

Modeliranje hitre tlačne razbremenitve v cevi Modelling of Fast Depressurization in the Pipe

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Ohranitvene enačbe programa RELAP5/MOD3.1 z zapiralnimi enačbami za mehurčkasti režim dvo faznega toka in z dopolnjeno obliko člena navidezne mase smo uporabili v računalniškem programu, ki smo ga poimenovali PDE (parcialne diferencialne enačbe).

Uporabljena numerična shema je še posebej primerna za simuliranje hitrih prehodnih pojavov z udarnimi in razredčitvenimi valovi. Shema je razmeroma stabilna in robustna in je temelj za sheme drugega reda natančnosti, ki upoštevajo udarne pojave.

Preizkus s hitro tlačno razbremenitvijo cevi, napolnjene z vročo vodo smo simulirali s programoma PDE in RELAP5/MOD3.1. Rezultati programa PDE so podobni rezultatom programa RELAP5. Dosegli smo tudi podobno ujemanje z meritvami.

Ključne besede: tok dvo fazni, tok mehurčkast, enačbe ohranitvene, modeliranje

RELAP5/MOD3.1 conservation equations with closure laws for bubbly regime of the two-phase flow and improved form of the virtual mass term were used in our own computer code named PDE (Partial Differential Equations).

The numerical scheme of the PDE code is especially suitable for the simulation of the fast transients with shocks and rarefaction waves. The scheme is relatively stable and robust, and presents a basis for the second order shock-capturing scheme.

The Edwards pipe experiment - fast depressurization in the pipe - has been modelled using the PDE and RELAP5/MOD3.1 codes. The PDE code results were similar to the RELAP5 results, and approximately the same agreement with the measurements was achieved.

Keywords: twophase flow, bubble flow, conservation equations, modelling

0 UVOD

Za opis splošnih dvo faznih tokov, v katerih je gibanje med fazne ploskve zaradi turbulence prehitro, da bi mu lahko sledili, uporabljamo povprečene enačbe, ki vsako od faz obravnavajo kot kontinuum. Modeli, ki jih današnji računalniški programi RELAP5 [2], TRAC in CATHARE uporabljajo za simuliranje več faznih tokov v jedrski tehniki, opisujejo dvo fazni tok s šestimi parcialnimi diferencialnimi enačbami. Osnovne enačbe programov so enodimenzionalne, kar zadošča za opis tokov v ceveh. Z omenjenimi programi lahko uspešno modeliramo širok razpon prehodnih pojavov, katerih karakteristično časovno lestvico določa hitrost toka. Programi so manj zanesljivi pri modeliranju hitrih prehodnih pojavov, kjer karakteristično časovno lestvico določa hitrost zvoka. To je posledica izkustvenih korelacij v osnovnih enačbah, ki na področju hitrih prehodnih pojavov niso preverjene, in numeričnih shem, ki so v osnovi namenjene simuliranjem nestisljivih tokov.

Preizkus "Edwardsova cev" [1] hitra tlačna razbremenitev v cevi - uporabljamo kot enega od osnovnih preizkusov za preverjanje programov za modeliranje dvo faznega toka zaradi njegove preproste geometrijske oblike in širokega spektra pojavov, ki jih preizkus obsegajo. Edwards in O'Brien sta 4 m dolgo vodoravno cev napolnila z vodo pri tlaku 7 MPa in temperaturi 502 K in nato hitro odprla en konec cevi. Merila sta tlak in prostorninski delež pare med iztekanjem v nekaj točkah vzdolž cevi. Pomembni opazovani pojavi so: razbremenilni val, samouparjanje, dvo fazni kritični tok in val parne faze.

0 INTRODUCTION

The general two-phase flows - where interphase surface geometry cannot be explicitly followed due to the rapid changes caused by turbulence - are described by averaged equations which treat each of the phases as a continuum. Today's computer codes RELAP5[2], TRAC and CATHARE, used for the simulations of the multiphase flows in nuclear engineering, describe a two-phase flow with six partial differential equations. The basic equations are one-dimensional and sufficient for the description of the flows through the pipes. The above codes can be successfully used for modelling the wide range of the transients, where fluid velocity determines the characteristic time scale. The codes are less reliable in the area of the fast transients where the characteristic time scale is determined by the sound speed. This is a consequence of the empirical correlations in the basic equations which are not verified in the area of the fast transients and numerical schemes based on the methods developed for incompressible flows.

The Edwards pipe experiment [1] is used as one of the basic benchmarks for the two-phase flow codes due to its simple geometry and the wide range of phenomena that it covers. Edwards and O'Brien filled a 4 m long pipe with liquid water at 7 MPa and 502 K and suddenly ruptured one end of the tube. They measured the pressure and void fraction during the blowdown at a few points along the pipe axis. Important phenomena observed included: pressure rarefaction wave, flashing onset, critical two-phase flow and void fraction wave. This experiment is used as a basic benchmark for all new versions of the RELAP code.

Ta preizkus zato pomeni osnovni testni primer za vse nove verzije programa RELAP. V našem delu smo rezultate meritev uporabili za preizkus dveh različnih numeričnih shem: shemo programa RELAP5 in shemo, uporabljen v programu PDE, ki temelji na karakteristični privetni diskretizaciji [3] in dodatno analizo modela dvofaznega toka iz programa RELAP5/MOD3.1 [2].

1 OSNOVNE ENAČBE

Osnovne enačbe dvofuidnega modela za opis dvofaznega toka v programu RELAP5 so enodimensionalne ohranitvene enačbe za maso, gibalno količino in energijo pare in kapljivine [3]:

$$\frac{\partial(1-\alpha)\rho_f}{\partial t} + \frac{1}{A} \frac{\partial(1-\alpha)\rho_f v_f A}{\partial x} = -\Gamma_s \quad (1)$$

$$\frac{\partial \alpha \rho_g}{\partial t} + \frac{1}{A} \frac{\partial \alpha \rho_g v_g A}{\partial x} = \Gamma_s \quad (2)$$

$$(1-\alpha)\rho_f \frac{\partial v_f}{\partial t} + \frac{1}{2}(1-\alpha)\rho_f \frac{\partial v_f^2}{\partial x} = -(1-\alpha) \frac{\partial p}{\partial x} - \frac{1}{2}(1-\alpha)\rho_f \frac{f_{fw}}{D} |v_f|v_f - \Gamma_s(v_f - v_g) + \frac{C_D}{8}\rho_c a_{sf} |v_r|v_r + C_{vm}\alpha(1-\alpha)\rho_m \frac{\partial v_r}{\partial t} \quad (3)$$

$$\alpha \rho_g \frac{\partial v_g}{\partial t} + \frac{1}{2}\alpha \rho_g \frac{\partial v_g^2}{\partial x} = -\alpha \frac{\partial p}{\partial x} - \frac{1}{2}\alpha \rho_g \frac{f_{gw}}{D} |v_g|v_g + \Gamma_s(v_g - v_f) - \frac{C_D}{8}\rho_c a_{sf} |v_r|v_r - C_{vm}\alpha(1-\alpha)\rho_m \frac{\partial v_r}{\partial t} \quad (4)$$

$$\frac{\partial(1-\alpha)\rho_f u_f}{\partial t} + \frac{1}{A} \frac{\partial(1-\alpha)\rho_f u_f v_f A}{\partial x} = -p \frac{\partial(1-\alpha)}{\partial t} - p \frac{\partial(1-\alpha)v_f A}{\partial x} + Q_{if} - \Gamma_s h_f + \frac{1}{2}(1-\alpha)\rho_f \frac{f_{fw}}{D} v_f^2 |v_f| \quad (5)$$

$$\frac{\partial \alpha \rho_g u_g}{\partial t} + \frac{1}{A} \frac{\partial \alpha \rho_g u_g v_g A}{\partial x} = -p \frac{\partial \alpha}{\partial t} - p \frac{\partial \alpha v_g A}{\partial x} + Q_{ig} + \Gamma_s h_g + \frac{1}{2}\alpha \rho_g \frac{f_{gw}}{D} v_g^2 |v_g| \quad (6)$$

Prvi dve enačbi sta kontinuitetni enačbi kapljivine in pare s členom Γ_s , ki opisuje medfazni prenos snovi. Enačbi (3) in (4) sta gibalni enačbi, členi na desni strani obeh enačb pa pomenijo: gradient tlaka, stensko trenje, izmenjavo gibalne količine zaradi izmenjave snovi, medfazno trenje in člen navidezne mase. Členi na desni strani enačb notranje energije (5) in (6) pomenijo spremembe notranje energije zaradi sprememb tlaka, hitrosti in prostorninskega deleža pare (prva dva člena), tretji in četrti člen popisujeta medfazno izmenjavo energije, zadnji člen pa popisuje naraščanje notranje energije zaradi stenskega trenja (glej seznam spremenljivk).

