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Analiza razredčitvenega vala v uparjalniku med nezgodo z izgubo hladiva**Analysis of the Rarefaction Wave in a Steam Generator During Loss of Coolant Accident**

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Razredčitveni valovi, ki jih povzroči hiter padec tlaka, lahko precej obremenijo cevovode in konstrukcije. Val v cevih uparjalnika jedrske elektrarne med veliko izlivno nezgodo smo modelirali:

- s programom RELAP5/MOD3 – to je enorazsežni šestenačni program za simuliranje termohidrauličnih pojavov dvofaznega toka,
- s homogenim ravnotežnim modelom HEM (HEM) s tremi parcialnimi diferencialnimi enačbami in z numerično shemo visoke ločljivosti, ki omogoča modeliranje strmih gradientov.

Za modeliranje pojava z lastnim programom smo uporabili model HEM, ker je izračun s programom RELAP5 napovedal zanemarljive neravnotežne pojave. Precejšnjo numerično difuzijo, ki jo je povzročila privetra diskretizacija konvektivnih členov v programu RELAP5, smo v lastnem programu uspešno zmanjšali z metodo za modeliranje strmih gradientov. Se stavili smo Roejevo matriko za enačbe modela HEM ter uporabili Roejevo metodo za približno reševanje Riemannovih problemov in Van-Leerov omejitveni faktor toka.

Natančnost izračunov programa RELAP5 in programa z modelom HEM ter z metodo za modeliranje strmih gradientov je mogoče izboljšati, če uporabimo Alamgir in Lienhardovo enačbo za opis padca tlaka pod nasičenje zaradi zakasnitve uparjanja.

Rarefaction waves caused by rapid depressurization can lead to significant loads on the piping and structures. The rarefaction wave in the nuclear power plant steam generator tubes during an accident involving a large loss of coolant has been modelled using:

- computer code RELAP5/MOD3 – one dimensional code for the simulation of the thermohydraulic phenomena in two phase flow with 6-equation two-fluid model,
- a 3-equation homogeneous equilibrium model (HEM) with high resolution shock-capturing numerical scheme.

The phenomenon has been modelled using HEM due to negligible nonequilibrium effects predicted by RELAP5 6-equation model. Significant numerical diffusion caused by RELAP5 first order upwind discretization of the convective terms has been successfully reduced by the high resolution shock-capturing method. The Roe matrix for the HEM has been constructed, and Roe approximate Riemann solver and Van-Leer flux-limiter were applied.

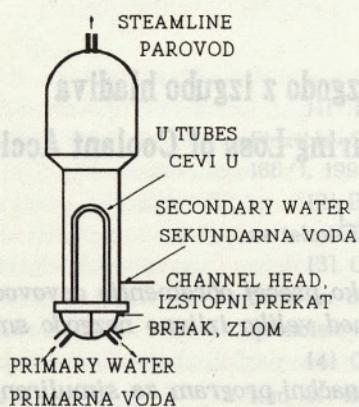
The accuracy of the RELAP5 and high resolution method results can be improved using the correlation of Alamgir & Lienhard for pressure undershoot due to the delayed flashing inception.

1 OPIS PREHODNEGA POJAVA

Razredčitveni val, ki bi se pojavi po 100-odstotnem zlomu hladne veje primarnega sistema tik ob uparjalniku, bi povzročil največjo obremenitev cevi U. Tlak v izstopni komori uparjalnika takoj po zlomu pade na tlak nasičenja, približno 6,7 MPa (sl. 1). Na mestu zloma se pojavi dvofazni kritični tok in hitro padanje tlaka se ustavi. Razredčitveni val potuje skozi cevi U s hitrostjo, ki je blizu zvočne hitrosti kapljevine (približno 1000 m/s) in po 0,016 s doseže vstopno komoro uparjalnika po najkrajši cevi U, po 0,02 s pa po najdaljši cevi. Zaradi različno dolgih časov potovanja valov se tlak v najdaljših cevih zmanjšuje tudi na

1 THE TRANSIENT DESCRIPTION

The rarefaction wave which would follow a 100% break on the steam generator primary side outlet nozzle would cause maximum loads on the U-tubes. Immediately after the break the pressure in the outlet channel head (Fig. 1) drops to a saturation pressure of approxim. 6.7 MPa. The critical flow appears on the break preventing further decreasing of the pressure. The rarefaction wave travels through the U-tubes at a speed close to the liquid sonic velocity (approx. 1000 m/s) and reaches the SG primary inlet channel head in 0.016 seconds through the shortest U-tubes and in 0.02 seconds through the longest U-tube. Due to the difference in travelling times, depressurization in the longest tubes would also



Notranji premer cevi U	Sekundarni tlak
U-tube inner diameter 16.87 mm	Secondary pressure 6.3 MPa
Dolžina cevi U	Število cevi U
U-tube length 14.4-18.5 m	Number of U-tubes 4575
Moč uporjalnika	Radij polkrožnega loka
SG power 940MW	Radius of curved section 57.2-1352.5 mm
Primarni tlak	Temperatura vstopajoče primarne vode
Primary pressure 15.5 MPa	Primary inlet T 598.1 K
Primarni pretok primarne vode	Temperatura izstopajoče primarne vode
Primary flow 4500 kg/s	Primary outlet T 561.6 K

Sl. 1. Poenostavljena shema Westinghousovega uporjalnika s cevmi U in najpomembnejši tehnični podatki

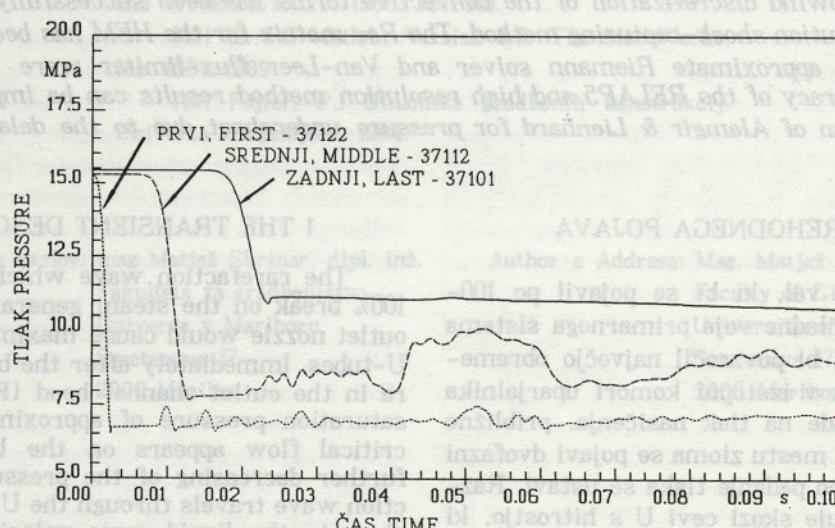
Fig. 1. Simplified scheme of the Westinghouse U-tube steam generator and main technical data

strani vstopne komore. Za natančen opis takega pojava bi morali modelirati veliko število cevi U z različnimi polmeri, za vstopno komoro pa bi morali uporabiti večrazsežni model. Ta pojav smo zanemarili, saj prispeva le k povečanju obremenitve ravneg dela cevi U, ta pa je razmeroma majhna v primerjavi s silami, ki se pojavit na polkrožnem delu cevi U.

