Strojniški vestnik - Journal of Mechanical Engineering 56(2010)4, 245-252
UDC 621.839:621.892

Paper received: 23.10.2009
Paper accepted: 30.04.2010

Influence of the Oil Churning, the Bearing and the Tooth Friction Losses on the Efficiency of Planetary Gears

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The efficiency of a planetary gear depends on many factors: mainly on the tooth- and bearing friction losses, but beside them, in certain circumstances, also the oil churning losses have to be taken into consideration. This theoretical investigation was made to determine the oil churning, the bearing and the tooth friction losses of planetary gears and evaluate the influence of these types of friction losses on their efficiency. Optimization of the design of the planetary gear drives can result the best parameters leading to their highest efficiency even in the case of compound planetary gears.

Keywords: tooth friction loss, bearing friction loss, oil churning loss, loss of expel, efficiency

0 INTRODUCTION

In some applications it is necessary to use transmission systems having high gear ratio, which can be solved by using planetary gears. There are simple planetary gear constructions which are able to result high gear ratio, for example the KB+B, the KB+K, the K+K, the B+B (Figs. 1 and 2).

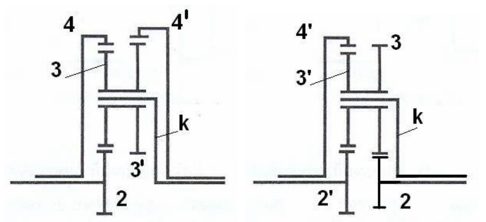

Fig. 1. *The planetary gear KB+B on the left and on the right planetary gear KB+K can be seen (K-external gear connection, B-internal gear connection)*

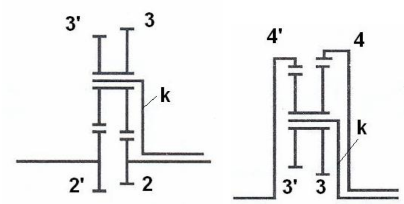

Fig. 2. *The planetary gear K+K on the left and on the right planetary gear B+B can be seen (K-external gear connection, B-internal gear connection)*

However, in this case the power flow of these types of planetary gear drives can be unbeneficial. It means that much higher rolling power can circulate inside the planetary gear than the input power increasing the friction loss and decreasing the efficiency. During design of a planetary gear having high gear ratio and a useless inner power circulation it is necessary to find compromise between the complexity of the construction and the allowable friction losses.

The efficiency of a planetary gear mainly depends on three factors: on the construction of the planetary gear drive determining the power flow, on the inner gear ratios and on the rolling efficiency. In practice the rolling efficiency is the multiplication of the tooth efficiencies. In some application and by some type of planetary gears it is not a correct procedure to calculate only the tooth friction losses while determining the efficiency of the gearbox. Investigating the methods for calculation the power losses of gears and gear drives, published in literature [1] to [8] we found that nearly every model concerns on the power loss model of simple gearboxes and not on the power loss model of planetary gear drives having special features. This fact pointed to our main goal creating a new mathematical model to predict the power losses of planetary gears.

1 POWER LOSS MODEL

1.1 Tooth Friction Losses

In practice the tooth friction loss of mating gears are taken into consideration mainly by using the known formulas of the tooth efficiency [3], [4], [6] and [8]. In our model the average coefficient of friction were calculated by using the equation developed by Eiselt and Ohlendorf

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[9], where the changes of parameters were also taking into consideration as a function of rotation angle of the pinion (φ):

$$\mu(\varphi) = 0.12 \cdot \sqrt[4]{\frac{w_t(\varphi) \cdot \tilde{R}_a}{\eta_M \cdot \Sigma v_t(\varphi) \cdot \rho_{ne}(\varphi)}} . \qquad (1)$$

The apparent power loss generated by the tooth friction along the contact line:

$$P_{fcs}(\varphi) = F_{1n} \cdot \mu(\varphi) \cdot v_{cs}(\varphi) . \qquad (2)$$

