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ANALYTICAL AND NUMERICAL STUDY OF DOUBLE DIFFUSIVE PARALLEL FLOW INDUCED IN A VERTICAL POROUS LAYER SUBJECTED TO CONSTANT HEAT AND MASS FLUXES

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ABSTRACT
Multiplicity of solutions induced by opposing thermal and solutal buoyancy forces in a vertical porous layer subject to horizontal fluxes of heat and mass is studied analytically and numerically using the Darcy model. The governing parameters for the problem are the Rayleigh number, R_T, the Lewis number, Le, the buoyancy ratio, N and the aspect ratio of the porous matrix, A. The analytical solution developed is based on the parallel flow approximation. The effect of the parameters Le and N on the multiplicity of solutions is studied. It is demonstrated, in this study, that numerical multiple steady state solutions, that agree very well with the analytical ones, are possible when the aspect ratio of the porous layer is large enough. It is also found that the heat and mass transfer depend strongly on the solution considered when multiple steady states are existing.

1. INTRODUCTION
Double-diffusive natural convection induced in a fluid-saturated porous medium is widely encountered in engineering applications. This phenomenon is of importance in many situations like the migration of moisture through air contained in fibrous insulations, food processing, contaminant transport in ground water, electrochemical processes, etc.

Past studies on double diffusive convection in a vertical porous enclosure indicate that the resulting flows can be very different from those driven by the temperature field solely, especially when the buoyancy forces are opposite. The literature review shows that a limited number of investigations were concerned with opposing double-diffusive natural convection in rectangular porous enclosures. Trevisan and Bejan (1985) used numerical methods to study heat and mass transfer in a square cavity subjected to temperature and concentration gradients in the horizontal direction. They observed that the Nusselt and Sherwood numbers were minimum in the vicinity of N = -1. It was demonstrated numerically by Alavyoon et al. (1994) that there is a domain of N (< 0) in which oscillating convection is obtained for a given set of the governing parameters. Outside this domain, the solution approaches steady-state convection. Also, for opposing forces, the existence of multiple steady solutions has been demonstrated analytically and numerically by Mamou et al. (1995a) and Amahmid et al. (1999a). For a given value of N, it was demonstrated by Mamou et al. (1995b) that both Lewis and Rayleigh numbers have an influence on the domain of existence of these multiple steady state solutions. For the case of opposing and equal buoyancy forces (N = -1) the rest state, in a vertical cavity with constant temperature and concentration on the vertical walls, is an exact solution of the problem. However it was demonstrated by Mamou et al. (1998a) and Charrier-Mojtabi et al. (1998), by mean of linear stability theory, that the onset of convection occurs when the Rayleigh number exceeds a critical value which depends on the aspect ratio of the cavity and the Lewis number. The case of a vertical cavity with constant gradients of temperature and concentration prescribed on the vertical walls of the enclosure has been also considered by Mamou et al. (1998b). A linear stability analysis was carried out to describe the oscillatory and the stationary instability in terms of the governing parameters of the problem. By using the parallel flow approximation it was demonstrated that there exists a subcritical Rayleigh number at which a stable convective solution bifurcates from the rest state through finite amplitude convection.

The objective of the present investigation is to study the effect of the Lewis number and the buoyancy ratio on the mulitiplicity of solutions in the case of opposing double diffusive natural convection in a vertical porous layer subject to horizontal gradients of heat and mass.

2. MATHEMATICAL MODEL
The studied configuration is a two dimensional rectangular porous matrix of height H' and width L'. The geometry of the enclosure is sketched in Fig. 1. The vertical sides of the enclosure are submitted to constant fluxes of heat, q', and

mass, j', while its horizontal sides are insulated and impermeable. The fluid saturated porous medium is assumed isotropic and homogeneous and the Darcy model is considered. Using the Boussinesq approximation and assuming constant properties, the dimensionless equations governing this problem are:

$$\nabla^2\Psi = -R_T\left(\frac{\partial T}{\partial x} + N\frac{\partial S}{\partial x}\right) \tag{1}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \nabla^2 T \tag{2}$$

$$u\frac{\partial S}{\partial x} + v\frac{\partial S}{\partial y} = \nabla^2 S \tag{3}$$

The velocity components are related to the stream function by

$$u = \frac{\partial \Psi}{\partial y}; \quad v = -\frac{\partial \Psi}{\partial x} \tag{4}$$

where Ψ is the stream function, T the temperature, S the concentration, R_T the thermal Darcy-Rayleigh number, N the buoyancy ratio and Le the Lewis number. These parameters are defined as:

$$R_T = \frac{g\beta_T K q' L'^2}{\lambda \alpha \nu}, \quad N = \frac{\beta_S j'/D}{\beta_T q'/\lambda} \quad \text{and} \quad Le = \frac{\alpha}{D}$$

where β_T and β_S are the thermal and solutal expansion coefficients, α and D are the thermal and solutal diffusivities and K is the permeability of the porous medium.