Ko razredčitveni val pride skozi cevi U, se vzpostavi navidezno stabilno stanje s precejšnjim tlačno razliko (približno 4 MPa) med vstopno in izstopno komoro zaradi različnih tlakov nasičenja. Na sliki 2 je potek tlaka, izračunan s programom RELAP5 za prvo, zadnjo in srednjo prostornino najdaljše cevi U slike 4.

occur from the inlet channel head side too. Description of such an effect would require modelling of a large number of the U-tubes with different radius, and a multidimensional model of the inlet channel head. This effect was neglected because it would have contributed only to the forces on the straight part of the U-tubes where loads are small compared with the forces on the curved parts of the U-tubes.

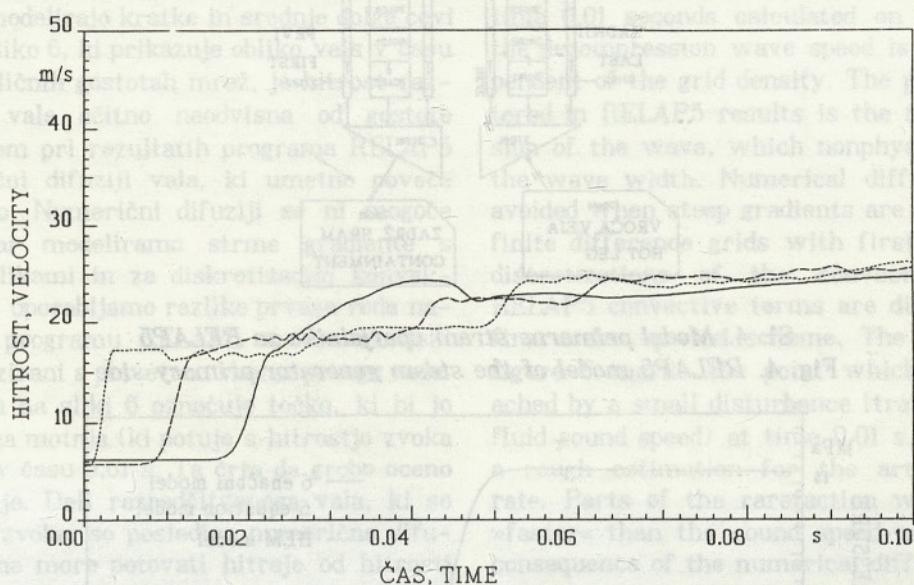
After the decompression wave has passed through the U-tubes a quasi steady-state is established with significant pressure difference (approx. 4 MPa) between inlet and outlet channel heads due to different saturation pressures. Figure 2 shows pressure histories predicted by RELAP5 in the first, last and middle volume of the longest U-tube from figure 4.



Sl. 2. Potek tlaka, izračunan s programom RELAP5 za prvo, zadnjo in srednjo prostornino najdaljše cevi U (izbira vozlišč na sliki 4)

Fig. 2. Pressure histories predicted by RELAP5 in the first, last and the middle volume of the longest U-tube (nodalization in Fig. 4)

Na sliki 3 je hitrost kapljivine v prvi, zadnji in srednji prostornini najdaljše cevi U. Razredčitveni val povzroči skok vseh fizikalnih parametrov: padec tlaka, povečanje hitrosti z začetnih 6 m/s na okoli 18 m/s in uparjanje – spremembo prostorninskega deleža pare z nič na $10^{-3} \text{ m}^3/\text{m}^3$. Delež pare za valom je zelo majhen. Obnašanje tlačnega vala opisujejo enačbe enofaznega toka, za valom pa je tok dvofazen.



Sl. 3. Hitrost kapljivine, izračunana s programom RELAP5 za prvo, zadnjo in srednjo prostornino najdaljše cevi U

Fig. 3. Fluid velocities in the first, last and middle volume of the longest U-tube predicted by RELAP5

2 REZULTATI IZRAČUNOV S PROGRAMOM RELAP5

Vhodni model za RELAP5 je standardni model jedrske elektrarne Krško [1], ki smo ga še poenostavili, ker se med potovanjem razredčitvenega vala skozi cevi U razmere v drugih delih primarnega in sekundarnega sistema ne spremenjajo in zato ne vplivajo na izračunane vrednosti. Poenostavljeni model primarne strani uparjalnika je prikazan na sliki 4. Robne pogoje za poenostavljeni model smo vzeli iz izračuna z obsežnim osnovnim modelom JE Krško. Rezultati na slikah 2 in 3 so bili dobljeni z modelom po sliki 4.

Ta novi model smo še poenostavili z zanemaritvijo prenosa toplotne na sekundarno stran, trenja ob stene cevi in težnosti. Končni model, ki smo ga uporabili za parametrične študije, je vodoravna cev z izmerami najdaljše cevi U, začetni in robni pogoji pa so vzeti iz celotnega modela JE. Podrobnosti o modelih in samem modeliranju s programom RELAP5 so opisane v [2].

Rezultati preračuna s programom RELAP5 na sliki 5 kažejo, da je mogoče narediti pomembno poenostavitev: zapleteni model dvofaznega toka, ki je v programu RELAP5 opisan s šestimi enačbami,

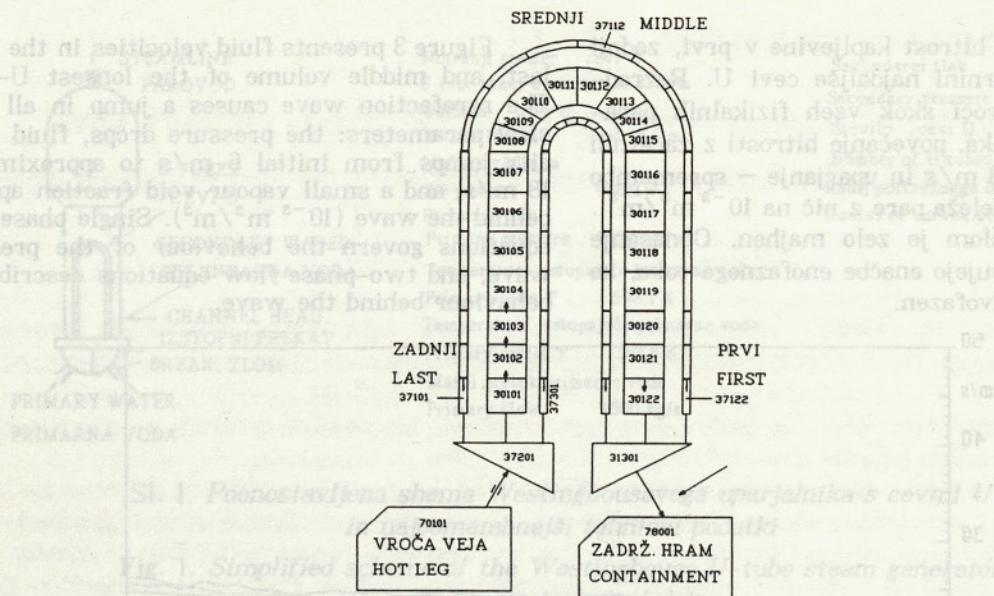
Figure 3 presents fluid velocities in the first, last, and middle volume of the longest U-tube. The rarefaction wave causes a jump in all physical parameters: the pressure drops, fluid velocity jumps from initial 6 m/s to approximately 18 m/s, and a small vapour void fraction appears behind the wave ($10^{-3} \text{ m}^3/\text{m}^3$). Single phase flow equations govern the behaviour of the pressure wave, and two-phase flow equations describe the behaviour behind the wave.

