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Napoved zbirnega števila okvar popravljivega izdelka na podlagi poteka delovanja

Prediction of the Cumulative Number of Failures for a Repairable System Based on Past Performance

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Napoved zbirnega števila okvar popravljivega izdelka je pomembna tema v teoriji zanesljivosti. Popravljiv izdelek je lahko po opravljenem popravilu v treh možnih stanjih: 'dober kot nov', 'slab kot star' in 'boljši kot star toda slabši kot nov'. Običajni verjetnostni modeli za napovedovanje pričakovanega števila okvar temeljijo na prvih dveh stanjih, zato so neprimerni za opis zadnjega stanja, ki je bolj pogosto v praksi. V prispevku je predstavljena robustna rešitev verjetnostnega modela, t.i. splošni obnovitveni proces (SOP), ki je omogoča predstavitev vseh treh stanj izdelka po popravilu. Raziskave kažejo, da je s SOP na osnovi mešanice m-tih Weibullovih porazdelitev omogoča splošen pristop k opisu kompleksnih popravljivih izdelkov in podaja uporabo algoritmom matematičnega pričakovanja (EM) za oceno neznanih parametrov. V prispevku je predstavljen tudi standardni SOP na podlagi dvoparametrične Weibullove porazdelitve. SOP na osnovi mešane porazdelitve z m komponentami in standardni SOP primerjamo z izračunom pričakovanega števila okvar in funkcije napake. Na podlagi rezultatov lahko sklepamo, da predlagan SOP na osnovi mešanice Weibullovih porazdelitev točno opisuje izmerjene vrednosti okvar in je primeren za napovedovanje okvar na podlagi predhodnega poteka delovanja izdelka, kljub omejenemu številu razpoložljivih podatkov o okvarah.

(Ključne besede: napovedi okvar, popravljivi izdelki, numerično modeliranje, ocenjevanje parametrov)

The prediction of the cumulative number of failures for a repairable system is an important topic in reliability theory. A repairable system may end up in one of the three possible states after a repair: 'as good as new', 'as bad as old' and 'better than old but worse than new'. Current probabilistic models used in repairable system analysis account for the first two states, but they do not properly apply to the last one, which is more common in practice. In this paper, a robust solution to a probabilistic model that is applicable to all of the three after repair states, called generalized renewal process (GRP), is presented. This research demonstrates that the GRP based on an m-fold Weibull mixture offers a general approach to modeling complex repairable systems and discusses application of the EM algorithm to estimation of the GRP parameters. This paper also presents a review of the standard GRP based on two-parameter Weibull distribution. The GRP with m mixture components distributions is compared to the standard GRP by calculating the expected cumulative number of failures and the error function. It is shown that the proposed GRP solution with a Weibull mixture accurately describes the failure data and it is suitable for predicting failures based on the past performance of the system, even when a small amount of failure data is available.

(Keywords: failure prediction, repairable systems, numerical modeling, parameter estimations)

0 UVOD

Napoved zbirnega števila okvar popravljivega izdelka je pomembna tema v teoriji

0 INTRODUCTION

The prediction of the cumulative number of failures of a repairable system is an important topic

zanesljivosti. Omogoča načrtovanje preventivnih vzdrževalnih posegov, stroškov kroga trajanja, ocenjevanje razpoložljivosti izdelka itn.

Popravljiv izdelek je lahko v enem izmed treh mogočih stanjih po opravljenem popravilu: 'dober kakor nov', 'slab kakor star' in 'boljši kakor star toda slabši kakor nov'. Trenutni verjetnostni modeli za analizo popravljivih izdelkov, kakršna sta obnovitveni proces (OP) in nehomogeni Poissonov proces (NHPP), so primerni za opis le prvih dveh stanj. Za prikaz zadnjega stanja izdelka po popravilu ni učinkovitega in dovolj natančnega postopka.

Kijima in Sumita [1] sta predlagala nov verjetnostni model, imenovan splošni obnovitveni proces (SOP), ki omogoča predstavitev vseh mogočih stanj izdelka po popravilu. Postopek s SOP sta izpopolnila Kaminskiy in Krivtsov [2] in predlagala približno rešitev na temelju metode Monte Carlo (MC).

Predpostavka, ki omogoča simulacijo SOP z metodo Monte Carlo, je poznavanje porazdelitve verjetnosti časa do prve okvare (ČDPO) in kakovosti popravila q oziroma možnost njune ocene na podlagi razpoložljivih podatkov ([2] in [3]). Čas popravila zanemarimo, tako lahko zaporedje okvar opazovanega izdelka obravnavamo kot naključni točkovni postopek. V glavnem je bil postopek Monte Carlo razvit za uporabo v primerih z veliko množico podatkov. Razpoložljivost večjega števila podatkov omogoča oceno porazdelitve verjetnosti ČDPO in q z visoko stopnjo natančnosti [4]. Seveda je težko zagotoviti zadostno količino podatkov v primerih, ko je na voljo omejeno število enakih izdelkov.

Postopek s simulacijo Monte Carlo ima pri SOP nekaj prednosti in pomanjkljivosti. Postopek omogoča rešitev za vse vrste porazdelitev, vključno z empirično, ki je nepristranska in dosledna. Na drugi strani postopek MC potrebuje veliko množico podatkov in je časovno zelo potraten. Ti razlogi omejujejo uporabo postopka Monte Carlo Kaminskega in Krivtsova zunaj avtomobilske industrije.

Krivtsov [3] se je zavedal zapletenosti in težavnosti razvoja matematično prilagodljivega verjetnostnega modela za SOP, zato je predlagal drugo pot na podlagi metode največje verjetnosti (MNV), vendar brez ustrezne rešitve za SOP. Kasneje je Yanez [4] na podlagi MNV izpeljal rešitev za oceno parametrov SOP. Razvoj MNV za

in reliability theory. It enables the planning of preventive maintenance actions and costs, an estimation of the system's availability, etc.

A repairable system may end up in one of three possible states after a repair: 'as good as new', 'as bad as old' and 'better than old, but worse than new'. Current probabilistic models used in repairable system analysis, such as the renewal process (RP) and the non-homogeneous Poisson process (NHPP), account for the first two states, respectively. However, no practical and accurate approach exists to address the remaining after-repair state.

A new probabilistic model to address all the after-repair states called the 'generalized renewal process' (GRP) has been proposed by Kijima and Sumita [1]. This GRP approach has been extended by Kaminskiy and Krivtsov [2] and they have offered a Monte Carlo (MC) based approximate solution.

The assumption that makes Monte Carlo simulation of the GRP possible is that the time to first failure (TTFF) distribution and the quality of the repair q are known and can be estimated from the available data ([2] and [3]). Furthermore, the repair time is assumed to be negligible so that the failures can be viewed as point processes. The Monte Carlo approach was developed mainly for cases where a large set of data is available. The availability of such data allows for an estimation of the TTFF distribution and q with a high degree of accuracy [4]. However, it would be difficult to obtain the same amount of data for cases where only a limited number of identical systems are present.