The boundary conditions for Eqs. (1)-(3) are:

$$\Psi = 0; \ \partial T/\partial x = \partial S/\partial x = 1 \quad \text{for} \ x = \pm 1/2 \tag{5}$$

$$\Psi = \partial T/\partial y = \partial S/\partial y = 0 \quad \text{for} \ y = \pm A/2 \tag{6}$$

where the aspect ratio of the enclosure is defined by $A = H'/L'$

The heat and mass transfers are evaluated in terms of the Nusselt and Sherwood numbers which are given respectively by:

$$Nu = \frac{1}{T(\frac{1}{2}, 0) - T(-\frac{1}{2}, 0)}; \quad Sh = \frac{1}{S(\frac{1}{2}, 0) - S(-\frac{1}{2}, 0)} \tag{7}$$

Note that, according to the boundary conditions (Eqs. (5) and (6)), the thermal buoyancy forces tend to induce a counterclockwise fluid circulation while the solutal buoyancy forces tend to induce a clockwise fluid circulation for N<0 (case of opposing flows) and a counterclockwise fluid circulation for N>0 (case of cooperative flows).

3. RESULTS AND DISCUSSION

For large aspect ratio (A >> 1) it has been demonstrated in the past by Alavyoon et al. (1994) that the present problem can be significantly simplified by the approximation of the parallel flow. With this approximation u = 0 and v = v(x) in the central part of the cavity, i.e., outside the end regions. The approximation allows the following simplifications:

$$\Psi(x,y) = \Psi(x), \ T = C_T y + \theta_T(x) \ \text{and} \ S = C_S y + \theta_S(x) \tag{8}$$

where C_T and C_S are unknown temperature and concentration gradients respectively in y direction. Those gradients are determined by imposing zero heat and mass transfer across any transversal section of the cavity. Using these approximations, together with the boundary conditions (5) and (6), Eqs. (1) to (4) are reduced to a set of ordinary differential equations which can be solved to yield a closed analytical solution. Depending on the sign of the parameter Γ defined by

$$\Gamma = R_T(NLeC_S + C_T) \tag{9}$$

two solutions are possible.

For $\Gamma>0$, we set $\Gamma=\Omega^2$ and the solution of the simplified governing equations (denoted Ω-solution), satisfying the boundary conditions in x-direction, is:

$$\Psi = A\cosh(\Omega x) + a \tag{10}$$

$$T = C_T y + (1 - aC_T)x - C_T A' \sinh(\Omega x) \tag{11}$$

$$S = C_S y + (1 - aLeC_S)x - LeC_S A' \sinh(\Omega x) \tag{12}$$

where $A=-a/\cosh(\Omega/2)$, $A'=A/\Omega$ and $a=R_T(N+1)/\Omega^2$

The expressions of C_T and C_S can be established by imposing zero heat and mass transfer across any transversal section which yields:

$$C_T = A_1/(1 + A_0); \quad C_S = LeA_1/(1 + Le^2 A_0) \tag{13}$$

Then, Nu and Sh are given by:

$$Nu = 1/(1 - C_T A_1); \quad Sh = 1/(1 - LeC_S A_1) \tag{14}$$

where

$$A_0 = a^2\frac{3}{2}\left[\frac{A_1}{a} - \frac{1}{3}(\tanh(\Omega/2))^2\right] \quad \text{and} \quad A_1 = a\left[1 - \frac{2}{\Omega}\tanh(\Omega/2)\right]$$

To compute the value of Ω, an equation of this parameter is established by combining Eqs. (9) and (13):

$$aA_2\Omega^4 - R_T^2(N+1)^2\left(A_0Le^2 + \frac{1+NLe^2}{1+N}\right)A_1 = 0 \quad 15)$$

where $A_2 = 1 + A_0 + A_0Le^2 + A_0^2Le^2$

Note that the Ω-solution consists of a unicellular flow circulating in counterclockwise direction for N>-1 and in clockwise direction for N<-1.