2 RELAP5 CODE RESULTS

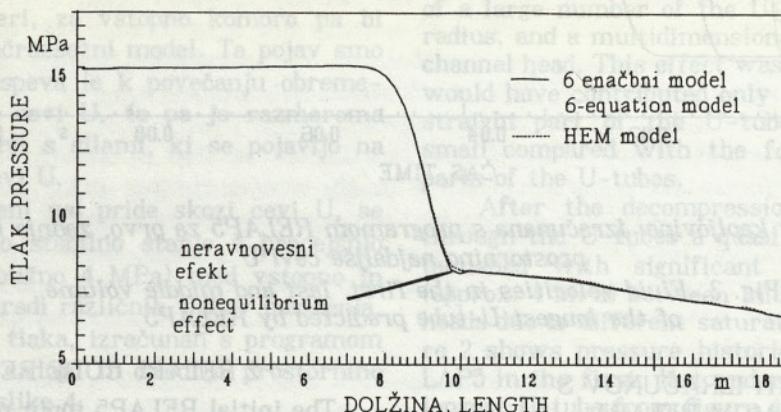
The initial RELAP5 input model was a standard input model of the Krško nuclear power plant [1]. This model was simplified due to the fact that during the travelling of the rarefaction wave through the U-tubes, the other plant systems – the whole secondary and primary system, except the affected steam generator primary side – remained undisturbed and were neglected without any effect on the calculated results. A simplified model of the steam generator primary side is shown in fig. 4. Boundary conditions for the simplified model were taken from the base model of the Krško NPP. The results presented in figs. 2 and 3 were calculated using the model from fig. 4.

This new model was further simplified neglecting heat transfer to the secondary side, wall friction, and gravitation. The final model, which has been used for the sensitivity studies, is a single horizontal tube with the dimensions of the longest U-tube with initial and boundary conditions taken from the complete model. Further details about the RELAP5 modelling are presented in [2].

The RELAP5 results presented in fig. 5 show that an important simplification is possible: the complicated 6-equation two-fluid model can be replaced by a simple homogeneous equilibrium



Sl. 4. Model primarne strani uparjalnika za RELAP5
Fig. 4. RELAP5 model of the steam generator primary side



Sl. 5. Tlak v cevi U 0,01 s po zlomu; primerjava rezultatov modela s šestimi enačbami iz programa RELAP5 in modela HRM

Fig. 5. Pressure profile in U-tube 0.01 s after the break; comparison of the RELAP5 6-equation model and RELAP5 HEM model

se da nadomestiti s preprostim homogenim ravnotežnim modelom (HRM) dvofaznega toka. Edini neravnotežni pojav nastane v zavetru, kjer tlak za kratek čas pada pod tlak nasičenja. Tega zmanjšanja tlaka RELAP5 ne napove pravilno. Do zmanjšanja tlaka pod stanje nasičenja pride zaradi zakasnjenega uparjanja. Ta pojav pregrete kapljevine ni vključen v enačbah programa RELAP5. Uparjanje se v programu RELAP5 zgodi takoj, ko tlak doseže stanje nasičenja. Padec tlaka pod tlak nasičenja, ki ga napove RELAP5 (sl. 5), je premajhen in nastane zaradi omejene rasti mehurčkov: mehurčki, ki nastajajo med uparjanjem, zaradi omejenega prenosa toplote in snovi med fazama ne rastejo dovolj hitro, da bi zmes kapljevine in pare obdržali v ravnotežju. Bolj natančno modeliranje pravilnega padca tlaka je opisano na koncu tega prispevka.

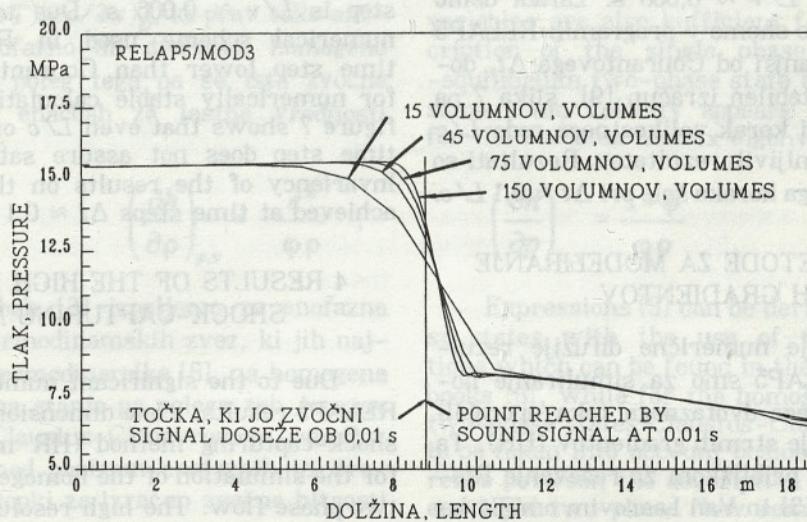
model (HEM) of the two phase flow. The only nonequilibrium effect appears behind the wave, where pressure drops under the saturation pressure for a short time. This pressure undershoot is not predicted correctly by RELAP5. The main mechanism of the pressure undershoot below the saturation pressure is delayed flashing inception which is not included in RELAP5 equations. The flashing onset predicted by RELAP5 equations appears as soon as the pressure reaches the saturation pressure. The pressure undershoot predicted by RELAP5 shown in figure 5 is too small and is a consequence of the limited bubble growth rate; bubbles created at the time of the flashing onset cannot grow fast enough to keep the fluid in equilibrium due to the limited heat and mass transfer between phases. An approach to the modelling of the correct pressure undershoot is described at the end of this paper.