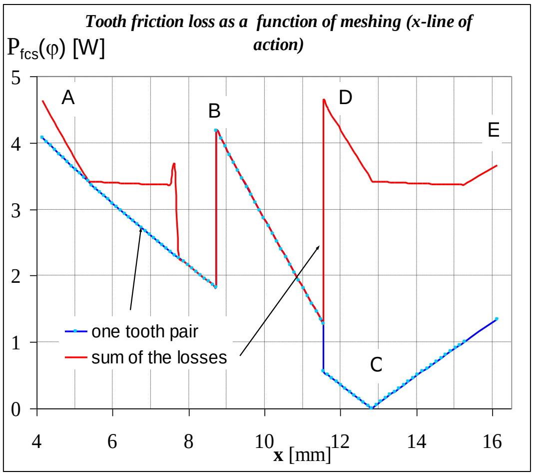

Fig. 3. *The apparent power loss generated by the tooth friction between sun gear and planet gear of a planetary gear K+K (Input power: P_{in}=500 W, tooth numbers: z_2=31, z_3=30, center distance: a_w=76.25 mm profile shift factor of sun gear x_2=0.5, profile shift factor of planet gear x_3=-0.5, gear ratio: i=15.7541, input speed: n_{in}=1440 1/min, point C is the pitch point)*

During calculating the sum of power losses generated by the tooth friction it has to be taken into consideration that thanks to the double tooth contact zone between points A and B and between D and E, the meshing teeth causes rolling and sliding friction losses between points A and B and at the same time between points D and E. Optimal tooth shift factors can lowering the power losses of meshing gears. With variation of the tooth profile shift factors the optimal tooth profile can be determined by the designer for a given application.

1.2 Bearing Friction Losses

To calculate the friction losses generated by the bearings the SKF model can be used, where the friction losses of the bearings are the functions of the average bearing diameters.

During an optimization of a planetary gearbox construction, it is difficult and sometimes impossible to select the average bearing diameters of every bearing of each gears and shafts from the main catalog. Therefore we set up a new model, where the average bearing diameter was calculated as a function of the load and the prescribed lifetime. Using this model the optimal type and size of the bearings of each shafts and gears can be determined for a given application.

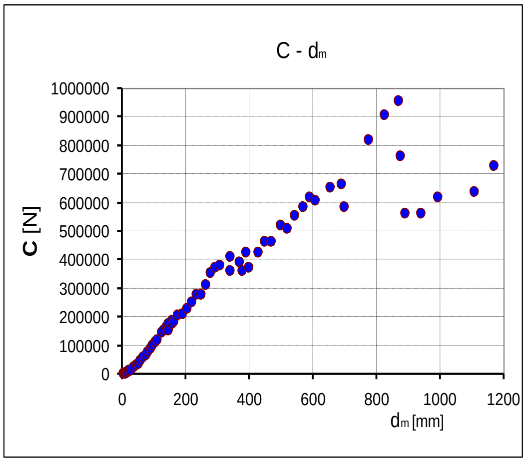

Fig. 4a. *The basic dynamic load rating as a function of the average diameter of deep groove ball bearings (Dimension series 22). Data were taken from SKF Catalog.*

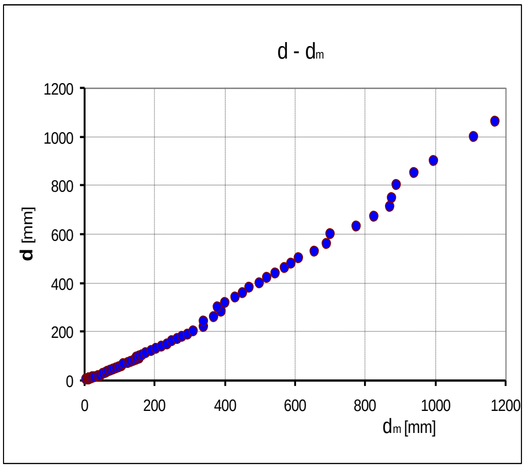

Fig. 4b. *The inner diameter as a function of the average diameter of deep groove ball bearings (Dimension series 22). Data were taken from SKF Catalog*