For $\Gamma<0$, we set $\Gamma=-\omega^2$ and the resulting solution (denoted iω-solution) can be deduced by setting $\Omega=i\omega$ in the above solution. Since $\sinh(i\omega)=i\sin(\omega)$ and $\cosh(i\omega)=\cos(\omega)$ it is readily found that the solutions are similar to those given by Eqs. (10)-(12) with the hyperbolic functions replaced by circular functions. The parameter ω can be computed from the following equation

$$bB_2\omega^4-R_T{}^2(N+1)^2(B_0Le^2+\frac{1+NLe^2}{1+N})B_1=0 \qquad (16)$$

where

$$B_2=1+B_0+B_0Le^2+B_0{}^2Le^2$$

$$B_0=b^2\frac{3}{2}(\frac{B_1}{b}+\frac{1}{3}(\tan(\omega/2))^2$$

$$B_1=b(1-\frac{2}{\omega}\tan(\omega/2)) \quad \text{and} \quad b=-R_T(N+1)/\omega^2$$

Analytical and numerical treatment of Eqs.(15) and (16) showed that the domain where multiple steady state solutions are possible in the plane (Le, N) is represented by regions 1 and 2 of Fig. 2. It is clear that these regions correspond to opposing thermal and solutal buoyancy forces and are included in the domain $-3<N<-1/3$.

By solving Eq. (16) it is found that three monocellular iω-solutions can be obtained for the same governing parameters in the small domain represented by region 1 in Fig. 2 when $R_{Tmin}\le R_T\le R_{Tmax}$. The limits R_{Tmin} and R_{Tmax} depend on Le and N. They can be determined numerically by solving Eq. (16). For example, these three solutions are possible when $-1\le N\le-0.8922$ for Le=10 and when $-1\le N\le-0.8897$ for $Le\ge1000$. Note that the fluid circulations corresponding to these three solutions are in the same direction (clockwise for Le<1 and counterclockwise for Le>1). The values of R_{Tmin} and R_{Tmax} that delimitate the range of R_T for which multiple solutions exist are functions of N and Le (not presented here). It is found that R_{Tmin} changes slightly with N while R_{Tmax} tends towards infinity when N approaches -1.

The presence of multiple monocellular iω-solutions in the region 1 of Fig. 2 is also demonstrated numerically by solving the full governing equations (1)-(4). The numerical technique used in this study is similar to that described by Amahmid et al. (1999b). The numerical results obtained are illustrated in Figs. 3 and 4 in terms of Nu and Sh as functions of R_T for Le=10 and N=-0.995. The computations were performed with aspect ratios A in the range $6\le A\le9$. It can be seen from these figures that three analytical solutions are observed for $2.22\le R_T\le7.8$. Two of these solutions were obtained numerically. Let ω_1, ω_2 and ω_3 denote the values of ω corresponding to these solutions such that $\omega_1\le\omega_2\le\omega_3$. It is noted that the two solutions obtained numerically correspond to ω_1 and ω_3. That corresponding to ω_2 was not obtained numerically despite the multiple favourable initial conditions tested. The difference between the two solutions obtained numerically in terms of Nu remains less than 25%. However, the Sherwood number changes considerably from one solution to the other. At $R_T=7$, for example, the solution ω_1 leads to Sh=1.002, while that corresponding to ω_3 leads to Sh=4.51, i.e., four times higher than the first value.

Multicellular iω-solutions consisting of (2m+1) cells (m=1, 2, 3,...), are also possible for values of N very close to -1 inside region 1 of Fig. 2. But these solutions could not be obtained numerically in the present work. It is noted that even when N is very close to -1, the changes observed in the number of existing solutions are extremely different from those found for N=-1 when R_T is increased. For example, for Le=10 and N=-0.995, there is a unique monocellular solution for $R_T<2.22$, the number of solutions passes to three monocellular solutions for $2.22\le R_T\le7.8$, drops to a single monocellular solution for $7.8<R_T<67.5$, passes again to three solutions (one monocellular and two three-cells solutions) for $67.5<R_T<459$, rises to five solutions (one monocellular, two three-cells solutions and two five-cells solutions) for $459<R_T<3958$, drop to three solutions (one monocellular and two three-cells solutions) for $3958<R_T<6385$ then to a single monocellular solution for $R_T>6385$. It is clear then that the number of iω-solutions undergoes increases and decreases when R_T increases. However, the number of solutions can only increase with R_T for N=-1 (Amahmid et al., 1999a). Also, when a flow mode m (i.e a flow consisting of (2m+1) cells) appears, it does not disappear by further increasing R_T for N = -1.