Enačbi 3 NUMERIČNA DIFUZIJA V PROGRAMU RELAP5

Rezultate programa RELAP5 smo preverili s parametrično študijo občutljivosti na gostoto mreže in velikost časovnega koraka. Pri teh preračunih smo odstranili prostornino 31301 (komora na dnu uparjalnika – slika 4), da smo izključili motnje, ki jih povzroča. Odstranili smo tudi prostornine, ki modelirajo kratke in srednje dolge cevi U. Glede na sliko 6, ki prikazuje obliko vala v času 0,01 s pri različnih gostotah mrež, je hitrost razredčitvenega vala očitno neodvisna od gostote mreže. Problem pri rezultatih programa RELAP5 je v numerični difuziji vala, ki umetno poveča njegovo širino. Numerični difuziji se ni mogoče izogniti, kadar modeliramo strme gradiante s končnimi razlikami in za diskretizacijo konvekcijskih členov uporabljam razlike prvega reda natančnosti. V programu RELAP5 so konvekcijski členi diskretizirani s privetrno shemo prvega reda. Navpična črta na sliki 6 označuje točko, ki bi jo dosegla majhna motnja (ki potuje s hitrostjo zvoka v kapljevinah) v času 0,01 s. Ta črta da grobo oceno stopnje difuzije. Deli razredčitvenega vala, ki so »hitrejši« od zvoka, so posledica numerične difuzije, saj val ne more potovati hitreje od hitrosti zvoka.

3 NUMERICAL DIFFUSION IN RELAP5 CODE

RELAP5 results were verified by sensitivity study on grid density and time step. In these calculations channel head volume 31301 from figure 4 has been removed in order to avoid the disturbances caused by it. Volumes representing short and middle U-tube have been also removed. According to figure 6, which shows the shape of the wave at time 0.01 seconds calculated on different grids, the decompression wave speed is obviously independent of the grid density. The problem encountered in RELAP5 results is the numerical diffusion of the wave, which nonphysically increases the wave width. Numerical diffusion cannot be avoided when steep gradients are modelled on the finite difference grids with first order accurate discretizations of the convective terms. In RELAP5 convective terms are discretized by the first order upwind scheme. The vertical line in figure 6 marks the point which would be reached by a small disturbance (travelling with the fluid sound speed) at time 0.01 s. This line gives a rough estimation for the artificial diffusion rate. Parts of the rarefaction waves which are »faster« than the sound speed are certainly the consequence of the numerical diffusion.

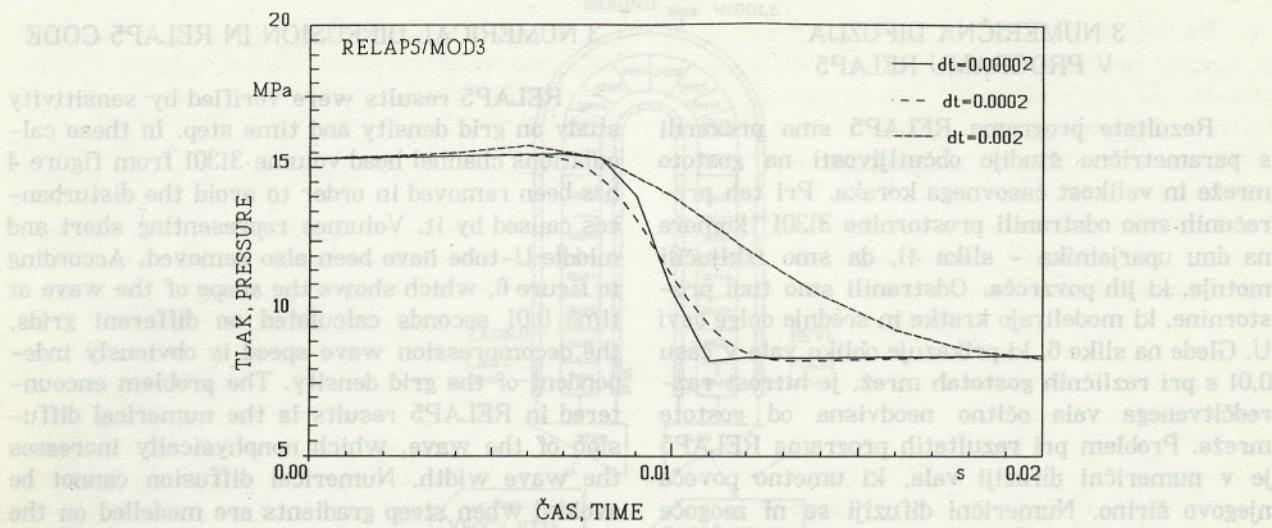


Sl. 6. Tlak v najdaljši cevi ob času 0,01 s, izračunan z različno gostimi mrežami

Fig. 6. Pressure in the longest U-tube at time 0.01 s, calculated with different grid densities

Slika 6 kaže stopnjo zmanjšanja difuzije s povečevanjem gostote mreže. Vidimo lahko, da je pri majhni gostoti mreže, ki se pogosto uporablja pri analizah sistema, širina vala predvsem posledica numerične difuzije. Slika 6 kaže tudi, da bi z nadaljnjjim drobljenjem mreže (več ko 150 prostornin) pridobili razmeroma malo, medtem ko bi se računalniški čas za preračun izredno podaljšal.

Figure 6 shows the rate of diffusion reduction with increasing grid density. One can see that at low grid density, which is often used for the system analyses, the wave width stems mainly from the numerical diffusivity. Figure 6 shows also that the further grid refinement (over 150 volumes) would mean a relatively small gain in accuracy for a significant increase in CPU time consumption.



Sl. 7. Potelek tlaka v srednji prostornini najdaljše cevi U, izračunan s programom RELAP5 z različnimi časovnimi koraki
Fig. 7. Pressure history in the middle volume of the longest U-tube, calculated by RELAP5 with different time steps

Opravili smo tudi preizkuse z različnimi časovnimi koraki. Slika 7 prikazuje zmanjševanje numerične difuzije ob zmanjševanju časovnega koraka. Razmerje med dolžino in zvočno hitrostjo je za izračune na sliki 7 $L/c \approx 0,0004$ s, Courantov časovni korak pa je $L/v \approx 0,006$ s. Zaradi delno implicitne numerične sheme v programih RELAP5 je časovni korak, manjši od Courantovega Δt , dovolj za numerično stabilen izračun [9], slika 7 pa kaže, da celo časovni korak velikostnega reda L/c ne zagotavlja sprejemljivih rezultatov. Rezultati so neodvisni od časovnega koraka šele pri $\Delta t \approx 0,1 L/c$.

