

Posplošeni model gibanja zraka v predoru A Generalised Model of Air Movement in a Tunnel

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V članku je prikazan pospoljeni model gibanja zraka v predoru. Razvit je splošen matematični model, ki obsegata različne možnosti vhodnih parametrov. Model je prirejen za predore s tekočim enosmernim prometom osebnih, tovornih in "mešanih" vozil. Pri tem je upoštevan tudi medsebojni vpliv vozil na hidrodinamične razmere v predoru. Posplošena diferencialna enačba omogoča izračun končne hitrosti v predorih poljubnih oblik pri različnih prometnih razmerah. To je tudi osnova za izračun koncentracij škodljivih snovi in količine zraka, ki zagotavlja, da te ne presežejo s standardi predpisanih vrednosti.

Ključne besede: predori, prezračevanje, modeli matematični, hitrosti

In the paper a generalised model of air movement in a tunnel is shown. A general mathematical model was developed, taking various inlet parameters into account. The model is prepared for a single-bore tunnel with a unidirectional flow of traffic, including cars, trucks and "mixed" vehicles. The effect between vehicles in hydrodynamic conditions in the tunnel is also considered. The general governing differential equation enabled the calculation of air velocity at the end of the tunnels with various forms. On this basis, the concentrations of the most serious contaminants and the air requirement for their diluting may be calculated, so that pollution values should not exceed the acceptable level prescribed by the standards.

Keywords: tunnels, ventilating, mathematical model, air velocity

0 UVOD

Predor lahko ponazorimo s cevjo, skozi katero se pomika skupina togih teles. Gibanje zraka je odvisno od različnih dejavnikov:

- tlačne, višinske in temperaturne razlike med vstopnim in izstopnim portalom,
- gibanja vozil,
- delovanja ventilatorjev.

Pri izdelavi modela smo upoštevali nekaj (pred)postavk. Vsi avtomobilski motorji so ogreti na delovno temperaturo, tako katalizatorji lahko opravljajo svojo funkcijo. Na koncu predora je morebitna hitrost vetra, usmerjenega proti izstopu vozila iz predora, manjša od hitrosti iz predora izstopajočega zraka. Zato jo upoštevamo kot povišanje tlaka. Upoštevan je povprečni koeficient zračnega upora in povprečna celna površina vozil. Prav tako sta upoštevana povečanje koeficiente zračnega upora v zaprtem prostoru ter njegovo zmanjšanje zaradi vožnje v koloni (v [3], dopolnjeno v [1]).

V predorih imamo, realno gledano, trirazsežne tokove, njihovo reševanje terja prostorsko analizo. Ker je pretakanje stisljivega fluida definirano z nelinearnimi parcialnimi diferencialnimi enačbami, težko dobimo natančno analitično rešitev za poljubne robne pogoje, razen v posebnih primerih, v katerih se enačbe močno poenostavijo. Zato velikokrat uporabljamo aproksimacijske metode, kar je bilo deloma raziskano v [5].

0 INTRODUCTION

A tunnel may be illustrated by a pipe and a group of firmly constructed vehicles, moving through it. The air movement is caused by:

- pressure, height and temperature or temperature differences between the inlet and outlet cross-sections,
- movement of vehicles,
- effect of fans.

When modelling, several suppositions were considered. All the engines have a working temperature, thus enabling the catalysers to function. At the end of the tunnel the velocity of wind in the opposite direction of a vehicle's drive direction is lower than the velocity of the air outlet. The average air resistance coefficient and the average face area of vehicles were also considered. In relation to [3] and to [1], the increasing air resistance coefficient in an enclosed space, and its reduction by driving in column were also considered.

To solve the problem of three-dimensional flow in the tunnel, three-dimensional analysis is needed. The flow of compressible fluid is defined by nonlinear partial differential equations and, except in some cases, it is difficult to obtain an exact analytical solution for optional boundary conditions; therefore we have used approximative methods investigated in [5].

V veliko primerih lahko probleme pri pretakanju dovolj natančno rešimo z uporabo enodimensijske analize, katere osnova je predpostavka, da so vsi parametri toka odvisni samo od ene koordinate v smeri pretakanja fluida. To je zlasti primerno za proučevanje tokov v cevih s konstantnim prečnim prerezom. V tem primeru variirajo parametri toka po prečnem prerezu. Enodimensijska analiza zanemarja te spremembe in spremi povprečne vrednosti parametrov v vzdolžnem prerezu, tako dobljeni rezultat je omejen le na ugotavljanje povprečnih vrednosti.

Enodimensijsko analizo lahko z dovolj veliko natančnostjo uporabimo pri tokovih, pri katerih so izpolnjeni naslednji pogoji:

- relativna sprememba prečnega prereza glede na dolžino toka je majhna, ko je $dD/dx \ll 1$, kjer je D značilna dimenzija toka,
- polmer krivine osi toka je veliko večji od širine toka,
- porazdelitev parametrov toka po prečnem toku se zelo malo spreminja.

Velika večina problemov izpolnjuje zahtevane pogoje. Strogo vzeto, takšni tokovi niso enodimensionalni, vendar jih lahko rešujemo z uporabo enodimensijske analize, zato jih imenujemo *kvazienodimensionalni* ali *kvaziparalelni* tokovi.

V idealiziranem primeru obravnavamo idealno pretakanje v izolirani cevi, brez upoštevanja trenja v fluidu, prenosa toplote in mehanskega dela. To je izentropno pretakanje fluida, katerega parametri se spremenijo samo zaradi spremembe prečnega prereza toka.

Pri dejanskem stanju se vedno pojavi notranje trenje, ki lahko v primeru spremembe tlaka močno vpliva na karakteristiko toka. Zaradi poenostavitev obravnavamo enodimensionalno pretakanje s trenjem v cevi s konstantnim prečnim prerezom. Takšno pretakanje je v praksi zelo pogosto (plinovodi, parovodi, motorji z notranjim zgorevanjem, cestni in železniški predori itn.).

1 MATEMATIČNI MODEL

1.1 Shema modela

Matematični model smo izdelali kot izvirno in splošno orodje za analizo dogajanja v predoru. Z njim prikažemo medsebojne odvisnosti opazovanih parametrov na koncu predora. Na sliki 1 so z močnejšo črto poudarjeni opazovani in v modelu zajeti parametri [6].

Na sliki 1 je prikazana shema modela, ki prikazuje celotno dogajanje v predoru. V članku podajamo rezultate modeliranja tistega dela, ki določa hitrost zraka kot posledico indukcije.