Iste enačbe smo uporabili v programu PDE, spremenili smo le člen navidezne mase. Člene v energijskih enačbah, ki popisujejo prenos toplote s tekočine na steno, smo zanemarili, ker je običajno njihov vpliv na hitre prehodne pojave zanemarljiv. Za popoln sistem enačb potrebujemo še dve enačbi stanja za vsako od obeh faz. Enačba stanja za fazo k je:

$$d\rho_k = \left(\frac{\partial \rho_k}{\partial p} \right)_{u_k} dp + \left(\frac{\partial \rho_k}{\partial u_k} \right)_p du_k \quad (7)$$

We used experimental data to examine two different numerical schemes: one from RELAP5/MOD3.1 code, and one from PDE code which was based on characteristic upwind discretization [3] and for an additional analysis of the RELAP5/MOD3.1 [2] six-equation two-phase flow model.

1 BASIC EQUATIONS

Basic equations of the two-fluid model taken from RELAP5 are one-dimensional mass, momentum and energy balances for vapour and liquid [3]:

The first pair of the equations are continuity equations, with the term Γ_s describing the interphase exchange of mass. Equations (3) and (4) are momentum equations, with terms on the right hand side: pressure gradient, wall friction, exchange of momentum due to the exchange of mass, interphase friction and virtual mass term. The terms on the right hand side of the internal energy Equations (5) and (6) present variations of the internal energy due to the changes of pressure, velocity and void fraction (first two terms); the third and fourth terms describe the interphase exchange of energy, while the last term describes the increase of the internal energy due to the wall friction (see Nomenclature).

The same equations were used in the PDE code; only the virtual mass term in momentum equations was different. The terms for wall-to-liquid heat transfer in energy equations were neglected since they are seldom important for the fast transients. Two additional equations of state for each phase are needed to close the system of equations. Equation of state for phase k is:

Odvode v enačbi (7) izračunamo iz osnovnih termodinamičnih razmerij:

$$\left(\frac{\partial \rho_k}{\partial p} \right)_{u_k} = \frac{c_{pk} \kappa_k \rho_k - T_k \beta_k^2}{c_{pk} - p \beta_k / \rho_k} \quad (8),$$

$$\left(\frac{\partial \rho_k}{\partial u_k} \right)_p = - \frac{\beta_k \rho_k}{c_{pk} - p \beta_k / \rho_k} \quad (9).$$

Spremenljivke na desni strani enačb (8) in (9) določimo z uporabo podprogramov za izračun lastnosti vode iz znanega tlaka in specifičnih notranjih energij, ki jih uporablja program RELAP5 [2].

Konstitutivne enačbe za zidno trenje, medfazno trenje, navidezno maso in medfazni prenos toplove in snovi smo prav tako povzeli po programu RELAP5. Uporabili smo korelacije za mehurčasti dvofazni tok, ki veljajo pri prostorninskih deležih pare, manjših od 0,5 in gostotah masnega pretoka, večjih od 3000 kg/m²s. Pri modeliraju preizkusa "Edwardsova cev" smo korelacije za mehurčast tok uporabili tudi pri večjih prostorninskih deležih pare, ki se pojavijo v zadnji fazi preizkusa.

Člene za popis zidnega trenja v enačbah (3) in (4) smo nekoliko poenostavili v primerjavi s programom RELAP5. Testiranja programa PDE so pokazala, da te spremembe nimajo znatnega vpliva na rezultate. Preostale konstitutivne enačbe smo uporabili v isti obliki kakor so uporabljene v programu RELAP5/MOD3.1.

V parcialnih diferencialnih enačbah RELAP5 (1) do (6) se pojavljajo samo prvi odvodi, medtem ko so difuzijski členi namesto z drugimi odvodi poenostavljeno zapisani v obliki nediferencialnih izkutvenih korelacij. V enačbah torej ni mehanizmov difuzije in jih po obliki zato lahko primerjamo z Eulerjevimi enačbami enofaznega toka [8] in lahko pričakujemo tudi podobno obnašanje rešitev, ki dopuščajo tudi nezveznosti (npr. udarni valovi). Difuzijo v rešitve prinesejo numerične metode, s katerimi te enačbe rešujejo računalniški programi, kakršen je RELAP5. V splošnem je zanesljivost enačb dvofuidnih modelov zaradi točnosti korelacij (tipične napake korelacij so 20%) precej manjša od zanesljivosti Eulerjevih enačb enofaznega toka, zaradi česar numerična difuzija pomeni pogosto sprejemljivo napako.

Specifičen problem modeliranja toka s samouparjanjem je prehod iz enofaznega v dvofazni tok. Za uspešnejše modeliranje prehoda iz enofaznega v dvofazno, zaradi hitrega padca tlaka, bi morali upoštevati padec tlaka pod tlak nasičenja zaradi zamude pri uparjanju. To lahko opišemo s korelacijo Alamgir-Lienhart [4]. V programih RELAP5 in PDE zamude pri uparjanju nismo eksplicitno modelirali. Prehod iz enofaznega v dvofazno se začne, ko so doseženi pogoji za nasičenje.

Derivatives from Eq. (7) can be calculated from the basic thermodynamic relations as:

$$\left(\frac{\partial \rho_k}{\partial p} \right)_{u_k} = \frac{c_{pk} \kappa_k \rho_k - T_k \beta_k^2}{c_{pk} - p \beta_k / \rho_k} \quad (8),$$

$$\left(\frac{\partial \rho_k}{\partial u_k} \right)_p = - \frac{\beta_k \rho_k}{c_{pk} - p \beta_k / \rho_k} \quad (9).$$

Variables on the right side of Eqs. (8) and (9) are determined by the RELAP5 water properties subroutines if the pressure and the specific internal energies are known [2].

Constitutive equations for wall friction, interphase drag, virtual mass and interphase heat transfer were also taken from RELAP5 code. We applied the correlations for the bubbly regime of two-phase flow. According to the RELAP5 authors [2], bubbly flow exists at mass fluxes larger than 3000 kg/m²s if the vapour void fraction is lower than 0.5. In the PDE model of the Edward's pipe experiment bubbly flow correlations were also used for higher vapour void fractions which appeared in the later stages of the experiment.

In PDE code wall friction terms in momentum Eqs. (3) and (4) were applied in a simplified way compared to the RELAP5. Our tests showed that this difference did not have a significant influence on the results. The other constitutive equations were applied in the same form as in the RELAP5/MOD3.1.

Basic RELAP5 partial differential equations (1)-(6) contain only first order derivatives. Diffusion terms with second order derivatives are simplified and written in the form of the empirical correlations. Since there is no diffusion in the equations they can be compared with Euler equations of the single phase flow [8], and we can also expect similar behaviour from the solutions which allow discontinuities (i.e. shock waves). Diffusion is introduced into solutions by the numerical method applied in RELAP5 code. The general reliability of the two-fluid equations is lower than the reliability of the Euler equations due to the accuracy of the correlations (typical uncertainty of the correlations is 20%); for this reason numerical diffusion often represents an acceptable error.

The specific problem of the flashing flow modelling is transition from single to two-phase flow. Successful modelling of the single to two-phase transition due to the fast depressurization requires some pressure undershoot below the saturation pressure as a result of the flashing delay. This can be modelled by using the Alamgir-Lienhard correlation [4]. In RELAP5 and PDE codes flashing delay has not been explicitly modelled. Single to two-phase transition in both codes starts when saturation conditions are achieved.

2 ČLEN NAVIDEZNE MASE

V členu navidezne mase v enačbah (3) in (4) so v programu RELAP5 uporabili samo časovne odvode relativne hitrosti $\partial v / \partial t$. Bolj popoln zapis člena navidezne mase se glasi [2]:

$$\frac{\partial v_g}{\partial t} + v_f \frac{\partial v_g}{\partial x} - \frac{\partial v_f}{\partial t} - v_g \frac{\partial v_f}{\partial x} \quad (10)$$

To obliko odvodov v členu navidezne mase smo uporabili v programu PDE.

Osnovne enačbe programa RELAP5 s poenostavljenim zapisom člena navidezne mase po-menojo nekorektno postavljen matematični problem [5], [6] s kompleksnimi lastnimi vrednostmi Jacobijeve matrike (matrika v (15)). Program PDE lahko rešuje samo korektno postavljeni hiperbolični matematični probleme. S členom navidezne mase (10) se izognemo nekorektnemu problemu za prostorninske deleže pare, manjše od približno 0,7 (sl. 1).