4 REZULTATI METODE ZA MODELIRANJE STRMIH GRADIENTOV

Zaradi precejšnje numerične difuzije rezultatov programa RELAP5 smo za simuliranje homogenega ravnotežnega dvofaznega toka uporabili metodo za modeliranje strmih gradientov (HR). Ta postopek z Roejevim približkom za reševanje Riemannovih problemov [3] in Van Leerovim omejitvenim faktorjem toka je kombinacija Lax-Wendroffove diskretizacije natančnosti drugega reda in privetrne diskretizacije natančnosti prvega reda. Privetrni delež toka je odvisen od gladkosti rešitev; čim bolj so rešitve »zlomljene«, tem večji je privetrni del. S tem se izognemo umetnim nihanjem, ki se običajno pojavi, ko strme gradiante modeliramo s shemami drugega reda. Podrobni opis uporabljenih numeričnih metod za tok idealnega plina je v [3]. To metodo smo razširili na homogeni, toplotno ravnovesni model (HRM) dvofaznega toka – zgradili smo torej Roejevo matriko za enačbe modela HRM.

Tests were also made with the different time steps. Figure 7 shows the decrease of the numerical diffusion as the time step is decreasing. The ratio between the volume length and the sonic speed for the calculations presented on figure 7 is $L/c \approx 0,0004$ s, and the Courant time step is $L/v \approx 0,006$ s. Due to a semi-implicit numerical scheme used in RELAP5 codes, a time step lower than Courant Δt is sufficient for numerically stable calculations [9], however figure 7 shows that even L/c order of magnitude time step does not assure satisfactory results. Invariancy of the results on the time step was achieved at time steps $\Delta t \approx 0,1 L/c$.

4 RESULTS OF THE HIGH RESOLUTION SHOCK-CAPTURING METHOD

Due to the significant numerical diffusion in RELAP5 results, a one dimensional high resolution shock-capturing method (HR method) was used for the simulation of the homogeneous equilibrium two phase flow. The high resolution scheme with Roe approximate Riemann solver [3] and Van Leer flux-limiter is a combination of a second order accurate Lax-Wendroff discretization and first order accurate upwind discretization. The portion of the »upwinding« is a function of the »smoothness«; on nonsmooth solutions more upwinding is used to avoid nonphysical oscillations which usually appear when steep gradients are modeled with second order schemes. A detailed description of the applied numerical method for ideal gas flow can be found in [3]. This method has been extended for the HEM flow i.e. Roe matrix for the HEM has been constructed.

Enačbi HRM brez virov toplotne in trenja ob stene sta v skalarni in vektorski obliki:

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho v \\ \rho e \end{bmatrix} + \frac{\partial}{\partial x} \begin{bmatrix} \rho v \\ \rho v^2 + p \\ \rho vh \end{bmatrix} = 0, \quad \frac{\partial \mathbf{u}}{\partial t} + \frac{\partial f(\mathbf{u})}{\partial x} = 0 \quad (1),$$

kjer so: ρ , v , p gostota, hitrost in tlak, e vsota specifične notranje in kinetične energije in h vsota specifične entalpije in specifične kinetične energije.

Enačbo stanja smo zapisali v diferencialni obliki, ki velja za enofazni tok in homogeni ravnotežni model dvofaznega toka. V obeh primerih lahko h zapišemo kot enolično funkcijo gostote ρ , tlaka p in hitrosti v :

$$dh = \left(\frac{\partial h}{\partial \rho} \right)_{p,v} d\rho + \left(\frac{\partial h}{\partial p} \right)_{\rho,v} dp + \left(\frac{\partial h}{\partial v} \right)_{\rho,p} dv \quad (2).$$

Iz definicije h ($h = (\text{spec. entalpija}) + v^2/2$) sledi $(\partial h / \partial v)_{\rho,p} = v$. Preostala dva parcialna odvoda v enačbi (2) bi lahko neposredno aproksimirali z razlikami in ju izračunali z uporabo funkcij za izračun lastnosti vode. V predstavljenem modelu smo parcialna odvoda zapisali v odvisnosti od zvočne hitrosti $c^2 = (\partial p / \partial \rho)_s$ in parametra $\varphi = 1/(\rho T)(\partial p / \partial s)_p$, ki prav tako enolično opisuje enofazno ali dvofazno homogeno ravnotežno stanje, poleg tega pa se ista zvočna hitrost pojavlja v enačbah za lastne vrednosti Jacobijeve matrike:

$$\left(\frac{\partial h}{\partial \rho} \right)_{p,v} = -\frac{c^2}{\varphi \rho}, \quad \left(\frac{\partial h}{\partial p} \right)_{\rho,v} = \frac{1+\varphi}{\varphi \rho} \quad (3).$$

Izraze v enačbah (3) izpeljemo za enofazna stanja z uporabo termodinamskih zvez, ki jih najdemo v učbenikih termodinamike [8], za homogena ravnotežna dvofazna stanja pa polega teh izrazov upoštevamo še Clausius-Clapeyronovo enačbo. Osnovna razlika med popisom enofaznih in dvofaznih stanj so postopki za izračun zvočne hitrosti c in parametra φ .

Za reševanje enačb (1) z metodo HR je treba sistem enačb napisati v t.i. nekonzervativni [4] obliki:

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho v \\ \rho e \end{bmatrix} + \begin{bmatrix} 0 \\ v^2(\varphi-1)-\varphi h+c^2 \\ v^3\varphi-vh(1+\varphi)+vc^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ (2-\varphi)v & \varphi \\ h-\varphi v^2 & v(1+\varphi) \end{bmatrix} \frac{\partial}{\partial x} \begin{bmatrix} \rho \\ \rho v \\ \rho e \end{bmatrix} = 0, \quad (4).$$

HEM equations without heat sources and wall shear are in scalar and vectorial form:

$$\frac{\partial \mathbf{u}}{\partial t} + \frac{\partial f(\mathbf{u})}{\partial x} = 0 \quad (1),$$

with ρ , v , p as density, velocity and pressure, e as a sum of the specific internal and kinetic energy, and h as a sum of the specific enthalpy and the specific kinetic energy.

The equation of state has been written in differential form which is valid for single-phase and homogeneous equilibrium two-phase flow. In both cases h can be expressed as an unique function of density ρ , pressure p and velocity v :

$$dh = \left(\frac{\partial h}{\partial \rho} \right)_{p,v} d\rho + \left(\frac{\partial h}{\partial p} \right)_{\rho,v} dp + \left(\frac{\partial h}{\partial v} \right)_{\rho,p} dv \quad (2).$$

From the definition of h ($h = (\text{spec. enthalpy}) + v^2/2$) follows $(\partial h / \partial v)_{\rho,p} = v$. Other partial derivatives in equation (2) could be directly approximated with differences and calculated with the water properties subroutines. However, in the presented model both partial derivatives were written as functions of the sonic velocity $c^2 = (\partial p / \partial \rho)_s$ and parameter $\varphi = 1/(\rho T)(\partial p / \partial s)_p$. New variables are also sufficient for the unique description of the single phase or homogeneous-equilibrium two-phase state and besides that the same sonic velocity appears in the expressions for the Jacobian matrix eigenvalues:

$$\left(\frac{\partial h}{\partial p} \right)_{\rho,v} = \frac{1+\varphi}{\varphi \rho} \quad (3).$$

Expressions (3) can be derived for single-phase states with the use of thermodynamic relations which can be found in thermodynamics textbooks [8], while for the homogeneous equilibrium two-phase states Clausius-Clapeyron equation has to be taken into account besides that. Basic difference between the description of the single-phase and HEM two-phase flow remains in the expressions for the sonic velocity c and parameter φ .