In some simple cases we may use one-dimensional analysis for solving the problems. In such cases we may say that all the parameters of flow are dependent on one coordinate only. This is convenient for observing the flow in a pipe with a constant cross-section. In this case the parameters of flow vary in the cross-section. The one-dimensional analysis, however, neglects these various parameters, and follows the average values of parameters in the longitudinal section, with the results being limited only to the establishment of average values.

For using the one-dimensional analysis the following conditions must be assured:

- the relative change of cross section in comparison with length must be small, if $dD/dx \ll 1$, where D is the significant dimension of flow,
- the radius of the stream must be much larger than the width of stream,
- the flow parameters distribution in cross flow must be approximately constant.

Many problems can be satisfied by adhering to these requirements. These flows are one-dimensional only approximately. Nevertheless, they may also be solved by one-dimensional analysis; therefore, we call them *quasi one-dimensional* or *quasi parallel* flows.

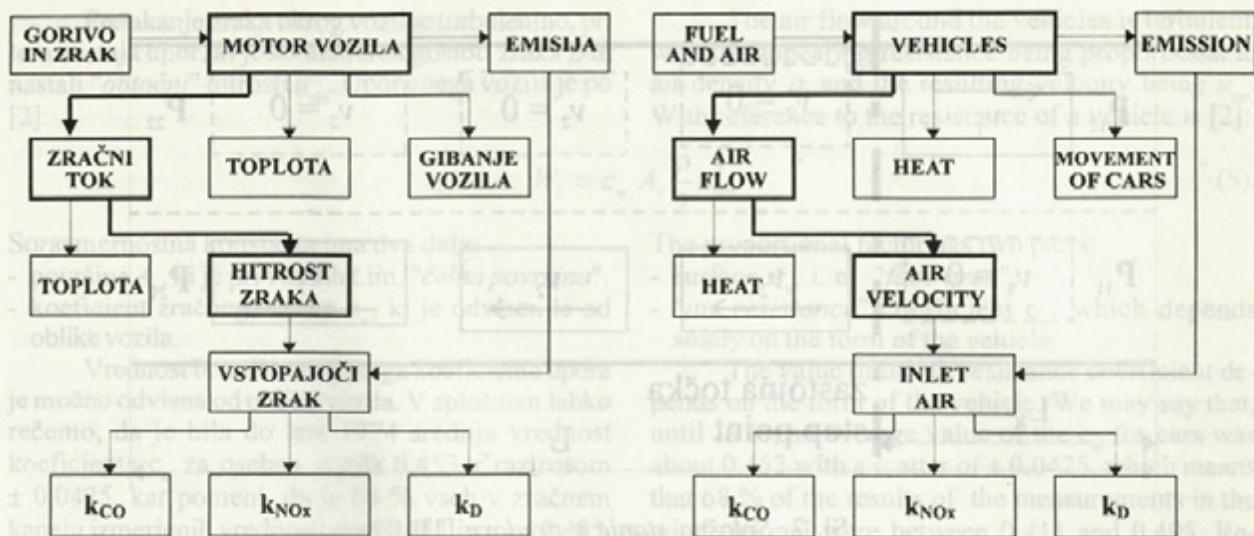
In an ideal case we consider the ideal flow in the insulated pipe, without friction in the fluid, without heat transfer, and without performing mechanical work. This is an isentropic fluid flow, the parameters of which change only because of the change of the cross-section.

In real circumstances, however, internal friction always occurs, and may influence the characteristics of flow in the case of pressure setting. In order to simplify matters, we concentrated on the one-dimensional flow in a pipe with a constant cross section. Such flow is frequent in real situations (gas pipelines, steam lines, internal combustion engines, road and railway tunnels, etc.).

1 MATHEMATICAL MODEL

1.1 Scheme of the Model

A mathematical model is an original and universal tool for analysing the conditions in the tunnel. It shows the reciprocal dependency on the observed parameters at the end of tunnel. In the model observed, the important parameters (Fig. 1) are shown in bold [6]. In Figure 1, the scheme of the model is presented, showing the events in the tunnel. In this paper, a presentation is given only of those results of a model part which determine air velocity based on induction.



Sl.1. Shema dogajanj v predoru

1.2 Splošno stanje v predoru

Vozča vozila povzročajo gibanje zraka, stoječa ali v nasprotni smeri vozeča to gibanje zavirajo. Razlika zračnih tlakov pri obeh portalih (pri vstopu in pri izstopu) gibanje bodisi pospešuje, bodisi zavira. Tako je časovno spremenljiva hitrost zraka poleg od časa, odvisna tudi od:

- trenja in tlačne razlike med portalni,
- števila stoečih vozil v predoru,
- števila in smeri vožnje gibajočih se vozil,
- hidrodinamičnih lastnosti vozil,
- strukture prometa,
- hitrosti vožnje,
- geometrijskih parametrov predora.

Analizirajmo najprej primer, ko poteka promet skozi predor v dveh smereh. Na poljubni razdalji L_1 imamo zastojno točko. V tej točki se pojavi ovira, ki zaustavi promet. V predoru si zamislimo področja: P_{11} , P_{12} , P_{21} in P_{22} .

$$P_{11}$$

$$v_1 = 0$$

N^s_1 - število stoečih osebnih vozil

N^s_{ts} - število stoečih tovornih vozil

$$P_{12}$$

$$v_1 > 0$$

N^{os}_1 - število vozečih osebnih vozil

N^{os}_{ts} - število vozečih tovornih vozil

$$P_{21}$$

$$v_2 = 0$$

N^s_2 - število stoečih osebnih vozil

N^s_{ts} - število stoečih tovornih vozil

$$P_{22}$$

$$v_2 > 0$$

N^{os}_2 - število stoečih osebnih vozil

N^{os}_{ts} - število stoečih tovornih vozil

Fig. 1. Scheme of circumstances in the tunnel

1.2 General State in the Tunnel

Vehicles driving in one direction produce air movement, whereas the vehicles driving in the opposite hold it back. Because of the difference between movement, the inlet and outlet air pressure, the air movement falls or rises. Thus the air velocity also depends on

- friction and pressure differences between entrances,
- the numbers of stationary vehicles in the tunnel,
- numbers and direction of moving vehicles,
- hydrodynamic properties of vehicles,
- structure of traffic,
- velocity of vehicles,
- geometrical properties of the tunnel.

