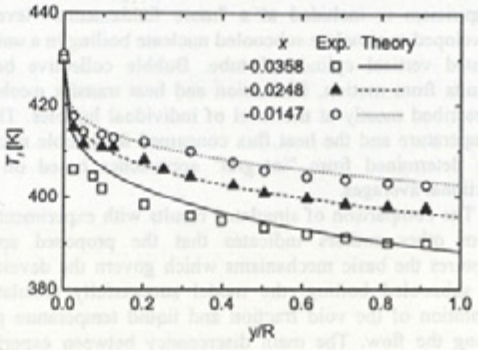

Figure 5. Simulated and experimental liquid temperature radial profiles (run 2, with data from Sekoguchi et al., 1980).

Both simulations were carried out with identical values of minimum relative overlapping (0.3), probability of turbulent dispersion (0.1), rest time (0.020 s) and ΔT_{cond} (10 K). Although turbulent dispersions of wake drift and of sticking bubbles were treated differently, equal probabilities of turbulent dispersion were prescribed for both phenomena to minimize the number of different parameter values. There is also a lack of information on rest times in turbulent flows, necessitating the use of what is in effect an adjustable parameter (Prince & Blanch, 1990).

The probability of bubble detachment depended on the ratio δ^* between average distance between nucleation sites δ and mean bubble departure diameter $d_{d,mean}$:

$$\delta^* = \delta / d_{d,mean} \tag{11}$$

If δ^* was larger than the threshold value 2.5, the probability of bubble detachment was equal 0.25. Otherwise, it was prescribed as a linear function of δ^*:

$$p_d = 1.9 - 0.66 \cdot \delta^* \tag{12}$$

The coefficient k_m in eq. (2) was set equal to 85 for run 1 and to 60 for run 2. More comparisons of simulated flows with experimental data are necessary before reaching conclusions on the correspondence between actual flow parameters and empirical factors which are used in modeling of bubble detachment and lateral migration

Figures 4 and 5 show simulated liquid temperature profiles. Experimental data are available only for run 2 (despite some flow conditions being stated slightly differently, it was presumed that data presented by Sekoguchi et al., 1980, correspond to run 2 from data presented in the work of Sekoguchi et al., 1981). The agreement appears to be good. The only major discrepancy is the somewhat steeper experimental temperature gradient near the tube wall at high subcooling.

CONCLUSIONS

A three-dimensional bubble-tracking model, in which empiricism is included at a "more fundamental" level, was developed to simulate subcooled nucleate boiling in a uniformly heated vertical cylindrical tube. Bubble collective behavior results from motion, interaction and heat transfer mechanisms prescribed mostly at the level of individual bubbles. The wall temperature and the heat flux consumed for bubble nucleation are determined from "integral" approaches based on cross-sectional averages.

The comparison of simulated results with experimental data from other authors indicates that the proposed approach captures the basic mechanisms which govern the development of subcooled boiling: the model successfully simulates the evolution of the void fraction and liquid temperature profiles along the flow. The main discrepancy between experimental and simulated results is the radial location of the void fraction peak near the tube wall.

ADDITIONAL NOMENCLATURE

a	ellipsoidal bubble large axis [m]
b	ellipsoidal bubble small axis [m]
d	bubble equivalent diameter [m]
h	specific enthalpy [J/kg]
	heat transfer coefficient [W/m^2·K]
j	volumetric flux [m/s]
l	bubble vertical chord length [m]
m^0	mass flux [kg/m^2·s]
p	probability [-]
r	radial coordinate [m]
x	thermodynamic equilibrium quality [-]
y	distance from tube wall [m]
z	axial coordinate [m]

Greek Letters

α	void fraction
δ	thermal boundary layer thickness [m]
	distance between nucleation sites [m]
ε	"pumping" factor

Subscripts

b	bubble
c	tube center-line
d	departure
i	i-th bubble
p	"pumping"

Other Symbols

$<>$	average over tube cross-section

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