Solving eqs. (1) with HR methods requires the system of equations to be written in the so called nonconservative [4] form:

$$\frac{\partial \mathbf{u}}{\partial t} + f'(\mathbf{u}) \frac{\partial \mathbf{u}}{\partial x} = 0 \quad (1),$$

Da lahko v tem sistemu uporabimo Roejev približek za reševanje Riemannovih problemov [3], je treba linearizirati Jacobijevu matriko $f'(u)$ med sosednjima točkama mreže i in $i+1$. Po Roeju mora za linearizirano matriko $A(u_i, u_{i+1})$ veljati naslednje:

– $A(u_i, u_{i+1})(u_{i+1} - u_i) = f(u_{i+1}) - f(u_i)$, (zagotavlja ohranitev snovi, gibalne količine in energije),

– $A(u_i, u_{i+1})$ je mogoče diagonalizirati in mora imeti realne lastne vrednosti,

– $A(u_i, u_{i+1}) \rightarrow f'(u_{ave})$, ko gresta $u_i, u_{i+1} \rightarrow u_{ave}$.

Matrika v enačbi (4) izpolnjuje te pogoje, če v, h, ρ nadomestimo z:

To apply the Roe approximate Riemann solver [3] for this system, the Jacobian matrix $f'(u)$ has to be linearized between neighbouring grid points i and $i+1$. According to Roe the following conditions should be imposed on the linearized matrix $A(u_i, u_{i+1})$:

– $A(u_i, u_{i+1})(u_{i+1} - u_i) = f(u_{i+1}) - f(u_i)$, (ensures the conservation of mass, momentum, energy),

– $A(u_i, u_{i+1})$ is diagonalizable with real eigenvalues,

– $A(u_i, u_{i+1}) \rightarrow f'(u_{ave})$, smoothly as $u_i, u_{i+1} \rightarrow u_{ave}$.

It was found that the matrix in eq. (4) satisfies these conditions if v, h, ρ are substituted by:

$$(3) \quad v_{ave} = \left(\sqrt{\rho_i} v_i + \sqrt{\rho_{i+1}} v_{i+1} \right) / \left(\sqrt{\rho_i} + \sqrt{\rho_{i+1}} \right) \quad (5)$$

$$h_{ave} = \left(\sqrt{\rho_i} h_i + \sqrt{\rho_{i+1}} h_{i+1} \right) / \left(\sqrt{\rho_i} + \sqrt{\rho_{i+1}} \right), \quad \rho_{ave} = \sqrt{\rho_{i+1} \rho_i}$$

in če enačbo stanja (2) lineariziramo med stanjema u_i in u_{i+1} :

$$(4) \quad h_{i+1} - h_i = -c_{ave}^2 / (\varphi_{ave} \rho_{ave}) (\rho_{i+1} - \rho_i) + (1 + \varphi_{ave}) / (\varphi_{ave} \rho_{ave}) (p_{i+1} - p_i) + v_{ave} (v_{i+1} - v_i) \quad (6).$$

Vrednosti c_{ave} in φ_{ave} sta funkciji stanja in ju izračunamo iz znanih ρ_{ave}, v_{ave} in h_{ave} . Vrednosti φ_{ave} nato popravimo tako, da je zadoščeno enačbi (6).

Linearizirani sistem enačb (4) nadalje spremeni v karakteristično obliko z izračunom lastnih vrednosti in lastnih vektorjev matrike $A(u_i, u_{i+1})$. Enočbe v karakteristični obliki diskretiziramo; določimo privetne tokove prvega reda in Lax-Wendroffove tokove drugega reda. Končni tok je kombinacija toka prvega reda in popravka drugega reda. Natančnost drugega reda je dosežena pri gladkih rešitvah, medtem ko se v bližini strmih gradientov, kjer rešitve niso gladke, uporabi večji delež privetrnega toka. Strmejši gradienti pomenijo manjši delež toka drugega reda. Popravke drugega reda določimo z Van Leerovimi omejitvenimi faktorji toka. Podrobnosti o numeričnih postopkih in omejitvenih faktorjih toka opisujeta [4] in [5].

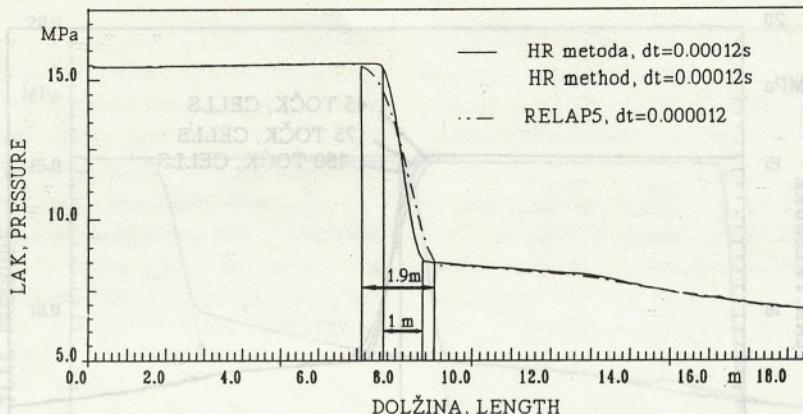
Slike 8 in 9 prikazujeta rezultate programa RELAP5 in metode HR. Pri slednji dosežemo pomembno izboljšavo rezultatov z vidika numerične difuzije. Razredčitveni val v času 0,0108 s je v programu RELAP5 skoraj dvakrat širši od vala, ki smo ga pri enaki mreži izračunali z metodo visoke ločljivosti.

and if the equation of state (2) is linearized between the states u_i and u_{i+1} :

Values of c_{ave} and φ_{ave} which are the functions of the state $\rho_{ave}, v_{ave}, h_{ave}$ are corrected with respect to the eq. (6).