Let us analyse this by the example of traffic moving in both directions in a tunnel. At length L_x there is a stop point. At this point an obstacle appears, suspending the flow of traffic. In the tunnel four areas are considered: P_{11} , P_{12} , P_{21} and P_{22} .

$$P_{11}$$

$$v_1 = 0$$

N^s_1 - numbers of standing cars

N^s_{ts} - numbers of standing trucks

$$P_{12}$$

$$v_1 > 0$$

N^{os}_1 - numbers of driving cars

N^{os}_{ts} - numbers of driving trucks

$$P_{21}$$

$$v_2 = 0$$

N^s_2 - numbers of standing cars

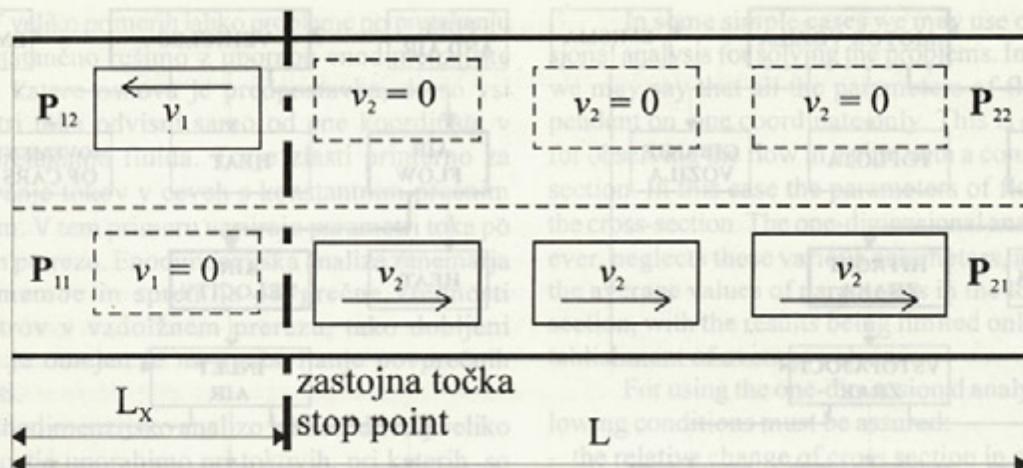
N^s_{ts} - numbers of standing trucks

$$P_{22}$$

$$v_2 > 0$$

N^{os}_2 - numbers of driving cars

N^{os}_{ts} - numbers of driving trucks



Sl. 2. Splošno stanje v predoru [1]
Fig. 2. General situation in the tunnel [1]

1.3 Splošna diferencialna enačba

Splošna diferencialna enačba za ta primer ima obliko:

$$\frac{du}{dt} = \pm A(u, t) \pm B(u, t) \pm C(u, t) \pm D(u, t) \pm E(u, t) \pm F(u, t) \quad (1)$$

Vpliv trenja je:

$$\Delta p = L \frac{\lambda}{D_h} \frac{\rho u^2}{2} + \Sigma \xi \frac{\rho u^2}{2} \quad (2)$$

ki jo reduciramo na dolžino L, tako dobi obliko (predznak minus pomeni zaustavljanje):

$$A = \frac{\Delta p}{L} = -\frac{\rho u^2}{2} \left(\frac{\lambda}{D_h} + \frac{\Sigma \xi}{L} \right) \quad (3)$$

Tlačno razliko med portali definiramo tako, da jo prikažemo kot funkcijo končne dolžine predora, kajti v vsakem primeru vpliva na končno hitrost zraka, ki pa se z dolžino predora zmanjšuje. Prav tako je pomembna gostota zraka, ki je funkcija temperature. S tlačno razliko med portali lahko upoštevamo tudi pozitivne ali negativne vplive vetra pri vstopu ali izstopu iz predora. Predznak je odvisen od razmer. Tako ima drugi člen diferencialne enačbe obliko:

$$B = \pm \frac{\Delta p}{L} = \pm \frac{\Delta H [\gamma(t_1) + \gamma(t_2)] \pm \Delta p_k}{L} \quad (4)$$

Pri enosmernem prometu lahko vpliv vozil na gibanje zraka določimo razmeroma preprosto, pri dvosmernem prometu pa moramo upoštevati vpliv nasprotne smeri vožnje, pri kateri je hitrost vozil negativna, če se vozila premikajo v nasprotnejši smeri, oziroma je enaka nič, če vozila stojijo.

The differential equation for this case has the general form:

The influence of friction is:

which is reduced to the length of tunnel L (minus sign means stopping), thus becoming

The pressure difference between the inlet and outlet is obtained by showing it as a function of the final length of the tunnel, since it always influences the air velocity at the end of the tunnel. The air velocity becomes reduced as the length of the tunnel increases. The air density is important, being a function of air temperature. With regard to pressure difference, the positive or negative influence of the wind at the inlet or outlet may also be considered. Thus the second section of the differential equation is:

In the case of uni-directional traffic the influence of vehicles on the air motion is defined in other circumstances; we therefore have to consider the influence of vehicles driving in the opposite direction, since the velocity of vehicles is negative, or zero, if they have stopped.

Pretakanje zraka okrog vozil je turbulentno, pri tem nastaja upor, ki je sorazmeren gostoti zraka ρ in nastali "obtočni" hitrosti u_{rez} . Upor enega vozila je po [2]:

$$W_z = c_w A_v \frac{\rho}{2} u_{rez}^2 \quad (5)$$

Sorazmernostna konstanta ima dva dela:

- površina A_v , ki je pri vozilih t.i.m. "čelna površina",
- koeficient zračnega upora c_w , ki je odvisen le od oblike vozila.

Vrednost brezdimenzijskega koeficiente upora je močno odvisna od oblike vozila. V splošnem lahko rečemo, da je bila do leta 1974 srednja vrednost koeficiente c_w za osebna vozila 0,453 z raztrosom $\pm 0,0425$, kar pomeni, da je 68 % vseh v zračnem kanalu izmerjenih vrednosti med 0,411 in med 0,495. Novejše raziskave (npr. [3]) pa kažejo, da je dosegljiva vrednost koeficiente zračnega upora za osebna vozila okrog 0,30, posebno oblikovana vozila pa dosegajo vrednosti celo okrog 0,15. Tako imajo osebna vozila koeficient c_w od 0,30 do 0,50, ter čelne površine od $1,75 \text{ m}^2$ do $1,90 \text{ m}^2$. V povprečju upoštevamo za osebna vozila: $c_w = 0,40$ in $A_v = 1,85 \text{ m}^2$. Pri tem predpostavljamo, da je smer zračnega toka enaka ali nasprotna smeri vožnje, torej ni stranskih vplivov, kar je v predoru dopustno. Podobno velja za tovorna vozila: $c_w = 1,00$ in $A_v = 9,00 \text{ m}^2$.