V priročniku programa RELAP5 [2], [7], v poglavju o kritičnem toku so podani aproksimativni izrazi za lastne vrednosti Jacobijeve matrike enačb s členom navidezne mase (10). Aproksimacije veljajo, če je relativna medfazna hitrost v majhna v primerjavi z zvočno hitrostjo v dvofaznem toku. Iz teh rešitev izhaja, da dve lastni vrednosti vsebujejo izraz:

$$\sqrt{(\rho_m C_{vm}/2)^2 - \alpha(1-\alpha)\rho_g\rho_f} \quad (11)$$

kjer je gostota ρ_m dvofazne zmesi in C_{vm} koeficient navidezne mase, ki je v programu RELAP5 izražen kot:

$$C_{vm} = \begin{cases} (1+2\alpha)/(2-2\alpha); & \alpha < 0.5 \\ (3-2\alpha)/(2\alpha); & \alpha > 0.5 \end{cases} \quad (12)$$

Vrednost (11) postane imaginarna, če je izraz pod korenem negativen. Če enačbo (11) delimo z ρ_f , lahko takšen izraz narišemo kot funkcijo α in ρ_g/ρ_f . Graf je prikazan na sliki 1, kjer je, kljub korekciji člena navidezne mase v primerjavi s programom RELAP5, jasno vidno območje prostorninskih deležev pare in razmerja faznih gostot, kjer so enačbe programa RELAP5 še vedno nekorektno postavljeni matematični problem.

Korektnost enačb smo popravili še s spremembami koeficiente navidezne mase:

$$C_{vm} = \begin{cases} (1+2\alpha)/(2-2\alpha); & \alpha < 0.5 \\ \sqrt{\left(\frac{3-2\alpha}{2\alpha}\right)^2 - \frac{(\alpha-1)(2\alpha-1)}{(1+\alpha\rho_g/\rho_f-\alpha)^2}}; & \alpha > 0.5 \end{cases} \quad (13)$$

Izraz, ki smo ga dodali, nima fizikalne podlage in le popravi korektnost postavljenih enačb. Dokaj zapleten izraz smo dodali tako, da ostanejo mejne

2 VIRTUAL MASS TERM

Virtual mass term in eqs. (3) and (4) is coded in RELAP5 only with time derivative of the relative velocity $\partial v / \partial t$. According to the RELAP5 manual [2] difference of the substantial derivatives should be applied instead of:

This form of the derivative in the virtual mass term was used in the PDE code.

Basic RELAP5 equations as coded in RELAP5 present an ill-posed problem [5], [6] with complex eigenvalues of the Jacobian matrix (matrix in (15)). The PDE code can solve only well-posed (hyperbolic) problems. With virtual mass term (10) ill-posedness of the equations is avoided for vapour void fractions lower than approx. 0.7 (Fig. 1).

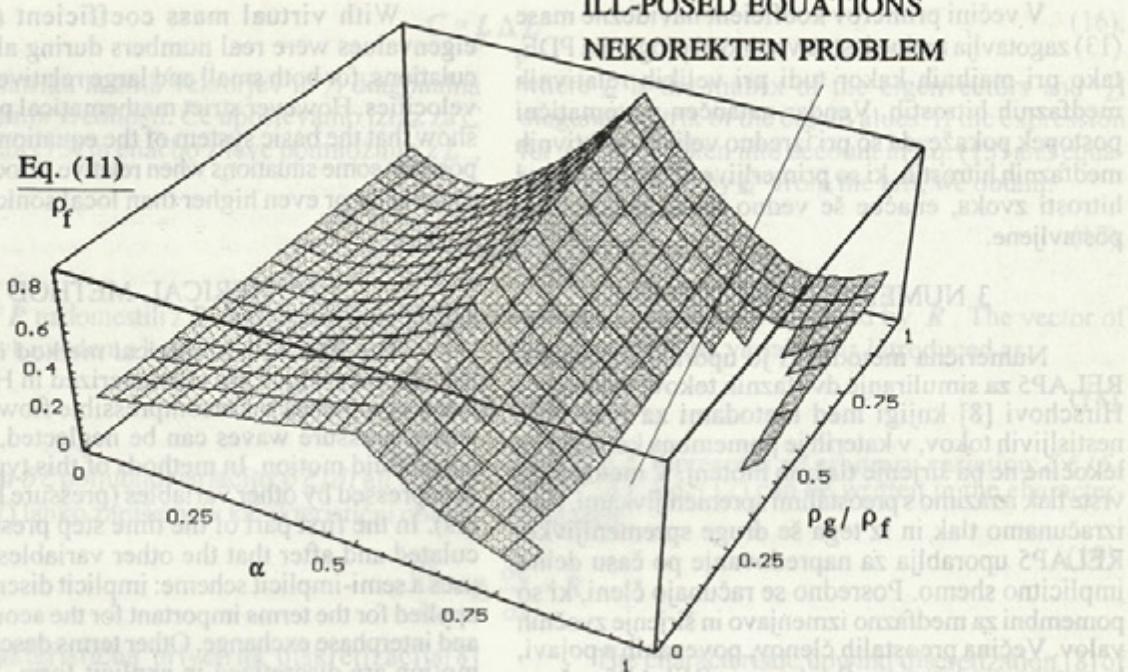
In the RELAP5 manual [2], [7], chapter "Special Process Models, Choked Flow" approximate analytical expressions for the Jacobian matrix eigenvalues are given. The results are valid for relative interphase velocities lower than mixture sonic velocity. From these solutions we can see that two approximate eigenvalues contain the term:

Term (11) becomes imaginary if the expression under the root is negative. If eq. (11) is divided by ρ_f , expression (11) can be plotted as a function of α and ρ_g/ρ_f . This plot is shown in Fig. 1 where the area of the vapour void fractions and phasic density ratios is clearly seen, where RELAP5 equations with complete virtual mass term are still ill-posed.

The well-posedness of the equations was improved by changing the virtual mass coefficient:

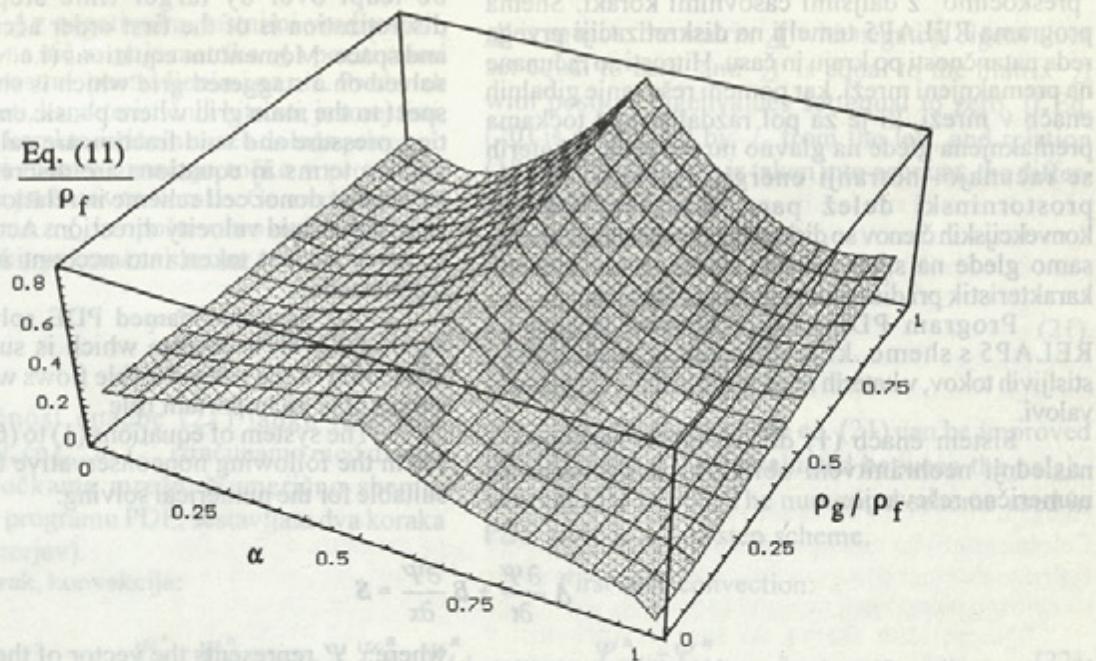
In practice, this change ensures the well-posedness of the equations, and does not have any underlying physical basis. Relatively complicated

ILL-POSED EQUATIONS NEKOREKTN PROBLEM

Eq. (11)

Sl. 1. Izraz (11) deljen z ρ_f s koeficientom navidezne mase (12), narisani kot funkcija α in ρ_g / ρ_f .

Fig. 1. Expression (11) divided by ρ_f with virtual mass coefficient (12) plotted as a function of α and ρ_g / ρ_f .

Eq. (11)

Sl. 2. Izraz (11) deljen z ρ_f s koeficientom navidezne mase (13), narisani kot funkcija α in ρ_g / ρ_f .

Fig. 2. Expression (11) divided by ρ_f with virtual mass coefficient (13) plotted as a function of α and ρ_g / ρ_f .

vrednosti koeficiente navidezne mase, ko gre α proti ena, nespremenjene. Da se izognemo nekorektno postavljenim enačbam moramo zapisati C_{vm} tudi kot funkcijo razmerja faznih gostot ρ_g / ρ_f . Graf izraza (11) kot funkcije α in ρ_g / ρ_f s koeficientoma videzne mase (13) je prikazan na sliki 2.

expression has been added in order to retain the limit value of the virtual mass coefficient as α approaches the level of one. In order to avoid ill-posedness, it is necessary to make C_{vm} also a function of the density ratio ρ_g / ρ_f . The plot of the expression (11) as a function of α and ρ_g / ρ_f with virtual mass coefficient (13) is presented in Fig. 2.