There are some advantages and disadvantages of using the Monte Carlo approach to the GRP. The approach offers a solution for all kinds of distributions, including empirical ones, which is unbiased and consistent. In contrast, besides the need for large amounts of data, the approach is extremely time consuming. For these reasons the direct application of Kaminskiy and Krivtsov's Monte Carlo approach outside the automotive industry would be limited.

Krivtsov [3] recognized the complexities and the difficulties of developing a mathematically tractable probabilistic model to the GRP, and discussed an alternative maximum-likelihood (ML) estimation approach to solve the GRP without offering any solution. Later, Yanez [4] developed a solution based on ML to estimate the GRP parameters. The

oceno parametrov SOP je odpravil potrebo po večji količini podatkov za izvedbo analize okvar (kakor v primeru postopka MC Kaminskega in Krivtsova pri določevanju SOP). Yanezov postopek z MNV je izpeljan s predpostavko, da porazdelitev ČDPO ustreza dvoparametrični Weibullovi porazdelitvi verjetnosti ter da naslednji časi med okvarami (ČMO) sledijo pogojni dvo-parametrični Weibullovi porazdelitvi.

Dvo- ali triparametrična Weibullova porazdelitev je najbolj splošno uporabna porazdelitev na področju modeliranja zanesljivosti, ker s parametroma β (oblika) in θ (velikost) omogoča zelo različne oblike. Zato je primerna za modeliranje raznolikih podatkov in obratovalnih značilk izdelka, t.i. porazdelitev ČDPO. Oblika porazdelitve funkcije dobe trajanja izdelka je pogosto sestavljena iz več osnovnih oblik, zato je smiselna vpeljava mešane porazdelitve kot osnovne porazdelitev SOP. Značilna pomanjkljivost, skupna vsem mešanim porazdelitvam, je težavna ocena neznanih parametrov.

Dvoparametrična Weibullova porazdelitev velja za zelo pomembno v teoriji zanesljivosti, zato je primerna njena uporaba kot komponente mešane porazdelitvene ČDPO okvare SOP. Veliko število prispevkov je na temo uporabnosti mešanice Weibullovih porazdelitev, posebej pri modeliranju zanesljivosti izdelka, obratovalni trdnosti in analizi preživetja. Predlagane so bile številne tehnike ocenjevanja neznanih parametrov ([5] do [9]).

Namen prispevka je ugotavljanje primernosti končne mešanice Weibullovih porazdelitev s pozitivnimi utežmi komponent kot osnovne porazdelitve ČDPO za SOP, kljub temu da 'dejanska' porazdelitev ni mešana porazdelitev ali prave uteži niso negativne. Dovolj so le podatki o časih okvar posameznih, če neznane parametre SOP ocenimo z ustrezno metodo. V ta namen je izpeljan postopek EM za SOP z osnovno mešanico m Weibullovih porazdelitev.

Prispevek ima naslednjo sestavo. SOP in funkcija največje verjetnosti sta podana v prvem poglavju. Drugo poglavje predstavlja model na temelju m-kratne mešanice Weibullovih porazdelitev in metodo za oceno parametrov na podlagi algoritma EM. Sledi tretje poglavje s prikazom uporabe in primerjave na primerih za predlagani model na podlagi mešane Weibullove porazdelitvene funkcije in osnovni triparametrični SOP na podlagi ocene parametrov z MNV. Četrto

development of the ML parameter estimation removes the need to have a large set of failure data in order to perform the analysis (as is the case with Kaminskiy and Krivtsov's Monte Carlo simulation approach to the GRP). Yanez's ML approach was derived using the assumption that the TTFF distribution is a two-parameter Weibull distribution and that the subsequent times between failures (TBFs) follow the conditional two-parameter Weibull distribution.

The two- and three-parameter Weibull distributions are some of the most commonly used distributions in reliability engineering because of the many shapes they attain for various values of the parameters β (shape) and θ (scale). It can, therefore, model a wide variety of data and life characteristics, i.e., the TTFF distribution. Since the shape of the life distribution is often composed of more than one basic shape, a reasonable step is to introduce the mixture distribution as the genuine distribution for the GRP. A significant difficulty common to all mixed distributions during their application is the estimation of unknown parameters.

Since the Weibull distribution is considered very important in reliability studies, it is natural to consider it as a base distribution in finite mixtures for the distribution of the TTFF in the GRP. There are a number of papers dealing with the usefulness of m-fold Weibull mixture distributions, especially in reliability, fatigue and survival analysis. In addition, several estimation techniques for unknown parameters have been proposed ([5] to [9]).

The aim of this article is to prove that a finite Weibull mixture, with positive component weights only, can be used as the underlying distribution of the TTFF of the GRP in spite of the 'true' distribution function of the TTFF not being a mixture distribution or the true weights being negative. In this way it is sufficient to have only the observed failure data set of the system, if the unknown parameters of the GRP are estimated by a proper method. For this purpose, the EM procedure is derived for the GRP with an underlying m-fold Weibull mixture.

The paper is structured as follows. The GRP and the likelihood function are given in Section 1. Section 2 presents a model based on the m-fold Weibull mixture and an estimation method based on the EM algorithm. Following this, two failure data sets are used in Section 3 to illustrate the application of the proposed GRP based on the Weibull mixture and to compare it with the standard three-parameter GRP based on the ML estimation.

poglavje sklenemo z razpravo ter s komentarji rezultatov.

We conclude with a discussion and some comments in Section 4.

1 OSNOVNA DOLOČITEV

Predpostavimo, da popravljiv izdelek prične delovati pri času t=0. Zaporedje časov delovanj $\{X_i\}_1^\infty$ sestavlja naključni točkovni obnovitveni postopek. Zaradi lažje nadaljnje izpeljave predpostavimo, da je pravilo takojšnje. Kijima [10] je vpeljal pojem navideznega časa delovanja. Če ima izdelek takoj po opravljenem (n-1)-tem popravilu navidezno dobo delovanja $V_{n-1}=y$, potem ima dolžina n-tega kroga oziroma čas med okvarama (ČMO) X_n naslednjo zbirno porazdelitveno funkcijo (ZPF), t.i. osnovna porazdelitev:

1 BASIC DEFINITION

Consider a repairable system that starts functioning at time t=0. A sequence of operating times $\{X_i\}_1^\infty$ forms a renewal-type stochastic point process. For simplicity, the repair is assumed to be instantaneous. Kijima [10] introduced the notion of virtual age. If a system has a virtual age of $V_{n-1}=y$ immediately after the (n-1)th repair, then the duration of the nth cycle or the time between failures (TBF) X_n has the following cumulative distribution function (CDF) (the so-called underlying distribution):

$$G(x)=\Pr\{X_n \le y \mid V_{n-1}=y\}=\frac{F(x+y)-F(y)}{1-F(y)}=1-\frac{R(x+y)}{R(y)} \quad (1),$$

kjer je $F(t)$ ZPF časa do prve okvare (ČDPO) novega izdelka in $R(t)$=1-$F(t)$ je zanesljivost pri posameznih časih. Vsoto:

where $F(t)$ is a CDF of the time to first failure (TTFF) of a new system and $R(t)$=1-$F(t)$ is the reliability at the respective time. The summation:

$$T_n=\sum_{i=1}^{n} X_i \quad (2),$$

s T_0=0 imenujemo dejanska doba delovanja izdelka. Predpostavimo, da n-to popravilo odpravi poškodbe, ki so se nakopičile pri obratovanju med (n-1)-to in n-to okvaro. S to predpostavko zapišemo navidezno dobo delovanja izdelka po n-tem popravilu z:

with T_0=0, is called the real age of the system. It is assumed that the nth repair would only compensate for the damage accumulated during the time between the (n-1)th and the nth failure. With this assumption the virtual age of the system after the nth repair is:

$$V_n=V_{n-1}+q\cdot X_n, \quad n=1,2,\ldots \quad (3),$$

kjer je q parameter učinkovitosti popravila (ali pomladitveni parameter) in V_0=0. Če opazujemo čase med zaporednimi okvarami, potem lahko navidezno dobo delovanja izrazimo kot [3]:

where q is the repair-effectiveness parameter (or the rejuvenation parameter) and V_0=0. If the times between the successive failures are considered, then the virtual age can be expressed as [3]:

$$V_n=q\cdot\sum_{i=1}^{n} X_i=q\cdot T_n \quad (4).$$

Glede na ta model in sliko 1, privzeta vrednost q=0 vodi do OP ('dober kakor nov'), medtem ko predpostavka q=1 vodi do NHPP ('slab kakor star'). Vrednosti parametra q iz obdobja 0<q<1 predstavljajo stanje izdelka po popravilu, ko je ta 'boljši kakor star, toda slabši kakor nov'.

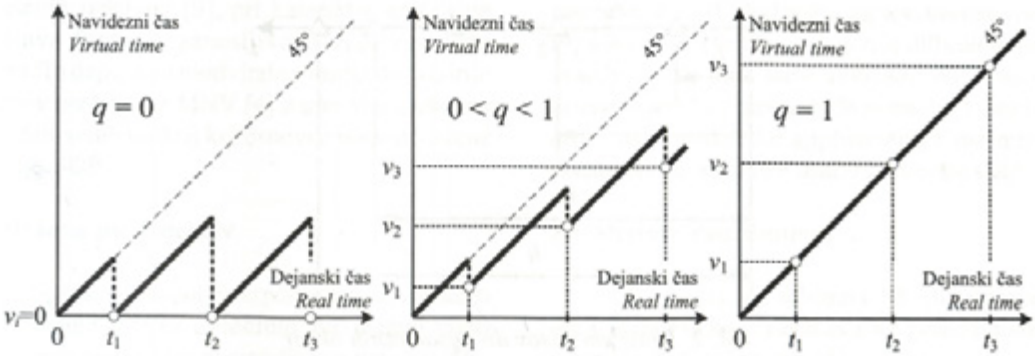
According to this model and Fig. 1, the result of assuming a value of q=0 leads to a RP (as good as new), while the assumption of q=1 leads to a NHPP (as bad as old). The values of q that fall in the interval 0<q<1 represent the after-repair state in which the condition of the system is better than the old but worse than the new.

1.1 Funkcija zanesljivosti vzorca

Če izvedemo oceno parametrov SOP za trenutek, ko se pojavi naslednja okvara, potem je

1.1 Likelihood function

If an estimation of the GRP is made for the moment when the very next failure occurs, then the



Sl. 1. *Primerjava navidezne in dejanske dobe delovanja za različne vrednosti parametra q*
Fig. 1. *Virtual age versus real age for varying q values*

funkcija zanesljivosti vzorca definirana z [11]:

likelihood function is defined by [11]:

$$L = f(x_1 \mid \beta,\theta)\prod_{i=2}^{n} g(x_i \mid \beta,\theta,q) \quad (5),$$

kjer sta $f(x)$ in $g(x)$ gostoti porazdelitve verjetnosti (GPV) za ČDPO in pogojna GPV za ČMO. Takoj je pomembno poudariti, da se prva okvara ne pokorava prej omenjeni pogojni verjetnosti. Torej lahko negativni logaritem funkcije zanesljivosti vzorca, imenovan tudi funkcija napake, podamo z izrazom:

where $f(x)$ and $g(x)$ are the probability density function (PDF) of the TTFF and the conditional PDF of the TBF. At this point it is important to recall that the first failure does not obey the conditionality mentioned previously. Therefore, the negative logarithm of the likelihood function, which can be regarded as an error function, is given as:

$$E = -\log(L) = -\left[\log\{f(x_1 \mid \beta,\theta)\} + \sum_{i=2}^{n}\log\{g(x_i \mid \beta,\theta,q)\}\right] \quad (6).$$

1.2 Triparametrični SOP

Običajni SOP je utemeljen na predpostavki, da je osnovna porazdelitev ČMO pogojna dvoparametrična Weibullova porazdelitvena funkcija [4].

Kakor je bilo predhodno omenjeno, se čas do prve okvare ne podreja pogojnosti ČMO. Dvoparametrična Weibullova ZPF za ČDPO ima obliko:

1.2 Three-parameter GRP

The standard GRP is based upon the assumption that the underlying TBF distribution is a conditional two-parameter Weibull probability distribution [4].

As we mentioned before, the time to first failure does not comply with the conditionality of the TBF. The two-parameter Weibull CDF for the TTFF is of the form:

$$F(t_1 \mid \beta,\theta) = 1 - \exp\left\{-\left(\frac{t_1}{\theta}\right)^{\beta}\right\} \quad (7),$$

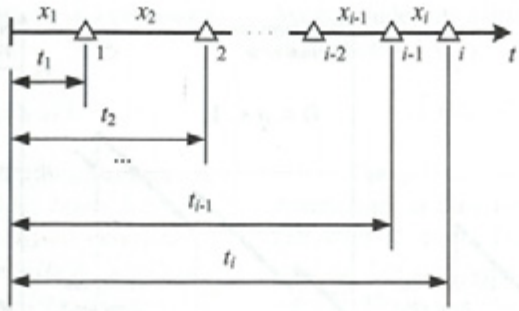
kjer sta θ in β parametra velikosti in oblike. Za naslednje okvare je enačba (1) podana kot:

where θ in β are the scale and shape parameters, respectively. For the subsequent failures Eq. (1) becomes:

$$G(x_i \mid \beta,\theta,q) = 1 - \exp\left\{\left(\frac{q\cdot t_{i-1}}{\theta}\right)^{\beta} - \left(\frac{q\cdot t_{i-1} + x_i}{\theta}\right)^{\beta}\right\} \quad (8),$$

kjer sta x_i čas med (i-1)-to in i-to okvaro in t_{i-1} je zbirni čas delovanja (dejanska doba delovanja) do (i-1)-te okvare (sl. 2):

where x_i is the period of time between the (i-1)th and the ith failure and t_{i-1} is the cumulative operating time (i.e., the actual age) up to the (i-1)th failure (see Fig. 2):



Sl. 2. *Diagram časov do posameznih okvar*
Fig. 2. *Time-to-failure diagram of the failure events*

$$t_{i-1} = \sum_{l=1}^{i-1} x_l$$

Z odvajanjem obeh ZPF in z nekaj algebre lahko odvod po velikostnem parametru θ logaritma funkcije zanesljivosti vzorca (enačba (6)) zapišemo v samostojni izraz samo z oblikovnim parametrom β in s parametrom učinkovitosti popravila q. S tem se bistveno poenostavi reševanje. Sistem dveh enačb za oceno parametrov β in q je rešujemo sočasno in je prikazan spodaj:

By differentiating the CDF functions and with some algebra the derivatives of the log-likelihood function (Eq. (6)) with respect to the scale parameter θ can be expressed in terms of the shape parameter β and the repair-effectiveness parameter q in a closed-form expression, thus simplifying the solution. The system of equations for the parameters β and q should be solved simultaneously and is shown below:

$$0 = \frac{-1}{\beta} - \frac{\log(x_1) + \sum_{i=2}^{n} \log(q \cdot t_{i-1} + x_i)}{n} + \frac{x_1^{\beta} \log(x_1) + \sum_{i=2}^{n} [(q \cdot t_{i-1} + x_i)^{\beta} \cdot \log(q \cdot t_{i-1} + x_i) - (q \cdot t_{i-1})^{\beta} \cdot \log(q \cdot t_{i-1})]}{x_1^{\beta} + \sum_{i=2}^{n} [(q \cdot t_{i-1} + x_i)^{\beta} - (q \cdot t_{i-1})^{\beta}]} \quad (9)$$

$$0 = (\beta - 1) \sum_{i=2}^{n} \frac{t_{i-1}}{(q \cdot t_{i-1} + x_i)} - \frac{n\beta \sum_{i=2}^{n} t_{i-1} \cdot [(q \cdot t_{i-1} + x_i)^{\beta-1} - (q \cdot t_{i-1})^{\beta-1}]}{x_1^{\beta} + \sum_{i=2}^{n} [(q \cdot t_{i-1} + x_i)^{\beta} - (q \cdot t_{i-1})^{\beta}]} \quad (10).$$

Enačbi (9) in (10) nista analitično rešljivi. Rešitev sistema enačb lahko najdemo z uporabo numerične metode [15]. Nato lahko z izračunanimi vrednostmi β in q določimo vrednost izraza za velikostni parameter θ:

Eqs. (9) and (10) do not have a closed-form mathematical solution. A numerical algorithm [15], can be used to solve them. Then, the expression for the scale parameter θ is updated with the values of β and q, thus:

$$\theta = \left\{ \frac{1}{n} \left(x_1^{\beta} + \sum_{i=2}^{n} [(q \cdot t_{i-1} + x_i)^{\beta} - (q \cdot t_{i-1})^{\beta}] \right) \right\}^{1/\beta} \quad (11).$$

2 SOP NA PODLAGI WEIBULLOVE MEŠANE PORAZDELITVE

Uporaba Weibullove porazdelitev kot modela zanesljivosti je mogoča tudi pri obravnavi

2 GRP BASED ON THE WEIBULL MIXTURE DISTRIBUTION

The application of the Weibull mixture distribution as a reliability model is also feasible in the

zapletenih izdelkov [9], pri katerih je analitična izpeljava funkcije zanesljivosti zelo zahtevna naloga. Ta dejstva so motivirala avtorje, da razširijo Yanezov postopek z MNV [4] z uporabo mešanice porazdelitvenih funkcij kot osnovne porazdelitvene funkcije SOP.

2.1 Mešana porazdelitev

Splošno lahko poljubno porazdelitveno funkcijo dobe trajanja izdelka določimo kot utežno vsoto posameznih komponent porazdelitvene funkcije [5]:

$$f(t_1 \mid \Theta) = \sum_{j=1}^{m} w_j f_j(t_1 \mid \theta_j) \quad (12),$$

kjer je $t_1=x_1$ čas od prve okvare izdelka, pri tem za parametre $\Theta=(w_1,...,w_m, \theta_1,..., \theta_m)$ velja, da je $w_j>0$ za $(j=1,...,m)$ in $\Sigma_{j=1}^{m} w_j = 1$. Stalnico w_j imenujemo utežni količnik in f_j je komponenta mešanice gostot porazdelitev verjetnosti ter je določena s parametri θ_j. Ker zavzema Weibullova porazdelitev pomembno mesto v teoriji zanesljivosti, je pri našem postopku zvezna gostota porazdelitve verjetnosti (GPV) časa do prve okvare modelirana kot mešanica dvoparametričnih Weibullovih porazdelitev:

$$f(t_1 \mid \Theta) = \sum_{j=1}^{m} w_j \frac{\beta_j}{\theta_j}\left(\frac{t_1}{\theta_j}\right)^{\beta_j - 1} \exp\left\{-\left(\frac{t_1}{\theta_j}\right)^{\beta_j}\right\} \quad (13),$$

kjer sta β_j in θ_j oblikovni in velikostni parameter komponente mešanice Weibullovih porazdelitev. Vrednosti parametrov so omejene glede na lastnosti Weibullove porazdelitve, tako da velja $\beta_j>0$ in $\theta_j>0$ za $(j=1,...,m)$.

Če imamo mešanico Weibullovih porazdelitev za osnovno GPV za ČDPO, potem je pripadajoča funkcija zanesljivosti določena kot utežna vsota posameznih komponent funkcije zanesljivosti [12]:

$$R(t \mid \Theta) = \sum_{j=1}^{m} w_j R_j(t \mid \theta_j) \quad (14).$$

Z upoštevanjem zgornjega izraza za funkcijo zanesljivosti lahko pogojno ZPF za čas med prvo in drugo okvaro, za čas med drugo in tretjo okvaro itn., zapišemo ob upoštevanju enačbe (1) kot:

case when we try to deal with complex structure systems [9] where the reliability function is difficult to derive exactly. These facts have motivated our interest in extending the ML approach discussed by Yanez [4] in order to consider the application of the mixture distribution as the TTFF distribution in the GRP.

2.1 Mixture distributions

Generally, an arbitrary life distribution of the system can be defined as a weighted sum of the component distributions [5]:

where $t_1=x_1$ is the time to first failure of the system and the parameters $\Theta=(w_1,...,w_m,\theta_1,...,\theta_m)$ are such that $w_j>0$ for $(j=1,...,m)$ and $\Sigma_{j=1}^{m} w_j = 1$. The constant w_j is called a weighting factor and f_j a component density function parameterized by θ_j. In our approach a continuous probability density function (PDF) of times to the first failure is modelled as a mixture of two-parameter Weibull distributions, because the Weibull distribution is considered to be very important in reliability studies:

where the constants β_j in θ_j stand for the Weibull shape and scale parameters of the component densities. They are limited according to the characteristics of the Weibull distribution $\beta_j>0$ and $\theta_j>0$ for $(j=1,...,m)$.