Zelo pomembna sta še dva vpliva, ki lahko močno spremeni vrednosti koeficiente zračnega upora:

- v zaprtem prostoru se po [4] koeficient zračnega upora poveča za okoli 15 %,
- pri vožnji v koloni se po [3] koeficient zračnega upora zmanjšuje.

Tako velja za tovorna vozila, katerih hitrost je 80 km/h , in je torej njihova medsebojna razdalja okoli 40 m , da se koeficient zračnega upora za drugo vozilo v koloni zmanjša za okoli 22 %, za tretje in vsa naslednja vozila pa je to zmanjšanje približno 35 %. Podobno velja tudi za osebna vozila. Vendar pa meritve kažejo, da je v predorih, predvsem iz psiholoških razlogov, razdalja med vozečimi vozili večja kakor na prostem, zato se koeficient zračnega upora zmanjša manj, kakor na prostem, upoštevamo okoli 70 % zmanjšanje v primerjavi z zmanjšanjem na prostem. Tako dobimo po [1]:

osebna vozila: $c_w^* A_v = 0,90$

tovorna vozila: $c_w^* A_v = 5,20$

Če v enačbo (5) vstavimo tako popravljene vrednosti ter upoštevamo število in hidravlične lastnosti vozil, dobimo tretji člen splošne diferencialne enačbe, ki upošteva vpliv stoječih vozil na časovno spremenjanje hitrosti zraka in ima zaradi zaviralnega učinka negativen predznak:

$$C = -\frac{1}{LA} \left(c_w^* A_v N_{os} + c_w^* A_v N_{ts} \right) \frac{\rho u^2}{2} \operatorname{sign}(u) \quad (6)$$

The air flow around the vehicles is turbulent, with the appearing resistance being proportional to air density ρ , and the resulting velocity being u_{rez} . With reference to the resistance of a vehicle is [2]:

$$W_z = c_w A_v \frac{\rho}{2} u_{rez}^2 \quad (5)$$

The proportional factor has two parts:

- surface A_v , i.e. "face area",
- "air resistance" coefficient c_w , which depends solely on the form of the vehicle.

The value of the air resistance coefficient depends on the form of the vehicle. We may say that, until 1974, an average value of the c_w for cars was about 0.453 with a scatter of ± 0.0425 , which means that 68 % of the results of the measurements in the wind channel were between 0.411 and 0.495. Recent investigations (referring to [3]) show that the reach value of the air resistance coefficient for cars is 0.30, in same examples 0.15. Thus the c_w coefficient for cars is between 0.30 and 0.50, and the face area between 1.75 m^2 and 1.90 m^2 . The average values are: $c_w = 0.40$ and $A_v = 1.85 \text{ m}^2$. Here we consider that the air flow direction is equal to or opposed to the drive direction, which means that we do not take into account side effects, which are allowable in the tunnel allowable. The values for trucks are: $c_w = 1.00$ and $A_v = 9.00 \text{ m}^2$.

Finally, there are two effects which influence the air resistance coefficient:

- when driving in a closed space (referring to [4]) the air resistance coefficient rises for approx. 15 %,
- when driving in column (referring to [3]) the air resistance coefficient reduces.

For trucks with a driving speed of 50 mph/s - which means that the distance between trucks is about 40 m - the reduction of the air resistance coefficient for the second truck in the column is about 22 %, and for the third truck (and for others trucks) the reduction is about 35 %. The same goes for cars. The measurements show that for psychological reasons the distance between cars in tunnels is greater than on the open road. Thus the reduction of the air resistance coefficient is lower, and only 70 % has to be taken into account. So, referring to [1] we obtain: cars: $c_w^* A_v = 0.90$
trucks: $c_w^* A_v = 5.20$

If these values are introduced in the equation (5), and we take into account the number and hydrodynamic properties of the vehicles, we obtain the third section of the general differential equation, which considers the influence of the stationary vehicles on the change of time-dependent air velocity. With the negative sign due to braking action:

Najpomembnejši je vpliv gibajočih se vozil. Uporabimo enaka izhodišča, kakor pri tretjem členu diferencialne enačbe, upoštevati pa moramo ustrezni predznak, kajti vozeča vozila gibanje zraka pospešujejo in je predznak pozitiven, če poteka promet v nasprotni smeri, pa je predznak negativen:

$$D = -\frac{1}{LA} \left(c_{wo}^* A_{vo} N_{o1} + c_{wt}^* A_{vt} N_{t1} \right) \left(1 - \frac{t}{L_x} |v_1| \right) \frac{\rho(v_1 + u)^2}{2} \operatorname{sign}(v_1 + u) \quad (7)$$

S četrtim členom zajamemo vpliv nasproti vozečih vozil, zato ima negativen predznak. Pri tem je pomembna tudi hitrost vožnje. Število vozil je odvisno od dolžine predora L ter od hitrosti vožnje. Pri enaki dolžini se število vozil nekaj časa veča, nato se začne zmanjševati, ker se zaradi hitrosti povečuje potrebna razdalja med vozili.

Če je predor poln, lahko izrazimo število vozil tudi kot funkcijo hitrosti vožnje. Tako lahko pri tekočem prometu po [1] zapišemo, da je največje število osebnih vozil v predoru:

$$N_{omaks} = \left(1 - \frac{P_t}{100} \right) M_{maks} \quad (8)$$

in največje število tovornih vozil

$$N_{tmaks} = \frac{P_t}{100} M_{maks} \quad (9)$$

Pri upoštevanju časovne odvisnosti hitrosti je pomemben člen:

$$\left(1 - \frac{t}{L_x} |v| \right)$$

ker se njegova vrednost z daljšanjem časa zmanjšuje.

Podobno velja tudi za vozila, ki vozijo v nasprotni smeri:

$$E = \frac{1}{LA} \left(c_{wo}^* A_{vo} N_{o2} + c_{wt}^* A_{vt} N_{t2} \right) \left(1 - \frac{t}{L-L_x} |v_2| \right) \frac{(v_2-u)^2 \rho}{2} \operatorname{sign}(v_2-u) \quad (10)$$

Tudi v tem primeru se vrednost člena

$$\left(1 - \frac{t}{L-L_x} |v_2| \right)$$

z daljšanjem časa zmanjšuje. To pomeni, ko vsa vozila v vseh smereh zapustijo predor, postane hitrost zraka enaka nič.