V večini primerov koeficient navidezne mase (13) zagotavlja realne lastne vrednosti programa PDE, tako pri majhnih kakor tudi pri velikih relativnih medfaznih hitrostih. Vendar natančen matematični postopek pokaže, da so pri izredno velikih relativnih medfaznih hitrostih, ki so primerljive ali celo večje od hitrosti zvoka, enačbe še vedno lahko nekorektno postavljene.

3 NUMERIČNA METODA

Numerična metoda, ki jo uporablja program RELAP5 za simuliranje dvo faznih tokov, najdemo v Hirschovi [8] knjigi med metodami za reševanje nestisljivih tokov, v katerih je pomembna konvekcija tekočine ne pa širjenje tlačnih motenj. V metodah te vrste tlak izrazimo s preostalimi spremenljivkami, nato izračunamo tlak in iz tega še druge spremenljivke. RELAP5 uporablja za napredovanje po času delno implicitno shemo. Posredno se računajo členi, ki so pomembni za medfazno izmenjavo in širjenje zvočnih valov. Večina preostalih členov, povezanih s pojavimi, ki potekajo s hitrostjo fluida, se v RELAP5 računa eksplicitno. Takšna numerična shema je primerna za računanje več ur trajajočih prehodnih pojavov, pri katerih hitrost zvoka, tlačni valovi in podrobnosti medfaznega prenosa niso pomembni in jih zato lahko "preskočimo" z daljšimi časovnimi koraki. Shema programa RELAP5 temelji na diskretizaciji prvega reda natančnosti po kraju in času. Hitrosti so računane na premaknjeni mreži, kar pomeni reševanje gibalnih enačb v mreži, ki je za pol razdalje med točkama premaknjena glede na glavno mrežo točk, v katerih se računajo: notranji energiji, gostoti, tlak in prostorninski delež pare. Krajevni odvodi konvekcijskih členov so diskretizirani s privetno shemo samo glede na smer hitrosti fluida. Smer zvočnih karakteristik pri diskretizaciji ni upoštevana.

Program PDE rešuje enačbe programa RELAP5 s shemo, ki je namenjena modeliranju stisljivih tokov, v katerih imajo pomembno vlogo tlačni valovi.

Sistem enačb (1) do (6) lahko zapišemo v naslednji neohranitveni obliki, ki je primerna za numerično reševanje:

$$\mathcal{A} \frac{\partial \Psi}{\partial t} + \mathcal{B} \frac{\partial \Psi}{\partial x} = \mathcal{S} \quad (14),$$

kjer pomenijo: Ψ – vektor neodvisnih spremenljivk $\Psi = (\rho, \alpha, v_f, v_g, u_f, u_g)$, \mathcal{A} in \mathcal{B} sta matriki sistema in \mathcal{S} je vektor z nediferencialnimi členi v enačbah. Enačba (14), pomnožena z \mathcal{A}^{-1} z leve, da:

$$\frac{\partial \Psi}{\partial t} + \mathcal{C} \frac{\partial \Psi}{\partial x} = \mathcal{P} \quad (15).$$

$\mathcal{C} = \mathcal{A}^{-1} \mathcal{B}$ je Jacobijeva matrika in vektor $\mathcal{P} = \mathcal{A}^{-1} \mathcal{S}$. Če poiščemo lastne vrednosti in lastne vektorje matrike \mathcal{C} , lahko Jacobijovo matriko zapišemo kot:

With virtual mass coefficient (13), PDE eigenvalues were real numbers during all PDE calculations, for both small and large relative interphase velocities. However strict mathematical proof would show that the basic system of the equations is still ill-posed in some situations when relative velocity is comparable to, or even higher than local sonic velocity.

3 NUMERICAL METHOD

The RELAP5 numerical method is based on the schemes which are characterized in Hirsch's [8] book as schemes for incompressible flows, i.e. flows where pressure waves can be neglected, compared to the fluid motion. In methods of this type pressure is expressed by other variables (pressure based solvers). In the first part of the time step pressure is calculated and after that the other variables. RELAP5 uses a semi-implicit scheme: implicit discretization is applied for the terms important for the acoustic waves and interphase exchange. Other terms describing fluid motion are discretized in explicit form. Such a numerical scheme is suitable for transients which are a few hours long, in which the short time scale phenomena: sound speed, pressure waves and details of the interphase exchange are not important and can be leapt over by larger time steps. RELAP5 discretization is of the first order accuracy in time and space. Momentum equations (i.e. velocities) are solved on a staggered grid which is shifted with respect to the main grid where phasic energies, densities, pressure and void fraction are calculated. Convective terms in equations are discretized with an upwind or donor cell scheme in relation to the direction of the fluid velocity direction. Acoustic characteristics are not taken into account in the upwind discretization.

Our program, named PDE solves RELAP5 equations with a scheme which is suitable for the modelling of the compressible flows where pressure waves play an important role.

The system of equations (1) to (6) can be written in the following nonconservative form which is suitable for the numerical solving:

where: Ψ represents the vector of the independent variables $\Psi = (\rho, \alpha, v_f, v_g, u_f, u_g)$, \mathcal{A} and \mathcal{B} are matrices of the system, and \mathcal{S} is a vector with nondifferential terms in the equations. Equation (14) multiplied by \mathcal{A}^{-1} from the left gives:

$\mathcal{C} = \mathcal{A}^{-1} \mathcal{B}$ is Jacobian matrix and vector $\mathcal{P} = \mathcal{A}^{-1} \mathcal{S}$. If the eigenvalues and eigenvectors of the matrix \mathcal{C} are found, the Jacobian matrix can be written as:

$$\mathcal{C} = L \Delta L^{-1} \quad (16)$$

kjer je L matrika lastnih vektorjev in Δ diagonalna matrika lastnih vrednosti. Če upoštevamo izraz za \mathcal{C} (16) v enačbi (15) in enačbo z leve pomnožimo z L^{-1} , dobimo:

$$L^{-1} \frac{\partial \Psi}{\partial t} + \Delta L^{-1} \frac{\partial \Psi}{\partial x} = R \quad (17)$$

kjer smo $L^{-1}P$ nadomestili z R . Vektor karakterističnih spremenljivk uvedemo kot:

$$\delta \xi = L^{-1} \delta \Psi \quad (18)$$

kjer pomeni $\delta \xi$ poljubno variacijo: $\delta \xi / \delta t$ ali $\delta \xi / \delta x$. Enačbo (17) lahko zapišemo v karakteristični obliki:

$$\frac{\partial \xi}{\partial t} + \Delta \frac{\partial \xi}{\partial x} = R \quad (19)$$

Karakteristična privetrna diskretizacija [8] enačbe (19) z eksplicitno shemo končnih razlik je:

$$\frac{\xi_j^{n+1} - \xi_j^n}{\Delta t} + (\Delta^*)_j^n \frac{\xi_j^n - \xi_{j-1}^n}{\Delta x} + (\Delta^*)_j^n \frac{\xi_{j+1}^n - \xi_j^n}{\Delta x} = R_j^n \quad (20)$$

Δ^* je matrika Δ z negativnimi lastnimi vrednostmi, postavljenimi na nič in Δ^- je matrika Δ s pozitivnimi lastnimi vrednostmi, postavljenimi na nič. Predznak lastnih vrednosti, karakterističnih hitrosti, je torej tisti, ki za vsako od karakterističnih enačb določa privetno stran. Spodnji indeks označuje točke v prostorski mreži, zgornji pa časovne korake. Če enačbo (20) z leve pomnožimo z L , in upoštevamo razmerje (18) v diskretni obliki, je koračna shema:

$$\frac{\Psi_j^{n+1} - \Psi_j^n}{\Delta t} + (\mathcal{C})_j^n \frac{\Psi_j^n - \Psi_{j-1}^n}{\Delta x} + (\mathcal{C})_j^n \frac{\Psi_{j+1}^n - \Psi_j^n}{\Delta x} = P_j^n \quad (21)$$

Natančnost enačbe (21) lahko nekoliko izboljšamo [8], če \mathcal{C}^+ in \mathcal{C}^- izračunamo med dvema sosednjima točkama mreže. Numerično shemo, uporabljeno v programu PDE, sestavlja dva koraka (razcep operatorjev).