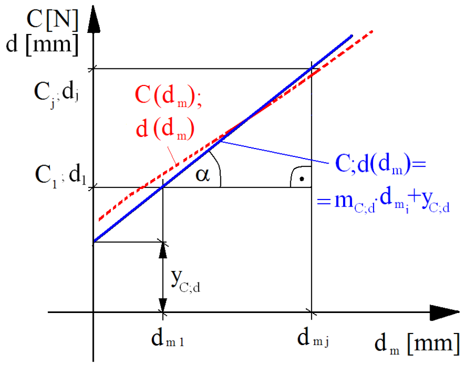

Fig. 5. *The linearization of the basic dynamic load and inner diameter functions*

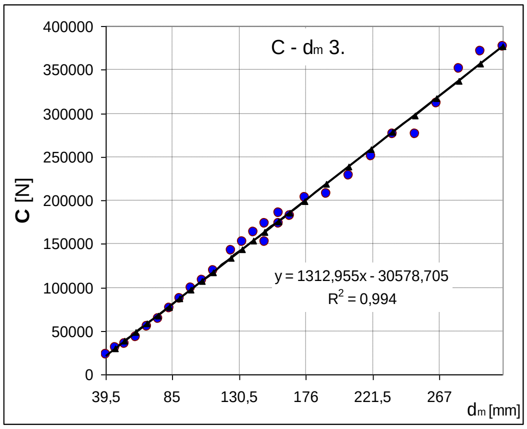

Fig. 6a. *The basic dynamic load rating as a function of the average diameter of deep groove ball bearings and the equations with* m_C*-* y_C *parameters; data were taken from [5]*

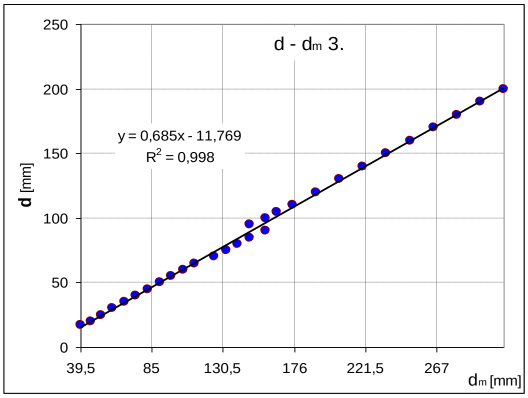

Fig. 6b. *The inner diameter as a function of the average diameter of deep groove ball bearings and the equations with* m_d*-* y_d *parameters; data were taken from [5]*

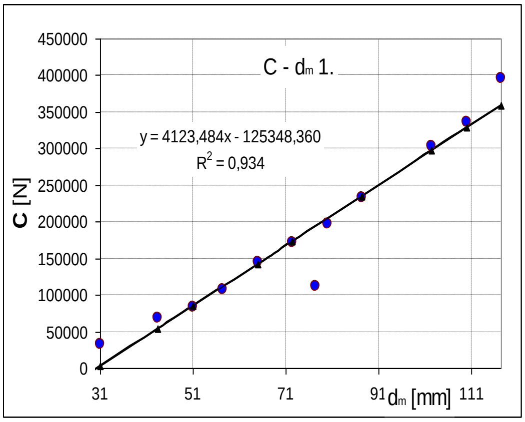

Fig. 7a. *The basic dynamic load rating as a function of the average diameter of cylindrical roller bearings and the equations with* m_C*-* y_C *parameters; data were taken from [5]*

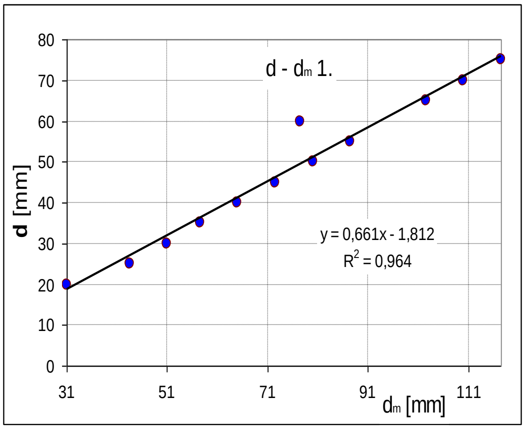

Fig. 7b. *The inner diameter as a function of the average diameter of cylindrical roller bearings and the equations with* m_d*-* y_d *parameters; data were taken from [5]*

The revealed connections between the bearing inner diameter (d) and its average diameter (d_m), and between the prescribe lifetime (L_h) and the average bearing diameter (d_m) were developed by using strength calculations. Factors m_i, y_i were developed by using [5] (Figs. 4 to 7).