V primeru, ko imamo prisilno prezračevanje, moramo upoštevati še vpliv ventilatorjev, ki glede na smer delovanja hitrost zraka pospešujejo ali zavirajo. Tako je:

$$F = \pm \frac{S_p}{LA} \quad (11)$$

Pri vseh členih upoštevamo še gostoto zraka, nato posamezne člene združimo v celotno diferencialno enačbo.

The influence of moving vehicles is the most important. We take into account the same base as by the third section of the differential equation, and the corresponding sign, because vehicles accelerate air velocity, and therefore the sign is positive in one direction, and negative in the opposite direction.

In the next section we consider the influence of vehicles driving in the opposite direction, so the sign is negative. The velocity of vehicles is also important. The number of vehicles depends on the tunnel length (L) and the velocity (v) of vehicles. The number of vehicles increases - and later decreases - by the same length due to the increasing velocity and separating distances between the vehicles.

If the tunnel is full, the number of vehicles is also a function of velocity. In terms of traffic flow, referring to [1], the highest number of cars in the tunnel is:

$$N_{omaks} = \left(1 - \frac{P_t}{100} \right) M_{maks} \quad (8)$$

and the highest number of trucks

$$N_{tmaks} = \frac{P_t}{100} M_{maks} \quad (9)$$

If we take into account the speed-time dependence, the following part is important:

because its value decreases with increase of time.

The same holds for vehicles which move in the opposite direction.

$$E = \frac{1}{LA} \left(c_{wo}^* A_{vo} N_{o2} + c_{wt}^* A_{vt} N_{t2} \right) \left(1 - \frac{t}{L-L_x} |v_2| \right) \frac{(v_2-u)^2 \rho}{2} \operatorname{sign}(v_2-u) \quad (10)$$

In this case the value of part

also decreases, if the time (t) increase - which means, that the air velocity vanishes - when all the vehicles leave the tunnel.

In the case of forced ventilation, we also consider the influence of fans; their effect on the air velocity is either negative or positive, which means that:

The air density is also considered, and all the parts are combined in the total differential equation.

Tako dobimo:

$$\frac{du}{dt} = -\left(\frac{\lambda}{D_h} + \frac{\Sigma \xi}{L}\right) \frac{u^2}{2} \operatorname{sign}(u) \pm \pm \frac{\Delta p}{\rho L} - \quad \text{A},$$

$$(1)$$

$$\text{So it is:}$$

$$\pm \frac{S_p}{\rho L A} \quad \text{B},$$

$$-\frac{1}{L A} \left(c_{wo}^* A_{vo} N_{os} + c_{wt}^* A_{vt} N_{is} \right) \frac{u^2}{2} \operatorname{sign}(u) - \quad \text{C},$$

$$-\frac{1}{L A} \left(c_{wo}^* A_{vo} N_{o1} + c_{wt}^* A_{vt} N_{i1} \right) \left(1 - \frac{t}{L_x} |v_1| \right) \frac{(v_1 + u)^2}{2} \operatorname{sign}(v_1 + u) + \quad \text{D},$$

$$+ \frac{1}{L A} \left(c_{wo}^* A_{vo} N_{o2} + c_{wt}^* A_{vt} N_{i2} \right) \left(1 - \frac{t}{L - L_x} |v_2| \right) \frac{(v_2 - u)^2}{2} \operatorname{sign}(v_2 - u) \pm \quad \text{E},$$

nadomestimo s simboli:

$$\pm \frac{S_p}{\rho L A} \quad \text{F (12).}$$

Enačba (12) je splošna in omogoča analizo hitrosti gibanja zraka, tako v predorih z dvosmernim kakor tudi v predorih z enosmernim prometom. Vsebuje elemente časovne in krajevne lestvice. Časovna odvisnost je definirana s časom t , oziroma z odvodom hitrosti po času, krajevna pa s končno dolžino predora L in delno dolžino L_x .

Diferencialna enačba (12) je sestavljena iz šestih členov. Prvi člen (A) zajema trenje, drugi člen (B) tlačno razliko med portalami, tretji člen (C) vpliv stoječih vozil, četrtni (D) in peti člen (E) vpliv vozečih vozil in smer gibanja, šesti člen (F) pa vpliv ventilatorjev. Pri tekočem prometu odpade vpliv stoječih vozil, člena D in E pa vplivata v nasprotnih smereh. Zato moramo v predorih z dvosmernim prometom zagotoviti gibanje zraka z ventilatorji, ali z drugimi besedami, naravno prezračevanje ni mogoče, razen če imamo dovolj veliko tlačno razliko med portalami. Vendar je ta razlika spremenljiva, tako po velikosti kakor tudi po smeri.

1.4 Enačba za predore z enosmernim prometom

Če se omejimo na predore s tekočim enosmernim prometom, odpadeta člena C in D. Če je prezračevanje *naravno*, odpade člen F. Če ni tlačne razlike med portaloma, odpade tudi člen B enačbe (12). Prav tako odpade označba sign, ker hitrosti ne spreminja svoje smeri.

Tako dobim enačba (12) za predore s tekočim enosmernim prometom, brez upoštevanja vpliva ventilatorjev in vetra, obliko:

$$\frac{du}{dt} = -\left(\frac{\lambda}{D_h} + \frac{\Sigma \xi}{L}\right) \frac{u^2}{2} + \frac{1}{L A} \left(c_{wo}^* A_{vo} N_o + c_{wt}^* A_{vt} N_i \right) \left(1 - \frac{t}{L - L_x} |v| \right) \frac{(v - u)^2}{2} \quad (13).$$

The equation (12) is general, and is useful for analysing air velocity in the tunnel with unidirectional traffic or with traffic in both directions. It contains the elements of local and time scales. The time dependence is obtained by the time (t), or by derivation of velocity per time, and the local dependence by the length of the tunnel (L) and partial length (L_x).

The differential equation (12) combines six parts. The first (A) contains friction, the second (B) the air pressure difference between the inlet and outlet, the third (C) the influence of standing vehicles, the fourth (D) and the fifth (E) the influence of moving vehicles and their direction, and the sixth (F) the influence of fans. For flowing traffic, the section (C) vanishes, the sections (D) and (E) have an influence in opposite directions. Therefore, for the air moving in a tunnel with traffic in both directions the fans are necessary or - better to say - the natural ventilation is not possible.

1.4 Equation for Tunnels with Undirectional Traffic

If we take into account a single-bore tunnel with two lanes of unidirectional traffic, sections (C) and (D) are superfluous, if the ventilation of the tunnel is natural, the section (F) is superfluous; and if the pressure difference between the inlet and outlet is zero, section (B) of equation (12) is superfluous. Also the mark sign is eliminated, because the direction of velocity does not change.