Prvi korak, konvekcija:

$$\frac{\Psi_j^* - \Psi_j^n}{\Delta t} + (\mathcal{Q})_{j-1/2}^* \frac{\Psi_j^n - \Psi_{j-1}^n}{\Delta x} + (\mathcal{Q})_{j+1/2}^* \frac{\Psi_{j+1}^n - \Psi_j^n}{\Delta x} = 0 \quad (22)$$

Drugi korak, integracija virov:

$$\Psi_j^{n+1} = \int_{t^*}^{t^{n+1}} P(\Psi_j(t)) dt \quad (23)$$

Vire P smo integrirali z Eulerjevo metodo z ločenimi časovnimi koraki, ki so lahko tudi več stokrat manjši od časovnega koraka konvekcije.

where L is the matrix of the eigenvectors and Δ diagonal matrix of the eigenvalues. If the expression for \mathcal{C} (16) is taken into account in Eq. (15) and equation multiplied by L^{-1} from the left, we obtain:

$$L^{-1} \frac{\partial \Psi}{\partial t} + \Delta L^{-1} \frac{\partial \Psi}{\partial x} = R \quad (17)$$

where $L^{-1}P$ has been replaced by R . The vector of the characteristic variables is introduced as:

$$\delta \xi = L^{-1} \delta \Psi \quad (18)$$

where $\delta \xi$ represents an arbitrary variation: $\delta \xi / \delta t$ or $\delta \xi / \delta x$. Eq. (17) can be written in the characteristic form:

$$\frac{\partial \xi}{\partial t} + \Delta \frac{\partial \xi}{\partial x} = R \quad (19)$$

The characteristic upwind discretization [8] of the Eq.(19) with the explicit finite difference scheme is:

Δ^* is equal to the matrix Δ with negative eigenvalues set equal to zero, and Δ^- is equal to the matrix Δ with positive eigenvalues set equal to zero. If Eq. (20) is multiplied by L , from the left, and relation (18) in discrete form is taken into account, the difference scheme is:

The accuracy of the eq. (21) can be improved [8] if \mathcal{C}^+ and \mathcal{C}^- are evaluated between the neighbouring grid points. The numerical scheme used in PDE code is a two-step scheme.

First step, convection:

Second step, integration of the sources:

The source terms P were integrated using the Euler method with separated time steps; these steps can be a few hundred times smaller than the time

Karakteristična časovna lestvica je namreč veliko manjša za izvore kakor za konvekcijske člene [3]. Časovni korak konvekcije omejuje pogoj CFL (Courant-Friedrics-Levy $\Delta t \leq \Delta x / \text{maks}(\lambda)$, $j = 1, 6$, medtem ko je časovni korak za integracijo izvorov spremenljiv. Največja relativna sprememba osnovnih spremenljivk v enem časovnem koraku integracije virov mora biti manjša od 0,1 odstotka. Če je relativna sprememba prevelika, se časovni korak za integracijo virov zmanjša, če pa se relativna sprememba virov manjša, se časovni korak za vire povečuje, vendar kvečjemu do časovnega koraka, ki se uporablja v hidrodinamiki.

Diagonalizacijo Jacobijevih matrik (16) smo izvedli numerično s podprogrami iz knjižnice EISPACK [9], ki poiščejo realne ali kompleksne lastne vrednosti in lastne vektorje poljubne matrike. Program PDE lahko rešuje samo probleme z realnimi lastnimi vrednostmi Jacobijeve matrike. Računanje se ustavi, če program najde kompleksne lastne vrednosti.

Glavna prednost predstavljenih numeričnih shem v primerjavi z delno posredno shemo programa RELAP5 je korektna obravnava privetrne diskretizacije. V shemi PDE določajo stran privetrne diskretizacije karakteristične hitrosti. V programu RELAP5 določata stran privetrne diskretizacije samo hitrosti obeh faz, druge karakteristične hitrosti pa niso upoštevane. V podzvočnem toku se smer ene od šestih karakterističnih hitrosti razlikuje od smeri preostalih, na primer, če sta hitrosti obeh faz v_r, v_s pozitivni, najmanjša karakteristična hitrost ostane negativna. To pomeni, da eno od šestih enačb RELAP5 rešuje z nestabilno odvetrno diskretizacijo. To nestabilnost RELAP5 sheme nadomesti numerična difuzija [2].

Numerična shema v programu PDE ni ohranitvena, kar pomeni, da ne zagotavlja numeričnega ohranjanja mase, gibalne količine in energije. Numerični poskusi v dvoafazni "šok-cevi" [8] (zaprti cev z različnimi začetnimi pogoji v vsaki polovici cevi) so pokazali praktično zanemarljiva nihanja celotne mase (relativne spremembe mase in energije 10^{-6}) kljub neohranitveni numerični shemi. Celotna relativna nihanja mase pri modeliranju hitre tlake razbremenitve v cevi so okoli 10^{-3} , kar je glede na splošno natančnost računov zanemarljivo.

Numerična shema, ki smo jo uporabili v programu PDE, pomeni osnovo za tako imenovane sheme drugega reda natančnosti za dvoafazni tok. Rešitve, izračunane s takšnimi shemami, imajo zelo majhno numerično difuzijo, kar pomeni ostro ločljivost pri modeliranju udarnih in razredčitvenih valov.

4 REZULTATI

Primerjava izmerjenih in izračunanih rezultatov kaže vplive numerične sheme in pomembnih fizikalnih parametrov na modeliranje preizkusa "Edwardsova

step for the hydrodynamics due to the characteristic time scale which is much smaller for the source term than for the convective terms [3]. The time step for the convection is limited by the CFL (Courant-Friedrics-Levy) condition $\Delta t \leq \Delta x / \text{max}(\lambda)$, $j = 1, 6$, while the time step for the integration of the sources may vary. Maximal relative change of the basic variables in a one "source" time step must be less than 0.1%. If the relative change is larger, "source" time step is decreased; if the relative change is smaller, the "source" time step is increased but cannot exceed the time step used for the hydrodynamics.

Decomposition of the Jacobian matrix (16) has been performed numerically with subroutines from the EISPACK library [9], which are able to find real or complex eigenvalues and eigenvectors of an arbitrary matrix. The PDE program is written only for problems with real eigenvalues of the Jacobian matrix. Calculation is interrupted if complex eigenvalues are found.

The main advantage of the numerical scheme presented, compared to the RELAP5 semi-implicit scheme, is correct treatment of the upwind discretization. In the PDE scheme the characteristic velocity determines the side of the upwind differencing for the corresponding characteristic equation. In RELAP5, the side of the upwind differencing is determined only by phasic velocities, while the directions of the other characteristic velocities are not taken into account. In the subsonic two-phase flow, the direction of one of the six characteristic velocities differs from the directions of the other five characteristic velocities. For example, if phasic velocities v_r, v_s are positive, the smallest characteristic velocity remains negative. This means that one of the six equations in RELAP5 is solved by unstable downwind discretization. This instability is compensated by numerical diffusion [2].

The numerical scheme applied in the PDE program is a nonconservative one. This means that the scheme does not ensure numerical conservation of mass, momentum and energy. Numerical experiments with the two-phase shock tube problem [8] (closed tube with different initial conditions in each half of the tube) have shown practically negligible fluctuations of the overall mass and energy in the tube (relative mass and energy changes around 10^{-6}) despite the nonconservative scheme. The relative changes of the mass during the Edwards pipe transient are around 10^{-3} , which is negligible if the overall accuracy of the simulation is taken into account.

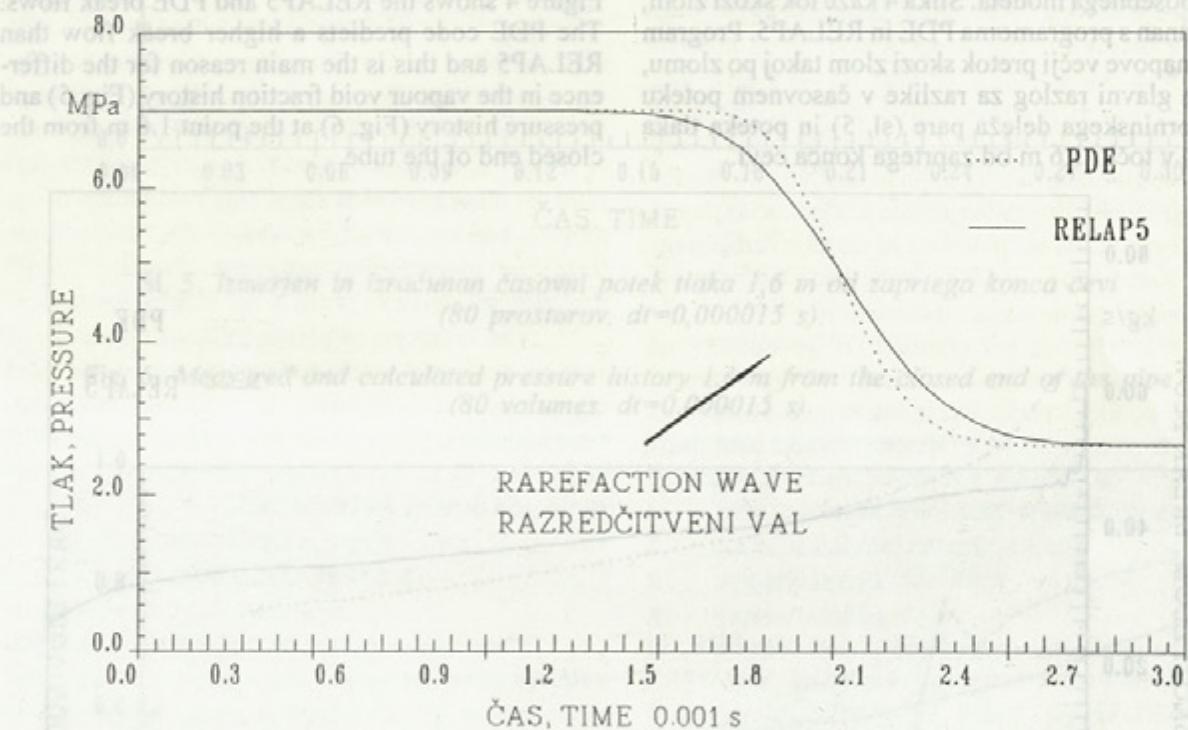
The scheme in the PDE code presents a basis for the second order accurate shock-capturing scheme for a two-fluid model of two-phase flow. Such a scheme will produce solutions with a very small amount of numerical diffusion, i.e. with sharp shocks and rarefaction waves.

