

# Practical Tracking Control of the Electropneumatic Piston Drive

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*According to the fundamental importance of the tracking theory on technical systems, the main goal of this paper is further development of the theory and the application of tracking, especially on the practical tracking concept.*

*The plant under consideration is a pneumatic cylinder supplied with pressurized air by an electropneumatic servovalve. This system is often applied as the final control element of the controller in automatic control systems. The correction device for the mentioned plant will be a digital computer. The pure inertial load of the pneumatic cylinder will be time variable. Therefore this plant belongs to the unstationary class of systems. For time varying desired output value the control algorithm will be synthesized.*

*The control algorithm is based on the self-adjustment principle. Structural characteristic of such a control system is existence of two feedbacks: global negative of the output value and local positive of the control value. Such a structure ensures synthesis of the control without the internal dynamics knowledge and without the measurement of disturbance values. The mentioned control forces the observed plant output to track the desired output value with the prespecified accuracy.*

*In this paper simulation results produced by the practical tracking control algorithm on an electropneumatic piston drive will be presented.*

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**Keywords:** exponential tracking, electropneumatic piston drive, self-adjustment principle

## 0 INTRODUCTION

The practical tracking concept is very important according to the technical viewpoint. Consideration of dynamics behavior of technical plants on limited time interval, with prespecified quality of that behavior, states practical request and necessities which can be placed to any technical plant. For many technical plants the most adequate tracking concept is the practical tracking concept. This concept most completely satisfied practical technical requirements on the dynamics behavior and on quality of the dynamics behavior. Practical tracking concept includes physically possible and realizable initial deviations of the output value, maximal permitted deviations of the output value in relation to the desired output value (according to desired accuracy), the set of expected and unexpected disturbances on such a time interval which is of the technical interest.

In this paper elementwise exponential tracking is considered. Each element of vector  $\mathbf{y}$  should exponentially approach appropriate element of vector  $\mathbf{y}_d$ . Elementwise exponential tracking was introduced in [1] to [3]. In those

papers Lyapunov approach to the exponential tracking study is used. That approach assumes the existence of the bound (the envelope of the output error vector) which limits the exponential evolution of output error vector, but that bound is not predefined. In this framework, bounds are predefined and determined with function set  $I_A(\cdot)$  and scalar  $\beta$ .

Nonuniforme practical exponential tracking is introduced in [9], where definitions, criterias and algorithms for such a tracking are presented for certain class of technical objects.

## 1 SYSTEM DESCRIPTION

The object considered can be described by the mathematical model expressed by the state and output equations:

$$\begin{aligned}\frac{d\mathbf{x}}{dt} &= \mathbf{Ax} + \mathbf{Bu} + \mathbf{Dd}, \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Fd}.\end{aligned}\tag{1}$$

Admitted bounds of the vector  $\mathbf{y}$  of the object real dynamic behavior are determined by the vector of desired dynamic behavior  $\mathbf{y}_d$  and sets  $E_I$  and  $E_A$  as follows:

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$$I_I(\mathbf{y}_{d0}; E_I) = \{\mathbf{y}_0 : \mathbf{y}_0 = \mathbf{y}_{d0} - \mathbf{e}_0, \mathbf{e}_0 \in E_I\},$$

$$I_A(t; \mathbf{y}_d(\cdot); E_A) = \{\mathbf{y} : \mathbf{y} = \mathbf{y}_d(t) - \mathbf{e}, \mathbf{e} \in E_A\}.$$

## 2 DEFINITION

**Definition 1** [9] The system (1) controlled by  $\mathbf{u}(\cdot) \in S_u$  exhibits *practical exponential tracking* with respect to  $\{\tau, \Lambda, \beta, I_I(\cdot), I_A(\cdot), S_d, S_z\}$  (Fig. 1) if  $[\mathbf{y}_0, \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in I_I(\mathbf{y}_{d0}) \times S_d \times S_z$  implies:

$$\mathbf{y}[t, \mathbf{y}_0, \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in I_A(t), \forall t \in R_\tau,$$

and for  $\forall i \in \{1, 2, \dots, n\}$  and  $\forall t \in R_\tau$  holds:

$$\begin{aligned} y_i[t, y_{i0}, \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] &\geq \\ y_{di}(t) - \alpha_i (y_{di0} - y_{i0}) e^{-\beta t}, \quad y_{i0} &\leq y_{di0}, \\ y_i[t, y_{i0}, \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] &\leq \\ y_{di}(t) - \alpha_i (y_{di0} - y_{i0}) e^{-\beta t}, \quad y_{i0} &\geq y_{di0}. \end{aligned}$$