We obtain the equation (12) with both directions of traffic, and without taking into account fans and the influence of wind, in the form of:

Ker je promet sestavljen iz osebnih in tovornih vozil in je delež tovornih vozil podan v odstotkih glede na celoten promet (p_t), lahko zapišemo za celotno število vozil v polnem predoru v odvisnosti od deleža tovornih vozil:

$$N = \left(1 - \frac{p_t}{100}\right) N_o + \frac{p_t}{100} N_t \quad (14)$$

Za izračun hitrosti zraka v posameznih predorih, torej na končnih dolžinah L moramo upoštevati tudi število vozil v posameznih točkah. Če je dolžina avtomobila l_o in dolžina predora L , je število vozil v točki (na koncu) predora:

$$N_o = \frac{N_{\max}}{L} l_o \quad (15)$$

Podobno velja tudi za tovorna vozila.

V dejanskih razmerah moramo upoštevati mešan promet skozi predor, kombinacijo osebnih in tovornih vozil, torej trenutno naključno stanje, ter tudi gostoto prometa. V tem primeru se hitrost spreminja od minimalne do maksimalne. Tako dobimo različne kombinacije, ki so za analizo trenutnih razmer primerne, problem pa nastane, če želimo izračunati povprečne količine zraka, ki je osnovni kriterij za odločitev, ali zadošča naravno prezračevanje.

Tako je po [1] za "mešano" ali "povprečno" vozilo:

$$P = \left(1 - \frac{p_t}{100}\right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \quad (16)$$

1.5 Hitrost zraka

Če v enačbi (13) za enosmerni promet upoštevamo še razmere v določenem trenutku, lahko predpostavimo kvazistacionarno stanje:

$$t = 0 \quad \text{in} \quad \frac{du}{dt} = 0$$

Tako iz enačbe (13) dobimo:

$$\frac{1}{LA} \left(c_{wo}^* A_{vo} N_o + c_{wt}^* A_{vt} N_t \right) \frac{(v-u)^2}{2} - \left(\frac{\lambda}{D_h} + \frac{\Sigma \xi}{L} \right) \frac{u^2}{2} \left(\pm \frac{\Delta p}{L\rho} \right) = 0 \quad (17)$$

Z uporabo enačbe (17) lahko določimo hitrost zraka na koncu predora, torej pri dolžini L v odvisnosti od števila vozil, pri tem pa moramo upoštevati ustrezno število vozil na celotni dolžini predora.

Za analizo hitrosti na koncu vsakega od množice predorov preuredimo enačbo tako, da pri znanih podatkih: dolžini in prerezu predora, hitrosti vožnje in številu vozil izračunamo ustrezno hitrost zraka. Tako dobimo z upoštevanjem "mešanega" vozila enačbo [1]:

The traffic is composed of cars and trucks; the part of the trucks is given in percentages (p_t), so the number of all the vehicles in the tunnel is function of part of the trucks

because vehicles accelerate air velocity, and therefore the sign is positive in one direction, negative in the opposite direction.

For computing the air velocity in tunnels - which means at the final lengths L - we have to consider the numbers of vehicles at single points in the tunnel. If the length of a car is l_o and the length of the tunnel L , then the number of cars at the point (at the end) of the tunnel is:

$$N_o = \frac{N_{\max}}{L} l_o \quad (15)$$

The same goes for the trucks.

In real circumstances, we have to consider mixed traffic through the tunnel, a combination of cars and trucks, which means a stochastic stand. In this case, the air velocity varies from minimum to maximum. Thus we obtain various combinations; however, the problem is to obtain the air velocity available for calculating the average air demand, this being a basic criterion for judgement, if the natural ventilation is sufficient.

Referring to [1], for a "mixed" or "average" vehicle, we obtain:

1.5 Air Velocity

If in the equation (13) for unidirectional traffic we consider the circumstances just at a particular moment, we may suppose that there exist quasi steady state conditions:

$$t = 0 \quad \text{and} \quad \frac{du}{dt} = 0$$

From equation (13) we obtain:

From equation (17) we may obtain the air velocity at the end of tunnel, which means at length L , as a function of the vehicle number; in which case we have to consider the numbers of vehicles in relation to the whole length of the tunnel.

If we take into account a single-bore tunnel with two lanes of undirectional traffic, with the tunnel length, cross section and traffic characteristics, the air velocity as an effect of induction, referring to [1] is:

$$\begin{aligned}
 u^2 & \left\{ \frac{1}{2LA} \left[\left(1 - \frac{p_t}{100} \right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \right] - 0.50 \left(\frac{\lambda}{D_h} + \frac{1.35}{L} \right) \right\} - \\
 & - u \left\{ \frac{1}{LA} \left[\left(1 - \frac{p_t}{100} \right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \right] v \right\} + \\
 & + v^2 \left\{ \frac{1}{2LA} \left[\left(1 - \frac{p_t}{100} \right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \right] \right\} \pm \frac{\Delta p}{L\rho} = 0
 \end{aligned} \quad (18)$$

Kvadratna enačba (18) omogoča izračun hitrosti zraka na koncu predora z enosmernim prometom kot funkcijo dolžine in prereza predora, števila vozil ter koeficiente trenja.

Če v enačbi (18) posamezne koeficiente nadomestimo s simboli:

$$f_1 = \left\{ \frac{1}{2LA} \left[\left(1 - \frac{p_t}{100} \right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \right] - 0.50 \left(\frac{\lambda}{D_h} + \frac{1.35}{L} \right) \right\} \quad (19),$$

$$f_2 = - \left\{ \frac{1}{LA} \left[\left(1 - \frac{p_t}{100} \right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \right] v \right\} \quad (20),$$

$$f_3 = \left\{ \frac{1}{2LA} \left[\left(1 - \frac{p_t}{100} \right) \frac{M}{L} l_o c_{wo}^* A_{vo} + \frac{p_t}{100} \frac{M}{L} l_t c_{wt}^* A_{vt} \right] v^2 \right\} \quad (21),$$

je rešitev enačbe (18) za hitrost zraka:

the solution of the equation (18) for air velocity is:

$$u = \frac{-f_2 + \left[\left(\frac{f_2}{2} \right)^2 - f_1 f_3 \right]^{0.50}}{f_1} \quad (22).$$

Vidimo, da vsebuje hitrost zraka naslednje funkcione povezave med parametri:

It may be observed that the velocity of air (u) is a function of several parameters:

$$u = u(L, A, D_h, \lambda, \sum \xi, \Delta p, p_t, c_{wo}^*, c_{wt}^*, M, l_o, l_t) \quad (23).$$

Iz izraza (23) vidimo, da je hitrost zraka na koncu predora funkcija številnih parametrov. Zato moramo za njen konkreten izračun zmanjšati število parametrov. To storimo tako, da izberemo množico enakih predorov, pri katerih se spreminja samo dolžina L , spremojamo pa npr. število vozil, ki vozijo skozi predor, pri tem pa se ne spreminja drugi, za promet značilni parametri. Ta postopek lahko ponavljamo za različne primere.