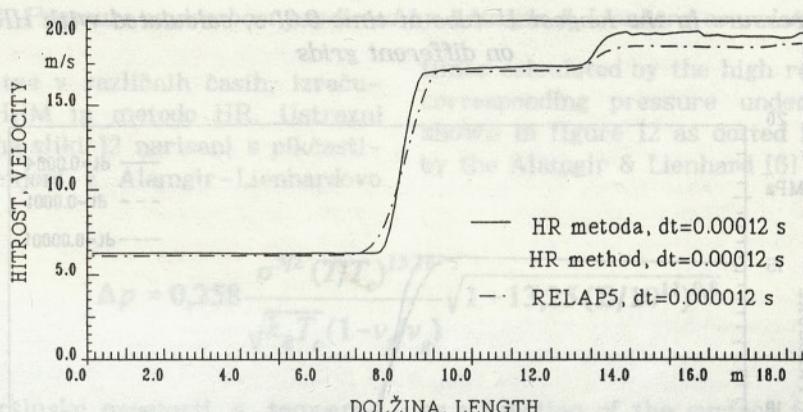
The linearized system of eqs. (4) is further transformed into the characteristic form by the evaluation of the eigenvalues and eigenvectors of the matrix $A(u_i, u_{i+1})$. Equations in characteristic form are discretized; first order upwind fluxes and second order Lax-Wendroff fluxes are determined. The final flux is the combination of the first order flux and second order correction. Second order accuracy is achieved on smooth solutions while more »upwinding« is used in the vicinity of the steep gradients. Nonsmooth solutions and steeper gradients mean a lower portion of the second order flux. Second order corrections are determined by Van Leer flux limiters. Details of the numerical method and flux limiters can be found in [4] or [5].

Figures 8 and 9 show results of the RELAP5 and high resolution method. Significant improvement of the results from the standpoint of the numerical diffusion is achieved with the high resolution model. The decompression wave at time 0.0108 s in RELAP5 results is almost twice as wide as the wave predicted by high resolution method on the same grid.



Sli. 8. Tlak v najdaljši cevi U v času 0,0108 s pri 150 celicah v mreži; primerjava metode HR in rezultatov programa RELAP5

Fig. 8. Pressure in the longest U-tube at time 0.0108 s, 150 grid cells; comparison of the HR method and RELAP5



Sli. 9. Hitrosti v najdaljši cevi U v času 0,0108 s; primerjava rezultatov metode HR in programa RELAP5

Fig. 9. Velocities in the longest U-tube at time 0.0108 s; comparison of the HR method and RELAP5

Na slikah 10 in 11 so rezultati parametrične študije vpliva gostote mreže in časovnega koraka za metodo HR. Slika 10 prikazuje, da je za majhno numerično difuzijo pri metodi HR potrebna čim gostejša mreža (podobno kakor za program RELAP5). Slika 11 pa kaže, da se najmanjša numerična difuzija pri metodi HR doseže pri časovnem koraku $\Delta t \approx L/c = 0,0004$ s, kjer se numerična difuzija zaradi prostorske diskretizacije kompenzira z numerično difuzijo zaradi časovnega koraka [5]. Izračuni z večjimi časovnimi koraki niso stabilni, izračuni s krajišimi časovnimi koraki, dva od njih sta na sliki 11, pa vnašajo večjo difuzijo.

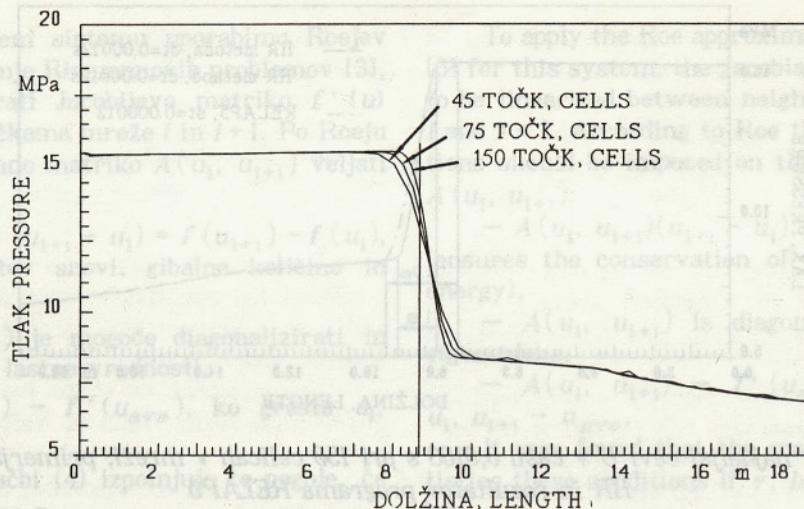
5 PADEC TLAKA POD STANJE NASIČENOSTI

Uparjanje pri hitri tlači razbremenitvi se po enačbah programa RELAP5 začne, ko pade tlak na tlak nasičenja. Zaradi tega s programom RELAP5

The results of the sensitivity study on grid density and time step are presented in figs. 10 and 11. Figure 10 shows that grid has to be as dense as possible as in RELAP5. Figure 11 shows that minimum of the numerical diffusion is reached at time step $\Delta t \approx L/c = 0,0004$ s, where numerical diffusion due to the spatial discretization is compensated by the numerical diffusion due to the time step [5]. Calculations with larger time steps are not stable, and calculations with lower time step are more diffusive.

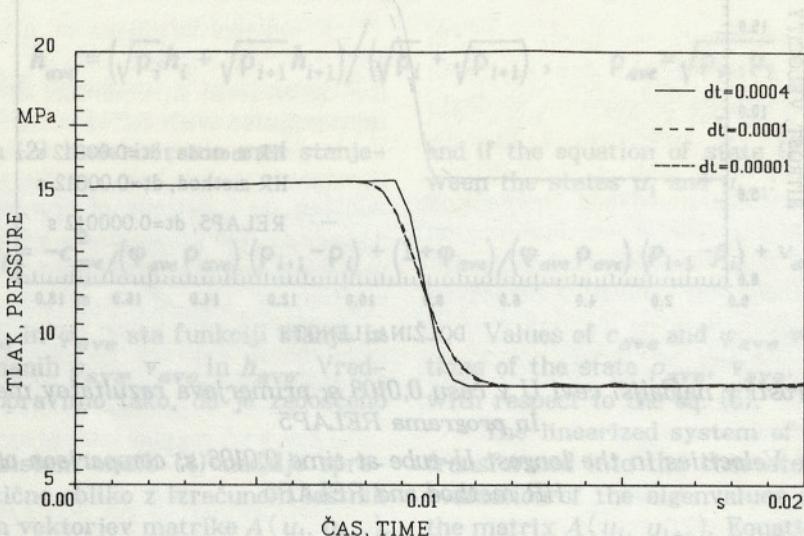
5 PRESSURE UNDERSHOOT BELOW THE SATURATION PRESSURE

Flashing onset at rapid depressurization in RELAP5 equations starts when the pressure drops to the saturation pressure. The consequence is that RELAP5 cannot be used for pressure undershoot calculation, and that RELAP5 and HEM



Sl. 10. Tlak v najdaljši cevi U ob času 0,01 s, izračunan z metodo HR na različnih mrežah

Fig. 10. Pressure in the longest U-tube at time 0.01 s, calculated with HR method on different grids