Average diameter for sun gear and ring gear necessary to carry the load:

$$d_{m_{2;4}}(d) = \frac{\left(\sqrt[3]{\dfrac{16 \cdot M_{2;4}}{\tau_m \cdot \pi}} - y_d \right)}{m_d} \tag{3a}$$

and for planet gear:

$$d_{m_3}(d) = \frac{\left(\sqrt{\dfrac{16 \cdot V}{\sqrt{3 \cdot \sigma_m^2} \cdot \pi}} - y_d\right)}{m_d}. \qquad (3b)$$

Average diameter necessary to reach the lifetime of L_{1h}:

$$d_m(L_h) = \frac{\left(\sqrt[p]{\dfrac{L_{1h} \cdot 60 \cdot n}{10^6 \cdot a_1}} \cdot F_r - y_C\right)}{m_C}. \qquad (4)$$

To carry the applied load and also to reach the prescribed lifetime the biggest average diameter has to be chosen from the calculated values whether the load carrying capacity or the lifetime is dominant. The biggest average diameter is the resultant average (ball or roller) bearing diameter which can be calculated with the following equation:

$$d_{m_{res}} = d_m(d) + \\ + \frac{\left(\left|\left(d_m(L_h) - d_m(d)\right)\right| + \left(d_m(L_h) - d_m(d)\right)\right)}{2}. \qquad (5)$$

After determined the average diameter (d_m), the friction loss of bearings can be determined by the methods suggested by the bearing manufacturers [5]. For example the load dependent friction torque of a bearing can be calculated with the following simple equation:

$$M_1 = f_1 \cdot P_1^a \cdot d_{m_{res}}^b. \qquad (6)$$

The load independent friction torque of a bearing can be calculated with the following equation:

$$\nu \cdot n < 2000$$

$$M_0 = 160 \cdot 10^{-7} \cdot f_0 \cdot d_{m_{res}}^3$$

and $\qquad (7)$

$$\nu \cdot n \geq 2000$$

$$M_0 = 10^{-7} \cdot f_0 \cdot (\nu \cdot n)^{\frac{2}{3}} \cdot d_{m_{res}}^3.$$

1.3 Oil Churning Losses

The total oil churning loss consists of the loss of expel, the power loss of splashing and the disc churning loss components.

1.3.1 The Power Loss of Expel

As the teeth of the gear or pinion turns into the tooth valleys of the gear, the redundant oil volume is expelled by the teeth from the tooth valleys. During the expel process the oil has to be accelerated and to squeeze it out, which needs energy causing power losses. Define coefficients that describe the oil volume in the tooth valleys of the pinion ($_1$) and gear ($_2$), the flow rates (V'_{oki}) as a function of time can be calculated with the following equations developed for external and for internal gears:

$$\dot{V}_{o_{k1}}(\varphi_\Omega) = \frac{l}{t_{fb_1}} \cdot \int_{r_{a1}-\Sigma y_1(\varphi_\Omega)}^{r_{a1}} s_{yt_1}(r_y)\, dr_y$$

and

$$\dot{V}_{o_{k2}}(\varphi_\Omega) = \frac{l}{t_{fb_2}} \cdot \int_{r_{a2}-\Sigma y_2(\varphi_\Omega)}^{r_{a2}} s_{yt_2}(r_y)\, dr_y \qquad (8)$$

$$\dot{V}_{o_{k1}}(\varphi_\Omega) = \frac{l}{t_{fb_1}} \cdot \int_{r_{a1}-\Delta y_1(\varphi_\Omega)}^{r_{a1}} s_{yt_1}(r_y)\, dr_y$$

and

$$\dot{V}_{o_{k2}}(\varphi_\Omega) = \frac{l}{t_{fb_2}} \cdot \int_{r_{a2}}^{r_{a2}+\Delta y_2(\varphi_\Omega)} s_{yt_2}(r_y)\, dr_y. \qquad (9)$$