If the Weibull mixture is considered as underlying the PDF of the TTFF, the corresponding reliability function can be defined as a weighted sum of component reliability functions [12]:

Considering the expression for the reliability function, the conditional CDF of the times between the first and second failure, the second and third failure, etc. can be written according to Eq. (1) as:

$$G(x_i \mid \Theta, q) = \Pr\left\{X_i \le x_i \mid V_{i-1} = qt_{i-1}\right\} = 1 - \frac{\sum_{j=1}^{m} w_j R_j(qt_{i-1} + x_i \mid \theta_j, q)}{\sum_{j=1}^{m} w_j R_j(qt_{i-1} \mid \theta_j, q)} \quad (15).$$

2.2 Algoritem EM

Algoritem EM je splošna metoda za iskanje parametrov porazdelitvene funkcije, ki dajo največjo vrednost funkcije zanesljivosti vzorca glede na izmerjene vrednosti. Standardni algoritem EM obravnava izmerjene podatke kot nepopolno informacijo o opazovanem naključnem pojavu. Pri algoritmu EM vstavimo izmerjene podatke v večji 'popoln' vzorčni prostor. Pripadajoča funkcija zanesljivosti vzorca zagotavlja celoten opis generacije oziroma vzorčenja podatkov glede na iskane parametre. Večji vzorčni prostor seveda ne moremo vzorčiti ali trenutno ni na voljo. Torej računamo pričakovano vrednost logaritma funkcije zanesljivosti vzorca celovitih podatkov, odvisno od izmerjenih vrednosti $\{x_i;i=(1,...,n)\}$ in trenutno oceno parametrov $\Theta^{(k)}$.

Pri SOP na podlagi mešanice Weibullovih porazdelitev je negativni logaritem funkcije zanesljivosti vzorca za analizo do zadnje zaznane okvare podan z:

2.2 EM algorithm

The EM algorithm is a general method for finding the maximum likelihood estimate of the parameters of an underlying distribution from a given data set [13]. The standard EM algorithm regards the measured data as incomplete information about the underlying stochastic process. For the EM algorithm, one embeds the measured data in a larger 'complete' data space. The corresponding likelihood function provides a complete description of the data generation, given the parameters in question. The larger data set, however, cannot be sampled or is not available at the moment. One computes, therefore, the expected value of the log-likelihood function of the complete data conditioned by the observed data $\{x_i;i=(1,...,n)\}$ and the current parameter estimates $\Theta^{(k)}$.

For the GRP based on the Weibull mixture distribution the failure-terminated negative logarithm of the likelihood function is defined by:

$$E=-\log(L)=-\left[\log\left\{f(x_1\mid\Theta)\right\}+\sum_{i=2}^{n}\log\left\{g(x_i\mid\Theta,q)\right\}\right] \quad (16),$$

kjer sta $f(x)$ in $g(x)$ GPV za ČDPO in pogojna GPV za ČMO. Izpeljavo naslednje enačbe je mogoče najti v sklicu [14]:

where $f(x)$ and $g(x)$ are the PDF of the TTFF and the conditional PDF of the TBF. The derivation of the following equation can be found in Ref. [14]:

$$\tilde{Q}(\Theta,\Theta^{(k)})=-\left[\sum_{j=1}^{m}f(j\mid x_1,\Theta^{(k)})\cdot\log(w_j)+\sum_{i=2}^{n}\sum_{j=1}^{m}g(j\mid x_i,\Theta^{(k)},q^{(k)})\cdot\log(w_j)\right.$$
$$\left.+\sum_{j=1}^{m}f(j\mid x_1,\Theta^{(k)})\cdot\log\left\{f_j(x_1\mid\theta_j)\right\}+\sum_{i=2}^{n}\sum_{j=1}^{m}g(j\mid x_i,\Theta^{(k)},q^{(k)})\cdot\log\left\{g_j(x_i\mid\theta_j,q_j)\right\}\right] \quad (17),$$

kjer smo vpeljali obrobne verjetnosti $f(j|x_1,\Theta^{(k)})$ in $g(j|x_1,\Theta^{(k)},q^{(k)})$, ki jih izpeljemo z uporabo Bayesovega pravila [14].

Vrednotenje zgornjega izraza imenujemo korak E algoritma. Pri drugem koraku (korak M) algoritma EM maksimiziramo pričakovanje, ki smo ga izračunali v prvem koraku, glede na parametre, tako da dobimo novo oceno parametrov $\Theta^{(k+1)}$. Maksimiziranje enačbe (17) lahko izvedemo z neodvisno maksimizacijo člena, ki vsebuje w_j, in člena, ki vsebuje θ_j, saj nista medsebojno povezana. Z nekaj algebre dobimo naslednje izraze:

where we introduce the posterior probabilities $f(j|x_1,\Theta^{(k)})$ and $g(j|x_1,\Theta^{(k)},q^{(k)})$, which can be expressed using Bayes's rule (see Ref. [14]).

The evaluation of this expectation is called the E-step of the algorithm. The second step (the M-step) of the EM algorithm is to maximize the expectation we computed in the first step with respect to the parameters to obtain new parameter estimations $\Theta^{(k+1)}$. To maximize Eq. (17), we can maximize the term containing w_j and the term containing θ_j independently since they are not related. After some algebra we get the next expressions:

$$0=\sum_{i=2}^{n}\sum_{j=1}^{m}g(j\mid x_i,\Theta^{(k)},q^{(k)})\cdot\left[\frac{(\beta_j^{(k+1)}-1)t_{i-1}}{q^{(k+1)}t_{i-1}+x_i}-\frac{\beta_j^{(k+1)}t_{i-1}}{\theta_j^{(k+1)}}\times\left\{(q^{(k+1)}t_{i-1}+x_i)^{\beta_j^{(k+1)}-1}-(q^{(k+1)}t_{i-1})^{\beta_j^{(k+1)}-1}\bar{R}_j(q^{(k+1)}t_{i-1}\mid\theta_j^{(k+1)},q^{(k+1)})\right\}\right] \quad (18)$$

$$0=\frac{-1}{\beta_j^{(k+1)}}-\frac{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})\log(x_1)+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})\log(q^{(k+1)}t_{i-1})}{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})}+\frac{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})x_1^{\beta_j^{(k+1)}}\log(x_1)+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})\ldots}{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})x_1^{\beta_j^{(k+1)}}+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})\ldots}$$

$$\frac{\ldots\times\left[(q^{(k+1)}t_{i-1}+x_i)^{\beta_j^{(k+1)}}\log(q^{(k+1)}t_{i-1}+x_i)-(q^{(k+1)}t_{i-1})^{\beta_j^{(k+1)}}\log(q^{(k+1)}t_{i-1})\bar{R}_j(q^{(k+1)}t_{i-1}\,|\,\boldsymbol{\theta}_j^{(k+1)},q^{(k+1)})\right]}{\ldots\times\left[(q^{(k+1)}t_{i-1}+x_i)^{\beta_j^{(k+1)}}-(q^{(k+1)}t_{i-1})^{\beta_j^{(k+1)}}\bar{R}_j(q^{(k+1)}t_{i-1}\,|\,\boldsymbol{\theta}_j^{(k+1)},q^{(k+1)})\right]} \quad (19)$$

$$\theta_j^{(k+1)}=\left\{\frac{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})\cdot x_1^{\beta_j^{(k+1)}}}{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})}+\frac{\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})\cdot(q^{(k+1)}t_{i-1}+x_i)^{\beta_j^{(k+1)}}}{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})}\right. \quad (20),$$

$$\left.+\frac{\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})\cdot(q^{(k+1)}t_{i-1})^{\beta_j^{(k+1)}}\bar{R}_j(q^{(k+1)}t_{i-1}\,|\,\boldsymbol{\theta}^{(k+1)},q^{(k+1)})}{f(j\,|\,x_1,\mathbf{\Theta}^{(k)})+\sum_{i=2}^{n}g(j\,|\,x_i,\mathbf{\Theta}^{(k)},q^{(k)})}\right\}^{\frac{1}{\beta_j^{(k+1)}}}$$

kjer

where

$$\bar{R}_j(q^{(k+1)}t_{i-1}\,|\,\boldsymbol{\theta}_j^{(k+1)},q^{(k+1)})=\frac{\exp\left\{-\left[(q^{(k+1)}\cdot t_{i-1})/\theta_j^{(k+1)}\right]^{\beta_j}\right\}}{\sum_{j=1}^{m}w_j^{(k+1)}\cdot\exp\left\{-\left[(q^{(k+1)}\cdot t_{i-1})/\theta_j^{(k+1)}\right]^{\beta_j}\right\}} \quad (21).$$