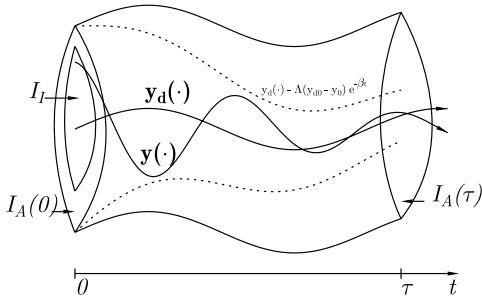


Fig. 1. Practical exponential tracking

## 2 CRITERIA

**Theorem 1** [9] In order for the system (1) controlled by  $\mathbf{u}(\cdot)$  exhibit *practical exponential tracking* with respect to  $\{\tau, \Lambda, \beta, I_I(\cdot), I_A(\cdot), S_d, S_z\}$  it is sufficient that control  $\mathbf{u}(\cdot)$  guaranties:

$$\dot{\mathbf{e}}(t, \mathbf{e}_0, \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)) = -\Gamma \mathbf{e},$$

$$\forall [t, \mathbf{e}_0, \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] \in R_\tau \times E_I \times S_d \times S_z,$$

where  $\gamma_i \in [\beta, +\infty], \forall i = 1, 2, \dots, n$ .

## 3 ALGORITHM

The algorithm is based on natural tracking control concept. Main characteristic of this concept, which follows from the self-adaptive principle [4] to [6], is existence of local positive feedback in the control  $\mathbf{u}$  (with possible derivates and/or integrals of  $\mathbf{u}$ ). The local positive feedback compensates influences of the disturbances and the internal dynamic of the controlled object, because during the control construction information about them are not used.

The main negative feedback loop in the output  $\mathbf{y}$  (with possible derivates and/or integrals of  $\mathbf{y}$ ) provides desired quality of the error evolution.

**Assumption 1** Values of all vector elements  $\mathbf{y}(t)$  and  $\dot{\mathbf{y}}(t)$  From the Eq. (1) are measurable in any time instant  $t \in R_\tau$ .

**Theorem 2** Let Assumption 1 hold, let  $S_u = \{\mathbf{u}(\cdot)\}$  and control  $\mathbf{u}(\cdot)$ :

$$\mathbf{u}(t) = \mathbf{u}(t^-) + G^T (GG^T)^{-1} [\dot{\mathbf{e}}(t) + \Gamma \mathbf{e}(t)],$$

$$\forall [t, \mathbf{e}_0, \mathbf{y}_d, \mathbf{d}(\cdot)] \in R_\tau \times E_I \times S_d \times S_z,$$

where  $G$  is arbitrary matrix satisfied  $\det(GG^T) \neq 0$ , and  $\gamma \in [\beta, +\infty], \forall i = 1, 2, \dots, n$ .

System (1) controlled by  $\mathbf{u}(\cdot)$  exhibits *practical exponential tracking* with respect to  $\{\tau, \Lambda, \beta, I_I(\cdot), I_A(\cdot), S_d, S_z\}$ .

## 4 ELECTROPNEUMATIC PISTON DRIVE

The plant under consideration is a linear control system with a single output and a single control variable, so the Eq. (1) can be written as  $(\mathbf{y} = y, \mathbf{y}_d = y_d, \mathbf{u} = u, \mathbf{e} = e)$ :

$$\begin{aligned} \frac{dx}{dt} &= A \mathbf{x} + \mathbf{b}u + \mathbf{D}\mathbf{d}, \\ y &= \mathbf{c}^T \mathbf{x} + \mathbf{f}^T \mathbf{d}. \end{aligned} \quad (2)$$

A symbolic functional diagram of an electropneumatic piston drive is shown in Fig. 2.