4 RESULTS

Comparison of the experimental and calculated results shows the influence of the numerics and important physical parameters on the modelling of

cev". Podoben razredčitveni val je izračunan s programa PDE in RELAP5/MOD3.1 (sl. 3). Padca tlaka pod tlak nasičenja zaradi zamude pri uparjanju na tej sliki ne vidimo. Razlog zato je, da v nobenem od programov PDE in RELAP5 ni člena za popis zamude pri uparjanju, ko enofazni tok prehaja v dvofaznega. Rahel padec tlaka pod tlak nasičenja bi lahko pričakovali zaradi omejene hitrosti rasti mehurčkov v prvih trenutkih po njihovem nastanku. Ta mehanizem je v enačbah programa RELAP5 zanemarljiv zaradi oblike korelacij za rast mehurčkov: koeficient topotne prestopnosti s parne faze na medfazno ploskev je umetno povečan v področju zelo majhnih prostorninskih deležev pare in velike pregretosti parne faze. To umetno povečanje koeficiente topotne prestopnosti povzroči hiter prehod parne faze v stanje nasičenja in prepreči padec tlaka pod tlak nasičenja.

the Edwards pipe experiment. The PDE code and RELAP5 predicted the same pressure rarefaction wave (Fig. 3). The pressure undershoot of the Edwards Pipe experiment is missing on this figure. The main reason for that is the absence of the flashing delay in the single to two-phase transition model in RELAP5 and PDE. Some pressure undershoot could be expected, also as a consequence of the limited bubble growth rate in the first moments after the flashing onset. This mechanism does not work in RELAP5 due to the form of the correlations for the bubble growth: the heat transfer coefficient is artificially increased in the area of the small vapour void fraction and in large vapour superheating. This artificial increase of the vapour heat transfer coefficient forces the vapour towards the equilibrium and prevents the appearance of the pressure undershoot.



Sl. 3. Izračunan časovni potek tlaka v točki 1,6 m od zaprtega konca cevi v prvih 0,01 s (80 prostorov, $dt=0,000015$ s). Razredčitveni val programa RELAP5 je zaradi večje numerične difuzije pri isti mreži in časovnem koraku nekoliko širši kakor val PDE.

Fig. 3. Calculated pressure history 1.6 m from the closed end of the pipe in the first 0.01 s (80 volumes, $dt=0.000015$ s). The rarefaction wave predicted by RELAP5 is slightly wider than the PDE wave, due to the larger numerical diffusion at the same grid density and time step.

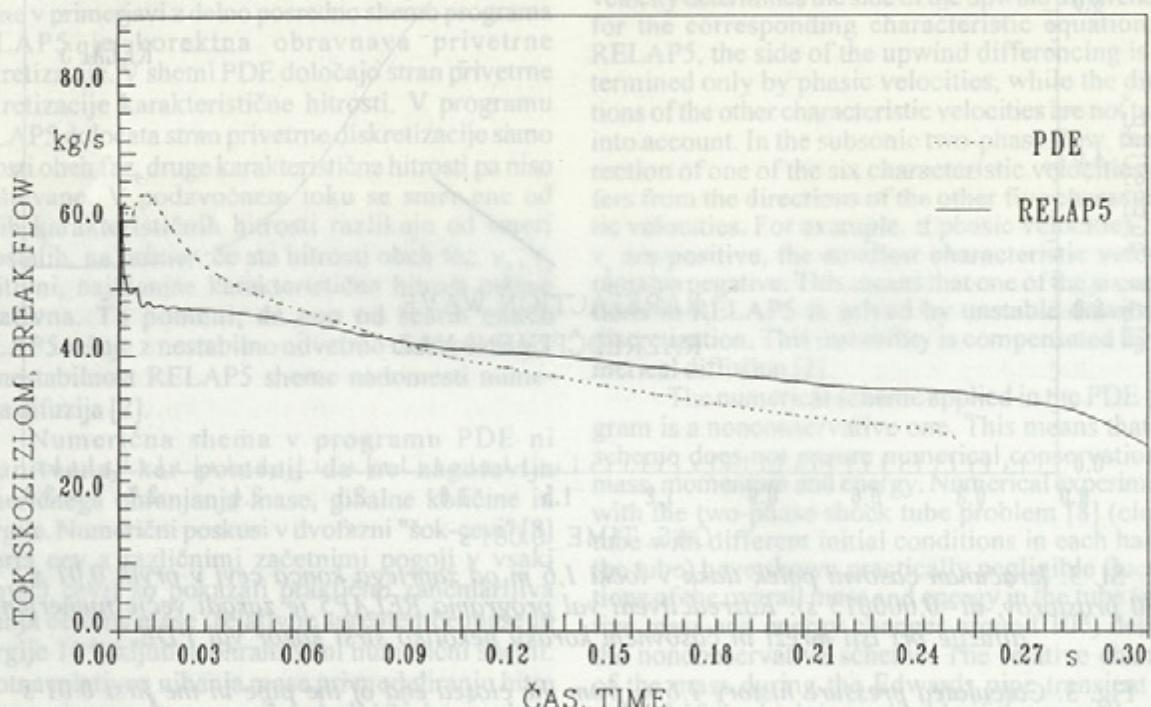
Padec tlaka pod tlak nasičenja, ki ga na videz lahko izračunamo s programom RELAP5, je numeričnega izvora in nastane kot posledica prevelikega časovnega koraka za integracijo izvorov. Če zmanjšamo časovni korak, izgine tudi padec tlaka pod tlak nasičenja [2].

Izbira koeficiente virtualne mase (12) ali (13) v programu PDE nima nobenega večjega vpliva na rezultate. Prednost korigiranega koeficiente (13) je, da se pri višjih prostorninskih deležih pare računanje ne ustavi zaradi kompleksnih lastnih vrednosti, ki se pojavijo z uporabo koeficiente (12).

Pressure undershoot - which can be achieved in some circumstances in the RELAP5 results - is a numerical artefact and is a consequence of the time step which is too large for the accurate integration of the sources. If the time step is decreased, pressure undershoot disappears [2].

Choice of the virtual mass coefficient (12) or (13) in the PDE code leaves the results almost unchanged. The significant advantage of the virtual mass coefficient (13) is that calculation is not interrupted at higher void fractions where equations with virtual mass coefficient (12) are ill-posed.

Razlike med rezultati programov PDE in RELAP5 izhajajo tudi iz različnega iztoka skozi odprt konec cevi. RELAP5 ne računa kritičnega toka iz osnovnih enačb (1) do (6) zaradi matematične nekorektnosti problema, ki pride na dan v razmerah kritičnega toka, in ker numerična shema tega ne zmore. Namesto tega uporablja poseben poenostavljeni model, ki je izpeljan iz poenostavljenih enačb dvofaznega toka in z nekaterimi dodatnimi empiričnimi popravki. Posebni model kritičnega toka se vključi, ko hitrost dvofazne mešanice doseže hitrost, ki je blizu lokalni zvočni hitrosti. Model na podlagi razlike tlakov pred zlomom in za njim napove masni tok skozi zlom. Medtem ko RELAP5 vključi v določenih razmerah ločen model kritičnega toka, zmore program PDE izračunati kritični tok neposredno iz osnovnih enačb brez posebnega modela. Slika 4 kaže tok skozi zlom, izračunan s programoma PDE in RELAP5. Program PDE napove večji pretok skozi zlom takoj po zlomu, kar je glavni razlog za razlike v časovnem poteku prostorninskega deleža pare (sl. 5) in poteka tlaka (sl. 6) v točki 1,6 m od zaprtega konca cevi.