Dobili smo sistem (2m+1) enačb z (2m+1) neznanimi parametri. Na podlagi Newton-Raphsonove metode [15] smo razvili numerično metodo reševanja sistema zahtevnih nelinearnih enačb.

Oceno novih parametrov $\mathbf{\Theta}^{(k+1)}$, $q^{(k+1)}$ dobimo s posodobitvijo enačb (18), (19) in (20) z vrednostmi parametrov iz prejšnjega koraka $\mathbf{\Theta}^{(k)}$, $q^{(k)}$. Koraka pričakovanja in največje vrednosti se izvajata simultano. Algoritem napreduje z uporabo novih parametrov kot začetnim približkom naslednje iteracije. Algoritem EM ponavlja korake, dokler se ne približa rešitvi.

We obtain a system of (2m+1) equations with (2m+1) unknown variables. We have developed a solution based on the Newton-Raphson method to solve these extremely complex equations [15].

Update Eq. (18), (19) and (20) for the estimation of the new parameters $\mathbf{\Theta}^{(k+1)}$, $q^{(k+1)}$, in terms of the old parameters $\mathbf{\Theta}^{(k)}$, $q^{(k)}$ perform both the expectation step and the maximization step simultaneously. The algorithm proceeds by using the newly derived parameters as the guess for the next iteration. The EM steps are iterated until the algorithm converges.

2.3 Ocena pričakovanega števila okvar

Dogodek okvare pri popravljivem izdelku lahko simuliramo z metodo Monte Carlo, pri kateri z uporabo zbirne porazdelitvene funkcije za ČDPO vnesemo prvo okvaro, medtem ko za naslednje okvare uporabimo pogojno zbirno porazdelitveno funkcijo za ČMO.

Predpostavimo, da je verjetnost pojava prve okvare $F(x_1) = P$ in naslednjih okvar $G(x_i) = P$ naključno število iz enakomerne porazdelitve na območju [0,1], potem lahko zapišemo enačbi za simuliranje časov okvar:

2.3 Estimation of the expected number of failures

The occurrence of failure in repairable systems can be simulated with a Monte Carlo method using the TTFF cumulative distribution function to generate the first failure, while for the subsequent failures, a conditional TBF cumulative distribution is used.

Assume that the probability of the first failure $F(x_1) = P$ and for subsequent failure times $G(x_i) = P$ are random numbers from a uniform distribution on the interval [0,1], then we get the equations for the generation of failure times:

$$1-\sum_{j=1}^{m} w_j \cdot R_j(x_1) = P \tag{22}$$

in

and

$$1-\frac{\sum_{j=1}^{m} w_j \cdot R_j(q \cdot t_{i-1} + x_i)}{\sum_{j=1}^{m} w_j \cdot R_j(q \cdot t_{i-1})} = P \tag{23}.$$

Predpostavimo obdobje opazovanja $[0,T]$, za katerega želimo oceniti pričakovano število okvar. Na začetku simulacije MC določimo enakomerno porazdelitev za nastajanje naključnih števil v mejah $[0,1]$ za P. Potem uporabimo numerično metodo, kakršni sta Newton-Raphsonova ali sekantna metoda, za rešitev enačb (22) in (23). Naključne vrednosti $\{x_i;i=(1,2,...)\}$ določimo, kakor je prikazano na sliki 3. Naključno vrednost x_i prištejemo vsoti predhodnih naključnih vrednosti ter primerjamo z dobo opazovanja T. Postopek ponavljamo, dokler vsota naključnih vrednosti x_i ne preseže T.

Vzemimo, da smo ta postopek ponovili k-krat, pri tem je n_j število zapisanih ali simuliranih okvar pri j-ti ponovitvi. Pričakovano število okvar (t.i. zbirna funkcija intenzivnosti CIF) pri času T je podana z:

Assume a time period $[0,T]$ as the period of interest to estimate the expected number of failures. The MC simulation starts with the definition of a uniform distribution that will generate random numbers in the range $[0,1]$ for P. Then, a numerical method, such as the Newton-Raphson or the secant one, is used to solve Eqs. (22) and (23). Random values of $\{x_i;i=(1,2,...)\}$ are generated, as depicted in Fig. 3. A random value for x_i is then added to the sum of the past random x_is and compared with the period of interest T. This procedure is repeated until the sum of x_is does not exceed T.

Assume that this procedure is repeated k times and n_j is the number of failures registered or generated at the jth repetition. The expected number of failures (i.e., the cumulative intensity function (CIF)) at time T is given by:

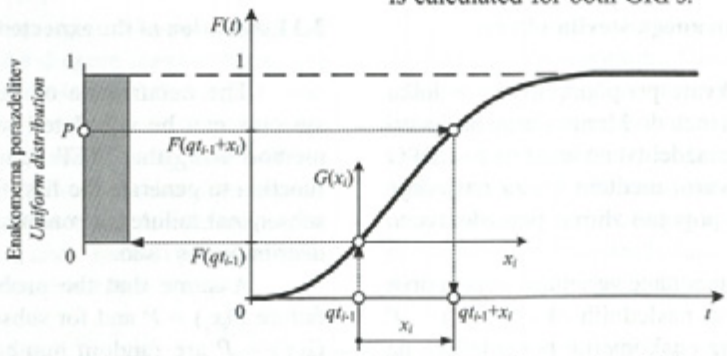
$$\mathrm{E}[N(T)] = \frac{\sum_{j=1}^{k} n_j}{k} \tag{24}.$$

3 NUMERIČNI PRIMERI

Na primeru iz literature in na simuliranih podatkih sta uporabljena dva SOP, običajni triparametrični model in model na temelju mešanice Weibullovih porazdelitev. Grafični prikaz rezultatov omogoča bralcu lažje razumevanje predstavljenih pojmov, razlag rezultatov in sklepov. Za oba predstavljena SOP je izračunana vrednost funkcije napake E [14].

3 NUMERICAL EXAMPLES

Two GRP, a standard three-parameter model and a model based on the Weibull mixture distribution, are applied in two examples, one from the literature and one that is simulated. The use of a graphical display enables the reader to gain a perspective on the various meanings and associated interpretations. In addition, the error function E [14] is calculated for both GRPs.