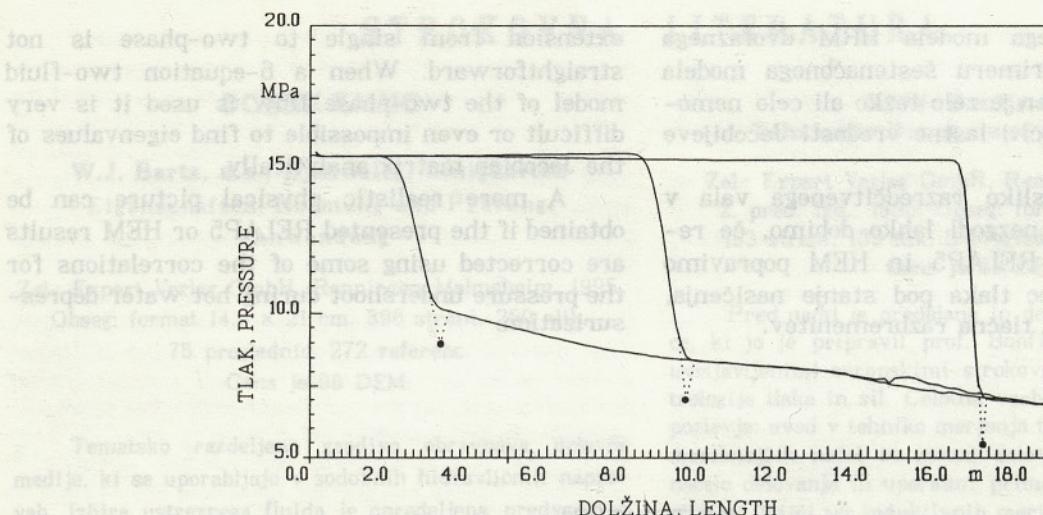


Sl. 11. Potelek tlaka v srednji prostornini najdaljše cevi U, izračunan z metodo HR z različnimi časovnimi koraki (75 točk)

Fig. 11. Pressure history in the middle volume of the longest U-tube calculated with HR method with different time steps (75 nodes)

ne moremo izračunati padca tlaka pod stanje nasičenja in so zato rezultati izračunov s programom RELAP5 zelo podobni rezultatom modela HRM. Ta pojav lahko empirično upoštevamo z Alamgir-Lienhardovo enačbo [6] ali z Bartakovo enačbo [7] za napoved padca tlaka pod stanje nasičenja kot funkcijo začetne temperature kapljivine in hitrosti tlačne razbremenitve. Začetna temperatura je znana spremenljivka, medtem ko je hitrost tlačne razbremenitve odvisna od numerične difuzije in strmine razredčitvenega vala. Bolj strm val pomeni hitrejšo tlačno razbremenitev in večji padec tlaka pod vrednost nasičenja. Na sliki 12 so

results are very similar. This effect can be empirically corrected using the Alamgir & Lienhard correlation [6] or Bartak [7] correlation for the prediction of the pressure undershoot as a function of the initial fluid temperature and the depressurization rate - pressure drop in unit of time. The initial temperature is a well known variable while the depressurization rate on the other side is directly affected by numerical diffusion and by the steepness of the depressurization wave. Steeper wave means higher depressurization rate and larger pressure undershoot. Figure 12 shows three pressure profiles at different



Sl. 12. Padec tlaka pod tlak nasičenja po Alamgir-Lienhardovi enačbi (7)

Fig. 12. Pressure undershoot predicted by Alamgir & Lienhard correlation (7)

tri tlačne porazdelitve v različnih časih, izračunane z modelom HRM in metodo HR. Ustrezni tlačni padci, ki so na sliki 12 narisani s pikčastimi črtami, so ocenjeni z Alamgir-Lienhardovo enačbo [6].

$$\Delta p = 0,258 \frac{\sigma^{3/2} (T/T_c)^{13,76}}{\sqrt{k_B T_c (1-v_f/v_g)}} \sqrt{1 + 13,25 (\Sigma/10^{11})^{0,8}} \quad (7)$$

in so funkcije površinske napetosti σ , temperaturre T , prostornine obeh faz v_p , v_g in hitrosti tlačne razbremenitve Σ (Pa/s). T_c je kritična temperatura vode, k_B pa Boltzmannova konstanta. Uporabili smo hitrost tlačne razbremenitve, dobljeno z modelom HRM na mreži s 150 točkami. Padec tlaka pod tlak nasičenja med potovanjem razredčitvenega vala skozi cevi U je 1 do 2 MPa.

6 SKLEP

Dobljeni rezultati kažejo, da RELAP5 lahko uporabimo za simuliranje tlačnih valov, če upoštevamo vplive numerične difuzije. Upoštevati je treba tudi dejstvo, da v programu RELAP5 ni modela zakasnitve uparjanja. Končni rezultati so zelo podobni rezultatom homogenega ravnotežnega medela (HRM).

Metode visoke ločljivosti so zelo uporabno orodje za modeliranje hitrih prehodnih pojavov s tlačnimi valovi, kakršni so skoki in razredčitve, vendar se večinoma uporabljajo pri enofaznem toku. Lahko jih uspešno uporabimo tudi pri dvo-faznem toku, pri čemer naletimo na številne težave. Prehod na dvofazni tok ni preprost niti v

times calculated by the high resolution HEM. The corresponding pressure undershoots which are shown in figure 12 as dotted lines are estimated by the Alamgir & Lienhard [6] correlation:

as a function of the surface tension σ , temperature T , specific volumes of both phases v_p , v_g and depressurization rate Σ (Pa/s). T_c is the critical temperature and k_B the Boltzmann constant. Depressurization rates predicted by high resolution HEM model on the grid of 150 cells were used. Pressure undershoots during the travelling of the decompression wave through the U-tubes are around 1 to 2 MPa.

6 CONCLUSION

The results presented show that RELAP5 code can be used for the simulations of the pressure waves if the effects of the numerical diffusion are taken into account. Absence of the flashing inception delay in the RELAP5 equations is another factor which has to be considered. The final results are very similar to the results of the homogeneous equilibrium model.

High resolution methods are a very suitable tool for modelling of the fast transients with pressure waves like shocks and rarefactions, but they are mainly used in single phase flow. Their application in two phase-flow can be very successful but is limited due to the fact that the

primeru trienačbnega modela HRM dvofaznega toka, kaj šele v primeru šestenačbnega modela dvofaznega toka, kjer je zelo težko ali celo nemogoče analitično določiti lastne vrednosti Jacobijeve matrike.

Bolj stvarno sliko razredčitvenega vala v ceveh U po izlivni nezgodi lahko dobimo, če rezultate programov RELAP5 in HEM popravimo z enačbami za padec tlaka pod stanje nasičenja, ki ga povzroči hitra tlačna razbremenitev.

extension from single to two-phase is not straightforward. When a 6-equation two-fluid model of the two-phase flow is used it is very difficult or even impossible to find eigenvalues of the Jacobian matrix analytically.

A more realistic physical picture can be obtained if the presented RELAP5 or HEM results are corrected using some of the correlations for the pressure undershoot during hot water depressurization.

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