This configuration is commonly used in fluid power systems. It is well known that, when used as position drives, they include integral action. The description of the state space model in the form (2) ensures that system matrix  $A$  is

singular. The trackability condition within a finite time interval according to [7] is:

$$-\mathbf{c}^T \mathbf{A}^{-1} \mathbf{b} \neq 0. \quad (3)$$

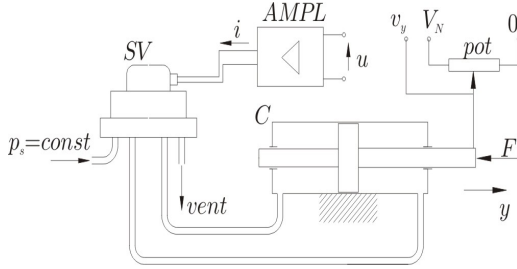


Fig. 2. Electropneumatic piston drive

The condition (3), for such a plant is not satisfied.

According to [10] it is possible to write the linearised equations of the electropneumatic piston drive in the block diagram form shown in Fig. 3.

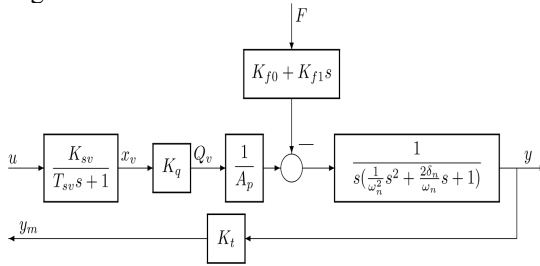


Fig. 3. Block diagram of electropneumatic piston drive

Typically, for the following system parameters apply [10]:

$$\begin{aligned} F_{pmax} &= 250 \text{ N}, & A_p &= 8.29 \cdot 10^{-4} \text{ m}^2, \\ m_p &= 2.5 \text{ Kg}, & A_{SVmax} &= 1.52 \cdot 10^{-6} \text{ m}^2, \\ C_2 &= 52.5 \text{ Ns/m}, & X_{vmax} &= 2.54 \cdot 10^{-4} \text{ m}, \\ H &= 0.05 \text{ m}, & P_s &= 5 \cdot 10^5 \text{ Pa}, \\ v_{max} &= 0.254 \text{ m/s} \end{aligned}$$

which give next values for constants from Fig. 3:

$$\begin{aligned} \delta_n &= 0.14, & K_{sv} &= 5.08 \cdot 10^{-5} \text{ m/V}, \\ T_{sv} &= 0.001 \text{ s}, & K_{f0} &= 1.722 \cdot 10^{-4} \text{ m/Ns}, \\ K_x &= 708.96 \text{ 1/s}, & K_{f1} &= 5.33 \cdot 10^{-5} \text{ m/N}, \\ \omega_n &= 86.71 \text{ 1/s}, & K_t &= 100 \text{ V/m}. \end{aligned}$$

## 5 DIGITAL COMPUTER SIMULATION

Digital computer realization of the algorithm requires the existence of the control value  $u(t)$  in time instant  $t$  and  $u(t^-)$  in time instant  $t$  infinitely close to the first one. In that case, the gain of this controller is infinity.

Symbolic block diagram of the control system is presented in Fig. 4.

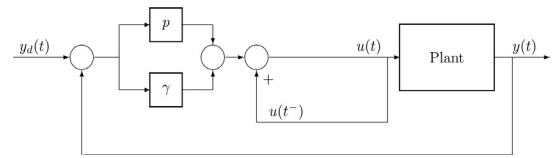


Fig. 4. System symbolic block diagram

A graphic of a disturbance change in terms of the external stall force acting on the rod ( $F$  in Fig. 2) is shown in Fig. 5.