Referring to expression (23), we see that the air velocity at the end of the tunnel is a function of more parameters. Therefore we have to minimise their number. By choosing a multitude of equal tunnels with different length only, we can change, e.g., the numbers of vehicles, while the other traffic parameters remain unchanged. This method is repeated for various cases.

2 HITROSTI ZRAKA NA KONCU PREDORA

2.1 Osnovni podatki

Za dolžino L izberemo vrednosti med 400 m in 3100 m. Vrednosti za M dobimo na podlagi predpostavljenega največjega števila vozil za dve stezi

2 AIR VELOCITIES AT THE END OF TUNNEL

2.1 Basic Data

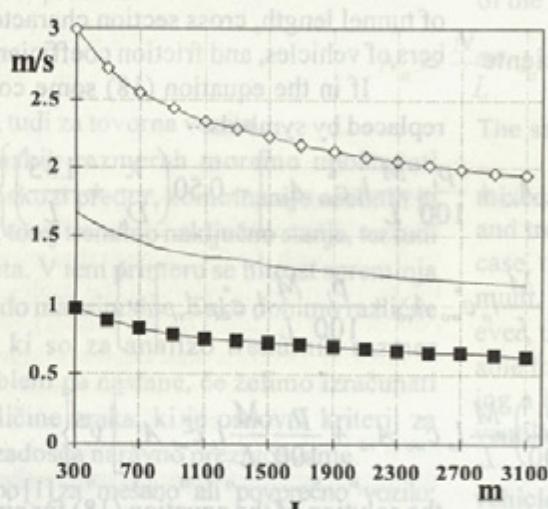
We choose values between 400 m and 3100 m length. Values for M we consider as maximum for two lanes ($M_{max} = 1000$ to 1800 per hour). For the

v predoru ($M_{\max} = 1000$ do 1800 na uro). Kot začetno vrednost izberemo $M_{\max} = 500 \text{ h}^{-1}$ (za eno stezo), delež tovornih vozil $p_t = 15\%$, hitrost vozil pa 60 km/h .

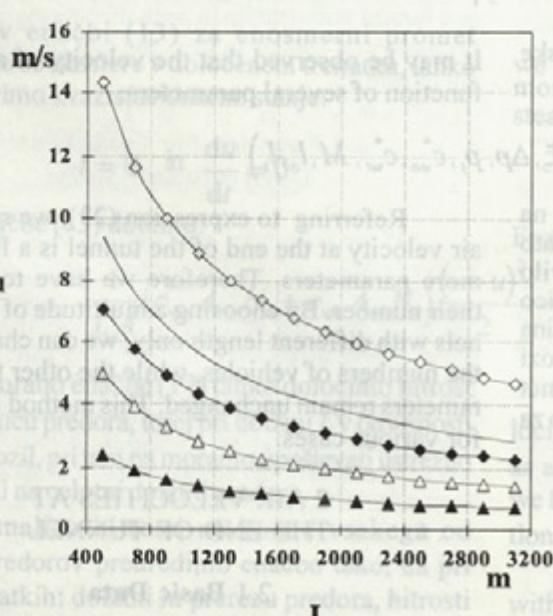
Če upoštevamo vpliv samo enega vozila, bodisi osebnega, tovornega ali "mešanega", dobimo vrednosti, prikazane na sliki 3. Tu ni upoštevan vpliv vseh vozečih vozil, temveč enega samega. Tako izračunane hitrosti so bistveno manjše, kakor pa če upoštevamo vpliv več vozil oziroma gostoto prometa. Pri tem upoštevamo vrednosti: $c_{wo} A_{vo} = 0,90$, $c_{wt} A_{vt} = 5,20$, za mešano vozilo pa izračunamo posamezne vrednosti po enačbi (18).

beginning, we consider $M_{\max} = 500 \text{ h}^{-1}$ (for one lane), part of trucks $p_t = 15\%$, and the velocity of vehicles $v = 60 \text{ km/h}$.

If we consider the influence of one vehicle, car, truck or "mixed" vehicle, we obtain the results shown in Figure 3. The influence of one vehicle only is also considered. Air velocities in this case are much lower, as they are in cases where we consider the influence of all the vehicles, or traffic density. The basic values are: $c_{wo} A_{vo} = 0.90$, $c_{wt} A_{vt} = 5.20$, for mixed vehicles, equation (18) used.



Sl. 3. Hitrosti zraka na koncu predorov, eno vozilo, $v = 60 \text{ km/h}$ [1]
Fig. 3. Air velocities at the end of tunnel, one vehicle, $v = 60 \text{ km/h}$ [1]



Sl. 4. Hitrosti na koncu predorov, $M_{\max} = 500 \text{ h}^{-1}$, $p_t = 15\%$, $v_1 = 30 \text{ km/h}$, $v_2 = 60 \text{ km/h}$, [1]
Fig. 4. Air velocities at the end of tunnel, $M_{\max} = 500 \text{ h}^{-1}$, $p_t = 15\%$, $v_1 = 30 \text{ km/h}$, $v_2 = 60 \text{ km/h}$ [1]

Če pa upoštevamo število vozil ($M_{\max} = 500 \text{ h}^{-1}$), dobimo vrednosti, prikazane na sliki 4. Tako izračunana hitrost se v primeru, ko je $p_i = 0\%$, prilega krivulji hitrosti zraka na koncu množice predorov za osebna vozila, ko je $p_i = 100\%$, se krivulja hitrosti prilega krivulji hitrosti zraka, povzročeni s tovornimi vozili, sicer pa leži nekje vmes.

Tako vidimo, da na povečanje hitrosti zraka na koncu predora vpliva zmnožek $c_{wo} A_{vo}$ pri osebnih, zmnožek $c_w A_v$ pri tovornih, oba zmnožka in odstotni delež tovornih vozil pri mešanem vozilu, ter gostota prometa in največje število vozil v vseh primerih. Čim se kateri izmed omenjenih faktorjev poveča, se krivulja hitrosti zraka na koncu predora pomakne navzgor.