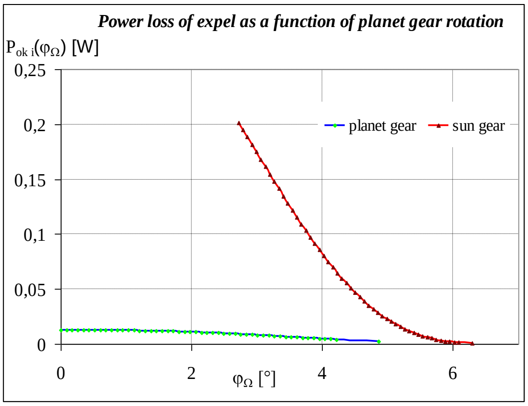

Fig. 8. *Power losses generated by oil expel in a planetary gear K+K (Input power: P_{in}=500 W, tooth numbers: z_2=31, z_3=30, z_2'=30, z_3'=31, center distance: a_w=76.25 mm, gear ratio: i=15.7541, input speed: n_{in}=1440 1/min, dip lubrication)*

The average power losses generated by the oil expel can be calculated with the following equations, taking into consideration the cross sections of the flow (A_{eyti}) and the flow rates as a function of time and the meshing conditions:

- if $\xi_i \geq \Omega_i$

$$\tilde{P}_{o_{ki}} = \frac{\rho}{\xi_i} \cdot \int_0^{\Omega_i} \frac{\dot{V}_{o_{ki}}^3(\varphi_\Omega)}{A_{e_{yti}}^2(\varphi_\Omega)} d\varphi_\Omega; \qquad (10)$$

- if $\xi_i < \Omega_i$

$$\tilde{P}_{o_{ki}} = \frac{\rho}{\Omega_i}\cdot \left(\int_0^{\Omega_i} \frac{\dot{V}^3_{o_{ki}}(\varphi_\Omega)}{A^2_{e_{yti}}(\varphi_\Omega)} d\varphi_\Omega + \int_{\Omega_i-\xi_i}^{\Omega_i} \frac{\dot{V}^3_{o_{ki}}(\varphi_\Omega)}{A^2_{e_{yti}}(\varphi_\Omega)} d\varphi_\Omega \right). \quad (11)$$