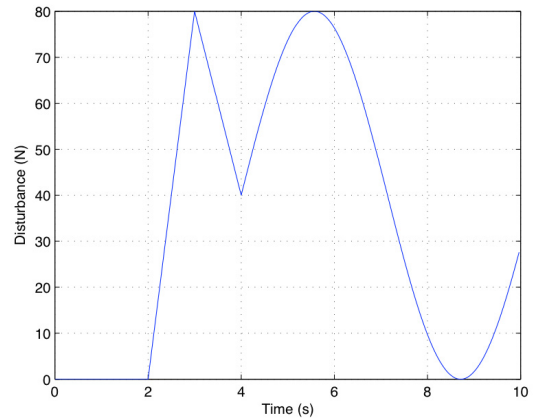


Fig. 5. External stall force

Let prespecified requirements are:

$$\begin{aligned} I_l &= \{y_0 : 0.0 \leq y_0 \leq 0.05\}, \\ I_A &= \{y : 0.0 \leq y \leq 0.05\}, \\ \tau &= 10, \\ \beta &= 1. \end{aligned} \quad (4)$$

Data used in simulation be:

$$y_0 = 0.0, e_0 = 0.023, \gamma = \beta = 1. \quad (5)$$

Since physical limitations of the control value ( $\pm 5 \text{ V}$ ) are not considered in the algorithm and plant is not natural trackable then the

achieved results are not appropriate as can be seen from the Figs. 6 to 8.

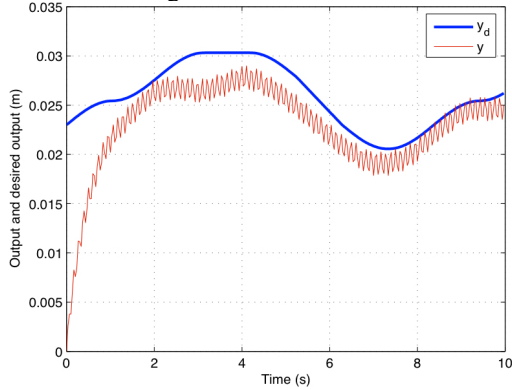


Fig. 6. Output and desired output

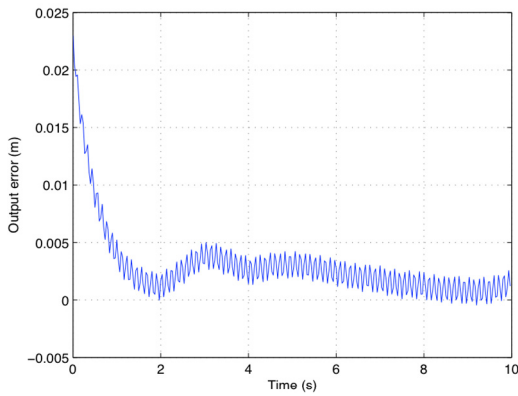


Fig. 7. Output error

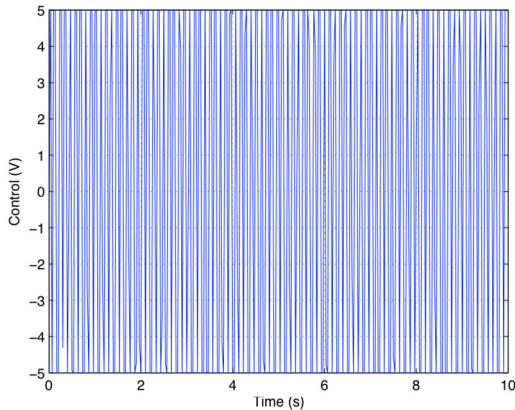


Fig. 8. Control

## 6 ALGORITHM MODIFICATIONS

For rectifying the problem another first order linear transition element is implemented. Its transfer function is as:

$$W(p) = \frac{1}{Tp + 1} \quad (6)$$

where  $T$  is a time constant.

Modified symbolic block diagram is presented in Fig. 9.