Sl. 3. *Simulacija MC naključnih časov med okvarami za SOP*

Fig. 3. *MC simulation of the random times between failures for the GRP*

Preglednica 1. *Ocenjeni parametri SOP za posamezne primere*
Table 1. *Estimated parameters for the GRP in the numerical examples*

SOP model *GRP model*	Ocenjeni parametri / *Estimated parameters* Primer 1 *Example No. 1* $n = 71$	 Primer 2 *Example No. 2* $n = 33$
Standardni SOP *Standard GRP*	$\beta_0 = 3{,}12$, $\theta_0 = 3649$ $q = 0{,}409$	$\beta_0 = 2{,}70$, $\theta_0 = 3{,}920$ $q = 0{,}160$
SOP z mešano porazdelitvijo *GRP based on mixture* ($m = 2$)	$w_1 = 0{,}923$, $\theta_1 = 2664$ $\beta_1 = 4{,}26$ $w_2 = 0{,}077$, $\theta_2 = 5006$ $\beta_2 = 4{,}26$ $q = 0{,}428$	$w_1 = 0{,}503$, $\theta_1 = 3{,}001$ $\beta_1 = 2{,}62$ $w_2 = 0{,}497$, $\theta_2 = 4{,}999$ $\beta_2 = 3{,}77$ $q = 0{,}159$

Najbolj ustrezne SOP za analizirane podatke okvar so izbrane glede na najnižjo vrednost E. Ocenjeni parametri in vrednosti E posameznih uporabljenih modelov so podani v preglednicah 1 in 2.

3.1 Primer 1

Kot prvi primer smo obravnavali podatke okvar tretjega glavnega pogonskega motorja bojne ladje U.S.S. Halfbeak [4]. Izvedli smo oceno parametrov in pričakovanega števila okvar za oba modela. Na sliki 4(a) je prikazano zbirno število okvar v odvisnosti od dejanskih časov do okvar skupaj s simulacijo MC pričakovanega števila okvar za oba modela. Primerjava dobljenih porazdelitvenih funkcij gostote verjetnosti za ČDPO je prikazana na sliki 4(b). S slike 4(a) je mogoče razbrati, da se rešitev z najmanjšo vrednostjo E zelo dobro prilega prvim 12 točkam. Vrednosti parametra učinkovitosti popravila q za obe rešitvi nakazuje, da je pogonski motor po popravilu v stanju 'boljši kakor star toda slabši kakor nov'.

3.2 Primer 2

Podatki za drugi primer so dobljeni s simulacijo okvar popravljivega izdelka, ki ima porazdelitev ČDPO definirano z mešanico Weibullovih porazdelitev s parametri (w_1=0,813, β_1=1,79, θ_1=1,91, w_2=0,187, β_2=3,21, θ_2=5,66) in s parametrom učinkovitosti popravila (q=0,23).

Na podlagi zgornjih parametrov so z metodo MC simulirani časi okvare izdelka, ki so na sliki 5(a) prikazani z zbirnim številom okvar v odvisnosti od dejanskih časov do okvar. Na

Preglednica 2. *Vrednosti E in AIC za SOP uporabljene v primerih*
Table 2. *E and AIC values for the GRP used in the numerical examples*

SOP model *GRP model*		Primer 1 *Example No. 1*	Primer 2 *Example No.*
Standardni SOP *Standard GRP*	E	460,814	31,938
SOP z mešano porazdelitvijo *GRP based on mixture* ($m = 2$)	E ΔE	458,471 -2,343	31,444 -0,494

The best GRP for the failure data, as determined by E, is the model with the lowest value of E. The estimated parameters obtained for the discussed models and the calculated E values are listed in Tables 1 and 2, respectively.

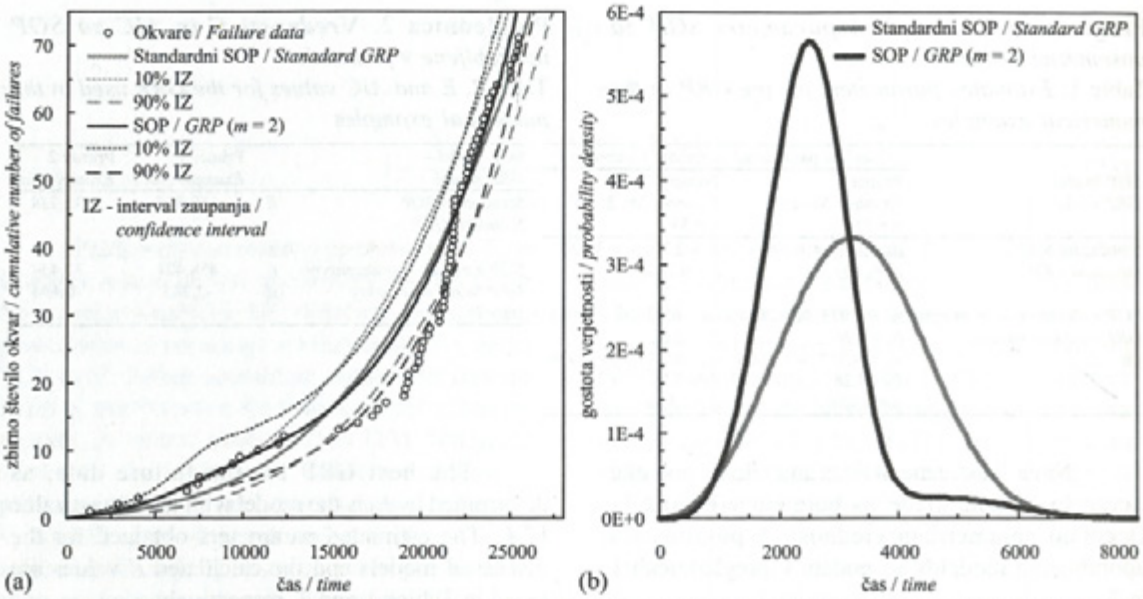
3.1 Example No. 1

As an example, consider the failure data related to the U.S.S. Halfbeak No. 3 main propulsion motor [4] used to perform a failure analysis. Both the parameter estimation and the expected number of failures were obtained. Fig. 4(a) shows the actual failure data in the form of a cumulative number of failures and a comparison of the MC-based prediction of the expected number of failures for both models. A comparison of the PDFs of the TTFF is presented in Fig. 4(b). As Fig. 4(a) depicts, when applying the mixture model with the lowest E value, the expected number of failures agrees with the observed data better in the first 12 points. For the standard model and mixture model the values of q suggest an equipment condition of better than old, but worse than new.