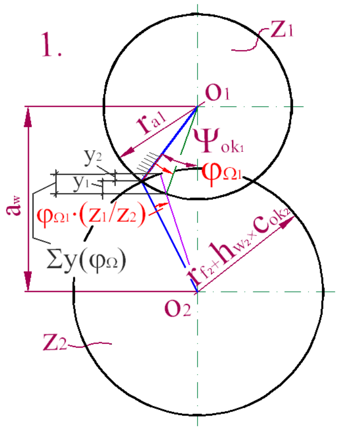

Fig. 9a. *Geometric model for calculation the oil expel for external gearing*

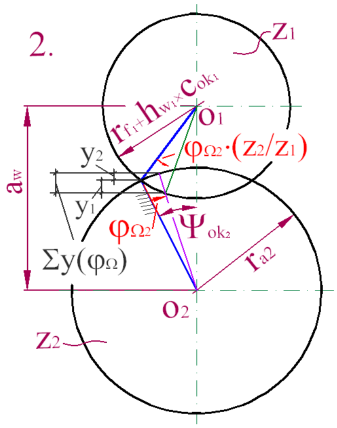

Fig. 9b. *Geometric model for calculation the oil expel for external gearing*

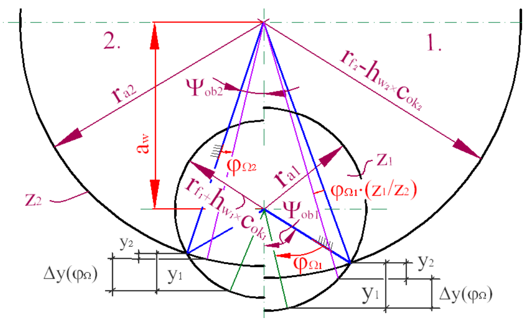

Fig. 10. *Geometric model for calculation the oil expel for internal gearing*

1.3.2 The Power Loss of Splashing

The planet gears submerge into the oil during revolving around the coaxial shaft of the transmission. Thanks to the revolving and the rotation of the planet gears, the gears submerge into lubricant a high tangential speed by some constructions. The wheel-body hits the lubricant surface and accelerates the oil getting in its tooth valleys. Determining the number of submerging teeth ($z_o(\varphi_o)$), tangential speed (v_{ra3}), the mass flux ($\dot{m}_{op}$) and the submerged area as a function of time, the power loss of splashing can be calculated with the following equation developed:

$$P_p(\varphi_o) = z_o(\varphi_o)\cdot \dot{m}_{op}\cdot v^2_{r_{a3}} = \frac{z_{3_i}\cdot\left(\hat{\alpha}_0(\varphi_o)+\omega_{3g}\cdot\dfrac{\hat{\varphi}_o}{\omega_k}\right)\cdot\rho\cdot c_{oe}\cdot A_{e_{yt3}}\cdot l\cdot v^2_{r_{a3}}}{2\cdot\pi\cdot t_k}. \quad (12)$$

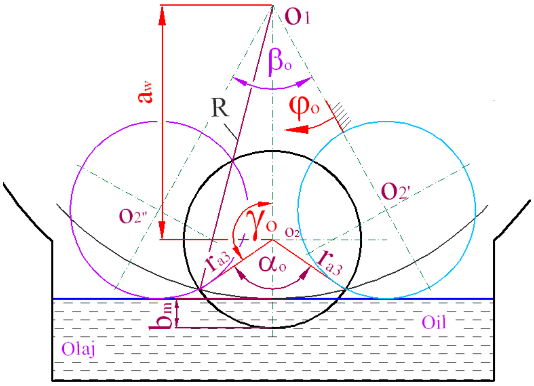

Fig. 11. *The model of a splashing planet gear*

The calculated result of the investigated planetary gear K+K (Fig. 15) is presented in Fig. 12.

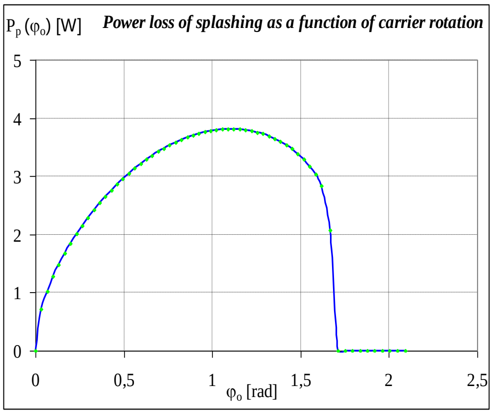

Fig. 12. *Calculated power loss caused by the splashing planet gears*

1.3.3 The Disc Churning Loss, the Windage Loss and the Friction Power of Seals

The power loss caused by a rotating disc submerged in oil can be calculated with equations published in the BS ISO/TR 14179/1 [3]. There are also some methods for calculation the windage [3] and seal losses [6], which are published in literature.

The calculated disc churning loss result for a planet gear of the investigated planetary gear K+K is presented in Fig. 13.

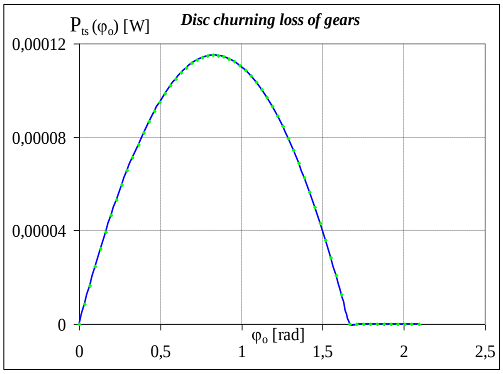

Fig. 13. *Disc churning loss of a planet gear*

1.4 Taking into Consideration the Number of Planet Gears

The number of planet gears influences the splashing and churning loss components.