For region 2 it is found that an Ω-solution is always existing regardless of the value of R_T. Furthermore, for adequate ranges of R_T, multiple iω-solutions (two or more) are possible in this region. It is found that this region satisfies the following equation:

$$-1\le N\le N_0 \quad \text{for} \quad Le<1 \quad \text{and} \quad N_0\le N\le-1 \quad \text{for} \quad Le>1$$

where N_0 is given by

$$N_0=\frac{3Le^2+2\sqrt{3}Le+1}{Le^2+2\sqrt{3}Le+3}$$

For any couple (Le, N) in region 2, it can be demonstrated analytically that there exist two values of R_T for which the velocity is zero on the vertical boundaries and the opposing effects of thermal and solutal buoyancy forces act such that the Darcy model satisfies the no-slip boundary conditions on the vertical boundaries. This condition is satisfied nowhere in the rest of the (Le, N) plane. The values of R_T for which this behavior is obtained are:

$$R_{T\pm}=4\pi^2\left[\frac{-d\pm\sqrt{d^2-12Le^2}}{3Le^2(N+1)^2}\right]^{1/2}$$

where

$$d=\frac{\left[(1+3Le^2+N(3+Le^2)\right]}{N+1}$$

Also it can be verified that the flow direction for any monocellular iω-solution depends only on the sign of (Le-1) (clockwise for Le<1 and counterclockwise for Le>1). While it is noted that the flow direction for the Ω-solutions depends on the sign of (N+1). This implies that the direction of the flow is imposed by the component (temperature or concentration) with the highest diffusivity in the case of iω-solutions and by the one with the highest buoyancy forces in the case of Ω-solutions.

The multiplicity of steady state solutions in region 2 is illustrated in Figs. 3 and 4 in terms of Nu and Sh variations with R_T for N=-1.6 and Le=10. The solutions presented in these figures are constituted by one Ω-solution and two iω-solutions. It can be seen from Figs. 3 and 4 that the Ω-solution is analytically possible for any given value of R_T while the iω-solutions are possible only in the range $3.49 \le R_T \le 92.7$. Furthermore, there exist values of R_T at which the iω-solutions pass from monocellular pattern to three-cells pattern. Numerically, it was demonstrated that it is possible to obtain only two monocellular solutions (one Ω-solution and one iω-solution) for $5 \le R_T \le 60$. If we focus our attention only on the solutions obtained numerically, it can be noted that the best heat and mass transfers are generated by the iω-solution for which Nu and Sh pass through maximum values when R_T increases. This means that, for this kind of opposing flow, the increase of R_T may lead to a decrease of Nu or Sh. For the Ω-solution, Nu and Sh have asymptotic evolutions (Nu≈1.006 and Sh≈1.609 at large R_T) and the heat transfer is always dominated by diffusive effects regardless of the value of R_T. It is noted that the analytical monocellular Ω and iω-solutions consist of clockwise and counterclockwise rotating cells, respectively. It should be noted that for $5 \le R_T < 20$, it was found that an aspect ratio A=4 was sufficient to predict numerically the analytical results for both Ω and iω-solutions. For this range of R_T, a secondary flow consisting of two small vortices confined inside the main cell are induced in the case of the numerical Ω-solution. The secondary cells are located in the upper and lower parts of the enclosure. However, for the same range of R_T, the secondary flow is not observed for the numerical iω-solution. It appears only when $R_T \ge 20$ and consists of two secondary clockwise rotating cells located in the upper right and lower left corners of the enclosure. The expansion of these cells in the vertical direction, when R_T increases, is found to affect gradually the parallel nature of the flow in the core region. In this case, higher values of A are required to predict the analytical results. Note that the parallel nature of the flow was not affected by the presence of the secondary cells in the case of Ω-solutions even at large values of R_T.

The above discussion shows that in region 2, up to three monocellular solutions are possible for the same set of the governing parameters (one Ω-solution and two iω-solutions). The flow rotation for the iω-solutions is opposite to that for Ω-solution. However, for region 1 the cells are rotating in the same direction for all the monocellular solutions

Analytically, it can be demonstrated in region 2 that there exists values of R_T for which the flow consists of n vertical cells rotating in the same direction (clockwise for Le<1 and counterclockwise for Le>1).