3.2 Example No. 2

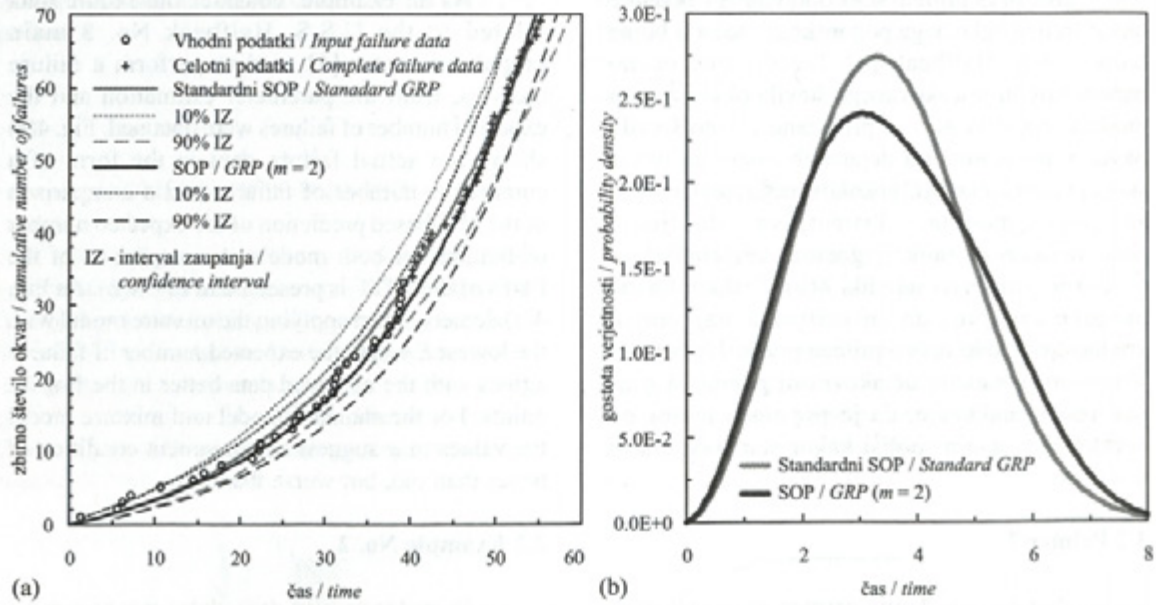
In order to simulate data, we assume a repairable system having a Weibull mixture distribution of the TTFF with parameters (w_1=0.813, β_1=1.79, θ_1=1.91, w_2=0.187, β_2=3.21, θ_2=5.66) and a repair-effectiveness parameter (q=0.23).

Based on the above parameters, a simulated failure data set is generated with the MC method and is given in Fig. 5(a) as the failure number plotted against the accumulated time to failure. This figure



Sl. 4. *Primerjava modelov SOP na podlagi dvoparametrične Weibullove PF in mešanice Weibulovih PF z dvema rešitvama: (a) pričakovano število okvar; (b) GPV za ČDPO*

Fig. 4. *A comparison of GRP models based on the 2-parameter Weibull DF and the Weibull mixture DF with two solutions: (a) expected number of failures; (b) PDF of TTFF*



Sl. 5. *Primerjava modelov SOP na podlagi dvoparametrične Weibullove PF in mešanice Weibulovih PF: (a) pričakovano število okvar; (b) PDF za TTFF*

Fig. 5. *A comparison of GRP models based on the 2-parameter Weibull DF and the Weibull mixture DF: (a) expected number of failures; (b) PDF of TTFF*

sliki je prikazana tudi simulacija MC pričakovanega števila okvar za oba modela. Ocena parametrov obeh SOP je na podlagi prvih 33 točk od možnih 70. Primerjava dobljenih porazdelitvenih funkcij gostote verjetnosti za

also shows a comparison of a Monte Carlo-based prediction of the expected number of failures for both models, where only the first 33 data points from a total of 70 values are considered to estimate the unknown parameters of both GRP models. Fig. 5(b)

ČDPO je prikazana na sliki 5(b). S slike 5(a) je mogoče razbrati, da SOP na podlagi mešane porazdelitve boljše opiše nagnjenje k naraščanju zbirnega števila okvar, kar tudi potrjuje najmanjša vrednost E (pregl. 2.). Povzetek ocenjenih parametrov obeh modelov je podan v preglednici 1.

depicts a comparison of the PDFs of the TTFF obtained for different models. As Fig. 5(a) shows, when applying the mixture model, the expected number of failures agrees with the observed data a little better than the standard model, which can be proved by the lower E value (see Table 2). A summary of the estimated parameters for both models is also provided (see Table 1).

4 SKLEPI

Model SOP na podlagi mešanice Weibullovih porazdelitvenih funkcij smo uporabili za modeliranje statistične odvisnosti zbirnega števila okvar izdelka.

Algoritem EM je ustrezna metoda za oceno parametrov SOP modela na podlagi mešanice Weibullovih porazdelitev za ČDPO. Z izbiro ustreznega števila komponent mešane porazdelitve je mogoče doseči dobro ujemanje med simuliranimi in izmerjenimi vrednostmi števila okvar. Algoritem EM ima tudi nekaj pomanjkljivosti. Rezultati in konvergenca iterativnga postopka za oceno neznanih parametrov mešane porazdelitve za ČDPO in parametra učinkovitosti popravila so zelo pogosto odvisni od začetnih vrednosti. Največji vpliv na rezultate in konvergenco algoritma EM ima začetna vrednost parametra q. Potrebni čas izračuna neznanih parametrov se občutno poveča z večjim številom komponent mešane porazdelitve. Večje število komponent porazdelitve zmanjšuje verjetnost konvergence algoritma in poveča število posameznih komponent.

Iz primerov lahko sklepamo, da so prednosti običajnega modela SOP numerična stabilnost, velika hitrost konvergence in robustnost podane numerične rešitve z Newton-Raphsonovo metodo.

Z ocenjenimi parametri je mogoče predstavljeni model uporabiti za nadaljnje izračune, to so ocena porazdelitvene funkcije verjetnosti za ČDPO, izračun pričakovanega števila okvar brez uporabe običajnih predpostavk o kakovosti popravila, kot sta 'dober kot nov' pri OP in 'slab kot star' pri NHPP. Parameter q lahko uporabljamo kot indeks učinkovitosti popravila.

4 CONCLUSIONS

The GRP model based on the mixture of Weibull distribution functions is used for modeling of the statistical dependency of the failure data.

The EM algorithm is an appropriate method for determining the parameters of the GRP based on the Weibull mixture distribution of the TTFF. With a proper choice of the number of underlying components, a good agreement between the numerically modelled and observed number of failures can be obtained. Nonetheless, the EM algorithm has some weaknesses as well. The result and convergence of the iterative procedure for an estimation of unknown parameters of the mixture distribution of the TTFF and the repair-effectiveness parameter very frequently depend on the initial conditions of the iteration. The biggest influence on the result and the convergence of the EM algorithm has a value of parameter q. The computing time needed for parameter estimation grows considerably with the increasing number of underlying components. If we increase the number of component distributions, the probability of the algorithm convergence decreases, and for the number of singular components it increases.

We can see from the presented examples that the major advantages of the standard model are numerical stability, high speed of convergence and robustness of the introduced numerical solution with the Newton-Raphson algorithm.

When the parameters are estimated it is possible to apply this model for further calculations, such as the prediction of the TTFF distribution and the expected number of failures without using the traditional assumptions of 'as good as new' and 'as bad as old', which are implicit in the RP and the NHPP, respectively. In addition, the parameter q can be used as an index of the repair's effectiveness.

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