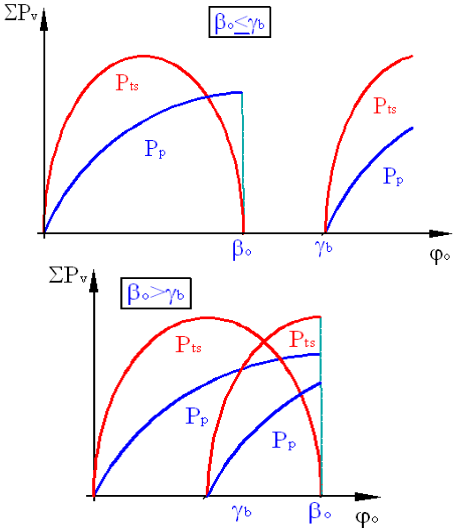

Fig. 14. *Distribution of the resultant churning losses as a function of revolution angle of planet carrier*

Taking into consideration the number of planet gears (Fig. 14), the average churning (P_{ts}) and splashing (P_p) loss can be calculated with the following equation developed:

- if: $\beta_o \le \gamma_b$

$$\tilde{P}_{pts} = \frac{1}{\gamma_b} \cdot \int_0^{\beta_o} \left(P_p(\varphi_o) + P_{ts}(\varphi_o)\right) d\varphi_o \,; \tag{13}$$

- if: $\beta_o > \gamma_b$

$$\tilde{P}_{pts} = \frac{1}{\beta_o} \cdot \left[\int_0^{\beta_o} \left(P_p(\varphi_o) + P_{ts}(\varphi_o)\right) d\varphi_o + \int_{\beta_o-\gamma_b}^{\beta_o} \left(P_p(\varphi_o) + P_{ts}(\varphi_o)\right) d\varphi_o \right] \tag{14}$$

The sum of average churning and splashing loss and average power loss of expel gives the total oil churning power loss of a planetary gearbox.

2 CONCLUSIONS

Heavy-duty planetary gears operate at high speeds and transmit high powers leading to considerable friction power losses. These power losses transform into heat, therefore the operating temperatures of the transmissions have to be calculated with taking into consideration all the power dependent and the power independent losses. Due to high power losses it would be important to know the heat load carrying capacity of the planetary gears in the early stage of designing. Using the presented method the oil churning loss, the bearing – and the tooth friction loss for all types of planetary gears can be calculated and the influence of the varying constituents of the power losses on the efficiency of the planetary gears can be evaluated. The results of such a calculation are presented in Fig. 16 for a K+K planetary gear.

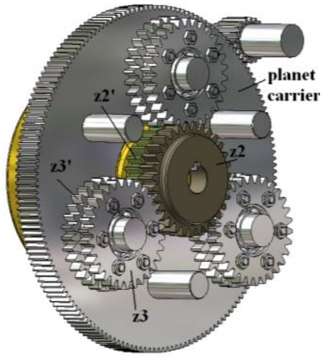

Fig. 15. *The investigated planetary gear K+K*

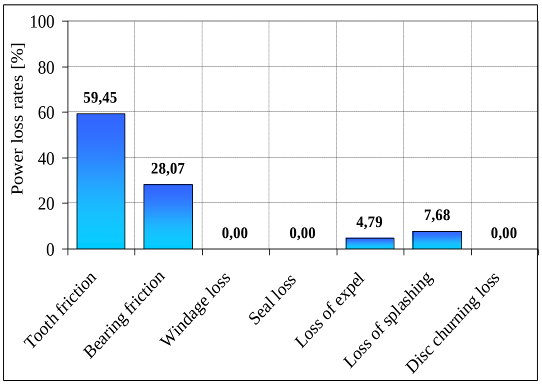

Fig. 16. *The constituents of the total power losses for a planetary gear K+K (Input power: P_{in}=500 W, tooth numbers: z_2=31, z_3=30, z_2'=30, z_3'=31, center distance: a_w=76.25 mm, gear ratio: i=15.7541, dip lubrication)*