If we consider the real number of vehicles ($M_{\max} = 500 \text{ h}^{-1}$), we obtain the results, shown in Figure 4. Air velocities in cases, when $p_i = 0\%$ fit the curve of air velocity at the end of the tunnel for cars, when $p_i = 100\%$, the air velocity curve fits the air velocity curve, caused by trucks; in other cases the curve lies somewhere in between.

It is therefore clear that the air velocity increasing at the end of tunnel influences the product of $c_{wo} A_{vo}$ by cars, product $c_w A_v$ by trucks, both products and part of trucks by mixed vehicles, and traffic density, and the greatest number of vehicles in all cases. If one of them increases, the curve of air velocities at the end of tunnel increases.

3 SKLEP

Prikazana diferencialna enačba (12) ter enačba (18), s katero lahko računamo hitrost na koncu predora, dajeta dovolj zanesljive rezultate. Obe enačbi sta univerzalni in ju je mogoče uporabiti za posamezne specifične primere. Prav tako je univerzalna in splošno uporabna enačba (17), ki je primerna za uporabo v predorih z enosmernim prometom, kar potrjujejo tudi analize [1].

3 CONCLUSION

The differential equation presented (12) and equation (18) - which enables computing the air velocity at the end of the tunnel - give satisfactory results. Both equations are universal, and it is possible to apply them to the various specific examples. Also the equation (17) is universal and applicable, and is useful for tunnels with unidirectional mixed traffic, as is also confirmed by the analysis, referring to [1].

4 OZNAČBE

4 NOMENCLATURE

prerez predora	A	m^2	cross-section of tunnel
čelna površina vozila	A_v	m^2	front section
koef. zračnega upora vozila	c_w		air resistance coefficient
hidravlični premer predora	D_h	m	hydraulic diameter
geodetska višina predora	H	m	geodetic height of tunnel
dolžina predora	L	m	length of tunnel
dolžina vozila	l	m	length of vehicle
število vozil v eni uri	M	h^{-1}	number of vehicles per one hour
število vozil v predoru	N		number of vehicles in tunnel
hitrost zraka	u	m/s	air velocity
hitrost vozil	v	km/h	velocity of vehicles
tlak	p	Pa	pressure
krajevna tlačna razlika	Δp_k		local pressure difference
čas	t	s	time

plaski enakomerno dvigajoči polprostor. Primer sta prvič obravnavala Carslaw in Jaeger [13]. Z obravnavno sumporno teplotnimi virov sta se jima pridružila še White [14] in Rykalin [15]. Bolj realistično, krožno simetrično obliko lasersko inducirane toplotnega vira sta prvič obravnavale Paek in Gagliano [16]. Pri tem sta se omecila na primeri z bodisi konstantno ali trenutno osvetlitvijo. Nekaj avtorjev je pri reševanju upoštevalo tudi temperaturno odvisnost snovnih parametrov, kakor so absorptivnost in toplotna kapaciteta ter prevodnost [17], drugi [18] pa so upoštevali nujenje absorpcije, in sicer kot eksponentno pojemanje moči toplotnega tokaz globino.

funkcija $I(r, t)$ pa opisuje toplotni vir, ki ga smemo v našem primeru zapisati kot produkt dveh krijevnih in časovne funkcije [19]:

$$I(r, t) = B \cdot e^{-\rho c_r r} \cdot e^{-\lambda r} P(t) \quad (3)$$

Tu so λ , ρ , c_r in w toplotna prevodnost, gostota, specifična toplota, absorpcijski koeficient in palmer svetlobnega snopa, B pa je poljubna konstanta. Ob začetnem pogoju $I(r, 0) = 0$ in zamevku $P(t) = 0$ se lahko rešitev temperaturne enačbe (1) za krožno simetrični primer (gl. [20]) zapisa kot Fourierova vrsta:

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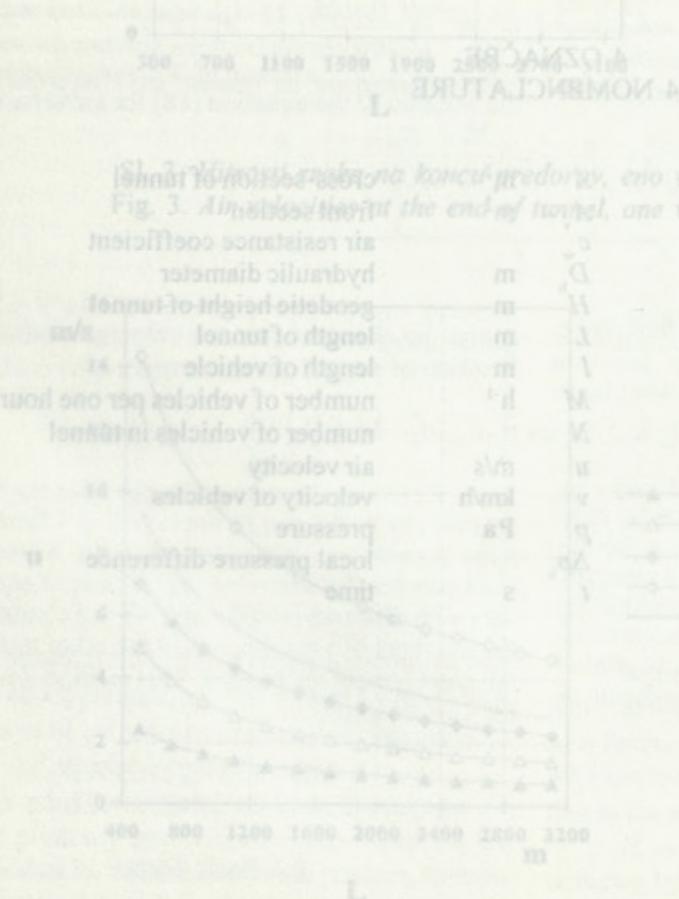
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Sl. 4. Hitrosti na koncu predorov, $M_{\text{pred}} = 500 \text{ h}^2$, $p_0 = 15\%$, $v_0 = 30 \text{ km/h}$, $v_1 = 60 \text{ km/h}$. [1]

Fig. 4. Air velocities at the end of tunnel, $M_{\text{pred}} = 500 \text{ h}^2$, $p_0 = 15\%$, $v_0 = 30 \text{ km/h}$, $v_1 = 60 \text{ km/h}$ [1].