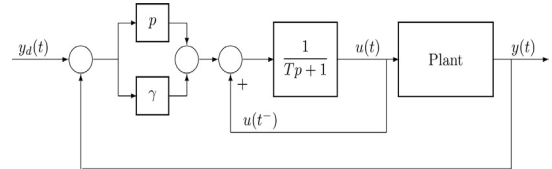


Fig. 9. Symbolic block diagram of modified system

Introduced element is obviously a low pass filter with the pole  $-1/T$  and the gain 1. Its purpose is to suppress higher order harmonics in the control signal produced by correction element.

For the illustration of the results achieved by the practical exponential tracking algorithm with first order low pass filter simulation results can be seen in Figs. 10 to 12.

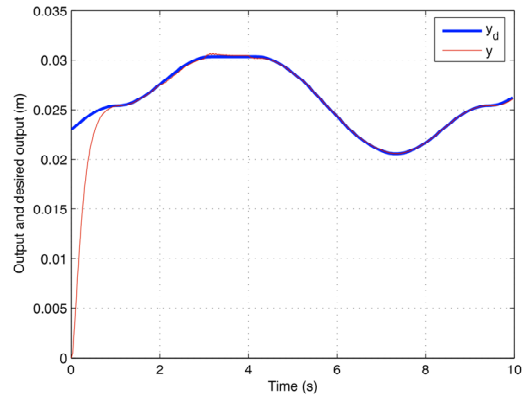


Fig. 10. Output and desired output

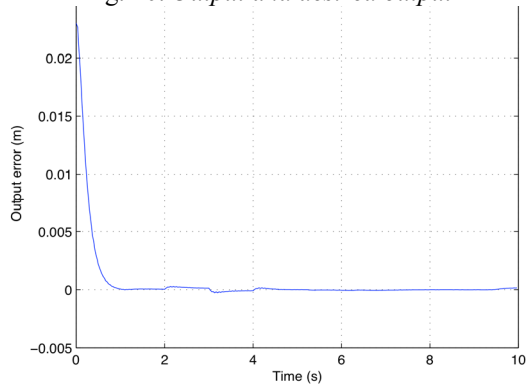


Fig. 11. Output error

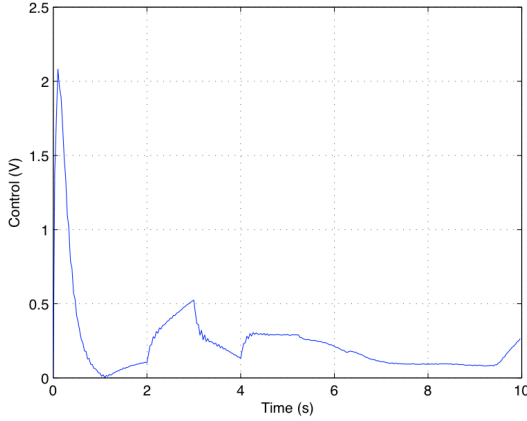


Fig. 12. Control

## 7 CONCLUSION

Simulation results obtained by control algorithm based on Fig. 4 do not satisfy Definition 1, as can be seen from the figures: Figs. 6 to 8. That is reasonable, because the plant is not trackable (conditions:  $\mathbf{c}^T \mathbf{b} \neq 0$  or  $\mathbf{c}^T \mathbf{A}^{-1} \mathbf{b} \neq 0$  are not satisfied), and physical limitations of the control were existing.

Simulation results obtained by the control algorithm based on the structure shown in Fig. 9 and prespecified requirements  $(\tau, I_I, I_A, \gamma)$  completely satisfy Definition 1.

## 8 NOTATION

$\mathbf{A} \in R^{q \times p}$  matrix

$\mathbf{B} \in R^{q \times r}$  matrix

$\mathbf{b} \in R^q$  vector

$\mathbf{C} \in R^{n \times q}$  matrix

$\mathbf{c} \in R^q$  vector

$\mathbf{D} \in R^{q \times p}$  matrix

$\mathbf{d}(\cdot): R \rightarrow R^p$  the disturbance vector function

$\mathbf{d}(t)$  The disturbance vector at time  $t$

$\mathbf{d} \in R^p$  the disturbance vector

$E_A \subset R^n$  the set of all admitted errors  $\mathbf{e}(t)$  on  $R_\tau$ ; closed connected neighborhood of  $\mathbf{0}_e$