3 NOMENCLATURE

a – exponent for bearing calculation [-]
a_1 – factor for bearing life correction [-]
Ae_{yti} – cross sections of the oil flow [m^2]
a_w – center distance [mm]
$\alpha_o(\varphi_o)$ – maximal immersion angle [°]
b – exponent for bearing calculation [-]
b_m – immersion depth [mm]
β_o – maximal splashing angle [°]
C – basic dynamic load [N]
c_{oe} – correction factor for splashing calculation [-]
d – inner diameter of bearing [mm]
$d_m(d)$ – average bearing diameter as a function of the inner diameter[mm]
$d_m(L_h)$ – average bearing diameter as a function of the prescribed bearing lifetime[mm]
$d_{m\ res}$ – resultant average bearing diameter [mm]
Δy – tooth immersion length for internal gear connection [mm]
η_M – viscosity at operating temperature [mPas]
F_{1n} – the normal force between the tooth profiles [N]
F_r – bearing radial load component [N]
f_0 – bearing coefficient [-]
f_1 – bearing coefficient [-]
φ – parameter, pinion or gear turn angle [°]
φ_o – carrier turn angle, parameter for splashing calculation [°]
φ_Ω – pinion or gear turn angle, parameter for oil expel calculations [°]
γ_b – planet gears minimal distribution angle [°]
i – gear ratio [-]
ξ – tooth distribution angle [°]
l – contact length of meshing teeth [mm]
L_{1h} – prescribed bearing lifetime, SKF rating life (99% reliability) [h]
M_0 – load independent friction torques of bearing [Nmm]
M_1 – load dependent friction torques of bearing [Nmm]
$M_{2;4}$ – sun or ring gear torque [Nm]
m_i, y_i – developed constants for linearization (bearing calculations)
$\dot{m}_{op}$ – splashed oil mass flux [kg/s]
$\mu(\varphi)$ – coefficient of tooth friction along the contact line [-]
n – bearing rotational speed [rpm]
n_{in} – input speed [rpm]
n_k – rotational speed of planet carrier [rpm]
ν – oil kinematical viscosity at operating temperature [mm^2/s]
ω_{3g} – planet gear angle velocity [rad/s]
ω_k – carrier angle velocity [rad/s]
Ω – tooth turn angle (loss of expel) [°]
P_1 – equivalent dynamic bearing load [N]
$P_{fcs}(\varphi)$ – tooth power loss along meshing [W]
P_{in} – input power [W]
$P_{oki}(\varphi)$ – power loss of oil expel [W]
$P_p(\varphi_o)$ – power loss of splashing [W]
$P_{ts}(\varphi)$ – disc churning loss [W]
R_a – average surface roughness (CLA)
r_{ai} – addendum radius [mm]
r_y – radius parameter [m]
ρ – oil density [kg/m^3]
$\rho_{ne}(\varphi)$ – effective curvature in pitch point [mm]
S_{yt} – tooth thickness [mm]
σ_m, τ_m – allowable normal and shear stress components [MPa]
$\Sigma v_t(\varphi)$ – sum of the tangential speeds of gear and pinion along the contact line [m/s]

Σy – tooth immersion length for external gear connection [mm]
t_{fb} – tooth turn time (loss of expel) [s]
t_k – planet gear splashing time [s]
V – shear load of planet gear pin [N]
v – entraining speed [m/s]
$v_{cs}(\varphi)$ – the sliding velocity between the tooth profiles [m/s]
v_{ra3} – tangential speed of planet gear [m/s]
$\dot{V}_{ok}(\varphi_\Omega)$ – flow rate, oil volume flux for oil expel calculations [m^3/s]
$w_t(\varphi)=F_{1n}/l$ – applied normal load/contact length of gear teeth l [N/mm]
x – tooth profile shift factor [-]
$z_o(\varphi_o)$ – number of submerging teeth [-]
z – tooth number [-]
$_2$ – index of sun gear
$_3$ – index of planet gear
$_4$ – index of ring gear
$_k$ – index of carrier

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