Sl. 4. Pretok skozi zlom, izračunan s programoma PDE in RELAP5. V začetnem delu prehodnega pojava napove PDE večji pretok od programa RELAP5, v katerem se na začetku pretok hitro zveča na 70 kg/s in nato hitro zmanjša na približno 40 kg/s.

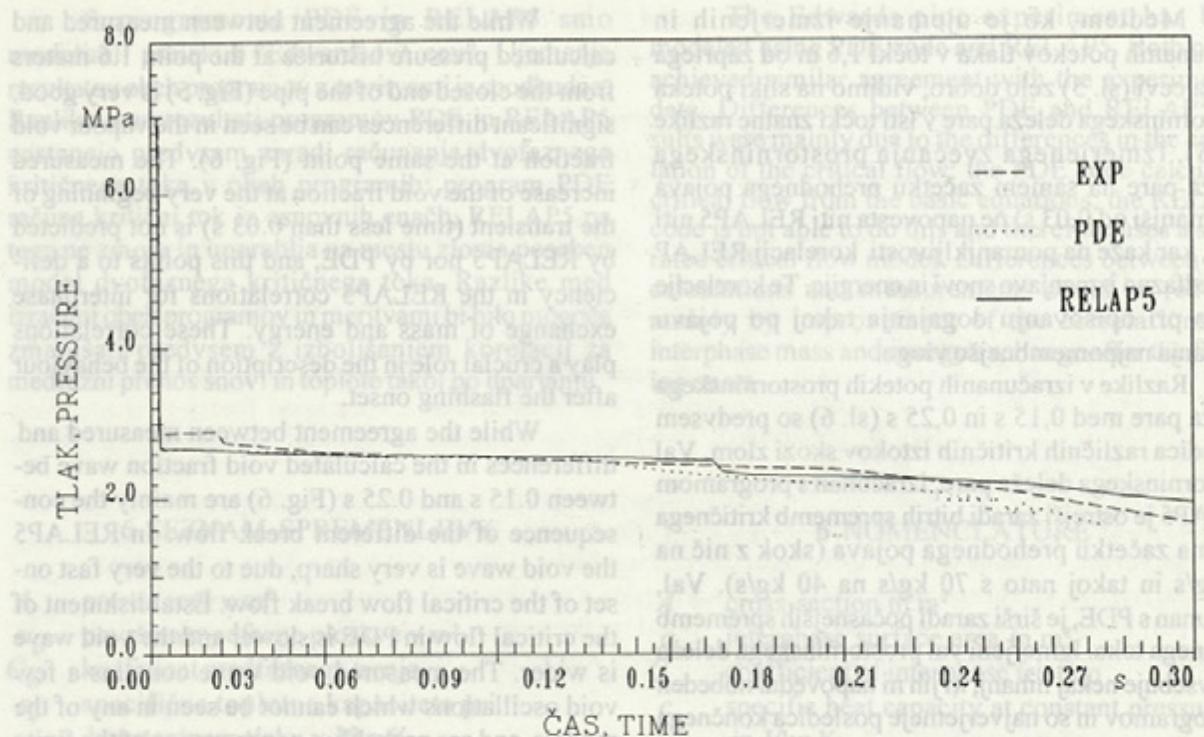
Fig. 4. Break flow predicted by PDE is higher than RELAP5 in the first part of the transient. In the RELAP5 results, the break flow rapidly increases upto 70 kg/s and then rapidly drops to approximately 47 kg/s.

Nekatere razlike izhajajo tudi iz omejitve korelacij za mehurčasti režim dvofaznega toka, ki so veljavne samo za prostorninske deleže pare, manjše od 0,5. Ker pa se najpomembnejši pojavi v preizkusu "Edwardsova cev" dogajajo pri nizkih prostorninskih deležih pare, to za program PDE ne pomeni resne omejitve.

Differences between RELAP5 and PDE results arise also from the different outflows through the rupture. RELAP5 does not calculate the critical flow from the basic equations due to the ill-posedness of the equations and the weakness of the numerical scheme in critical flow conditions. Instead, a simplified model is used which is derived from simplified mathematical models and empirical correlations. When certain phasic velocities are reached which are close to the local sonic velocity, the separated critical flow model is switched on; this predicts the flow for the given pressure difference upstream and downstream of the break. The critical flow model in RELAP5 is switched on when certain criteria are fulfilled, while the PDE code predicts the critical flow from the basic equations without any special model. Figure 4 shows the RELAP5 and PDE break flows. The PDE code predicts a higher break flow than RELAP5 and this is the main reason for the difference in the vapour void fraction history (Fig. 5) and pressure history (Fig. 6) at the point 1.6 m from the closed end of the tube.

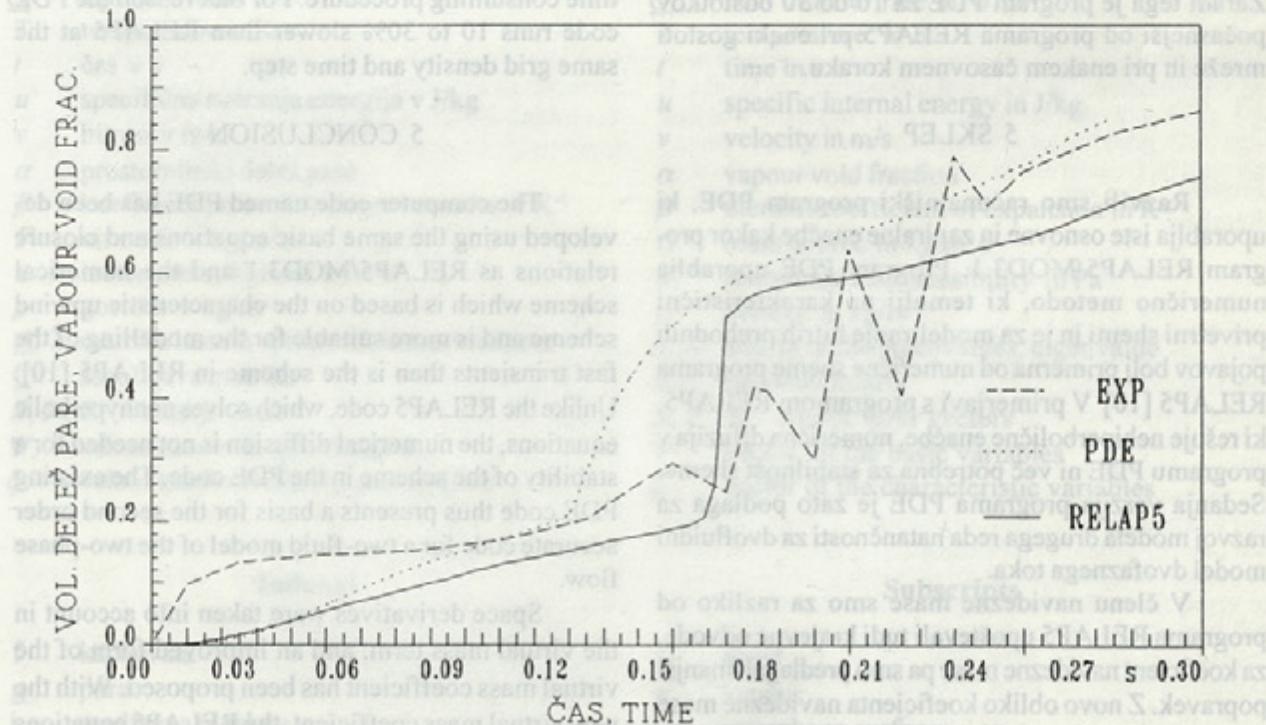
For the corresponding characteristics in RELAP5, the direction of the upstream flow is determined only by phasic velocities, while the directions of the other characteristic velocities are not taken into account. In the subsonic range, the direction of one of the six characteristic velocities differs from the directions of the other five characteristic velocities. For example, if v_x and v_y are positive, the smaller characteristic velocity v_z is negative. This means that the numerical scheme in the PDE program is a non-conservative one. This means that the scheme does not fully numerical conservation of mass momentum and energy. Numerical coefficients with the two initial values in the problem in (closed tube) are different initial conditions in each half of the tube. Thus, the two halfs numerically propagate different initial conditions. A more detailed analysis of the numerical scheme is given in the paper [1].

Some of the differences are also the consequence of the limitations of the PDE code with bubbly flow correlations (valid for vapour void fraction less than 0.5). Since the most important parts of the Edwards pipe transient happen at low void fraction this is not a severe limitation for the PDE code.



Sl. 5. Izmerjen in izračunan časovni potek tlaka 1,6 m od zaprtega konca cevi
(80 prostorov, $dt=0,000015$ s).

Fig. 5. Measured and calculated pressure history 1.6 m from the closed end of the pipe
(80 volumes, $dt=0.000015$ s).



Sl. 6. Izmerjen in izračunan potek prostorninskega deleža pare 1,6 m od zaprtega konca cevi
(80 prostorov, $dt=0,000015$ s).