4. CONCLUSION

Double-diffusive natural convection in a vertical porous layer submitted to constant fluxes of heat and mass on its vertical sides is studied analytically and numerically. It is found that multiple steady state solutions are possible when the thermal and solutal buoyancy forces have opposing effects. The values of the buoyancy ratio for which this phenomenon is observed are always included in the range $-3<N<-1/3$ for any value of the Lewis number. This means that the multiplicity of solutions is possible only when the thermal and solutal buoyancy forces have opposing effects and are of the same order of magnitude. The analytical study shows that, for adequate ranges of R_T, three analytical monocellular iω-solutions, rotating in the same direction, are possible in region 1 (Fig. 2). While in region 2 two monocellular iω-solutions, rotating in the same direction, and one Ω-solution, rotating in the opposite directions, are possible. The rotation direction corresponding to monocellular iω-solutions is imposed by the component (temperature or concentration) having the highest diffusivity (i.e depends on Le) while that corresponding to Ω-solutions is imposed by the component having the highest buoyancy forces (i.e depends on N). Numerically, two of the three monocellular solutions were obtained. Also for the same set of governing parameters, two different solutions (when these exist) can induce considerable differences in terms of Nu and Sh. For region 2 there are values of R_T such that the flow velocity is zero on the vertical boundaries for the iω-solutions meaning that the opposing buoyancy forces act as though the Darcy model satisfies the no-slip condition on the vertical boundaries.

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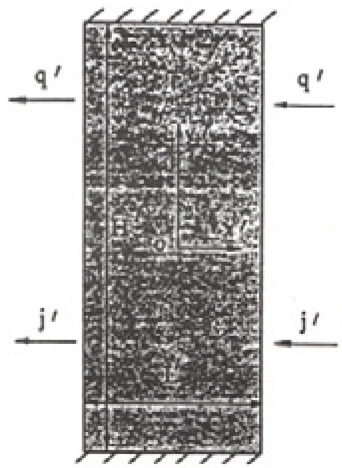

Fig. 1. The studied system

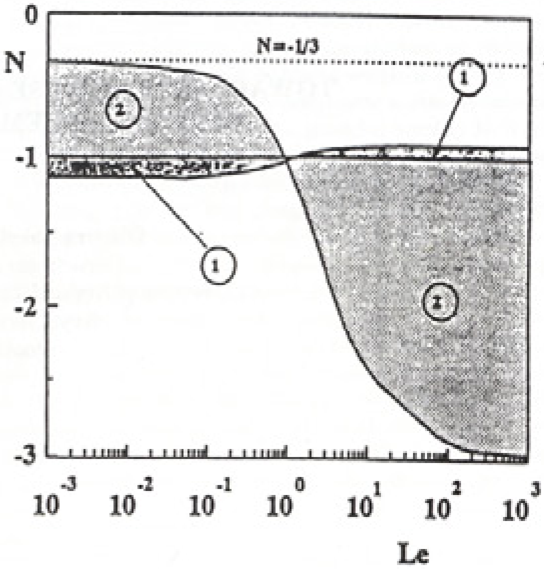

Fig. 2. Regions corresponding to multiple steady-state solutions

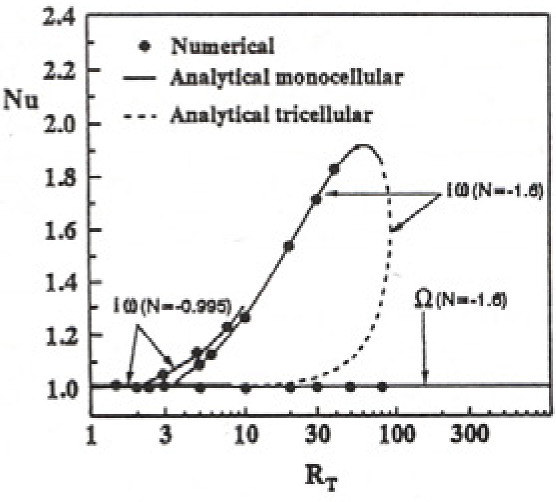

Fig. 3. Nu variation with R_T for Le=10: regions 1 and 2

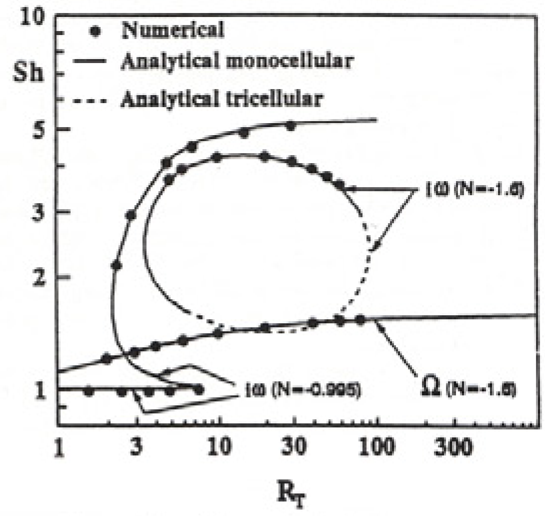

Fig. 4. Sh variation with R_T for Le=10: regions 1 and 2