$E_I \subset R^n$  the set of all admitted initial errors  $\mathbf{e}(0) = \mathbf{e}_0$ ; closed connected neighborhood of  $\mathbf{0}_e$

$\mathbf{e}[\cdot; \mathbf{e}_0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{z}(\cdot)]: R \rightarrow R^n$  the output error response, which at time  $t$  represents the output error vector  $\mathbf{e}(t)$  at the same time  $\mathbf{e}[t; \mathbf{e}_0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{z}(\cdot)] = \mathbf{e}(t)$

$\mathbf{e} \in R^n$  the output error vector,  $\mathbf{e} = \mathbf{y}_d - \mathbf{y}$

$\mathbf{F} \in R^{n \times p}$  matrix

$\mathbf{f} \in R^p$  vector

$\mathbf{G} \in R^{n \times q}$  arbitrary matrix

$I_A(\cdot): R \times R^n \times 2^{R^n} \rightarrow 2^{R^n}$ , the set function of all admitted vector functions  $\mathbf{y}(\cdot)$  on  $R_\tau$  with respect to  $\mathbf{y}_d(\cdot)$  and  $E_A$

$I_A(t) = I_A[t; \mathbf{y}_d(\cdot); E_A]$ , the set value of the set function  $I_A(\cdot)$  at time  $t$ , with respect to  $\mathbf{y}_d(\cdot)$  and  $E_A$

$I_I(\mathbf{y}_{d0}; E_I)$  the set value of the set function  $I_I(\cdot)$  at time  $t$ , with respect to  $\mathbf{y}_{d0}$  and  $E_I$ ; if  $\mathbf{y}_{d0}$  is chosen  $\Rightarrow I_I(\mathbf{y}_{d0}; E_I) = I_I$

$R_+ = ]0, +\infty[ = \{t: t \in R, 0 \leq t < +\infty\}$

$R^+ = ]0, +\infty[ = \{t: t \in R, 0 < t < +\infty\}$

$R_\tau = [0, \tau[$

$S_d \subset R^n$  the set of all admitted  $\mathbf{y}_d(\cdot)$  on  $R_\tau$ ;  $\mathbf{y}_d(\cdot) \in S_d \Rightarrow \mathbf{y}_d(t) \in C(R_\tau, R^n)$

$S_z \subset R^p$  the set of all admitted  $\mathbf{d}(\cdot)$  on  $R_\tau$

$t$  time

$\mathbf{u}(\cdot): R \times \dots \rightarrow R^r$  the vector function which describes evolution of the control vector

$\mathbf{u}(t)$  the value of the function  $\mathbf{u}(\cdot)$  at time  $t$

$\mathbf{u} \in R^r$  the control vector

$\mathbf{x} \in R^q$  the state vector

$\mathbf{y}[\cdot; \mathbf{y}_0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)]$  the real output response, which at time  $t$  equals the real output vector at same time,

$$\mathbf{y}[t; \mathbf{y}_0; \mathbf{u}(\cdot), \mathbf{y}_d(\cdot), \mathbf{d}(\cdot)] = \mathbf{y}(t)$$

$\mathbf{y} \in R^n$  the real output vector

$\mathbf{y}_d(\cdot): R \rightarrow R^n$  the desired output vector function

$\mathbf{y}_d(t)$  the desired output vector at time  $t$

$\mathbf{y}_d \in R^n$  the desired output vector

$$\beta \in R^+$$

$$\Gamma \in R^{n \times n}, \Gamma = \text{diag}\{\gamma_1 \gamma_2 \dots \gamma_n\}, \\ \gamma_i \in [\beta_i, +\infty[, \forall i = 1, 2, \dots, n$$

$$\Lambda \in R^{n \times n}, \Lambda = \text{diag}\{\alpha_1 \alpha_2 \dots \alpha_n\}, \\ \alpha_i \in [1, +\infty[, \forall i = 1, 2, \dots, n$$

$\tau \in ]0, +\infty]$  the final moment

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