Fig. 6. Measured and calculated vapour void fraction 1.6 m from the closed end of the pipe
(80 volumes, $dt=0.000015$ s).

Medtem, ko je ujemanje izmerjenih in izračunanih potekov tlaka v točki 1,6 m od zaprtega konca cevi (sl. 5) zelo dobro, vidimo na sliki poteke prostorninskega deleža pare v isti točki znatne razlike (sl. 6). Izmerjenega zvečanja prostorninskega deleža pare na samem začetku prehodnega pojava (čas manjši od 0,03 s) ne napovesta niti RELAP5 niti PDE, kar kaže na pomanjkljivosti korelacij RELAP za medfazno izmenjavo snovi in energije. Te korelacije imajo pri opisovanju dogajanja takoj po pojavi uparjanja najpomembnejšo vlogo.

Razlike v izračunanih potekih prostorninskega deleža pare med 0,15 s in 0,25 s (sl. 6) so predvsem posledica različnih kritičnih iztokov skozi zlom. Val prostorninskega deleža pare, izračunan s programom RELAP5 je ostrejši zaradi hitrih sprememb kritičnega toka na začetku prehodnega pojava (skok z nič na 70 kg/s in takoj nato s 70 kg/s na 40 kg/s). Val, izračunan s PDE, je širši zaradi počasnejših sprememb kritičnega toka. Izmerjeni val prostorninskega deleža pare vsebuje nekaj nihanj, ki jih ni napovedal nobeden od programov in so najverjetneje posledica končnega časa, ki je ob času nič potreben za razbitje razpočnega diska. V modelih RELAP5 in PDE smo predpostavili, da je čas odstranjevanja diska enak nič.

Šibka točka programa PDE je numerično računanje lastnih vrednosti in lastnih vektorjev matrike 6×6 v vsaki točki mreže, kar je časovno zelo zahtevno. Zaradi tega je program PDE za 10 do 30 odstotkov počasnejši od programa RELAP5 pri enaki gostoti mreže in pri enakem časovnem koraku.

5 SKLEP

Razvili smo računalniški program PDE, ki uporablja iste osnovne in zapiralne enačbe kakor program RELAP5/MOD3.1. Program PDE uporablja numerično metodo, ki temelji na karakteristični privetnici shemi in je za modeliranje hitrih prehodnih pojavov bolj primerna od numerične sheme programa RELAP5 [10]. V primerjavi s programom RELAP5, ki rešuje nehiperbolične enačbe, numerična difuzija v programu PDE ni več potrebna za stabilnost sheme. Sedanja verzija programa PDE je zato podlaga za razvoj modela drugega reda natančnosti za dvofluidni model dvofaznega toka.

V členu navidezne mase smo za razliko od programa RELAP5 upoštevali tudi krajevne odvode, za koeficient navidezne mase pa smo predlagali manjši popravek. Z novo obliko koeficiente navidezne mase so enačbe programa RELAP5 skoraj hiperbolične in predstavljajo korektno postavljen matematični problem. Kompleksne lastne vrednosti se lahko pojavijo samo pri zelo velikih relativnih medfaznih hitrostih, ki so primerljive z zvočno hitrostjo v dvofaznem toku.

While the agreement between measured and calculated pressure histories at the point 1.6 meters from the closed end of the pipe (Fig. 5) is very good, significant differences can be seen in the vapour void fraction at the same point (Fig. 6). The measured increase of the void fraction at the very beginning of the transient (time less than 0.03 s) is not predicted by RELAP5 nor by PDE, and this points to a deficiency in the RELAP5 correlations for interphase exchange of mass and energy. These correlations play a crucial role in the description of the behaviour after the flashing onset.

While the agreement between measured and differences in the calculated void fraction wave between 0.15 s and 0.25 s (Fig. 6) are mainly the consequence of the different break flow. In RELAP5 the void wave is very sharp, due to the very fast onset of the critical flow break flow. Establishment of the critical flow in PDE is slower and the void wave is wider. The measured void wave contains a few void oscillations which cannot be seen in any of the models, and are probably a consequence of the finite time interval needed to rupture the tube. In RELAP5 and PDE this time interval is assumed to be zero.

The weak point of the PDE program is numerical evaluation of the 6×6 matrix eigenvalues and eigenvectors at each point of the grid, which is a very time consuming procedure. For that reason the PDE code runs 10 to 30% slower than RELAP5 at the same grid density and time step.

5 CONCLUSION

The computer code named PDE has been developed using the same basic equations and closure relations as RELAP5/MOD3.1 and the numerical scheme which is based on the characteristic upwind scheme and is more suitable for the modelling of the fast transients than is the scheme in RELAP5 [10]. Unlike the RELAP5 code, which solves nonhyperbolic equations, the numerical diffusion is not needed for a stability of the scheme in the PDE code. The existing PDE code thus presents a basis for the second order accurate code for a two-fluid model of the two-phase flow.

Space derivatives were taken into account in the virtual mass term, and an improved form of the virtual mass coefficient has been proposed. With the new virtual mass coefficient, the RELAP5 equations present a well-posed problem in almost all situations. Complex eigenvalues can appear at very high interphase relative velocities which are comparable to the two-phase sound speed.

S programoma PDE in RELAP5 smo modelirali preizkus "Edwardsova cev". Ujemanje rezultatov obeh programov z meritvami je spodbudno. Razlike med rezultati programov PDE in RELAP5 nastanejo predvsem zaradi računanja dvofaznega kritičnega toka v obeh programih: program PDE računa kritični tok iz osnovnih enačb, RELAP5 pa tega ne zmore in uporablja na mestu zloma poseben model dvofaznega kritičnega toka. Razlike med izračuni obeh programov in meritvami bi bilo mogoče zmanjšati predvsem z izboljšanjem korelacij za medfazni prenos snovi in toplotne takoj po uparjanju.

The Edwards pipe experiment has been modeled using PDE code and RELAP5. Both codes achieved similar agreement with the experimental data. Differences between PDE and RELAP5 results were mainly due to the differences in the calculation of the critical flow: the PDE code calculates critical flow from the basic equations; the RELAP5 code is not able to do this and therefore uses a separated critical flow model. Differences between both calculations and measurements could be reduced mainly by improvement of the correlations for interphase mass and energy exchange after the flashing onset.

6 SEZNAM SPREMENLJIVK

A	prerez cevi v m^2
a_{sf}	površina medfazne ploskve v m^2
C_D	koeficient medfaznega trenja
c_p	specifična toplotna kapaciteta pri konstantnem tlaku v $J/kg K$
C_{vm}	koeficient navidezne mase
D	premer cevi v m
f_w	koeficient zidnega trenja
h	specifična entalpija v J/kg
p	tlak v N/m^2
Q_i	toplota v W/m^3
T	temperatura v K
t	čas v s
u	specifična notranja energija v J/kg
v	hitrost v m/s
α	prostorninski delež pare
β	koeficient prostorninskega raztezka v K^{-1}
Γ	vir mase v $kg/m^3 s$
κ	izotermna stisljivost v Pa^{-1}
ρ	gostota v kg/m^3
Δ, λ	matrika lastnih vrednosti, lastna vrednost
\mathcal{C}	Jacobijeva matrika
S, P, R	vektorji virov
Ψ	vektor osnovnih spremenljivk
ξ	vektor karakterističnih spremenljivk

Indeksi

f	kapljevina
g	para
i	medfazna ploskev
j	krajevni indeks
m	zmes
n	časovni indeks
r	označba relativne medfazne hitrosti

6 NOMENCLATURE

A	cross-section in m^2
a_{sf}	interphase surface area in m^2
C_D	coefficient of interphase friction
c_p	specific heat capacity at constant pressure in $J/kg K$
C_{vm}	virtual mass coefficient
D	tube diameter in m
f_w	wall drag coefficient
h	specific enthalpy in J/kg
p	pressure in N/m^2
Q_i	heat transfer rate in W/m^3
T	temperature in K
t	time in s
u	specific internal energy in J/kg
v	velocity in m/s
α	vapour void fraction
β	thermal coefficient of expansion in K^{-1}
G	mass source in $kg/m^3 s$
κ	isothermal compressibility in Pa^{-1}
ρ	density in kg/m^3
Δ, λ	matrix of the eigenvalues, eigenvalue
\mathcal{C}	Jacobian matrix
S, P, R	source term vectors
Ψ	vector of the basic variables
ξ	vector of the characteristic variables

Subscripts

f	liquid
g	vapour
i	interphase surface
j	local index
m	mixture
n	time index
r	description of the relative velocity

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veloped using the same basic numerical procedures as RELAP5-4D and uses a numerical scheme which is based on the implicit Euler and wind scheme with implicit finite difference for fast transients than is the scheme in RELAP5-1D. Unlike the RELAP5 code, which has implicitly coupled equations, the养 of the code is done separately for stability of the numerical scheme. Thus, using PDP code thus presents a basis for the second order accurate code for a two-fluid model of the two-phase